MINISTRY OF EDUCATION OF THE REPUBLIC OF BELARUS

INSTITUTION OF EDUCATION
«BREST STATE TECHNICAL UNIVERSITY»

APPLIED MECHANICS

Structural Mechanics

Part 1: STATICALLY DETERMINABLE FRAMEWORKS

Recommended by the University Council
as a manual
on discipline "Structural mechanics"
for students of building specialties

Brest 2020

УДК 624.04
ББК 38.112я73
И 26

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И 26 Structural mechanics : a manual. – Brest : Educational edition BSTU, 2020. – 74 p.

ISBN 978-985-493-484-6

The manual outlines the basic mechanics of plane grid framework, including the basic concepts and principles of structural mechanics, the kinematic analysis of structures, the determination of internal forces and displacements in statically determinable frames and beam systems, and the calculation of forces in statically determinable trusses. Examples of calculations and problems for independent solutions with answers are given.

The manual is intended for students of construction specialties.

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1. BASIC CONCEPTS

1.1. Structural Mechanics and Its Tasks

Structural mechanics is the science of determining structures strength, rigidity and stability.

The main goal of the structural mechanics is to determine the stress-strain state of structures, that is, the definition of internal forces (forces) and displacements that arise in the elements of structures from actions and influens. In general case action are devided accordance with EN 1990 (load arrangement): indirect actions (displacement of supports, temperature and shrinkage deformation), direct actions – mechanical forces.

The tasks of structural mechanics also include the study of the principles of the formation of structures, the study of the conditions for their stability and behavior of structures under various moving and dynamic loads.

1.2. Design Diagram for Structures Loads and Actions

Select the design diagram is the very important and difficult stage of the calculation – design diagram should be chosen so to accommodate all the main features of this structure and thus to facilitate the calculation.

The degree of accuracy of the reflection of the actual work of construction is associated with accounting in the design diagram:

– the actual geometry of construction elements and their compounds;

– Physico-mechanical properties of materials of structural member of structure;

– The use of calculation methods, computer programs and computing techniques to perform the calculation with the required accuracy.

1.2.1 Structural Idealization

The main idea of structural idealization is to make a mathematical model of the real construction convenient for analisis and calculation.

After we will know the idealization of different joint and supports, we will take care about whole structure idealization.

Structural mechanics is the computation of deformations, deflactions, and internal forces (as an effect of action of external forces) or stresses (strein equivalent) within structures, either for design or for performance evaluation of existing structures. It is one subset of structural analysis. However, the calculation of real structures with an accurate account of all its features is the complex and, in most cases, almost impossible task. Therefore, this calculation is simplified by replacing the actual construction of its design diagrum.

A design diagram for structures is a simplified, idealized scheme of the buildings entered into the calculation, which reflects the main properties and neglected the minor properties and minor details, slightly affect the structure.

The selection of the design diagram largely determines the complexity of calculation and the correctness of the obtained results. To determine the design diagrum engener must have experience in analysis of structures, a good understanding of the question of the structure and its individual elements, the principles of interaction of construction elements with each other.

Definition of structure (simplest). A structure refers to a system with connected parts used to maintain loads. There are a few types of structure: Three-space structures; Frame structures: trusses, three-hinged frame, frames, plane structures; Surface structures.

All the structures as usual are three-space structures. Often, however, if it is possible to make the structure of the buildings, the three-space structures is divided into plane structures – in this case, their calculation is greatly simplified. This approach can be applied, if in a three-space structures it is possible to allocate a plane load-bearing elements (slabs) connected by transverse bracing.

The structures are isolated elements (rods, plates, shells and solids), which are inter-connected in uniform system by means of bar connection: pin-connected joint (pin joint) or fixed connected joint (fixed joint), and rely on the earth (ground) by supporting structures, or structural supports (supports).

The rod elements are rectilinear or curvilinear three-space elements, in which one dimension (length) considerably larger than the other two (transverse dimensions). On the design diagrams the above mentioned elements are replaced with the axial lines (straight, curved or broken) and are called rods. In the calculations take into account the parameters of cross-sections of these elements through their corresponding characteristics (square cross-sections, moments of inertia, etc.), given to the centers of the cross sections.

If the structure consists only of the rod elements, it is a rod system.

1.2.2 Support Idealization

Actual supporting structures in the design diagram are replaced with the ideal schemes. A support transmits load from structures on the base associated with the earth.

The main types of supports and their characteristics are presented in table 1.1. (The essence of the concepts "kinematic relations" and "degree of freedom" – see below).

1.2.3 Joint Idealization

Joint can be pin-connected joint and fixed connected joint or torsional spring joint.

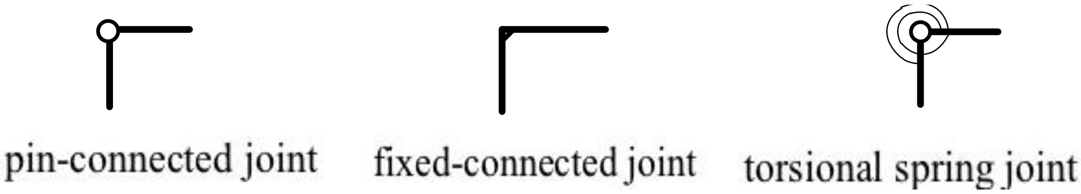
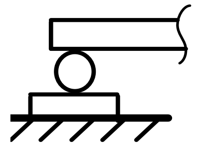
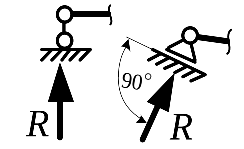
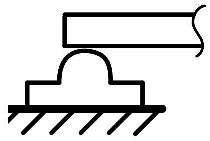
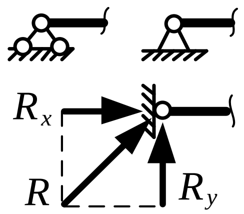
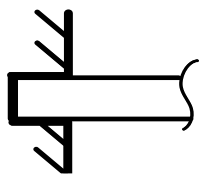
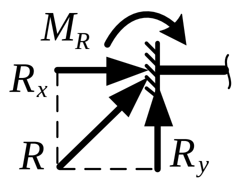
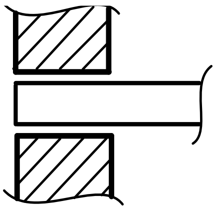
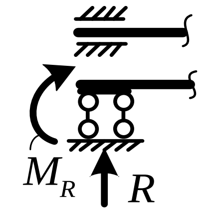


Fig 1.0

Table 1.1

The main types of supports

№	Name support	Possible constructive scheme of support	The image on the desing schemes	The number of kinematic joint connections	The number of reactions	The number of degrees of freedom
1	Roller support (**hinged movable support**)			1	1	2
2	Pin (pinned) support **(hinged immovable support)**			2	1(2)	1
3	Rigid restraind (pinching) (**fixed end**)			3	2(3)	0
4	Spring support			2	2	1

Pin-connected joint (hinge) is seen as a device allowing a mutual rotation of the connected elements relative to the center of hinge. On the design diagram of the hinge is indicated by a circle. The friction forces in the hinge are usually neglected.

The fixed connected joint (fixed joint) is completely eliminates mutual displacement, and the angle between the axes of the rods it does not change when the deformation of the system.

The division of joints on a pin and fixed joint is not always true. Often joints give and allow mutual displacement of connected elements (rotations, shifts) in dependent with arising in the joints of internal forces. On the design diagram pliable joints stipulate or represent with elastic connections – line (Fig. 1.1, *a*) or angular (Fig. 1.2, *a*). The internal forces in a accordant joint associated with the mutual displacement of the connected elements. For example, the value of bending moment at joint in Fig. 1.2,a will depend on the values of mutual rotation angle of the connected rods and can be expressed through the value of elastic compliance of a joint C that represents the magnitude of the bending moment generated during mutual rotation of the connected elements on a single angle value (Fig. 1.2, *b*).

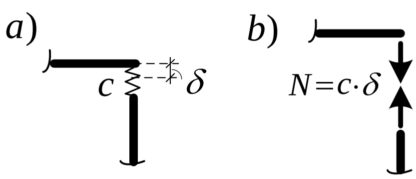


Fig. 1.1

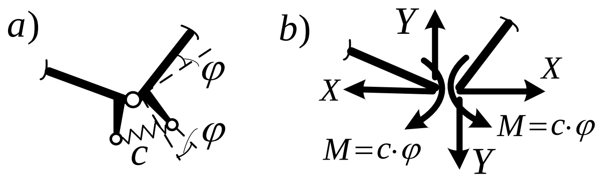


Fig. 1.2

In actiual structures all the elements (racks, beams, plates, shells, etc.) always have some kind of imperfactions from design shapes, sizes, properties of the materials used, which occur during their manufacture, transport, assembly and on which the current schemes are often not taken into account. It is impossible to imagine perfectly straight racks, which appear in the current schemes as straight rods.

Hinges are considered perfect, that is, it is considered that there are no friction forces, and the forces are transmitted through the centers of the pins (hinges), which in practice is difficult to achieve.

1.2.4 Idealization for Loads and Actions

Even more approximations associated with the loads and actions, to determine the exact magnitude of which is in some cases almost impossible. Characteristic values for snow and wind loads are calculated on the basis of statistical processing of the values of loads the results of many years observations. Division of loads into concentrated forces and moments, uniformly distributed loads is also quite conditional.

All the **actions on structures** can be divided into a direct and undirect; forces (or displacements) as has been mentioned.

The actions can be *static* and *dynamic*.

Static action is a load, which increases from zero to a finite value so slowly that the acceleration a points of system which may cause inertia forces (if the deformation is small) can be neglected.

Dynamic is an action, which give to the masses of buildings significant acceleration and, accordingly, this is cause of inertial forces, and their influence should be taken into account.

Example of dynamic actions is vibration, exposure to blast waves, seismic effects.

Forces (direct actions) as a mechanical load: point moments, distributed loads constant or variable intensity can be mobile and stationary. Static load acts constantly in one place. Moving load is move through the system (for example, transport across the bridge).

1.3. Classification of Structures

Classification of structures and their design schemes can be performed according to various criteria, some of which are presented below.

All the structures are *spatial*. However, as already indicated, often they can be calculated and is calculated as a *planar* system. This manual further describes only planar system.

On geometric attributes distinguish:

1. Frameworks (consisting of rod – beams, trusses, frames, arches, combined system).
2. Structures of plates and shells.
3. Massive structures (retaining walls, dams).

On working structures distinguish:

1. Beam construction.
2. Arch construction.
3. Frame – *framework system* with rigid connection of members in all or some joints.
4. Truss – system with all members which are work only tensile-compression.
5. Suspended system – basically, flexible elements wich only work for stretching.
6. The combined system, combination of 1–5.
7. Plates. 8. Shells.

From the point of view of *statics* all system are divided into:

1. Statically determinate systems – with no "extra" links and, consequently, the calculation of which can be performed using only the equilibrium equations (equations of statics).
2. Statically indeterminate systems – having the "extra" reactions at the supports, to calculate which is necessary to attract additional equations.

In the directions of the reactions at the supports there are:

1. The non-spreading structure – structures that have load one direction (e.g., vertical) causes the support reactions are the same direction (only vertical). Example: simple and multi-span beams.
2. Spacer structure – under load in one direction occur, the support reactions in other directions. Examples of push systems are fully articulated arches and frames.

1.4. The Main Assumptions, Principles and Concepts

The basis of the classical methods of structural mechanics based on the following main assumptions, principles and concepts:

1. The material of all elements of structures is a continuous, isotropic and homogeneous. Isotropic material is a material whose properties are the same in all directions.
2. Body considered perfectly elastic. These bodies recover their original shape and size after removal of external loads.
3. Considered materials satisfy Hooke's law, according to which the dependence between load and displacement and “stress – strein” is linear;
4. It is assumed that for structures fair the principle of superposition of forces. It may be stated as follows; the total displacement or internal forces at a point in structure which subjected to several external loads (actions) can be determinate by adding together the displacements or internal forces by each of the external loads acting separately.
5. Preconditions about displacement and deformations: we assume that the dis-

placements are small with respect to the dimansions of the elements and deformations are small compared with unit. This precondition allows as to write equilibrium conditions for the ininial shape of structure and also to neglect the small displacement of structure.

2. Kinematic Analysis of Structures

Building structures are designed to perceive acting on them loads, while maintaining a predetermined shape, that is, they must be *invariable* (geometrically) *system* (stable system).

Invariable system is called such a system where, shape change and mutual arrangement of elements are possible only due to the deformation of its elements.
An example of a simple invariable system is the three rod hinged triangle (Fig. 2.1).

Invariable system is a system the change of the shape and mutual arrangement of elements which is possible even in the case that all its elements are to be considered absolutely rigid. The simplest example of an unstable (inperfect) system is the rectangle hinged rod (Fig. 2.2).

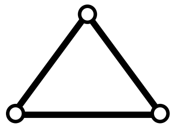


Fig. 2.1

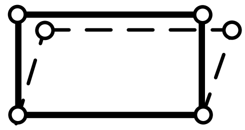


Fig. 2.2

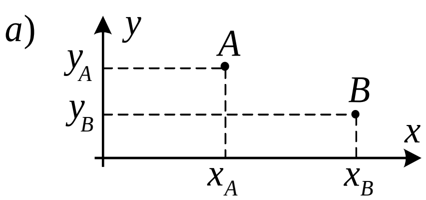
Unstable systems in buildings is unacceptable, given that they can dramatically change the form and therefore can not perceive acting on them load.

To clarify the geometric invariability or unstable of the systems is the *kinematic analysis* of the structures which is performed in two stages:

1) determination of the degree of freedom of the system;
2) Geometric analysis of the structure of the system.

<u>*The degree of freedom of a system*</u> is the number of independent geometrical parameters (coordinates, displacements) that determine the position of all elements of the structure on the plane or in space.

For example, a point on the plane has two degrees of freedom, since its position is described by two coordinates (Fig. 2.3, *a*).



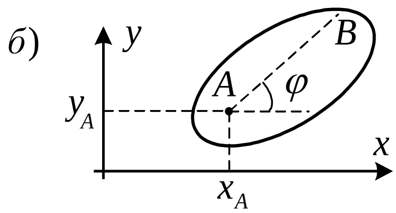


Fig. 2.3

Point in space has three degrees of freedom.

Any intentionally invariable (perfect) body, structure or its part will be called a disk.

Examples disk is shown in Fig. 2.4. Disk may be gotten if jointing of a number of rectilinear rods with the formation of the branched framework (see figure 2.4). Given that a rigid jointing can be gotten by connection of three elements (perfect way) in triangular structure.

Any disk in the plane has three degrees of freedom – the position in the plane determined by three parameters (x_A, y_A, φ) (Fig. 2.3, *b*).

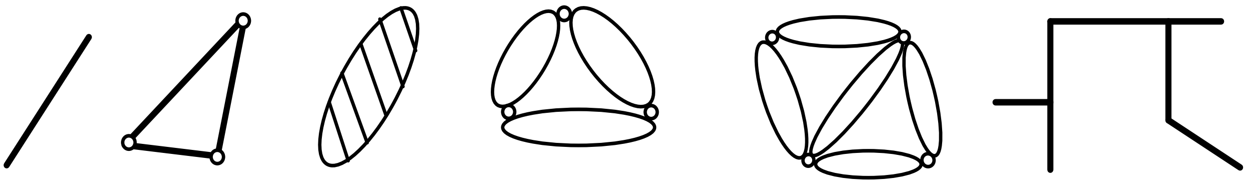


Fig. 2.4

In the space the disk has six degrees of freedom – three coordinates of a point and three angles of rotation about axes *x*, *y*, *z*.

Any body, which takes back from another body one degree of freedom, is called *kinematic* or *simple restriction*.

Construction on the design schemes, as already noted, can be represented in the form of systems consisting of a disk, connected by *hinge joint,* and resting on the base (on the ground) by means of supports.

Can be distinguished: – *hinge joint (joint)*
– *a simple or single hinge joint (single joint).*

A single joint – is a joint connecting two disks.

In the hinge joint that connects a number of disks, can be joined a few simple hinge joints, which can be determined by the formula: $n_{h.} = D_j - 1$,

where D_j – the number of disks connecting to the joint.

For example:

1 *s. h.* 2 *s. h.* 1 *s. h.* 3 *s. h.*

Fig. 2.5

Each simple joint prevents any mutual linear displacement of the connecting elements, leaving the possibility of their mutual rotation with respect to each other, respectively, has two kinematic constraints. And at its cutting in it there are two internal forces two reactions of interaction of the connected disks (Fig. 2.6).

<u>*Determination of the degree of freedom of the system*</u> can be performed according to the formulas:

a) *b*)

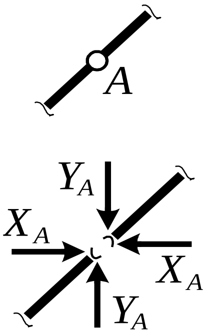


Fig. 2.6

1) $$W = -(3L - H), \tag{2.1}$$

where: L – the number of closed loops in the system;

H – the number of simple, single joints in the system, including hinge joints between the disks (rods) and a base (ground).

Under closed loop we mean a closed contour which formed by the series of rigidly or hingedly connected of disks , one of which may be ground (Fig. 2.7, *b*).

a) *b*) *c*)

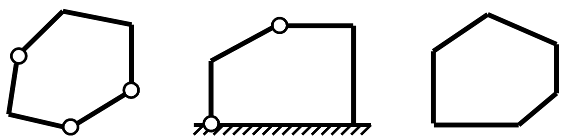
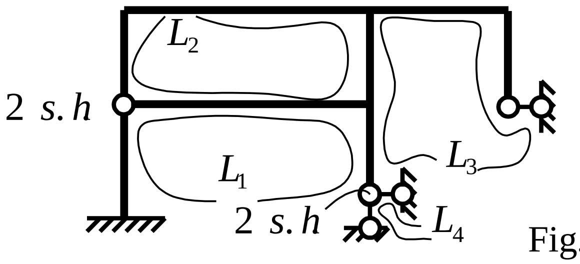


Fig. 2.7

If in closed loop all the elements are connected to each other only tough (rigidly), it will be called a *rigid closed loop* (Fig. 2.7, *c*).

Formula (2.1) can be applied to any planar systems.

For example, for the system depicted in Fig. 2.8, we get:



$$W = -(3L - H) = -(3\cdot 4 - 8) = -4.$$

Fig. 2.8

or

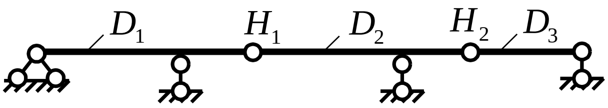
2) $$W = 3D - 2H - C_0\,, \tag{2.2}$$

where: D – the number of disks in the system; H – the number of single joints by which are performed the connecting the disks D; C_0 – the number of kinematic restrictions (*reactions at the supports*) of the system.

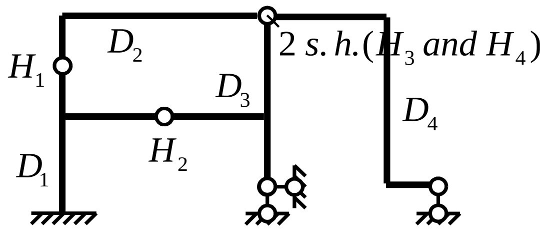
This formula can be used for all systems; exept those which contain completely rigid closed contours (Fig. 2.7, *c*).

Examples of the application of the formula:

1)



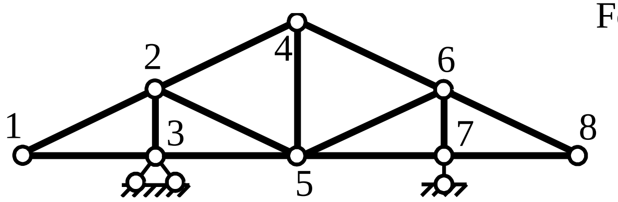
2)



$W = 3D - 2H - C_0 = 3\cdot 3 - 2\cdot 2 - 5 = 0$; $W = 3D - 2H - C_0 = 3\cdot 4 - 2\cdot 4 - 6 = -2$.

3. For truss: $$W = 2J - R - C_0, \tag{2.3}$$

where: J – the number of hinge joints in the truss; R – the number of rod in the truss; C_0 – the number of kinematic restrictions of the system.



For the truss shown in Fig. 2.9, will receive:

$$W = 2J - R - C_0 = 2\cdot 8 - 13 - 3 = 0.$$

Fig. 2.9

Depending on the number of degrees of freedom for systems, there are three qualitatively different from each other result:

1. $W>0$ – the unstable system is a mechanism as it does not have enough connections.

2. $W=0$ – the system has a sufficient number of connections to be invariable (perfect) and statically determinate.

3. $W<0$ – the system has a redundant connection, is statically indeterminate and must be invariable.

Conditions $W<0$ and $W=0$ – are necessary but not sufficient to claim that the system is invariable, as geometric invariability depends not only on links but also on their location, that is, from the structure of the system. In some cases the system may has a sufficient number of connections but to be unstable system. This illustrates a simple system, shown in Fig. 2.10, *a*. Beam as the disk has three degrees of freedom, which would seem out of the three available restrictions. But since all these restrictions are vertical, none of them fixes the beam from the horizontal displacement that is possible and accordingly (consequently) the system is unstable. If one of the supports to move, for example, set it horizontally as shown in Fig. 2.10, *b*, the system becomes invariable variant.

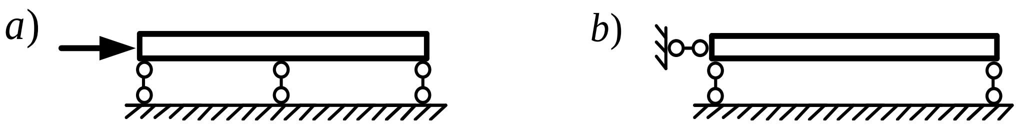


Fig. 2.10

As you can see, the invariability of the system to a large extent determined by the location of their elements, conditions of their restriction to each other and the disposition of the supports, that is, the structure of the systems.

Therefore, to determine whether the structure is (geometrically) invariable, or it is instantaneously variable system, it is necessary to perform geometric analysis of the structure of buildings, which usually performs on the basis of known previously (or in advance):

<u>Principles of Formation of Geometrically Invariable Systems</u>:

1. *Three disks connected to each other sequentially by three hinge joints are not collinear, form an invariable system, that is, the whole system is the disk* (Fig. 2.11, criterion 1).

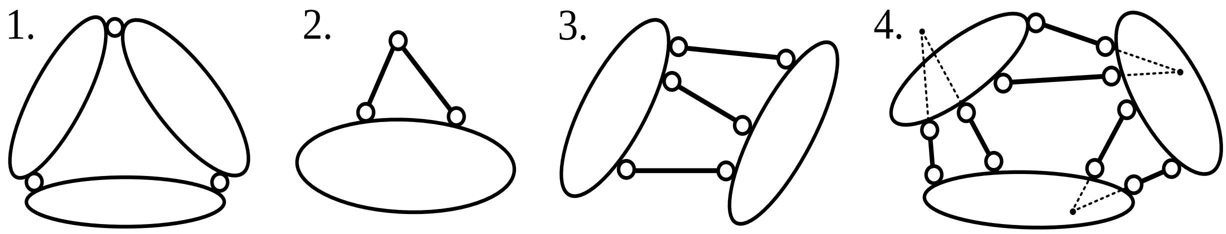


Fig. 2.11. The principles of formation of geometrically invariable systems

2. *If the point is attached to the disk (e. g. through two rods), not lying on one straight line, then the whole system invariable, i.e., is a disk* (Fig. 2.11, criterion 2).

3. *Two disks, united to each other by three rods that are not parallel to each other and not intersecting in one point, form an invariable system, that is, the whole system is a disk* (Fig. 2.11, criterion 3).

Consider the connection of two disks at some point (by help of two intersecting rods) (Fig. 2.12, *a*). Analysis of this system shows that the disks can be rotated each relative to the other with respect to the point of intersection of the rods C, (this point is the joint). Such a joint is called an *imaginary hinge*, and connection of the two disks can then be represented as shown in Fig. 2.12, *b*.

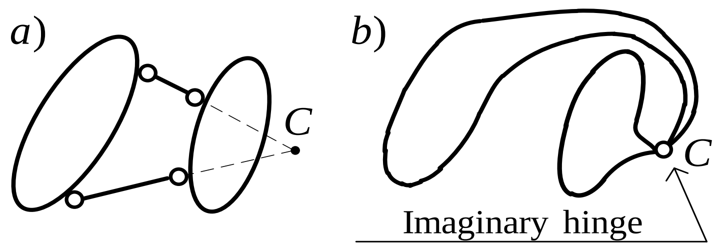


Fig. 2.12

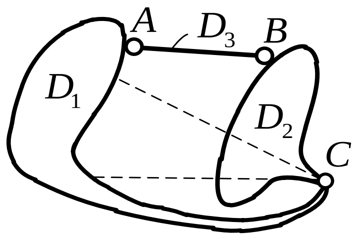


Fig. 2.13

Based on the case (Fig. 2.11, criterion 3, which was shown above) connecting three disks can be represented as shown in Fig. 2.13, which corresponds to the successive connection of the three disks three joints of which do not lie on a straight line (or the first criterion of invariability).

4. *Three disks linked successively to each other in pairs of rods, the points of intersection which lie on the same line, form an invariable system, and the system as a whole is the disk* (Fig. 2.11, criterion 4).

If a pair of intersecting rods to replace here imaginary hinges (joints located at the points of intersection of these pairs of rods), then again, we have three disks, connected to each other sequentially by three hinges (though imaginary hinges), not lying on one straight line.

<u>The Concept of Instantaneously Variable System</u>

Instantaneously variable system represents an exceptional case of geometrically invariable systems in which the mambers of a system get infinitely small displacements. For example, consider the system shown in Fig. 2.14, *a*.

The rightmost point of the disk D_1 (joint C) in this state can move vertically upward or downward (when happens the rotation of the disk D_1 with respect to a point A the displacement vector of this point will be perpendicular to AC radius); arguing similarly, we get that the leftmost point of the disk D_2 also has the possibility of vertical desplasement, which will be identical to the previous vertical desplasement. Thus, the point C in the system can move vertically. But as soon as this point moves to the some value, the system will satisfy of the first criterion of invariability – three disks to be linked successively to each other by three joint, not lying on one straight line become invariable.

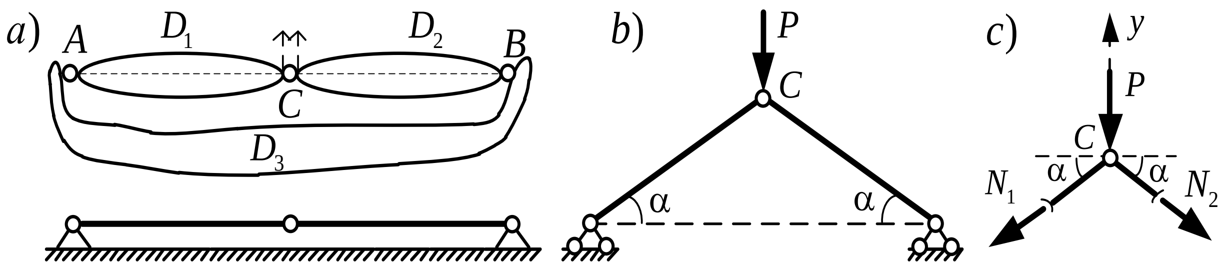


Fig. 2.14

This is the essence of instantaneously variable system. The possibility of displacements in instantaneously variable system, however small, does not allow their use in building constructions.

Instantaneously variable system can be obtained from the system shown in Fig. 2.14, *b*, if the angle α tends to zero ($\alpha \to 0$). When the jointing load only acts, this system will only work in compression and longitudinal forces can be found, cutting out the joint *C* (Fig. 2.14, *c*):

$$\sum Y = 0; \; -2N_1 \sin\alpha - P = 0; \quad N_1 = -\frac{P}{2\sin\alpha}; \text{ With } \alpha \to 0 \text{ we get: } N_1 = -\frac{P}{0} = \infty.$$

This calculation shows that for instantaneously variable system internal forces can take very large values, and accordingly the cross section elements should also be large up to enfinity, this is evidantly imposible. Thas instantaneously variable system may not be used in building constructions.

Criterias of Instantaneously Variable System

1. *If three disks are linked to each other sequentially by three hinges lying on a straight line, the system is instantaneously variable* (Fig. 2.14,*a*).

2. *If two disks are linked to each other by three rods parallel to each other, this system instantaneously variable* (Fig. 2.15, *a*).

3. *If two disk are connected to each other by three rods that intersect in one point, this system is instantaneously variable* (Fig. 2.15, *b*).

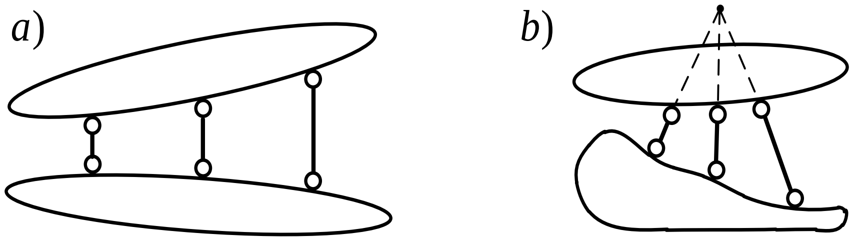


Fig. 2.15

Perform kinematic analysis of several systems.

Example 1. Consider the system shown in Fig. 2.16, *a*. We define the degree of freedom of the system by formulas (2.1) (Fig. 2.16, *a*) and (2.2) (Fig. 2.16, *b*):

$$W = -(3L - H) = -(3\cdot 4 - 12) = 0;$$

$$W = 3D - 2H - C_0 = 3\cdot 5 - 2\cdot 5 - 5 = 0.$$

The results, as you can see, the same, which, of course, it should be.

Proceed to the second stage of the kinematic analysis, geometric analysis of the structure of the system. Note that the base (ground) is a holistic object that is why it may be considered as disk too.

The procedure of geometric analysis:

1) Disks D_2, D_3 and D_4 are linked by three hinge joints *B, S, C,* non-collinear and respectively form an invariable system, that is, the disk (the first critarion of invariability (Fig. 2.16, *c*);

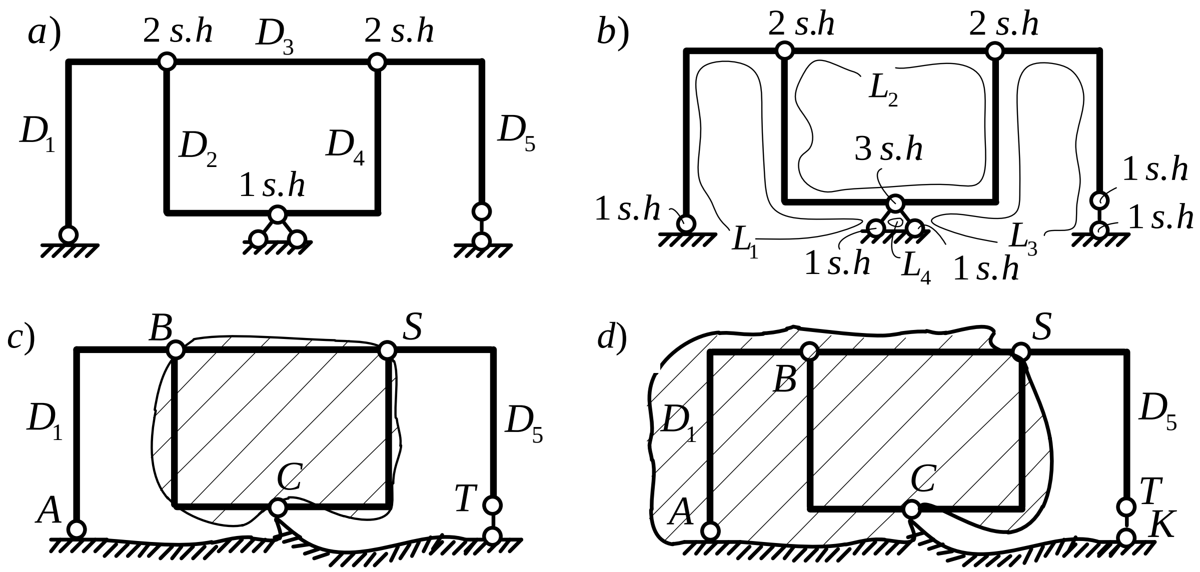


Fig. 2.16

2) The hinge joint C is attached to the disk of the earth the two rods do not lie on a straight line, they form with it (with earth) an invariable system (a second characteristic of invariability) (Fig. 2.16, *c*);

3) disk D_1, the disk of the earth and disk *BSC* are linked by three hinge joints *A*, *B* and *C*, non-collinear, forming an invariable system (the first critarion of invariability) (Fig. 2.16, *d*);

4) the last disc (together with the earth), disk D_5 and the support rod *TK* (and this is also the disk) linked to each other sequentially by three hinge joints *S*, *T* and *K*, not lying on one straight line (Fig. 2.16, *d*) and thus form a generally invariable system (the first critarion of invariability).

Thus, we conclude that the considered system is statically determinable and invariable geometrically.

Therefore, performing geometric analysis of the structure of the system represents the structural-logical problem which it is necessary to solve with use the above cases. The result of solution of it may be recapitulation about the structure of the system (connections of elements); is designed system has reliability to use farther or not? This should be done from point of view of usage of concepts: geometric invariability or the instantaneous variability (or concept of unstable structure).

Let's represent some quantity of systems which demand from you implementation of the kinematic analysis.

Tasks for independent solutions:

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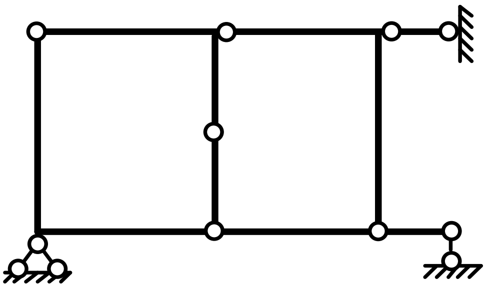

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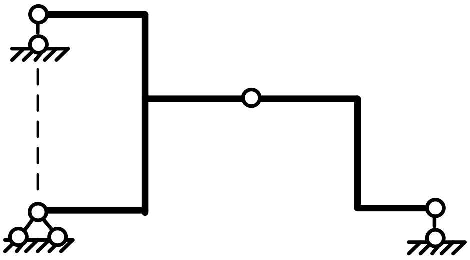

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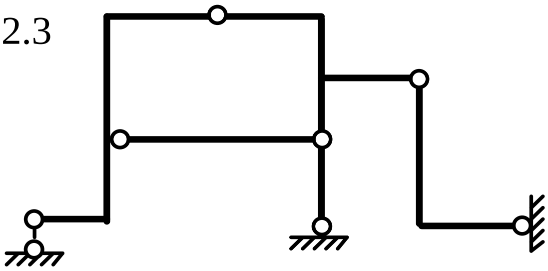

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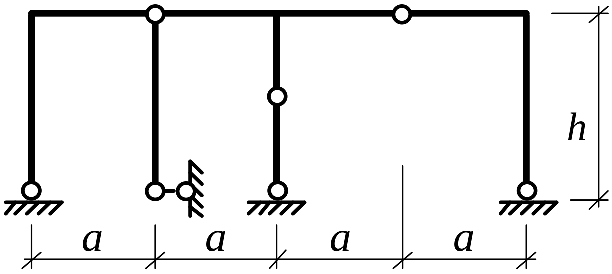

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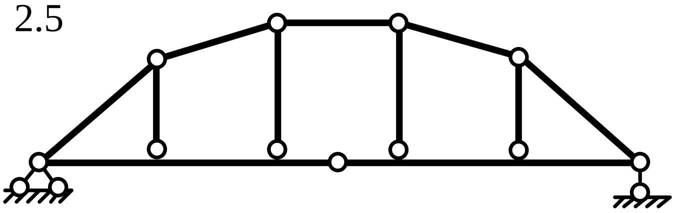

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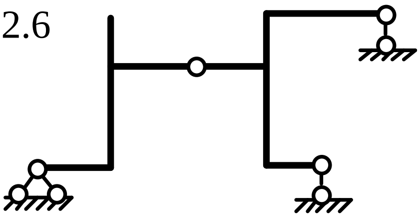

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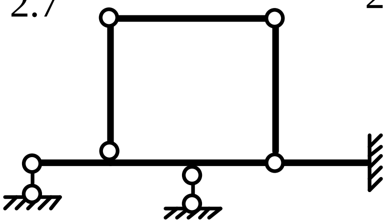

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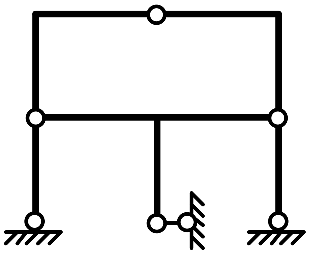

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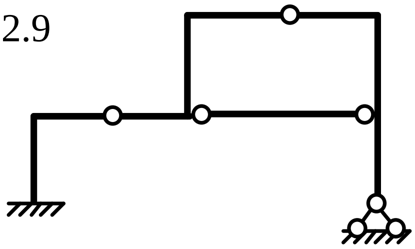

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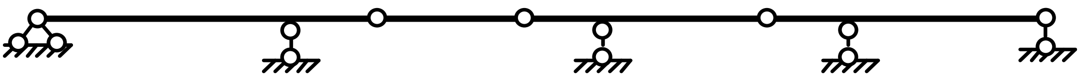

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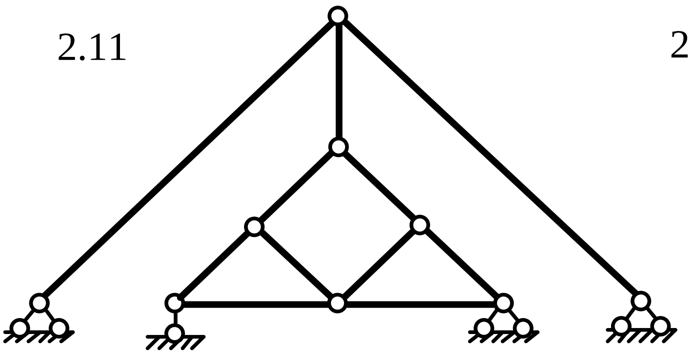

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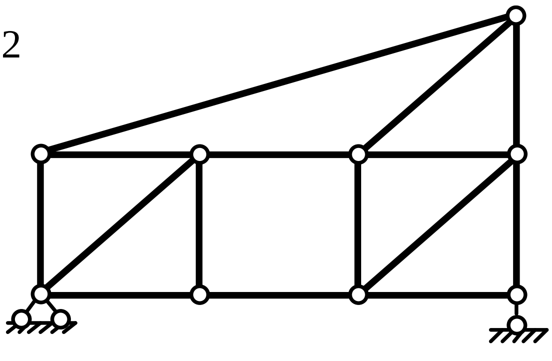


The answers to these tasks are presented in the end of the book in the section "Answers to tasks for independent solution" (p. 67).

3. Calculation of Statically Determinable Framworks

3.1. Internal Forces and Determination of Them

At the action onto the framework statically determinat system of external loads in each cross section (in the plane) may occur three types of internal forces (Fig. 3.1):

– *the bending moment M* acting in the plane of the structure with respect to (w.r.t.) the central axis of the cross section of the element (rod) paralel to the plane O*xy*;

– *The shear force Q* acting in the plane of the cross section in the direction of the central axis of the cross section element (rod), which lies in the plane of the structure;

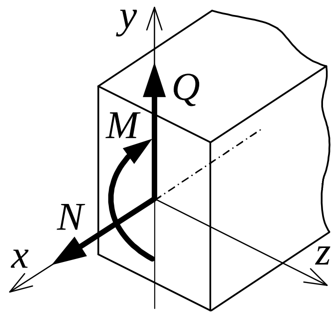


Fig. 3.1

– *The longitudinal force N* acting in perpendicular to the cross section (along the axis of the rod) and applies to the center of gravity of a section.

The determination of the forces of *M*, *Q*, *N* in the cross sections of statically determinate rod systems is performed by the *method of sections*. The method is based on the fact that if the same system is in equilibrium, that is equilibrium any part of it. In the set place of determining the forces a cross section need to divide the element into two parts. Then examine the balance of one of the parts of the system. Thus the discarded fraction at the considered part is replaced by a forces are equal to the internal forces in the section. These forces act on the remaining part, as an external forces. From equilibrium equations, the number of which is equal to three (corresponding to the number of unknown internal forces) the forces in the system section are determined.

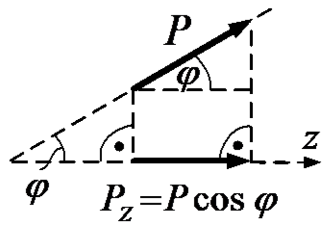
A method of sections allows us to formulate **rules for determining internal forces** in cross sections of framework:

(!) ***Bending moment*** in cross-section is numerically equal to algebraic sum of moments of all external forces (including support reactions), which is to parts of the framework on one side of the section, w.r.t. to the center of gravity of a section.

(!) ***The shear force*** in the cross-section is numerically equal to algebraic sum of projections of external forces (including support reactions), which is to part of the framework with one side of the section, on the axis perpendicular to the axis of the rod.

(!) ***The longitudinal force*** in cross-section is numerically equal to algebraic sum of projections of external forces (including support reactions), which is applied to part of the framework on one side of the section, on the axis that is tangent to the axis of the rod (a straight rod on the axis of the rod).

Recall that the projection of force on the axis is equal to the product of the magnitude of the force on the cosine of the angle between the *line of action* (l. a.) of the force and the axis $P_z = P \cos \varphi$. (see figure to the right).



<u>Moment of a force about a point</u> is equal to the product of the magnitude of this force on its arm (lever) about this point. Where the moment arm of a force about a point (h_P) is defined as the length of the perpendicular from this point to the line of action of the force. For example, $M_A(P) = P{\cdot}h_P$. in (Fig. 3.2, *a*)

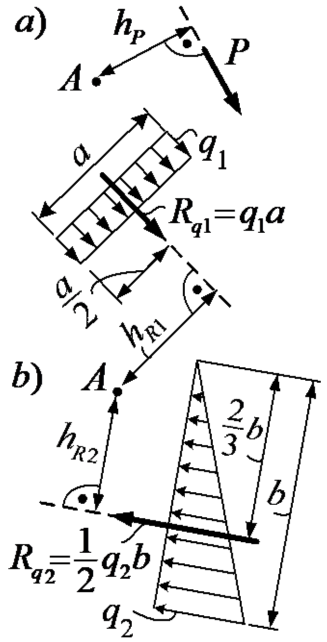


Fig. 3.2

According to the definition can easily be calculated the moment about any point and moment of projection of force on any axis, moment of loads distributed by any laws if these loads lead to resultant forces (R_q). The magnitude of the moment about a point A a uniformly distributed load q_1 (Fig. 3.2, *b*) (the resultant R_{q1} which is applied in the middle of the plot, on which acts the load) is equal to:

$$M_A(q_1)=R_{q1}h_{R1}=(q\,a)h_{R1},$$

and moment of load q_2, changing by the triangular law will be determined by the expression:

$$M_A(q_2)=R_{q2}h_{R2}=\left(\frac{1}{2}q_2b\right)h_{R2}.$$

Note that the moment about any point from the action of the concentrated moment equal to the value of the moment itself and its projection on any axis is equal to zero.

For visual presentation the change of internal forces in framework sections the diagrams of forces are plotted. **The diagram of forces** (*M*, *Q*, *N*) called a graph (diagram), reflecting the dependence of this forces to the lengths of all elements (rods) of a system.

Note some of the rules are used to plot internal forces:

1. Axis (base) on which to plot the diagram of forces, always choose so that it to be parallel to the axis of the rod or simply coincids with it.

2. The ordinate of the diagram of forces set aside from the basic axis is perpendicular to it. Each of the ordinate of the diagram represents the force of a certain scale.

3. To hatch the diagram of forces taken perpendicular to the base axis (each of the hatch lines is also the ordinate of the diagram of forces).

4. Ordinate the forces are laid off at a given scale; in specific points are put the values of the ordinates of force in the certain fields of diagram. Usually are put signs ordinate the diagram of forces (plas or minus) into the circles.

In determining shear and longitudinal forces and plot their diagrams in structural mechanics usually accept the following **rules of signs**:

– **The shear force** in the cross-section is defined by the above rule: is positive if when its infinitesimal distance from the section under consideration from the main part seeks to turn this part w.r.t. to the cross section clockwise, and **negative** if it tends to rotate the part w.r.t. to the section in a counterclockwise direction – see Fig. 3.3.

For example, when considering the cut off part in Fig. 3.3, *c* the shear force in the cross-section K is equal to the force P and is positive.

– **The longitudinal force** in the section is **positive** if it causes tension in the rod frame (is directed from the cross-section), and **negative** if it causes compression (is directed to the section) – see Fig. 3.4.

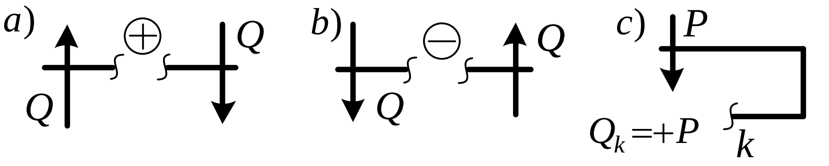


Fig. 3.3

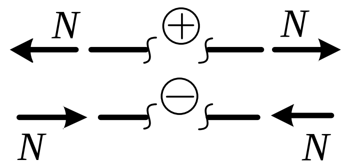


Fig. 3.4

Note that to plot Q and N (are the ordinates of the shear and longitudinal forces) can be plotted from any side of the base axles of rods. It should be guided with a point of view of best visibility of diagrams and, of course, on the same plot, which is a continuation of each other in a straight line, advantageously (beneficially), and preferably, the ordinates of the same sign to put off into one side. For bending moment diagram special rules are not set, the ordinates are put aside at the stretched fibers (from the stretched fibers). In determining the values of the bending moments signs they can be taken at your own discretion (consideration). In this book the bending moment is taken positive if it acts on the cross section clockwise. The stretched fiber in cross-section in this case are defined as follows. In that part of the system from equilibrium of which the bending moment determined, to allocat the infinitesimal element of the rod, adjacent to the section (in the diagrams, this infinitesimal element of the rod, for clarity depicted as element of finite length). Consider then that in the design cross-section of the specified element has a fixed-ended condition, put to it previously computed for this cross-section bending moment. Analyzing now the bending of the considered element of the system (frame) is easy to determine which side of the rod fibers are stretched and some are compressed.

Diagram of the bending element of the system, selected by the section 1–1 to the right side shown in Fig. 3.5, *b*. From the analysis of this scheme is visible that stretched in the cross-section 1–1 will be the lower fiber (hereinafter in the diagrams of the bending elements the stretching fibers will be denoted by dashed lines).

Note that for systems in equilibrium, stress in any cross-section obtained when considering one and (or) the other parts of frame w.r.t. this section, will be certenly equal to each other (the values of the bending moments in this case use of the known of the rule of signs will be obtained with opposite signs – but stretched fiber, defined by them, will be with the same side).

For example, for the system in Fig. 3.5, *a*, must be complied with equality:

$$M_{1\text{-}1}^{(left)} = -M_{1\text{-}1}^{(\text{right})}; \quad Q_{1\text{-}1}^{(left)} = Q_{1\text{-}1}^{(\text{right})}; \quad N_{1\text{-}1}^{(left)} = N_{1\text{-}1}^{(\text{right})}.$$

This should be used to verify the correctness of calculation of forces in sections of the system.

Here are some examples of computing forces in section 1–1 (in general) the system

b)

shown in Fig. 3.5, *a*, which we assume equilibrium:

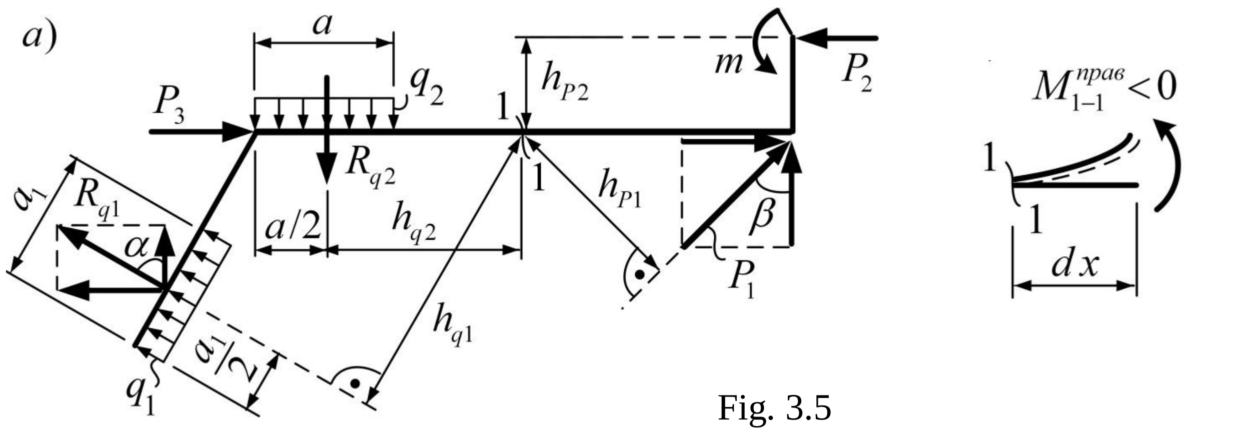


Fig. 3.5

$$M_{1-1}^{(left)} = R_{q1} \cdot h_{q1} - R_{q2} \cdot h_{q2} + P_3 \cdot 0 = q_1 a_1 h_{q1} - q_2 a h_{q2};$$

$$M_{1-1}^{(right)} = -P_1 \cdot h_{p1} - P_2 \cdot h_{p2} - m;$$

$$Q_{1-1}^{(left)} = R_{q1} \cdot \cos\alpha + P_3 \cdot \cos 90^0 - R_{q2} \cdot \cos 0^0 = q_1 a_1 \cos\alpha - q_2 a \;;$$

$$Q_{1-1}^{(right)} = -P_1 \cdot \cos\beta + P_2 \cdot \cos\ 90^0 = -P_1 \cos\beta;$$

$$N_{1-1}^{(left)} = R_{q1} \cdot \sin\alpha - P_3 \cdot \cos 0^0 - R_{q2} \cdot \cos 90^0 = q_1 a_1 \sin\alpha - P_3;$$

$$N_{1-1}^{(right)} = -P_2 + P_1 \sin\beta.$$

Here: $M_{1\text{-}1}^{(left)}$, $Q_{1\text{-}1}^{(left)}$, $N_{1\text{-}1}^{(left)}$ – forces to cross-section 1-1, obtained from consideration of the left part of the system with respect to this section; $M_{1\text{-}1}^{(right)}$, $Q_{1\text{-}1}^{(right)}$, $N_{1\text{-}1}^{(right)}$ – the same forces is obtained from consideration of the right part of the system with respect to cross-section 1-1.

3.2. Statically Determinable Frames, Their Types

Frames call system consisting of straight members connected together rigidly (and hinged joint), and supported by means of support on the base.

The statically determinate frames are usually divided into simple, three-hinged joint frame and composite frame.

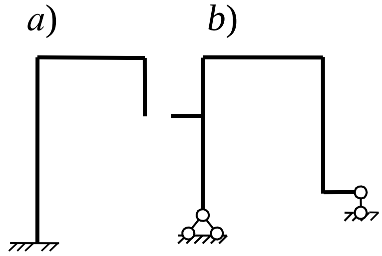


Fig. 3.6 Simple frame

A simple frame is a system (Fig. 3.6), consisting of a single disk as a broken branched rods connected to the base by three restrictions typically using three main types of supports are movable hinged support (or roller support), an immovable hinged support (or pin), and absolutely rigid restraint (or pinching) (table. 1.1).

Three-hinged frame (Fig. 3.7, *a*, *b*) – is a system, consisting of three disks (a broken branched rods), connected together sequentially by three hinge joints are not collinear (that is, in principle, three disks). One of the disks in this case may be the base (Fig. 3.7, *a*). Three-hinged frame belong to the class of thrust systems.

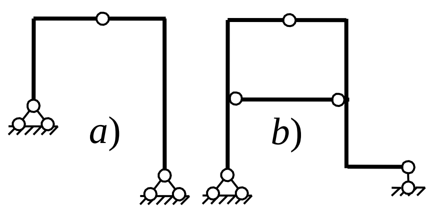


Fig. 3.7

Composite frame is called frame, which consists of several interconnected simple and (or) three-hinged frames (Fig. 3.7, *c*).

Calculation of composite frames is performed by calculating the separate simple and three-hinged frames that can be separated composite frame, taking into account their interaction with each other.

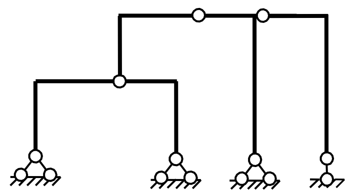


Fig. 3.7

3.3. The Calculation of Simple Frames

Consider the frame shown in Fig. 3.8. From the analysis of a frame show (it is visible) that the determination of the forces of its sections is impossible to do without knowing about the reactions at the supports. Consequently, the calculation of the frame must begin with their determination.

The reactions at the supports are determined from the equilibrium equations of the frame as a whole, which in general case can be in three different versions:

1) a sum of projections of forces on two arbitrary non-parallel to each other axis and a sum of the moments of force with respect to some point on the plane. So, we can find all unknowns with the help of the following system of equations ($\Sigma X = 0$; $\Sigma Y = 0$; $\Sigma M_T = 0$);

2) As a sum of projections of forces on an arbitrary axis and the two sums of moments about any points on the plane not lying on the same perpendicular to the mentioned axis of projection, for example ($\Sigma X = 0$; $\Sigma M_A = 0$; $\Sigma M_B = 0$);

3) Three sums of moments about three points, not lying on one straight line ($\Sigma M_A = 0$; $\Sigma M_B = 0$; $\Sigma M_C = 0$).

Note that the equations of equilibrium to determine reactions at the supports must be selected so that in each of them, if possible, there will be only one unknown reaction at the support that has not been previously defined. They must be selected using, for example, the equation of moments about the points of intersection of other unknown reactions or the sum of projections of forces on axis, perpendicular to the line connecting two moment points (see calculation frame in Fig. 3.8).

After determining the reactions at the supports it is always necessary to make the verification (or – confirm the results) of their calculation, we should use the equation of equilibrium, which has not previously been used, and which would include all previously calculated reactions.

Determine the reactions at the supports for the frame (Fig. 3.8). Here we use the second variant of the equilibrium equations:

$$\Sigma X = 0; \qquad R_A - 4 = 0; \qquad R_A = 4 \text{ кN};$$

$$\Sigma M_L = 0; \qquad -4\cdot3 + 4\cdot2 + 8 + 2\cdot4\cdot2 - R_C\cdot 4 = 0; \qquad R_C = 5 \text{ кN};$$

$$\Sigma M_D = 0; \qquad -4\cdot7 + 4\cdot2 + 8 + R_B\cdot4 - 2\cdot4\cdot2 = 0; \qquad R_B = +\,7 \text{ кN}.$$

If the value of the result is negative, this indicates that we assumed its direction incorrectly and it will be the opposite. In this case, it is recommended to immediately redirect the direction of the reaction.

Check the correctness of the determination of reactions at the supports:

$\Sigma M_K = 0$; $4{\cdot}4 - 2{\cdot}4 + 8 - 7{\cdot}3 + 2{\cdot}4{\cdot}5 - 5{\cdot}7 = 0$; $+64 - 64 = 0$; $0 = 0$.

For the convenience of further calculations it is recommended to show actual values of the calculated reactions on the design diagram of the frame (see Fig. 3.8).

Go to the determination of internal forces and the plotting of their diagrams. It is easy to see that any frame can be divided into separate parts (segments of the rods), each of which changes a shape or the internal force is described (within this site) by one equation. The boundary points of these sites in which there is a transition from one dependence changes to another, will be named the characteristic points.

Characteristic points are usually:

– the points at which the concentrated loads (forces, moments) or reactions are applied externally at the supports;

– The points of the beginning, middle and end of distribution of loads application;

– A salient point of the branching rods.

For the frame in Fig. 3.8 point's *A, T, K, B, C* will be characteristics and the frame can be divided into four design parts:

$$\mathrm{I} \to AT, \quad \mathrm{II} \to TK, \quad \mathrm{III} \to KB \text{ and part IV} \to BC.$$

Consider first part 1. Let's take the arbitrary cross section 1–1. The internal forces in any section of the frame can be determined from the w.r.t. the right or/and vice versa (lower and upper parts of the frame as well); however, these values should be equal to each other (see section 1); they can and should be used to check the correctness of the calculation of forces in the cross sections. Note that it is more convenient to produce the determination of internal forces in cross sections from the consideration of the cut out part of the frame, which have a smaller number of forces. For the section I–I consider the upper part of the frame (this example will show the considering part of the frame separately – Fig. 3.9, *a*). The distance from the upper extreme point (point *A*) to the cross section 1–1 is denoted as x_1. Then the expressions for determining internal forces in an arbitrary cross section 1–1 (force zone 1: $0 \le x_1 \le 2$) will have the form:

$$M_{\mathrm{I}}^{top} = R_A{\cdot}x_1 + P{\cdot}0 = 4x_1; \qquad Q_{\mathrm{I}}^{top} = +R_A = 4; \qquad N_{\mathrm{I}}^{top} = -P = -4.$$

Similarly, we determine the internal forces on part II ($0 \le x_2 \le 4$) (Fig. 3.9, *b*):

$$M_{\mathrm{II}}^{top} = 4{\cdot}(2 + x_2) - 4{\cdot}0 - 4{\cdot}x_2 = 8 \text{ (at any value of } x_2\text{)};$$

$$Q_{\mathrm{II}}^{top} = 4 - 4 = 0; \qquad N_{\mathrm{II}}^{top} = -4,$$

and on part III ($0 \le x_3 \le 3$) (Fig. 3.9,*c*): $M_{\mathrm{III}}^{left} = 4{\cdot}4 - 4{\cdot}2 - 4{\cdot}x_3 = 8 - 4{\cdot}x_3$

[at $x_3 = 0$ (section 5) – $M_5 = 8$ kN·m; at $x_3 = 3$ (section 6) – $M_6 = -4$ kN·m];

$$Q_{\mathrm{III}}^{left} = -4; \qquad N_{\mathrm{III}}^{left} = 0.$$

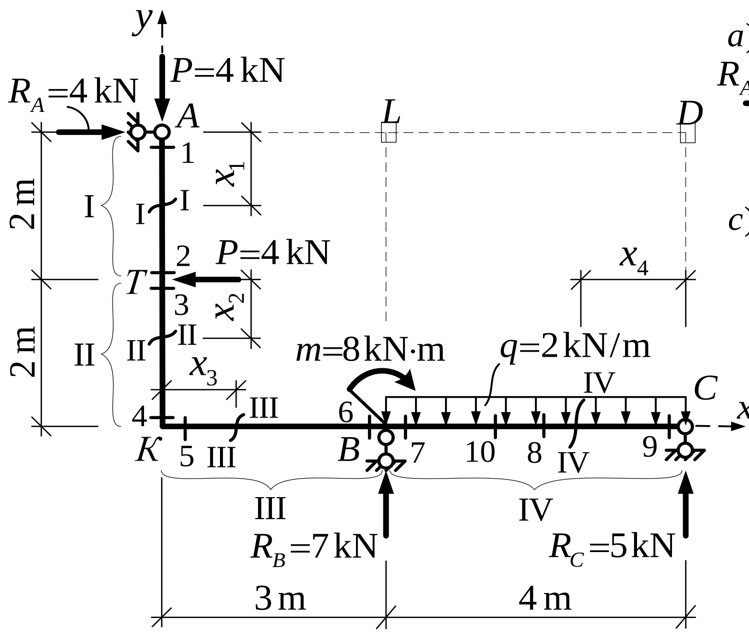


Fig. 3.8

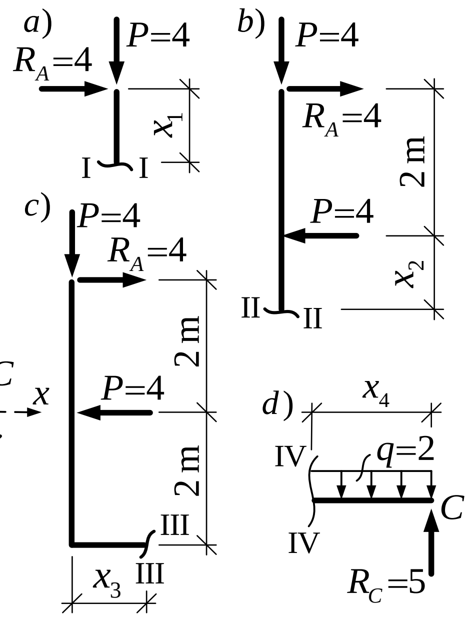


Fig. 3.9

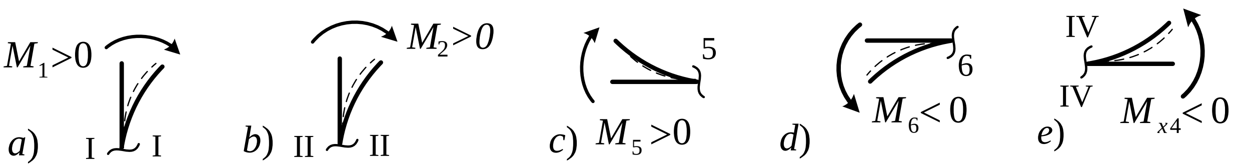


Fig. 3.10. Definition of stretched fibers

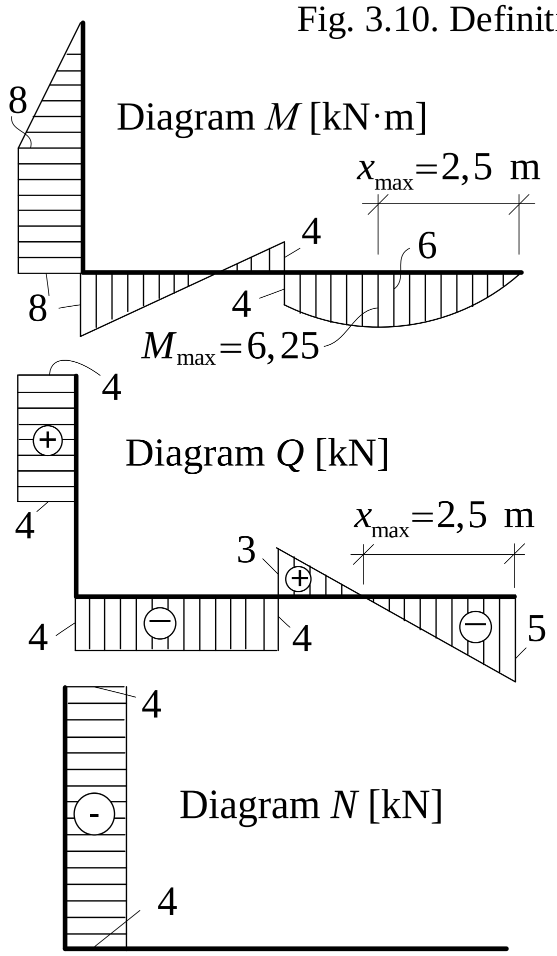
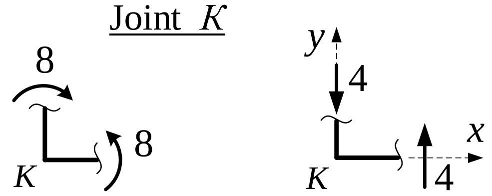


Fig. 3.11

<u>*a*) Checking the balance of joint</u>

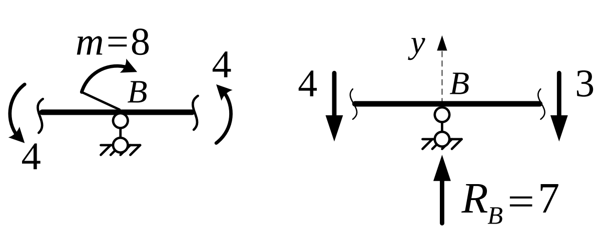
<u>Joint *K*</u>



$\Sigma M_K = 0$; $\quad \Sigma X = 0;\ \ 0 = 0;$

$8 - 8 = 0;$ $\quad \Sigma Y = 0;\ \ -4 + 4 = 0.$

<u>*b*) Supporting joint *B*</u>



$\Sigma M_B = 0;\quad 8 - 4 - 4 = 0;\quad 8 - 8 = 0;$

$\Sigma X = 0;\quad 0 = 0;$

$\Sigma Y = 0;\quad -4 - 3 + 7 = 0;\quad -7 + 7 = 0.$

Fig. 3.12

The bending moment at part 1 varies linearly and the shear and longitudinal force – is constant. Substituting values x_1 for the extreme cross-sections into the expression for the bending moment, we will find:

$M_1 = M_A = 4{\cdot}0 = 0$;

$M_2 = 4{\cdot}2 = +8$ kN·m (section 2 is located at an infinitely small distance from the top of the point T – the point of application of force P). Stretched fibers section 1-1 (and in this case for the whole part 1) is defined in Fig. 3.10, *a*.

Somehow determined forces are more complicated on part IV. Having an arbitrary cross section IV–IV, consider the right part of the frame (Fig. 3.8). Expressions for the internal forces on the section will have the form:

$$M_{\mathrm{IV}}^{(right)} = -5{\cdot}x_4 + 2{\cdot}x_4{\cdot}x_4/2 = x_4^2 - 5{\cdot}x_4; \qquad Q_{\mathrm{IV}}^{(right)} = -5 + 2{\cdot}x_4; \qquad N_{\mathrm{IV}}^{(right)} = 0.$$

It is seen that the bending moment on part IV varies in a parabolic dependence, and the shear force is linear (but not uniform unlike parts I, II, III). To plot M in this area, therefore, it is necessary to calculate the values of the bending moments in at least three points – for example, the edges of the site (in sections 7 and 9) and in the middle of it (section 8):

sec. 7 – $x_7 = 4$ м; $M_7 = 4^2 - 5{\cdot}4 = -4$ kN·m;

sec. 8 – $x_8 = 2$ м; $M_8 = 2^2 - 5{\cdot}2 = -6$ kN·m; sec. 9 – $x_9 = 0$; $M_9 = 0$.

The stretched fibers at part IV are defined by the obtained values of the bending moments in it (Fig. 3.10, *e*), and the diagram of M is presented in figure 3.11.

To plot the shear forces on part IV, it is sufficient to calculate the values of Q in two sections (because you can always draw a straight line through two points) – usually these values are calculated in the extreme cross sections schemes:

sec. 7 – $x_7 = 4$ m; $Q_7 = -5 + 2{\cdot}4 = +3$ kN;

sec. 9 – $x_9 = 0$; $Q_9 = -5 + 2{\cdot}0 = -5$ kN.

It should be borne in mind that the parts of action of uniformly distributed loads, where bending moments vary according to the parabolic dependences, the curve M can have the extreme (maximum or minimum), which is important characteristics of diagrams should be determined additionally. If we analyze the expressions for M_{IV} and Q_{IV}, given the condition of extreme functions (according to which the extremum of the function is the location where its first derivative equals zero) and a known differential dependence $Q = dM/dx$, it is easy to see that the extreme values of the bending moments are taken in sections in which the shear force is zero. These cross-sections can be determined from the expressions for Q (in our example – $Q_{\mathrm{IV}} = 2{\cdot}x_4 - 5 = 0$; $x_{max}=2{,}5$ m), or Q from geometric considerations ($5/x_{max}=3/(4-x_{max})$; $x_{max}=2{,}5$ m). The maximum bending moment of the considered frame on part IV thus takes place in section 10 ($x_{max} = 2{,}5$ m) and is equal to: $M_{10} = M_{\mathrm{IV}max} = 2{,}5^2 - 5{\cdot}2{,}5 = -6{,}25$ kN·m. Final diagrams M, Q, N are shown in Fig. 3.11

All joints of the frame, including a supporting joint, must be in equilibrium.

When we say “joint”, we mean a salient point or the point of branching rods of the frame and the point of junction of the rods through the hinge joints. So after plotting the frame diagrams *M*, *Q* and *N* t it is advisable to check the balance of their joint. For this purpose, the joints are cut out and drawn away from the frame (show them separately on fig 3.12,b). To make it clear, let’s show the elements adjacent to the joints of the rods and apply to them the forces occurring in the cross-sections infinitely close to the joints, and the external loads – forces and moments acting on the joints (if they are). Then let’s make the equations of equilibrium of all forces applied to the joints ($\Sigma M_y = 0$; $\Sigma X = 0$; $\Sigma Y = 0$) and check their implementation. For the considered frame checking the balance of joints are shown in Fig. 3.12.

(!) Analyzing the diagrams of internal forces allows us to set a **number of general regularities in the change of the diagrams *M, Q, N***, which should always be observed for rod systems:

1) on straightforward no-load segment the diagram of the bending moments is always linear and can be constructed with two coordinates (usually for the extreme sections of the diagram), and diagram *Q* and *N* are uniform (the same in all sections);

2) at the segment of the action of a uniformly distributed load the diagram *M* is always changed according to the parabolic law, and should be plotted using at least three coordinates (usually for outer and middle sections on the segment; if necessary, there is no difficulty to find *M* for additional sections); the convexity of the diagram of *M* is always plotted in the direction of the action of a uniformly distributed load; the diagram *Q* on this segment can be plotted for two ordinates (for cross sections);

3) at the point of application of some concentrated force the diagram M always has a break, pointing in the direction of the force, the diagram *Q* – the jump (discontinued) equal to the product of this force by the cosine of the angle between the force and the axis normal to the axis of the rod and the diagram. The diagram *N* – the jump equal to the product of this force by the sine of the angle between the force and the axis normal to the axis of the rod; if the external force is perpendicular to the axis of the rod, the jump on the diagram of *Q* is equal to the magnitude of the force, and on the diagram *N* there will be no jump;

4) at the point of application of concentrated moment *M* the diagram always has a jump (discontinuity) on the magnitude of this moment;

5) in the hinge the bending moment is always zero (not to be confused with the cross section and hinge section, infinitely close to the hinge; so if the section is infinitely close to the hinge, the applied concentrated moment *M* on the diagram in this section according to the previous situation will jump from zero at the hinge to the value of the concentrated moment at the point of application);

6) at the site of action of the distributed load in the cross section where the shear force is zero the bending moment always has an extremum (minimum, maximum);

Diagrams *M, Q, N* can be plotted to compute the values in the characteristic sections of the frame.

Consider the frame shown in Fig. 3.13.

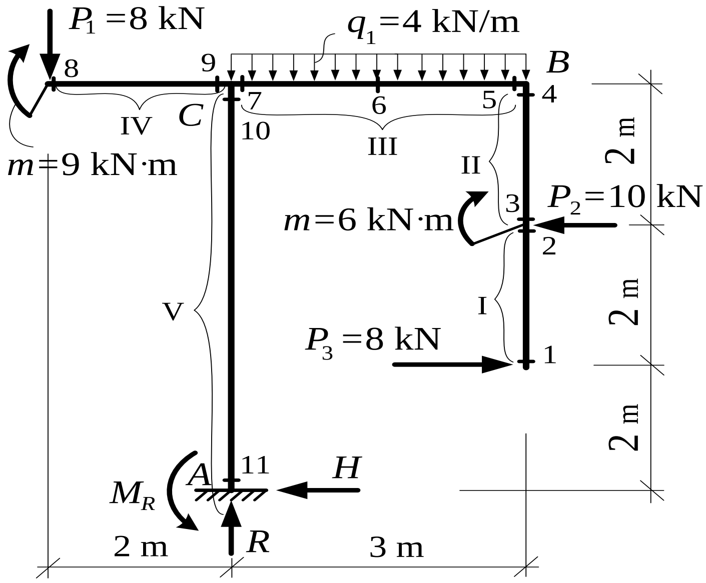


Fig. 3.13. Design scheme of the frame

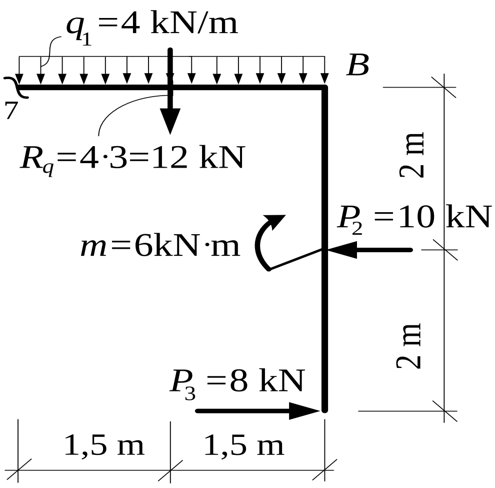


Fig. 3.14

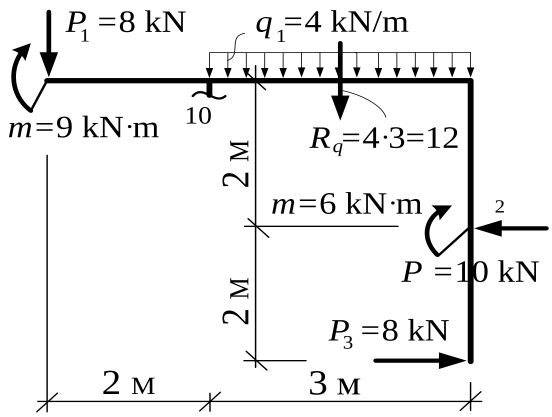


Fig. 3.15

The frame is a console tipe, and in the calculation of internal forces in any of its sections it is possible to do without determining reactions at the support. If we consider for all sections balance of cut off cantilever parts of the frame. Thus, to plot internal forces in these frames reactions at the support can not determine, if it is not explicitly required. On the other hand, knowing the reactions, we always have the opportunity to check previous calculations (considering the equilibrium of cut off part of the frame from support side and the balance of the support joint).

For plotting diagrams *M*, *Q* and *N* consider the frame (Fig. 3.13). It should be broken down into 5 parts. To plot the bending moments take into account that in parts I, II, IV, V of frame *M* plot will vary linearly and for its construction enough to know the values of the bending moments at the extreme points of these sections, that is sections 1, 2, 3, 4, 8, 9, 10, 11. On part III, which is subjected to action uniformly distributed load, the curve *M* will vary according to the parabolic law and for its construction it is necessary to calculate bending moments in sections 5, 6, 7. Perform the calculation internal forces in these cross sections, considering the equilibrium of cantilever parts of the frame (Fig. 3.14 and 3.15 shows the corresponding parts for 7 and 10 sections):

$M_1 = -8 \cdot 0 = 0$; $M_2 = -8 \cdot 2 = -16$ kN·m; $M_3 = -8 \cdot 2 + 6 = -10$ kN·m;

$M_4 = M_5 = -8\ 4 + 10 \cdot 2 + 6 = -6$ kN·m; $M_6 = -8 \cdot 4 + 10 \cdot 2 + 6 + 4 \cdot 1{,}5 \cdot 0{,}75 = -1{,}5$ kN·m;

$M_7 = -8 \cdot 4 + 10 \cdot 2 + 6 + 4 \cdot 3 \cdot 1{,}5 = 12$ kN·m; $M_8 = +9 - 8 \cdot 0 = 9$ kN·m;

$M_9 = +9 - 8 \cdot 2 = -7$ kN·m; $M_{10} = +9 - 8 \cdot 2 - 8 \cdot 4 + 10 \cdot 2 + 6 + 4 \cdot 3 \cdot 1{,}5 = -7$ kN·m;

$M_{11} = +9 - 8 \cdot 2 + 8 \cdot 2 - 10 \cdot 4 + 6 + 4 \cdot 3 \cdot 1{,}5 = +6$ kN·m.

Final plot of the bending moments is based on the stretched fibers, according to the rules outlined above. The plot itself is shown in Fig. 3.16, *a*.

Calculate the values of Q and N in the same section:

$Q_{\mathrm{I}} = Q_1 = Q_2 = -8$ kN; $Q_{\mathrm{II}} = Q_3 = Q_4 = -8 + 10 = +2$ kN;

$Q_{\mathrm{IV}} = Q_8 = Q_9 = -8$kN; $Q_{\mathrm{V}} = Q_{10} = Q_{11} = +8 - 10 = -2$ kN;

$N_{\mathrm{I}} = N_1 = N_2 = N_{\mathrm{II}} = N_3 = N_4 = N_{\mathrm{IV}} = N_8 = \mathrm{N}_9 = 0$; $N_{\mathrm{V}} = N_{10} = \mathrm{N}_{11} = -8 - 4 \cdot 3 = -20$ kN.

Compute Q in two sections – 5 and 7: $Q_5 = 0$; $Q_7 = 4 \cdot 3 = +12$ кN. $N_{\mathrm{III}} = -2$ кN.

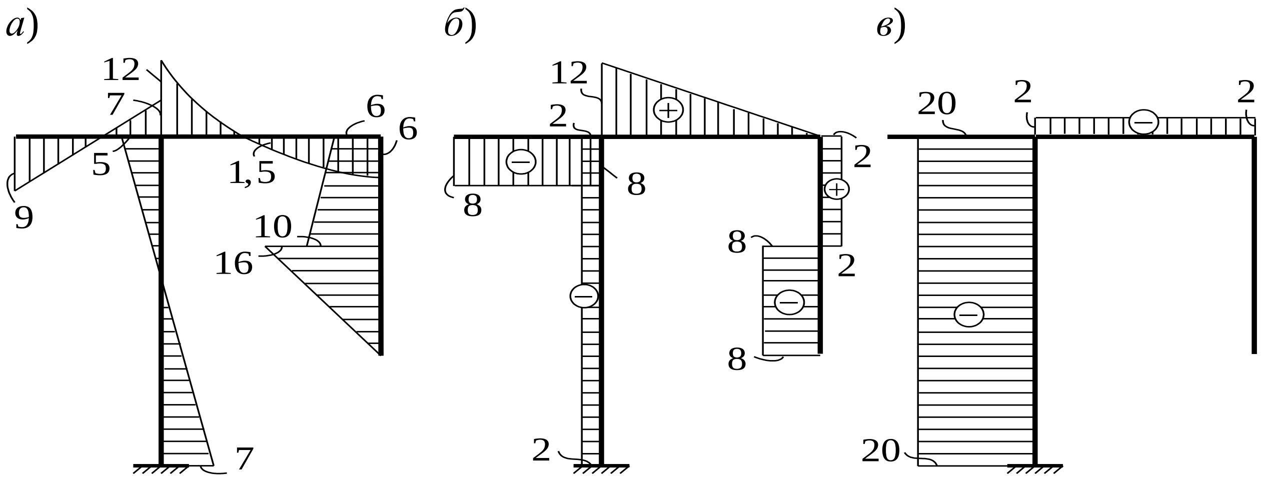


Diagram M [kN·m]   Diagram Q [kN]   Diagram N [kN]

Fig. 3.16. Final diagram of internal forces

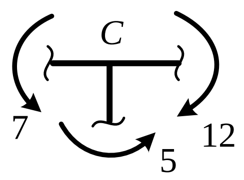
After plotting the final diagrams of M, Q and N, the system is checked to the balance of their joints. There are shown for the considered frame in Fig. 3.17.

a) on diagram *M*:

<u>Joint *C*</u>

$\Sigma M_C = 0$;

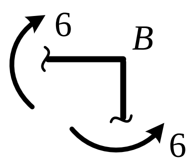
$12 - 7 - 5 = 0$;



<u>Joint *B*</u>

$\Sigma M_B = 0$;

$6 - 6 = 0$;



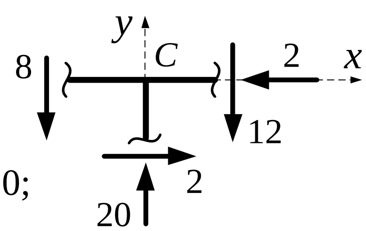
b) on diagram Q and *N*:

<u>Joint *C*</u>

$\Sigma X = 0$; $2 - 2 = 0$;

$\Sigma Y = 0$; $-8 + 20 - 12 = 0$;

$20 - 20 = 0.$



<u>Joint *B*</u>

$\Sigma X = 0$; $2 - 2 = 0$;

$\Sigma Y = 0$; $0 = 0.$

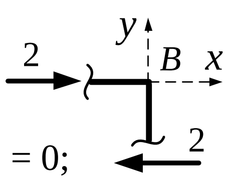


Fig. 3.17. Checking the balance of joints

The calculation of the consol frame is finished if we are not interested in the reaction of fixed end (pinching support). It should be noted that in the design practice of constructions calculation of value of support reactions is usually necessary to know. So here also fulfill the determination of reactions of fixed end, for which consider the equilibrium of the frame in general:

$\Sigma X = 0;$ $\quad 8 - 10 - H = 0;$ $\quad H = -2$ kN;

$\Sigma Y = 0;$ $\quad R - 8 - 4{\cdot}3 = 0;$ $\quad R = 20$ kN;

$\Sigma M_A = 0;$ $\quad 9 - 8{\cdot}2 + 4{\cdot}3{\cdot}1{,}5 - 10{\cdot}4 + 8{\cdot}2 - M_R = 0;$ $\quad M_R = -7$ kN·m.

It is easy to see by analyzing the final diagrams of M, Q and N (Fig. 3.16) that the values of the calculated reactions are equal to the corresponding internal forces at support joint 11. This suggests that the support joint is in equilibrium and that the calculation of frame is done correctly.

(!) The analysis of above examples allows suggesting the following order of plotting diagrams of the internal forces of M, Q and N in frame and truss and beam systems:

1. Show assumed directions of the reactions at the supports in the system.
2. Write the equilibrium equations of the system. By solving these equations the values of support reactions can be determined.
3. Perform verification of calculated reactions at the supports.
4. The system divides into calculated parts (areas between the characteristic points) and are identifyed section where nacessary to calculate the internal forces M, Q and N to plot diagram.
5. The values of internal forces M, Q and N comput for these sections (based on the rules set out above) for plotting of diagram of internal forces. In the areas of action of uniformly distributed loads comput extreme values of the bending moments (if they are exist)).
6. Are checked of the balance of the joints and is observed the general regularities of change in diagram of internal forces.

Perform the calculation of another simple frame shown in Fig. 3.18, *a*.

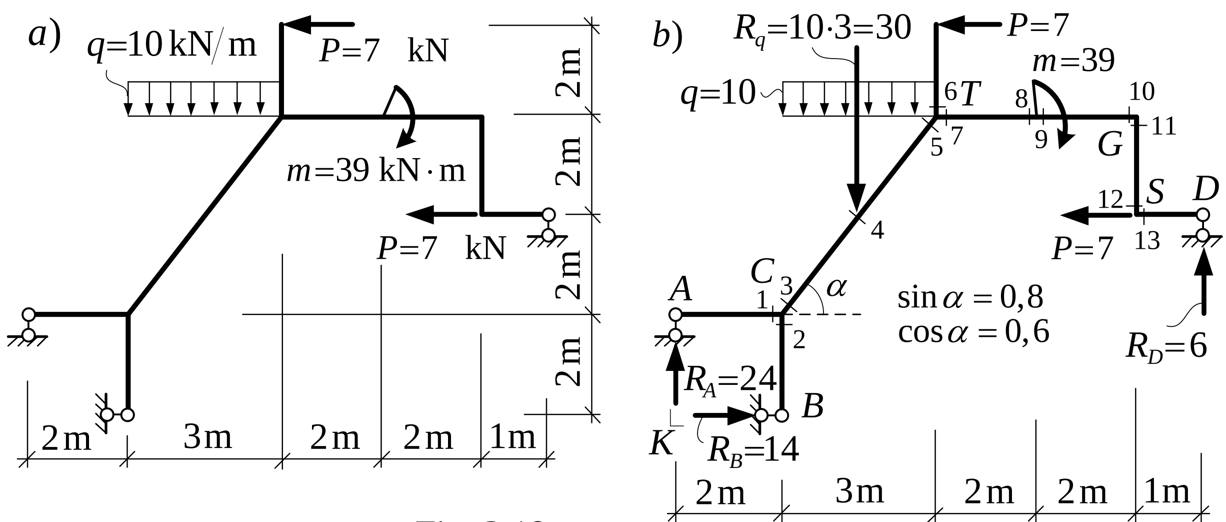


Fig. 3.18

From the point of view of kinematic analysis frame is a single disk (a broken branched rod) connected to ground (earth) by three rods (jointed by movable hinges), not parallel to each other and intersecting at one point.

Determine of the reactions at the supports (Fig. 3.18, *b*):

$$\Sigma M_K = 0; \quad (10\cdot3)\cdot3{,}5 - 7\cdot8 + 39 - 7\cdot4 - R_D\cdot10 = 0; \qquad R_D = 6 \text{ kN};$$

$$\Sigma Y = 0; \quad R_A - 30 + 6 = 0; \qquad R_A = 24 \text{ kN};$$

$$\Sigma X = 0; \quad R_B - 7 - 7 = 0 \qquad R_B = 14 \text{ kN}.$$

The checking of the calculations of reactions at the supports:

$$\Sigma M_T = 0; \quad 24\cdot5 - 14\cdot6 - (10\cdot3)\cdot1{,}5 - 7\cdot2 + 39 + 7\cdot2 - 6\cdot5 = 0; \quad 159 - 159 = 0.$$

For plotting diagrams M, Q and N the frame should be broken down into eight parts (Fig. 3.18, *b*), on which select the 13 sections for determining the shape of diagrams of internal forces. Omitted cross-sections in which the forces can easily be calculated orally and where bending moments equal to zero (are the cross-sections at the hinge joints and at the end of the console). It is recognized that the diagram of the bending moments at the site of action of a uniformly distributed load is changed by a parabolic dependence, and for its plotting it is necessary to calculate bending moments, at least, in three sections – 3, 4, 5; in other parts of the frame diagram *M* will be changed linearly and to plot diagram enough to know the values of the bending moments in the two extreme points of these sites.

Calculation of values of bending moments and determination of the stretched fibers in the character cross sections (Fig. 3. 18, *b*):

$M_1^{left} = 24\cdot2 = +48$ kN·m; $\quad M_2^{low} = -14\cdot2 = -28$ kN·m;

$M_3^{low} = 24\cdot2 - 14\cdot2 = +20$ kN·m;

$M_4^{low} = 24\cdot3{,}5 - 14\cdot4 - (10\cdot1{,}5)\cdot0{,}75 = +16{,}75$ kN·m;

$M_5^{low} = 24\cdot5 - 14\cdot6 - (10\cdot3)\cdot1{,}5 = -9$ kN·m; $\quad M_6^{top} = -7\cdot2 = -14$ kN·m;

$M_7^{right} = +39 + 7\cdot2 - 6\cdot5 = +23$ kN·m;

$M_8^{right} = 39 + 7\cdot2 - 6\cdot3 = +35$ kN·m; $\quad M_9^{right} = 7\cdot2 - 6\cdot3 = -4$ kN·m;

$M_{10}^{right} = 7\cdot2 - 6\cdot1 = 8$ kN·m; $\quad M_{11}^{low} = 7\cdot2 - 6\cdot1 = 8$ kN·m;

$M_{12}^{low} = 7\cdot0 - 6\cdot1 = -6$ kN·m; $\quad M_{13}^{right} = -6\cdot1 = -6$ kN·m;

The final diagram of the bending moments for the considerad frame is shown in Fig. 3.20, *a*.

In the same way are reasoning also at plotting of diagrams of shear and longitudinal forces.

At the area of action of uniformly distributed loads diagrams the shear and longitudinal forces will be linear. Compute the values of Q and N in the two sections 3 and 5:

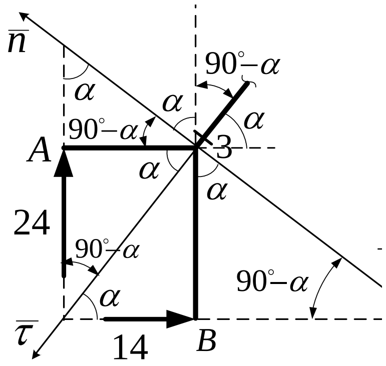


Fig. 3.19

$$Q_3^{low} = 24\cdot\cos\alpha - 14\cdot\sin\alpha = 24\cdot0{,}6 - 14\cdot0{,}8 = 3{,}2 \text{ kN};$$

$$Q_5^{low} = 24\cos\alpha - 14\sin\alpha - (10\cdot3)\cos\alpha = 24\cdot0{,}6 - 14\cdot0{,}8 - (10\cdot3)\,0{,}6 = -14{,}8 \text{ kN};$$

$$N_3^{low} = -24\cdot\sin\alpha - 14\cdot\cos\alpha = -24\cdot0{,}8 - 14\cdot0{,}6 = -27{,}6 \text{ kN};$$

$$N_5^{low} = -24\cdot\sin\alpha - 14\cdot\cos\alpha + (10\cdot3)\sin\alpha = 24\cdot0{,}6 - 14\cdot0{,}8 + (10\cdot3)\,0{,}8 = -3{,}6 \text{ kN}.$$

In other sections, shear and longitudinal forces will be uniform. For plotting diagrams of Q and N enough get the values of one's for each part section:

$$Q_1^{left} = 24 \text{ кН};\quad Q_2^{low} = -14 \text{ kN};\quad Q_6^{top} = -7 \text{ kN};\quad Q_7^{right} = Q_8 = Q_9 = Q_{10} = -6 \text{ kN};$$

$$Q_{11}^{top} = Q_{12} = +7 \text{ kN};\quad Q_{13}^{right} = -6 \text{ kN};\quad N_1^{left} = 0;\quad N_2^{low} = 0;\quad N_6^{top} = 0;$$

$$N_7^{right} = N_8 = N_9 = N_{10} = -7 \text{ kN};\quad N_{11}^{top} = N_{12} = -6 \text{ kN};\quad N_{13}^{right} = 0.$$

Plotted according to the given data diagrams Q and N are depicted in Fig. 3.20, *b*, *c*,

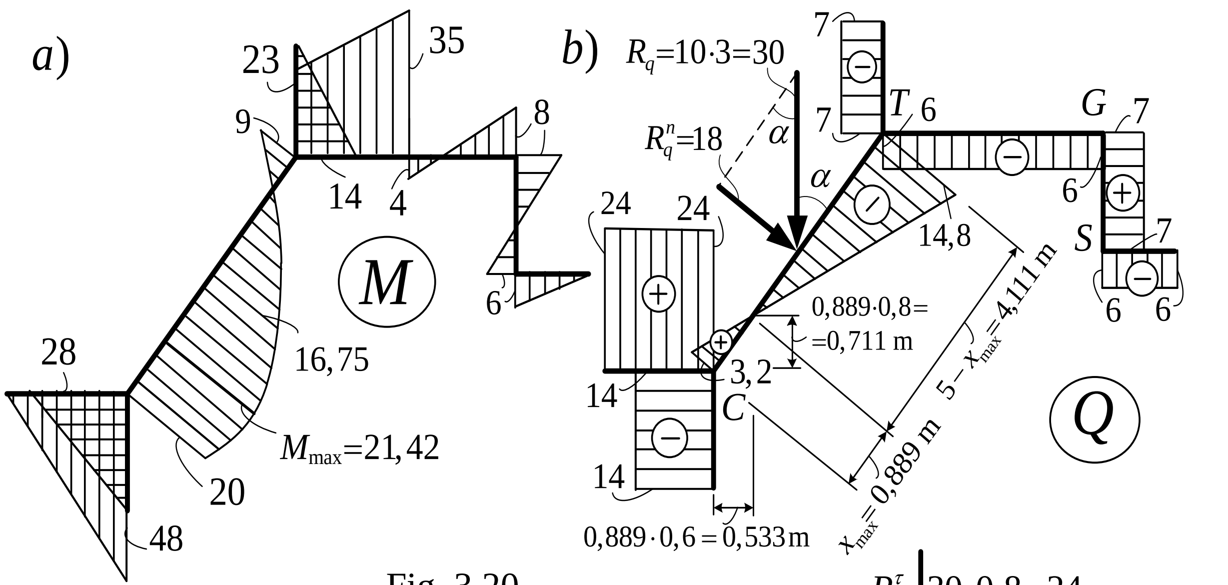
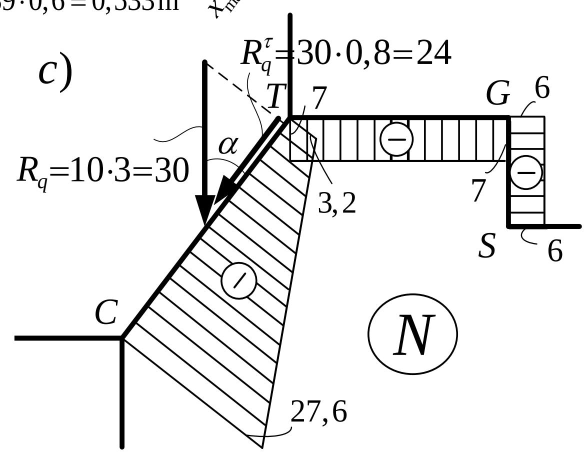


Fig. 3.20



At the part of action of a uniformly distributed load q on the diagram there is a section where the shear force is zero. In this section the bending moment will have the maximum value:

$$M_{max}^{left} = 24\cdot(2+0{,}533) - 14\cdot(2+0{,}711) - $$
$$-(10\cdot0{,}533)\cdot0{,}5\cdot0{,}533 = 21{,}42 \text{ kN}\cdot\text{m},$$

where is the location of the section is determined from the ratio:

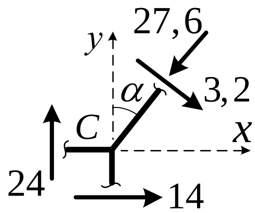
$$\frac{3,2}{14,8}=\frac{x_{\max}}{5-x_{\max}};\quad 3,2\cdot\left(5-x_{\max}\right)=14,8\cdot x_{\max};\quad 18\cdot x_{\max}=16;\ x_{\max}=0,889\ \text{м}.$$

After plotting the final diagrams of the internal forces of M, Q and N, in the system is checked the balance of the joints where bending moments, shear and longitudinal forces act.

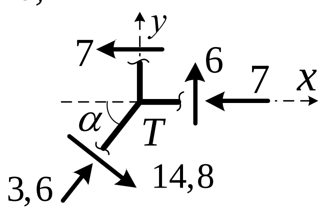
Checking the balance of joints on the diagram M:

$\Sigma M_S = 0;$
$6 - 6 = 0.$

$\Sigma M_C = 0;$ $\quad \Sigma M_T = 0;$ $\quad \Sigma M_G = 0;$
$48 - 28 - 20 = 0;$ $\quad 23 - 9 - 14 = 0;$ $\quad 8 - 8 = 0;$

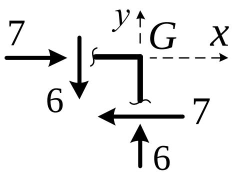


Checking the balance of joints for diagrams *Q* and *N*:
oint *C*:
$\Sigma X = 0;\ -27,6\cdot 0,6 + 3,2\cdot 0,8 + 14 = 0;\ -16,56 + 16,56 = 0;$
$\Sigma Y = 0;\quad -27,6\cdot 0,8 - 3,2\cdot 0,6 + 24 = 0;\quad -24 + 24 = 0;$

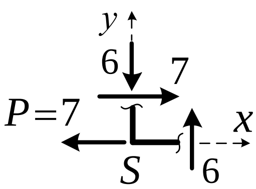


Joint *T*:
$\Sigma X = 0;\ 3,6\cdot 0,6 + 14,8\cdot 0,8 - 7 - 7 = 0;\ 14 - 14 = 0;$
$\Sigma Y = 0;\ 3,6\cdot 0,8 - 14,8\cdot 0,6 + 6 = 0;\ 8,88 - 8,88 = 0;$

Joint *G*:
$\Sigma X = 0 \quad 7 - 7 = 0;$
$\Sigma Y = 0;\ -6 + 6 = 0;$



Joint *S*:
$\Sigma X = 0;\quad -7 + 7 = 0;$
$\Sigma Y = 0;\quad -6 + 6 = 0.$



3.4. Features of the Calculation of Three-Hinged Frames

Three-hinged frames (Fig. 3.7) usually have more than three external support reactions (Fig. 3.7, *a*) or closed loop (Fig. 3.7, *b*), no "cutting" of which it is impossible completely determine the internal forces in such systems.

Therefore, to calculate three-hinged frames the three equilibrium equations of the entire system are not enough. Must be compiled additional equation of equilibrium of certain parts of these systems and determined along with the external reactions at the supports the internal forces (forces at joints). After finding reactions at the supports diagrams of internal forces *M, Q, N* in these systems can be plotted on the same principles as in simple frames.

Below we consider possible schemes for determining the external reactions at the supports, and some of the internal forces for a series of frames, the knowledge of which is sufficient to plot diagrams of the internal forces in these systems (external load on the frames can be any and their combinations for frames is not shown; features associated with the application of loads, will be discussed separately):

***I. Three-Hinged Frame With Restrictions at the Same Level* (Fig. 3.21):**

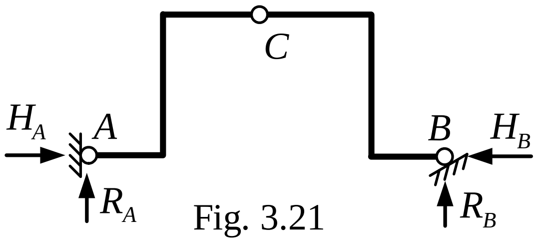


Fig. 3.21

Possible order of calculation:

1) $\Sigma M_A = 0;$ $R_B = ...$
2) $\Sigma M_B = 0;$ $R_A = ...$
3) $\Sigma M_C^{left} = 0;$ $H_A = ...$
4) $\Sigma X = 0;$ $H_B = ...$

Check: $\Sigma M_C^{right} = 0;$ ...

***II. Three-Hinged Frame with Restrictions at Different Levels* (Fig. 3.22):**

The peculiarity of this frame is that it is impossible to make an equations of equilibrium, which would include only one unknown. To determine the reactions at the supports should be combined the system of equations:

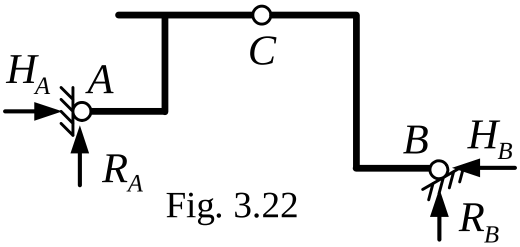


Fig. 3.22

$$\left\{\begin{array}{ll} 1)\ \Sigma M_A = 0; & (R_B,\ H_B) \\ 2)\ \Sigma M_C^{right} = 0; & (R_B,\ H_B) \end{array}\right. \Rightarrow \begin{array}{l} R_B = ... \\ H_B = ... \end{array}$$

3) $\Sigma X = 0;$ $H_A = ...$
4) $\Sigma Y = 0;$ $R_A = ...$

Check: $\Sigma M_C^{left} = 0;$ ...

***III. Three-Hinged Frame with Joining Beam* (Fig. 3.23, *a*):**

Feature of frame is that in its structure a closed loop (*CDK*), which does not allow to determine of internal forces in a closed loop sections. For their determination it is necessary to cut the contour in the frame with joining beam. This can be made by cutting joining beam.

Joining beam is a straight rod, connected with other parts of the system at the ends of the hinge joints and working in the absence of a load only in tension-compression.

Consider the frame shown in Fig. 3.23, *a*. Let the rod *DK unloaded*. Cut out it and consider it balance:

$\Sigma M_D = 0;$ $Y_K \cdot 1 = 0;$ $Y_K = 0;$ $\Sigma M_K = 0;$ $Y_D \cdot 1 = 0;$ $Y_D = 0;$

$\Sigma X = 0;$ $H_D - H_K = 0;$ $H_D = H_K = H.$

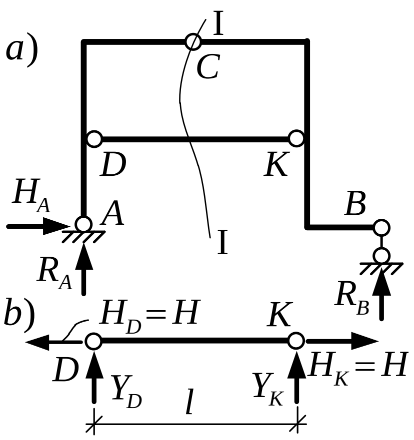


Fig. 3.23

We find that in the rod DK occurs only longitudinal force, and it accordingly works only in tension-compression, that is, the rod DK is *joining beam*, axial force, which we denote by H (fig 3.23, *b*).

Then frame with a unload joining beam can make the following calculation procedure:

1) define the external reactions at the supports (which in this frame – Fig. 3.23, *a* – three, as in the simple frames), for example, from the equations:

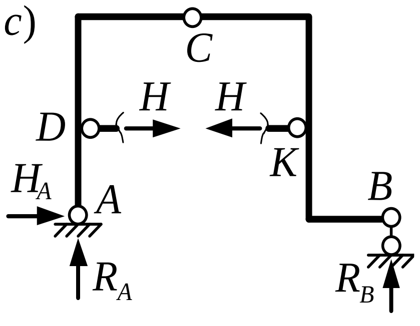


Fig 3.23

$\Sigma M_A = 0$; $R_B =$... $\Sigma M_B = 0$; $R_A =$ ...

$\Sigma X = 0$; $H_A =$... Check: $\Sigma Y = 0$;

2) Take section 1–1 through the joint C and joining beam *DK* (Fig. 3.23, *a*), the force let's denote by H; the frame takes the form shown in Fig. 3.23, *c*;

3) To determine the force at joining beam, consider the equilibrium of one of the half-frame:

$$\Sigma M_C^{right} = 0; \qquad H = \ldots$$

4) To check use the equation of equilibrium to another part of the frame: $\Sigma M_C^{left} = 0$.

***Three-Hinged Frame with Loaded Joining Beam* (fig. 3.24, a):**

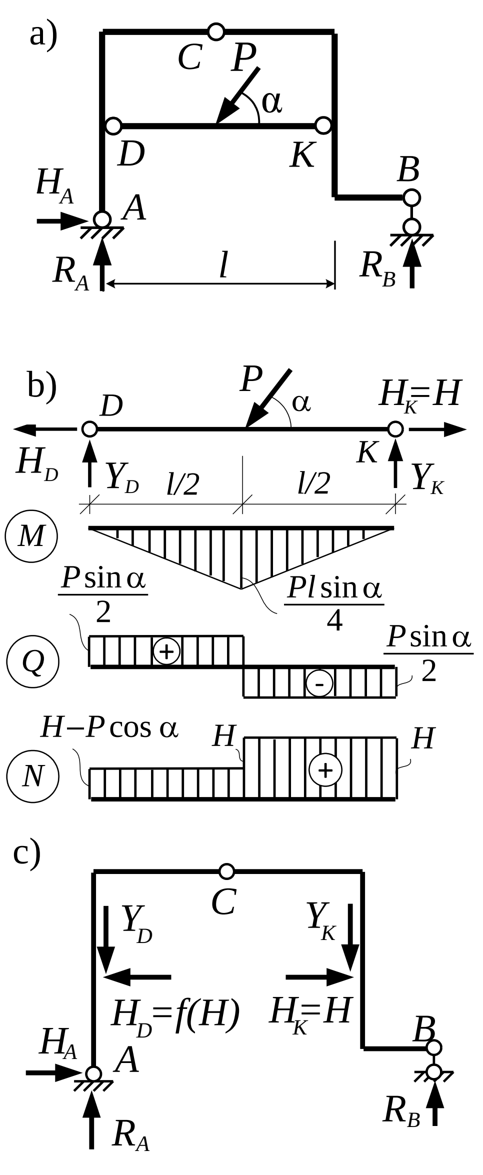


Fig. 3.24

Schemes for the calculation can be as follow:

1) external reactions at the supports R_A, H_A and R_B are defined the same as for the three-hinged frame with unload joining beam (Fig. 3.23, *a*):

$\Sigma M_A = 0$; $R_B =$ ... $\Sigma X = 0$; $H_A =$ ...

$\Sigma M_B = 0$; $R_A =$ ... Check: $\Sigma Y = 0$;

2) cut of joining beam DK and consider it balance along with the act on it a loadings (Fig.3.24, *b*);

From the equilibrium equations $\Sigma M_D = 0$ and $\Sigma M_K = 0$ determine a vertical reactions Y_D and Y_K in joints *D* and *K*, and from the equation $\Sigma X = 0$ find the relationship between H_D and H_K. Let us denote one of these unknown by *H*.

For example, for load on joining beam shown in (Fig. 3.24, *a*) we obtain:

$Y_D = Y_K = 0{,}5 \sin\alpha$; $H_D = H_K - P\cos\alpha = H - P\cos\alpha$.

Having these values, we can plot in the joining beam diagrams of the internal forces *M*, *Q* and *N* with a precision of parameter *H*, (Fig. 3.24, *b*);

3) consider the frame of ABC without any joining beam, but given the transmitted from it to the frame (in reverse directions) forces at joints D and K – Y_D , Y_K, which are already known, and actions H_D, H_K, which are known with a precision of parameter *H* (Fig. 3.24, *c*).

Find *H* (Fig. 3.23, *c*)

$\Sigma M_C^{right} = 0$; $H =$ ... Check: $\Sigma M_C^{left} = 0$;

IV. Three-Hinged Frame with Two Parallel Joining Beam (Fig. 3.25)

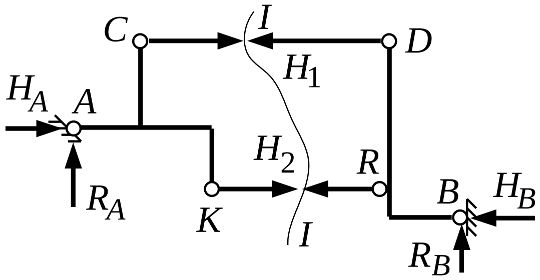


Fig. 3.25

A possible calculation scheme:

1) take section I–I and consider the equilibrium of left and right parts of the frame:

$\Sigma Y^{\text{left}} = 0;\quad R_A = ...\quad \Sigma Y^{\text{right}} = 0;\quad R_B = ...$

2) consider the equilibrium of the frame in general: $\Sigma M_A = 0;\quad H_B = ...$
$\Sigma M_B = 0;\quad H_A = ...$

3) considering now the balance left (or right) part, we find the forces for joining beam: $\Sigma M_K^{left} = 0;\quad H_2 = ...\qquad \Sigma M_C^{left} = 0;\quad H_1 = ...$

4) perform the check on the correctness of reactions: $\Sigma M_B^{right} = 0; ...$

The Example of Calculation of Three-Hinged Frame

Consider the calculation of the frame shown in Fig. 3.26, *a*.

First, determine the reactions at the supports:

$\Sigma M_A = 0;\qquad 10{\cdot}4 - 10 - 20{\cdot}4 - 10 + R_D{\cdot}10 = 0;\qquad R_D = 6$ kN;

$\Sigma M_C^{right} = 0;\quad 20{\cdot}2 + 6{\cdot}7 - 10 - R_B{\cdot}6 = 0;\qquad R_B = 12$ kN;

$\Sigma M_K = 0;\qquad 10{\cdot}4 - 10 - 20{\cdot}4 - 10 + R_A{\cdot}10 = 0;\qquad R_A = 6$ kN;

$\Sigma M_C^{left} = 0;\quad 6{\cdot}3 - 10{\cdot}2 - 10 + {\cdot}6 = 0;\qquad H_A = 2$ kN;

Check: $\Sigma X = 0;\quad -2 + 10 - 20 + 12 = 0;\quad 22 - 22 = 0;\qquad \Sigma Y = 0;\quad -6 + 6 = 0.$

For plotting diagrams *M*, *Q* and *N* the frame will be divided into eight parts (Fig. 3.26, *b*). At the part of action of a uniformly distributed load diagram of bending moments will vary according to the parabolic law and for its plotting it is necessary to calculate bending moments, at least, in three sections – in sections 9, 10, 11. For other sections diagram of *M* will vary according to a linear law and to plot of it enough to know the values of the bending moments at the extreme points of these sections, i.e. in sections 1, 2, 3, 4, 8, 12, 13, 14 (here omitted a sections in which the calculation of internal moments are easily done even verbally, and in which the bending moments are clearly equal to zero, for example, sections of the joints).

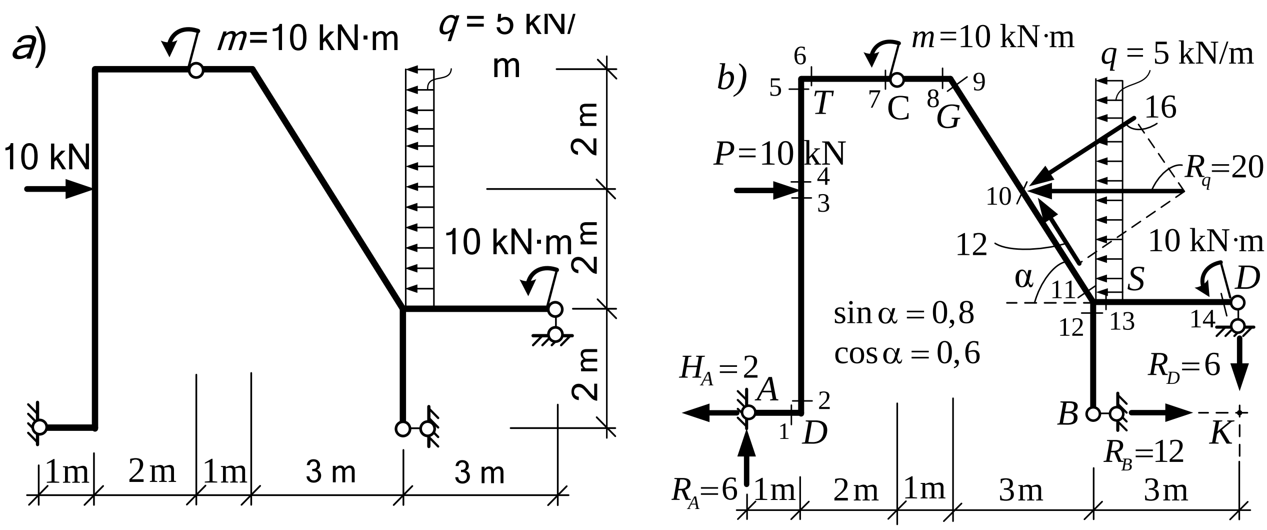


Fig 3.26

The calculation of internal forces in the given cross-sections and determination of the location of the stratched fibers:

$M_1^{left} = 6\cdot 1 = +\,6$ kN·m; $\qquad M_2^{low} = 6\cdot 1 + 2\cdot 0 = +\,6$ kN·m;

$M_3^{low} = M_4 = 6\cdot 1 + 2\cdot 4 = +14$; $\qquad M_5^{low} = 6\cdot 1 + 2\cdot 6 - 10\cdot 2 = -2$ kN·m;

$M_6^{left} = M_5^{low} = -2$ kN·m; $\qquad M_7^{left} = 6\cdot 3 + 2\cdot 6 - 10\cdot 2 = +10$ kN·m;

$M_8^{left} = M_9^{left} = 6\cdot 4 + 2\cdot 6 - 10\cdot 2 - 10 = +\,6$ kN·m;

$M_9^{right} = (5\cdot 4)\cdot 2 - 10 + 6\cdot 6 - 12\cdot 6 = -\,6$ kN·m;

$M_{10}^{right} = (5\cdot 2)\cdot 1 - 10 + 6\cdot 4{,}5 - 12\cdot 4 = -\,21$ kN·m;

$M_{11}^{low} = -10 + 6\cdot 3 - 12\cdot 2 = -16$ kN·m; $\qquad M_{12}^{low} = -\,12\cdot 2 = -\,24$ kN·m;

$M_{13}^{right} = -10 + 6\cdot 3 = +\,8$ kN·m; $\qquad M_{14}^{right} = -10 + 6\cdot 0 = -\,10$ kN·m;

Finally the bending moments diagrams in the frame under consideration based on the results of calculations presented in Fig. 3.27, *a*.

Similarly are plotted of diagrams of shear and longitudinal forces.

At the area of action of a uniformly distributed load the shear and longitudinal forces will be linear and for plotting of these (diagrams of *Q* and *N*) calculate the values of these forces, for two sections – 9 and 11 (Fig. 3.26, 3.27, *b*):

$$Q_9^{low} = +\,(5\cdot 4)\,\sin\alpha - 12\cdot\sin\alpha + 6\cdot\cos\alpha = (20 - 12)\cdot 0{,}8 + 6\cdot 0{,}6 = +\,10\ \text{kN};$$

$$Q_{11}^{low} = -\,12\cdot\sin\alpha + 6\cdot\cos\alpha = -\,12\cdot 0{,}8 + 6\cdot 0{,}6 = -\,6\ \text{kN};$$

$$N_9^{low} = -\,(5\cdot 4)\cdot\cos\alpha + 12\cdot\cos\alpha + 6\cdot\sin\alpha = (-\,20 + 12)\cdot 0{,}6 + 6\cdot 0{,}8 = 0;$$

$$N_{11}^{low} = 12\cdot\cos\alpha + 6\cdot\sin\alpha = 12\cdot 0{,}6 + 6\cdot 0{,}8 = +12\ \text{kN}.$$

In the other sections, shear and longitudinal forces will be constant and to plotting of their diagrams is sufficient to calculate the values of *Q* and *N* in one of the set sections:

$Q_1^{left} = +\,6$ kN; $\qquad Q_2^{low} = Q_3 = +\,2$ kN; $\qquad Q_4^{low} = Q_5 = +2 - 10 = -8$ kN;

$Q_6^{left} = Q_7 = Q_8 = 6$ kN; $\qquad Q_{12}^{low} = -12$ kN; $\qquad Q_{13}^{right} = Q_{14} = +\,6$ kN;

$N_1^{left} = 0$; $\qquad N_2^{low} = N_3 = N_4 = N_5 = -\,6$ kN; $\qquad N_{12}^{low} = 0$;

$N_6^{left} = N_7 = N_8 = +2 - 10 = -8$ kN; $\qquad N_{13}^{right} = N_{14} = 0.$

Plotted diagrams *Q* and *N* according to the data is depicted in Fig. 3.28.

At the area of action of a uniformly distributed load q on the diagram of shear force there is a special section where shear force is zero. In this section the bending moment acquires the maximum value. The position of this cross section we find from the ratio (Fig. 3.28, *a*):

$$\frac{10}{5-s_{\max}}=\frac{6}{s_{\max}};\quad 10\cdot s_{\max}=6\cdot\left(5-s_{\max}\right);\qquad 16\cdot s_{\max}=30;\quad s_{\max}=1{,}875\ \text{m}\,.$$

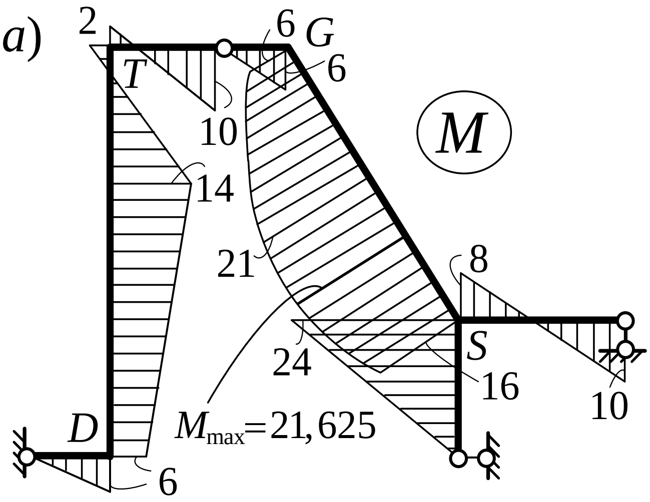


Fig. 3.27

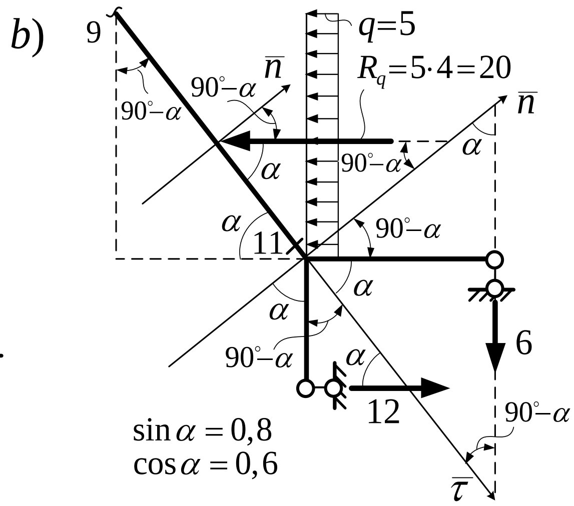


Fig. 3.27

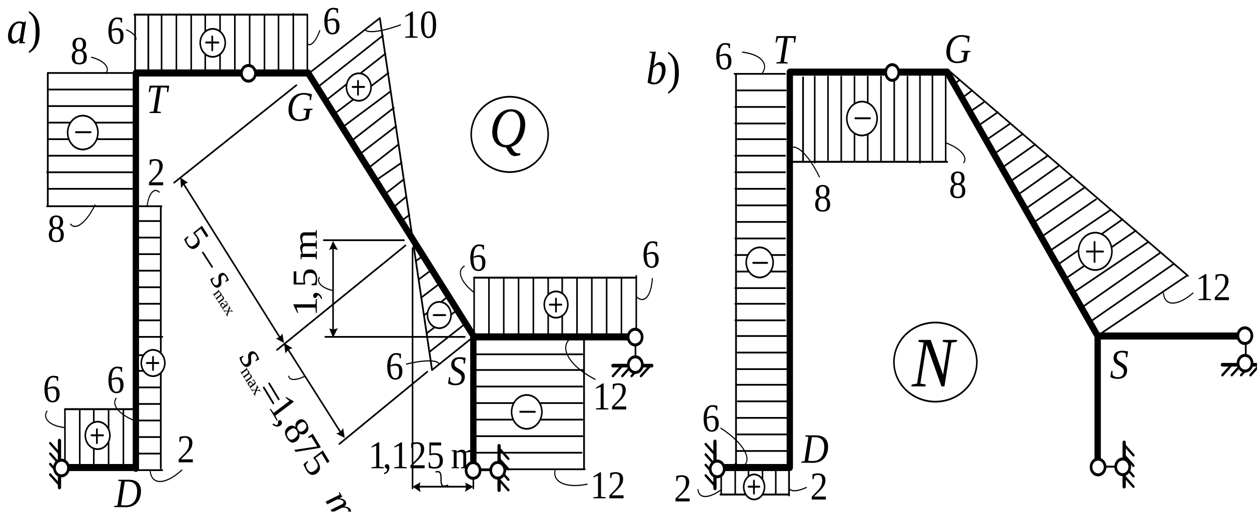


Fig. 3.28

The horizontal and vertical dimensions of the position of the cross section will be equal:

$$x_{\max}=1{,}875\cdot 0{,}6=1{,}125\ \text{m};\qquad y_{\max}=1{,}875\cdot 0{,}8=1{,}5\ \text{m}\quad (\text{Fig. 3.28, } a).$$

The maximum bending moment is equal:

$$M_{\max}^{right}=(5\cdot 1{,}5)\cdot 0{,}75-10+6\cdot(3+1{,}125)-12\cdot(2+1{,}5)=-\,21{,}625\ \text{kN}\cdot\text{m}.$$

Checking the balance of joint for final diagrams *M, Q* and *N*:

a) <u>on diagram *M*:</u>

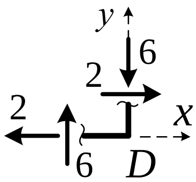
$\Sigma M_D = 0;$ $\quad 6 - 6 = 0;$ $\qquad \Sigma M_T = 0;$ $\quad 2 - 2 = 0;$ $\qquad \Sigma M_G = 0;$ $\quad 6 - 6 = 0;$ $\qquad \Sigma M_S = 0;$ $\quad 16 + 8 - 24 = 0.$

<u>b) On diagrams Q and N:</u>

<u>Joint *D*:</u>

$\Sigma X = 0;\ \ -2 + 2 = 0;$

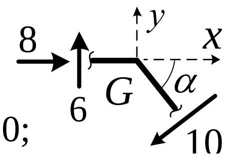
$\Sigma Y = 0;\ \ 6 - 6 = 0;$



<u>Joint *G*:</u>

$\Sigma X = 0\ \ 8 - 10 \cdot 0{,}8 = 0;\ \ 8 - 8 = 0;$

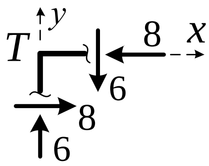
$\Sigma Y = 0;\ \ 6 - 10 \cdot 0{,}6 = 0;\ \ 6 - 6 = 0;$



<u>Joint *T*:</u>

$\Sigma X = 0;\ \ 8 - 8 = 0;$

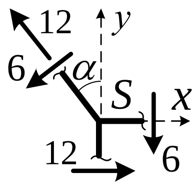
$\Sigma Y = 0;\ \ 6 - 6 = 0;$



<u>Joint *S*:</u>

$\Sigma X = 0;\ \ 12 - 12 \cdot 0{,}6 - 6 \cdot 0{,}8 = 0;\ \ 12 - 12 = 0;$

$\Sigma Y = 0;\ \ 12 \cdot 0{,}8 - 6 \cdot 0{,}6 - 6 = 0;\ \ 9{,}6 - 9{,}6 = 0;$



3.5. Calculation of Composite Frames

The frame is called composite if it consists of several *three-hinged frames* and (or) simple frames (Fig. 2.3, *c*; 3.29, *b*).

During the calculation of such frames as well as the multi-span should be divided (in hinge joints) into a separate three-hinged frame and (or) simple frame, some of which will have to rely on others, and the calculation of which we know how to perform; these simple and three-hinged frame here can also be divided into *main* (major) and *secondary* (minor). The calculation, of course, should begin with the secondary frame (top level), gradually moving to the calculation of the bottom frames and transmitting their reactions (in reverse directions) of the upper located frames. Full diagram of forces for a composite frame will be obtained by composing the respective diagrams for the separate frames.

For example, the calculation of the composite frame (Fig. 3.29, *b*) should be performed in the following sequence.

1. We cut out the top part Section I–I, which is a three-hinged frame *CDO* with support at different levels (in the joint *C* and *imaginary hinge O*) and determine the reactions at the hinge joint *C* and at the supports *S* and *R*.

2. The right half of the frame *DO* of the considered above frame *CDO*, represents itself as a three-hinged frame with a joining beam *FK*. Therefore, for breaking the closed loop we need to make the cross-section II–II (Fig. 3.29, *c*) and find the force *H* in the *FK* joining beam, considering the balance either left or right of the frame.

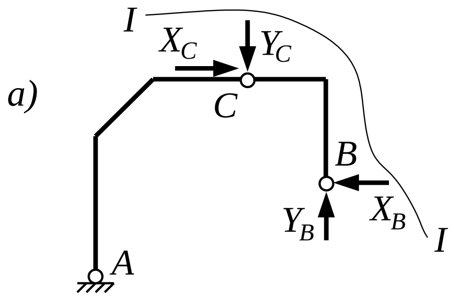


Fig. 3.29

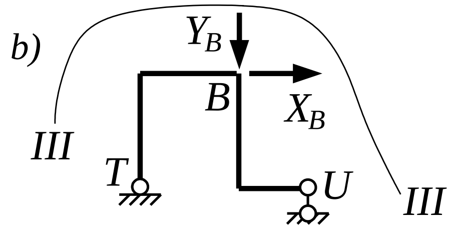


Fig. 3.29

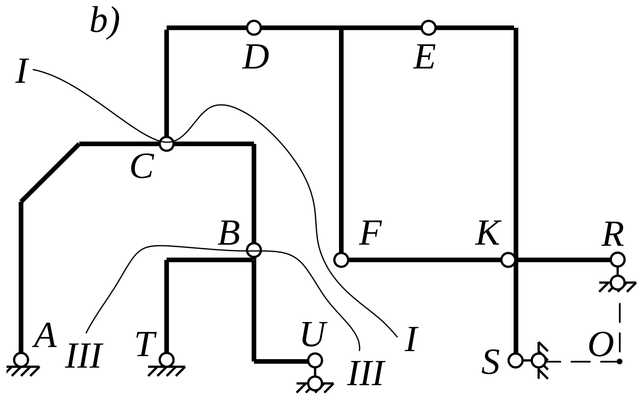


Fig. 3.29

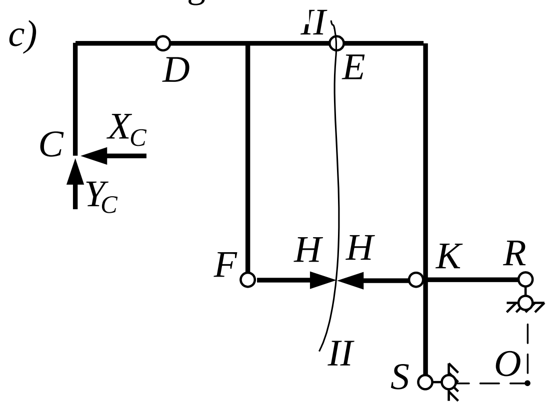


Fig. 3.29

3. Make the cross-section *III – III*, and considering separately the frame *ABC*, which is a three-hinged frame with supports at different levels (*A* and *B*).

4. Finally we consider the simple frame TBU.

Below we will consider an example of calculating the composite frame.

Frame, that is shown in Fig. 3.30, can be divided into three-hinged frame *DCT* and two simple console beams *AD* and *BT* (Fig. 3.31), while structurally the three-hinged frame *DCT* relies on the rods *AD* and *BT*.

Let us first consider the three-hinged frame *DCT, hinges at the supports* of which lie at different levels. Therefore, to determine the reactive forces at joints *D* and *T* we must solve the system of equations:

$$\begin{array}{l} 1) \\ 2) \end{array} \begin{cases} \Sigma M_D = 0; \quad +22 - 6\cdot 2 - (4\cdot 3)\cdot 2{,}5 + X_T + 4\cdot Y_T = 0; \\ \Sigma M_C^{right} = 0; \quad +(4\cdot 3)\cdot 1{,}5 - 3\cdot X_T + 2\cdot Y_T = 0; \end{cases} \Rightarrow \begin{array}{l} X_T = 8 \text{ kN}; \\ Y_T = 3 \text{ kN}; \end{array}$$

3) $\Sigma X = 0;\ \ +X_D + 8 - 4\cdot 3 = 0;\quad X_D = 4$ kN;

4) $\Sigma Y = 0;\quad + Y_D - 6 - 3 = 0;\quad Y_D = 9$ kN.

<u>Check:</u> $\Sigma M_C^{left} = 0;\quad +22 + 9\cdot 2 - 6\cdot 4 - 4\cdot 4 = 0;\quad 40 - 40 = 0.$

We transfer the found forces at joints *D*, and T in the reverse direction on the console rods *AD* and *BT* (Fig. 3.31). After that, the computation of forces in the three-hinged frame sections *DCT* and console rods *AD* and *BT* is not very difficult. Final bending moment diagrams, shear and longitudinal forces in the frame under consideration is presented on Fig. 3.32 – 3.34.

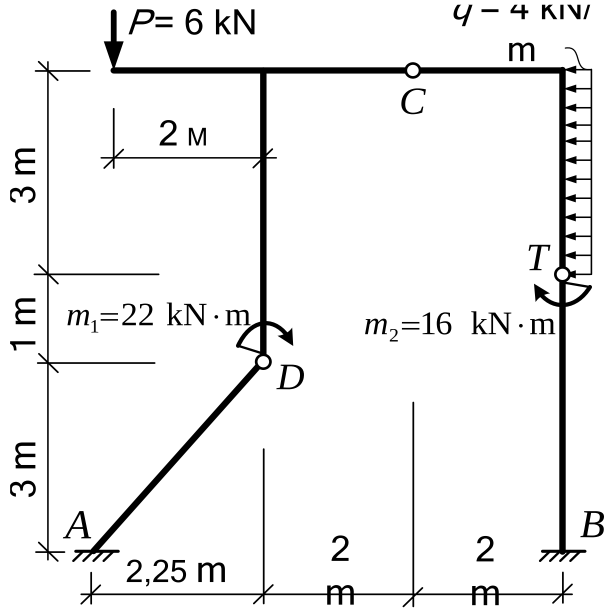


Fig. 3.30

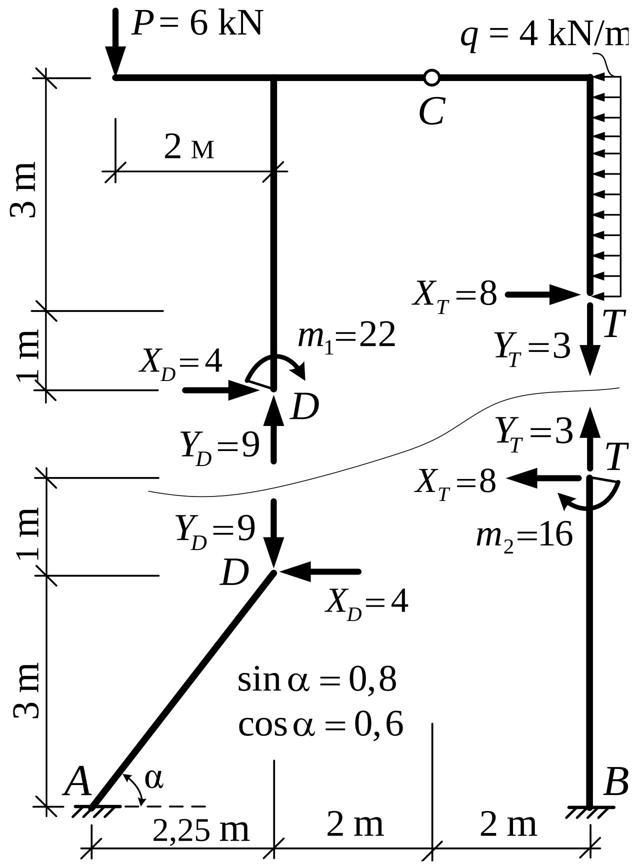


Fig. 3.31

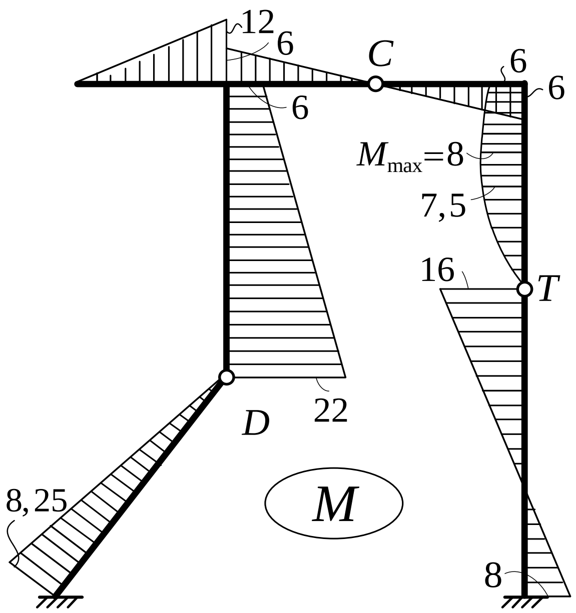


Fig. 3.32

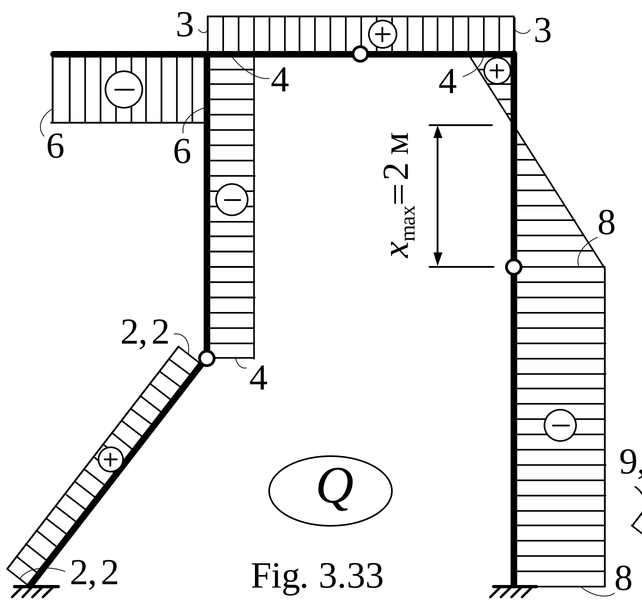


Fig. 3.33

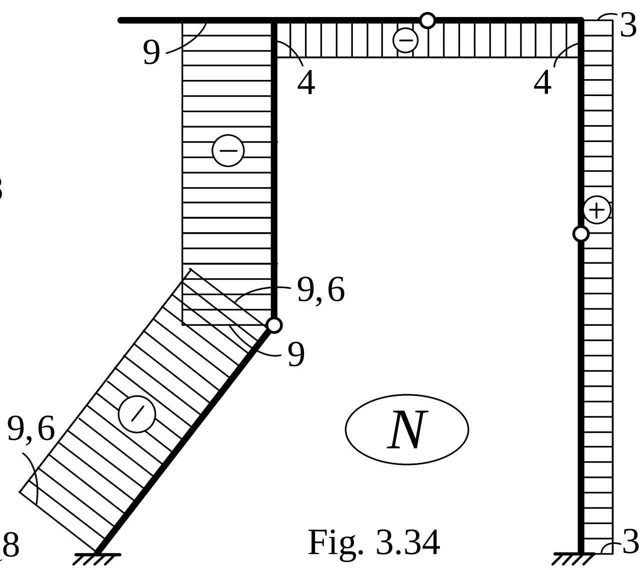


Fig. 3.34

3.6. Calculation of Statically Determinable Composite Beams

Beams are called *composite* and *statically determinate* if they consist of several simple beams connected at the ends by hinge joints and, as a rule; do not coincide with the supports.

The kinematic analysis of such systems is convenient to perform using the formula $W = (3D - 2H - C_0)$, where: D – the number of disks in the system, which are the simple beams; H – the number of single (simple) joints connecting the beams; C_0– the number of restrictions at the supports in the system.

Geometric structure analysis of composite beams allows introducing the concept of *floor-by-floor schemes* of the beam. Floor scheme of the composite of statically determinate beams is a scheme of the interaction of simple beams, where they unite into composite one (see Fig. 3.35). Thus among these simple beams it is possible to identify the main and secondary beams.

The main are called the simple beams, which after cutting of the composite beam by joints connecting the simple beams can carry the load by themselves (invariable). We consider the beam as a disk having three degrees of freedom and the main beam must have three support restrictions it means that the main beam will be simple beam or beam with pinching (fixed end). The main beam in the composite system can also be the simple beam having two vertical reactions at the supports, considering that the third link for them – the horizontal beams are adjacent (which confirms the geometric analysis of the corresponding system).

Secondary are called the simple beams, which can not carry the load by themselves because they are unstable. These beams rely on the adjacent joints. Also some of the secondary beams can rely on the other that means that among the secondary beams there is a certain hierarchy and, accordingly, their level of minarity may be different. The most secondary beams will be the beams that are located just above on the floor scheme.

Work analysis of statically determinable composite beams allows us to identify the **regularitys** of their work and to formulate their possible **order of calculation**:

– it is convenient to perform the calculation of composite statically determinable beams by the calculation of the simple beams that containts the compound one;

– the calculation must start from the top of floor-by-floor scheme, gradually moving to the calculation of the bottom beams and sequantly passing the reactions on them from the upper beams to the lower, in the opposite directions; The main beams are calculated the last;

– The forces from the load on floor-by-floor schemes are transferred only to the underlying beams and are not transferred to the overlying;

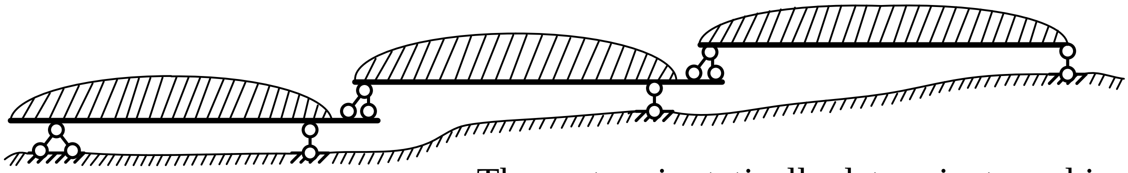
– The calculation of the simple beams is similar to the calculation of simple frames;

– During the action of only vertical loads on the statically determinate composite beam, the longitudinal force will be missing;

– Final diagrams of internal forces in a composite beam are plotted by combining the diagrams of these forces, obtained by the calculation of simple beams.

Let’s perform the calculation of statically determinat beam, represented on Fig. 3.35.

Kinematic analysis of the system: $W = 3D - 2H - C_0 = 3\cdot3 - 2\cdot2 - 5 = 0.$



The system is statically determinate and invariable.

Determination of reactions at the supports and plot the internal forces diagram:

1) The calculation begins from the beam *EF*, the top floor on the scheme:

$\Sigma M_E = 0;$ $-R_F\cdot 6{,}6 + 2\cdot4{,}4\cdot2{,}2 + 5\cdot8{,}8 = 0;$ $R_F = 9{,}6$ kN;

$\Sigma M_F = 0;$ $R_E\cdot 6{,}6 - 2\cdot4{,}4\cdot4{,}4 + 5\cdot2{,}2 = 0;$ $R_E = 4{,}2$ kN;

Check: $\Sigma Y = 0;$ $4{,}2 - 2\cdot4{,}4 + 9{,}6 - 5 = 0;$ $0 = 0.$

The calculation of the ordinate of the diagram *M*: $M_1^{left} = 0;$

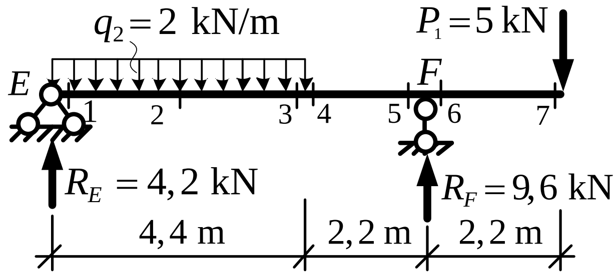
$M_2^{left} = 4{,}2\cdot2{,}2 - 2\cdot2{,}2\cdot1{,}1 = 4{,}4$ kN·m;

$M_3^{left} = 4{,}2\cdot4{,}4 - 2\cdot4{,}4\cdot2{,}2 = -0{,}88$ kN·m;

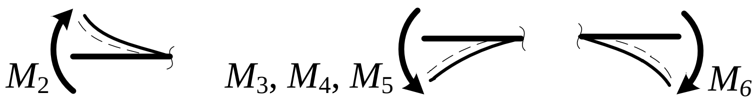
$M_4^{left} = -0{,}88$ kN·m;

$M_5^{left} = 4{,}2\cdot6{,}6 - 2\cdot4{,}4\cdot4{,}4 = -11$ kN·m;

$M_6^{left} = 5\cdot2{,}2 = 11$ kN·m; $M_7^{left} = 5\cdot0 = 0$ kN·m.



Determination of stretched fibers:



Calculation of the ordinates of the diagram *Q*: $Q_1^{left} = 4{,}2$ kN;

$Q_3^{left} = Q_4^{left} = 4{,}2 - 2\cdot4{,}4 = -4{,}6$ kN; $Q_5^{right} = -9{,}6 + 5 = -4{,}6$ кN; $Q_6^{right} = Q_7^{right} = 5$ kN.

Calculation of extreme values of the bending moment on the section 1-2:

$$\frac{x_1}{4{,}2} = \frac{4{,}4 - x_1}{4{,}6} \Rightarrow x_1 = 2{,}1 \text{ m}; \qquad M_{\text{мах}} = 4{,}2\cdot2{,}1 - 2\cdot2{,}1\cdot1{,}05 = 4{,}41 \text{ kN·m}.$$

Diagrams *M* and *Q* are depicted in the general scheme of composite beam (Fig. 3.35).

2) Then we calculate the second beam *CDE*:

$\Sigma M_C = 0;$ $9{,}4\cdot1{,}55 + 9{,}4\cdot3{,}1 - R_D\cdot4{,}65 + 4{,}2\cdot6{,}85 = 0;$ $R_D = 15{,}587$ kN;

$\Sigma M_D = 0;$ $4{,}65R_C - 9{,}4\cdot3{,}1 - 9{,}4\cdot1{,}55 + 4{,}2\cdot2{,}2 = 0;$ $R_C = 7{,}413$ kN;

$\Sigma M_D = 0;$ $4{,}65R_C - 9{,}4\cdot3{,}1 - 9{,}4\cdot1{,}55 + 4{,}2\cdot2{,}2 = 0;$ $R_C = 7{,}413$ kN.

Check: $\Sigma Y = 0;$ $7{,}413 - 9{,}4 - 9{,}4 + 15{,}587 - 4{,}2 = 0;$ $0 = 0.$

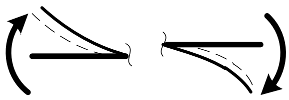
The calculation of the diagram of ordinates of bending moment:

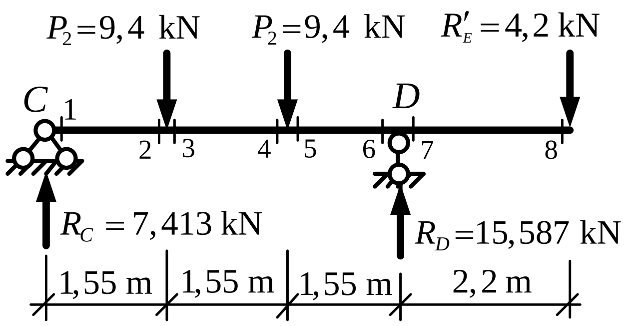
$M_1^{left} = 0;$ $M_2^{left} = 7{,}413\cdot1{,}55 = 11{,}49$ kN·m; $M_3^{left} = 11{,}49$ kN·m;

$M_4^{left} = 7{,}413\cdot3{,}1 - 9{,}4\cdot1{,}55 = 8{,}41$ kN·m; $M_5^{left} = 8{,}41;$ $M_6^{right} = -4{,}2\cdot2{,}2 = 9{,}24$ kN·m;

M_7^{right} = 9,24 kN·m; M_8^{right} = 0.

Determination of stretched fibers:

M_2, M_3, M_4, M_5  M_6, M_7



The calculation of the ordinate of the diagram Q:

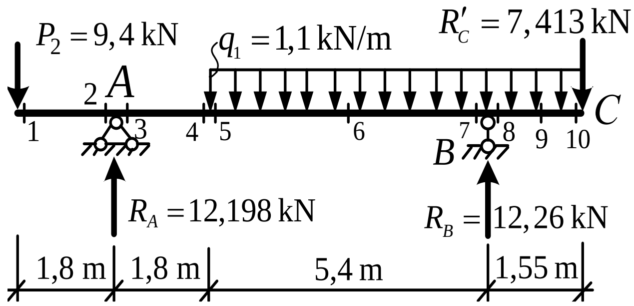
Q_1^{left} = 7,413 kN; Q_2^{left} = 7,413 kN;

$Q_3^{left} = Q_4^{left}$ = 7,413 – 9,4 = – 1,987 kN;

$Q_5^{right} = Q_6^{right}$ = – 15,587 + 4,2 = – 11,387 kN; $Q_7^{right} = Q_8^{right}$ = 4,2 kN.

Diagrams M and Q in the beam depicted in the general scheme of composite beam (Fig. 3.35).

3) We calculate the main beam ABC in the end: $\Sigma M_A = 0$;



– 9,4·1,8 + 1,1·6,95·5,275 – R_B·7,2 + + 7,413·8,75 = 0; R_B = 12,26 kN;

$\Sigma M_B = 0$;

– 9,4·9 + R_A·7,2 – 1,1·6,95 1,925 + + 7,413·1,55 = 0; R_A = 12,198 kN.

Check: $\Sigma Y = 0$; – 9,4 + 12,198– 1,1·6,95 + 12,26 – 7,413 = 0; 0 = 0.

The calculation of the ordinates of bending moment diagrams:

M_1^{left} = 0; M_2^{left} = – 9,4·1,8 = – 16,92 kN·м; $M_3^{left} = M_2^{left}$ = – 16,92 kN·м;

$M_4^{left} = M_5^{left}$ = – 9,4·3,6 + 12,198·1,8 = – 11,884 kN·m;

M_6^{left} = – 9,4·(3,6+2,7) + 12,198·(1,8+2,7) – 1,1·2,7·1,35 = – 8,325 kN·m;

$M_7^{right} = M_8^{right}$ = 7,413·1,55 – 1,1·1,55·0,775 = 12,812 kN·m;

$M_9^{right} = 7{,}413\cdot\frac{1}{2}\cdot1{,}55 - (1{,}1\cdot0{,}775)\cdot\frac{1}{2}\cdot0{,}775 = 6{,}076$ kN ·m; M_{10}^{right} = 0.

Determination of stretched fibers: M_2, M_3, M_4, M_5, M_6  M_7, M_8

The diagram of the bending moments and shear forces is shown in Fig. 3.35.

The calculation of the ordinate of the diagram of shear forces Q:

$Q_1^{left} = Q_2^{left}$ = – 9,4 kN; Q_3^{left} = – 9,4 + 12,198 = 2,798 kN;

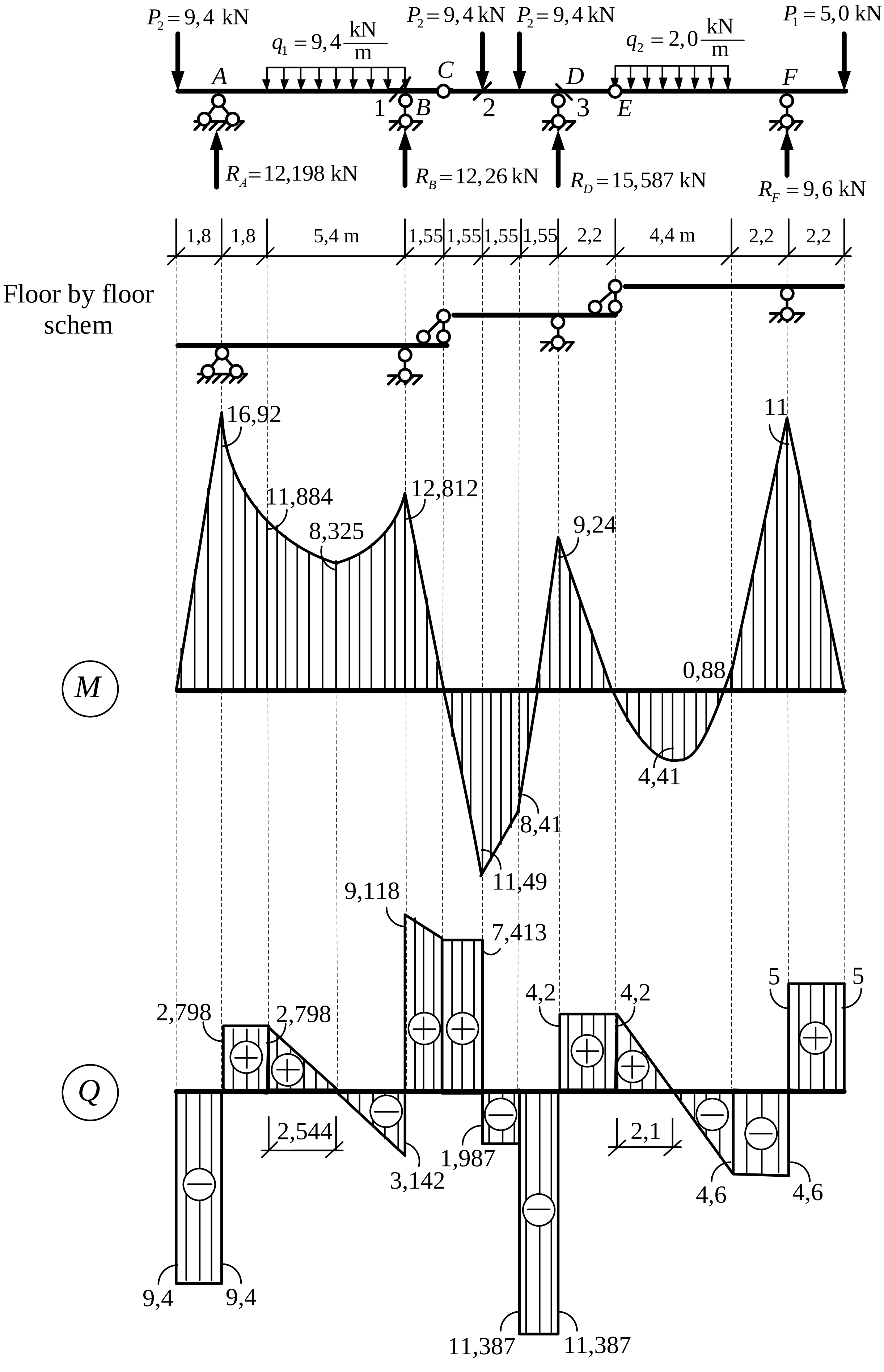
$Q_4^{left} = Q_5^{left} = Q_3^{left}$ = 2,798 kN; Q_7^{right} = 7,413 + 1,1·1,55 – 12,26 = – 3,142 kN;

Q_8^{right} = 7,413 + 1,1·1,55 = 9,118 kN; Q_9^{right} = 7,413 kN.

The calculation of the extreme values of the bending moments in the section 5–7:

$$\frac{x_2}{2{,}798} = \frac{5{,}4 - x_2}{3{,}142} \Rightarrow x_2 = 2{,}544 \text{ m};$$

$M_{\text{мах}}$ = – 9,4·6,144 + 12,198·4,344 – 2,544·1,1·2,544/2 = – 8,325 kN·m.



P2 = 9,4 kN
q1 = 9,4 kN/m
P2 = 9,4 kN
P2 = 9,4 kN
q2 = 2,0 kN/m
P1 = 5,0 kN
A
C
D
F
1
B
2
3
E
RA = 12,198 kN
RB = 12,26 kN
RD = 15,587 kN
RF = 9,6 kN
1,8
1,8
5,4 m
1,55
1,55
1,55
1,55
2,2
4,4 m
2,2
2,2
Floor by floor schem
16,92
11
11,884
12,812
8,325
9,24
M
0,88
4,41
8,41
11,49
9,118
7,413
5
5
4,2
4,2
2,798
2,798
Q
2,544
2,1
3,142
1,987
4,6
4,6
9,4
9,4
11,387
11,387

Fig. 3.35

3.7. Influence Lines in The Beams Systems

3.7.1. The Concept of Influence Lines

Influence line for reaction at the support is a diagram represents of changes in the reaction in a particular element (section) of the structure when moving alongwith the construction of a *unit dimensionless force* (*unit force*) of constant direction.

During the plotting of the influence lines of some reactions or internal forces we consider an arbitrary position of the unit force on the structure. For this state we make the equations of equilibrium, which displays the functional dependence of the considered forces from the abscissa of the unit force position. The diagram of this relation represents the influence line.

The unit force is taken as dimensionless. Therefore, the dimensions of the influence lines are determined by the expression:

$$[\text{Dimension of the influence lines}] = \left[\frac{\text{dimension of internal force}}{\text{dimension of force}}\right]$$

Accordingly, the ordinates of the influence lines of reactions, shear and longitudinal forces will be dimensionless (N/N) and the dimension of ordinates of moments-influence lines will be equal to the dimension of length (N·m/N=m).

Note the differences between influence lines in comparison with the diagrams of internal forces.

Plot internal forces is a diagram showing values of the internal forces (bending moment, shear force, longitudinal force, etc.) in all cross-sections of the considered structure from the effects of a particular set of fixed loads (concentrated forces, moments, distributed loads). The ordinates of the diagram the internal forces show the values of internal forces in the structures in which they are laid off. Any change in position and the load values, appearing (new) of loads, causes the internal forces to change, and the diagrams of internal forces should be replotted.

Influence lines is a diagram showing the changes of the pointed out concrete force in one pointed section of the structure depending on position of unit concentrated dimensionless force, moving across the structure. The ordinate of the influence line shows the value of force in one particular section of the structure at the position of a unit force in the place where the ordinate is depicted. When plot the influence lines diagram the ordinate is depicted under the point of application of a unit force. This influence line can not say anything about the change force in other sections of the structure.

For influence lines M and Q in beams the following rule of signs are used: the ordinates of the influence line M are positive, if stretched the bottom fibers of the beams, and for the ordinate of the influence lines Q also applies the same rule of signs that for plots Q.

Influence lines of forces allow us:

– to determine the values of reactions or internal forces from a variable action;

– To find the most unfavorable position of the systems of variable actions (loads) and temporary loads in order to determine in a particular element of the structure of extreme (maximum or minimum) internal forces;

– To determine the internal forces from stationary system of loads, this is especially important during the multiple variants loading of the stucture.

3.7.2. Plotting of the Influence Lines in Simple Beams

Let's consider the static method of plotting influence lines diagram for forces in one of the sections firstly for the simplest of beams – console beam (Fig. 336, *a*).

The dependencies for the determination of the reactions at the supports during the desplacement of the unit force on the beam can be found from the equilibrium equations of the beam.

The horizontal reaction can be found from the condition of equality of projections of the forces acting on the beam, on the horizontal axis $\sum X = 0$. Considering that a unit movable vertical force while moving the beam does not change its direction, we get that $H_A = 0$. Influence line of horizontal reaction in the console beam will have zero ordinate.

We determine the vertical reaction from the equation $\sum Y = 0$. Which means that regardless of the position of the load $R_A = 1$, the vertical reaction at the console beam will be constant, and equal to the unit in the whole area of load movement. Diagram of the influence line of R_A is represented in Fig. 3.36, *b*

The reactive moment of the fixed end will be determined from the condition of equality to zero of the sum of the moments about point *A*:

$$\sum M_A = 0; \quad M_{RA} + 1 \cdot x_{FA} = 0;$$

$$M_{RA} = -x_{FA}.$$

Reactive moment varies linearly. To plot a straight line it will be enough to calculate the ordinates at two points: at the beginning and at the end of the beam: when $x_{FA} = 0$ $M_{RA} = 0$; when $x_{FA} = l$ $M_{RA} = -l$.

Connecting these points of a straight line, we will get the influence line of the reactive moment.

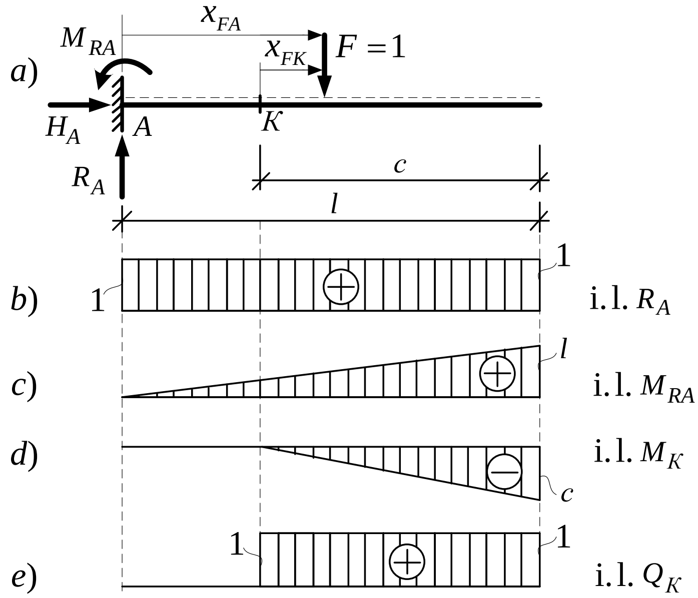


Fig. 3.36

Bending moment and shear force in cross-section K will be determind from the equilibrium equations of the right side of the beam.

If the load F=1 is located to the right from the cross-section K:

$$M_K^{right} = -1 \cdot x_{FK} \text{ (When } x_{FK} = 0,\ M_K = 0;\ \text{When } x_{FK} = c,\ M_K = -c); \quad Q_K^{right} = +1.$$

During the movement of the load to the left from the section K from the equations $\sum M_K^{right} = 0$ and $\sum Q_K^{right} = 0$ it follows that $M_K = 0$, $Q_K = 0$. Influence lines M_K and Q_K are shown on Fig. 3.36, *d*, *e*.

The highest modulo value of the bending moment in the cross section K occurs when the unit loads position at the end of the console. If the load is located to the right from the section K the top fiber of the beam will be stretched in this section, so the ordinates of the influence line of bending moment will be negative. The shear force in the cross section K at the same position of the unit load is positive (rotates the beam element in clockwise direction) and equal to one. In the section K, the influence line for the bending moment has a break, and the influence line for shear forces – the jump on the unit.

Let's plot the influence lines of R_A and R_B for simple beam with the console (Fig. 3.37, *a*). From the equilibrium equations of the beam we can see:

$$\sum M_A = 0; \qquad 1 \cdot x - R_B\, l = 0; \qquad R_B = x/l; \tag{3.1}$$

$$\sum M_B = 0; \qquad -1 \cdot (l - x) + R_A\, l = 0; \qquad R_A = (l - x)/l. \tag{3.2}$$

These dependencies represent the equations of the straight lines, which we will plot on the two ordinates: when $x = 0$ $R_A = 1$, $R_B = 0$; when $x = l$; $R_A = 0$, $R_B = 1$.

Additionally, we will compute the values of the reactions at the supports at the load position in specific points:

a) at the end of the left console – when $x = -l_{k1}$: $R_A = (l - l_{k1})/l$; $R_B = l_{k1}/l$;

b) In cross section K between the supports – when $x = a$: $R_A = (l - a)/l$; $R_B = a/l$;

c) At the end of the right console – when $x = l + l_{k2}$ $R_A = -l_{k2}/l$; $R_B = (l + l_{k2})/l$.

Influence lines of support reactions, plotted according to the dependencies and ordinates is shown in Fig. 3.37, *b*, c.

Internal forces in cross-section K, located in the span of the beam (between the supports) can be identified from consideration of equilibrium in the left and right sides of the beams w.r.t. the cross-section K. Expedient to consider the part where there is no load. In this case, to determine the internal forces we will get more simple equations.

During the movement to the left from the section K we will obtain the bending moment M_K from the equation of equilibrium of the right part of the beam:

$$M_K^{right} = R_B\, b. \tag{3.3}$$

This expression leads to a linear dependence:

$$M_K = \frac{xb}{l}; \tag{3.4}$$

That is true for the left part of the beam, where the unit load is situated.

Left direct influence line M_K can be plotted by multiplying all ordinates of influence line R_B by the amount b:

i.l. M_K = (i.l. R_B) b.

Similarly, the motion of the load to the right of the section K will have:

i.l. M_K = (i.l. R_A) a.

That means that right direct influence line of M_K can be plotted, by increasing the ordinates of i. l. R_A on a times (Fig. 3.37, *d*). Note that the left and right branches of influence line of M_K intersect at section K.

Shear-influence line in the cross section K is plotted similarly. During the movement of the unit load to the left from the cross-section let's consider the right part of the beam. From the equation $\sum Y^{right} = 0$ we obtain (the left influence line):

$$Q_K = -R_B \qquad \text{or} \qquad \text{i.l. } Q_K = -\,(\text{i.l. } R_B).$$

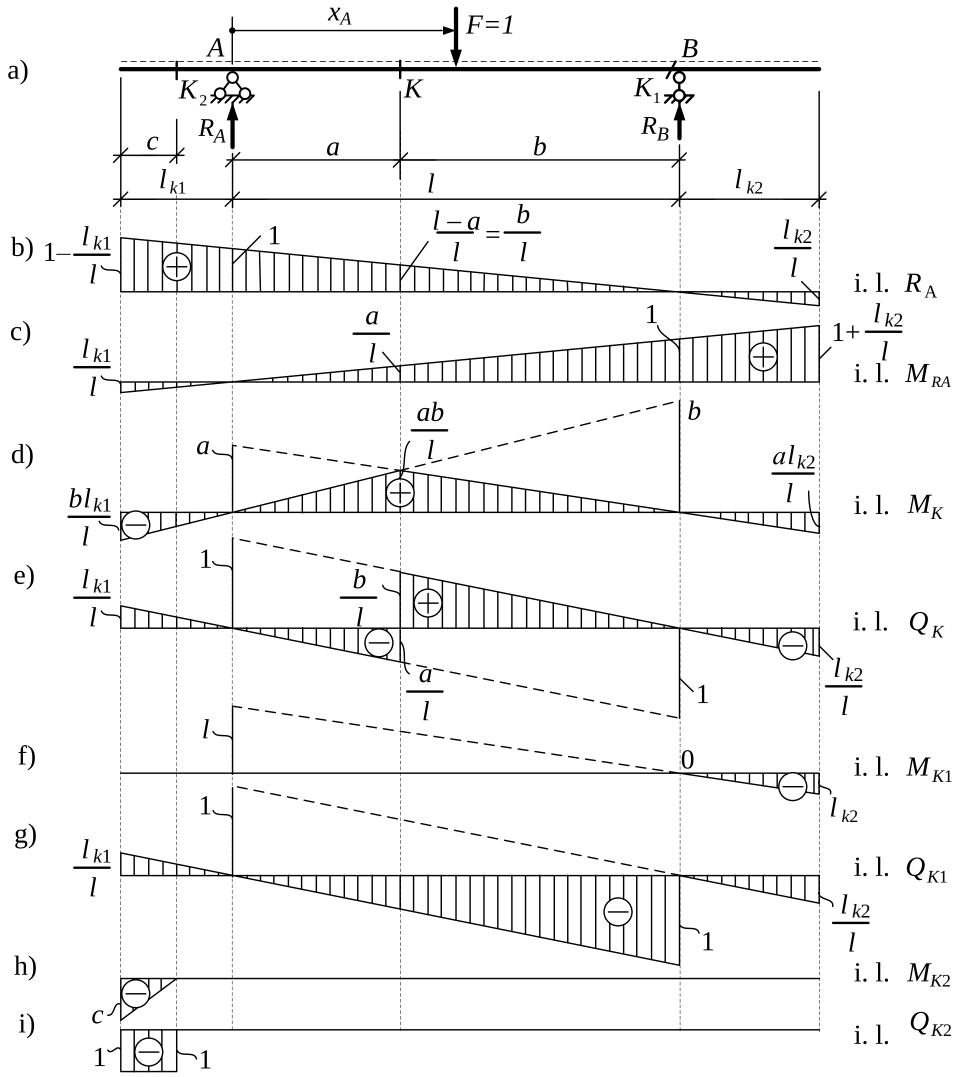


Fig. 3.37

During the movement of force to the right from the cross-section, examining the left part of the beam will give the direct line for the right part of the beam: $Q_K = R_A$ or i.l. Q_K = i.l. R_A.

Influence line Q_K, plotted in accordance with these dependencies, is shown in Fig. 3.37, *e*. Under the section K it has the jump by the value equal to one ($a / l + b / l = 1$). In Fig. 3.37, *f*, g influence lines are shown that have been plotted up on the same principles of of bending moment and shear force in the cross-section K_1, infinitely close to the support B.

Influence lines in the sections on consoles of semple beam are being plotted (Fig. 3.37, *h*, *i*) as in section of console beams (Fig. 3.36, *d*, *e*).

3.7.3. Plotting the Influence Lines in Multi-Span Beams

At plotting the influence lines in multi-span beams, necessary to remamber that in this case there is only one force (load) that moves through the system. Procedure of calculation starts with the movement of load by the simple beam, to which the required action is applied. Plotting of the influence lines in the simple beams was discussed above. For movement of the load on the remaining beams we can use the interaction conditions of simple beams in the system of many-span beams, which are easy to identify from the analysis of such systems:

– during the transition of force through the hinge joint connecting the simple beams, all the loads in multi-span beam remain unchanged (equal), because the effect of the load is not changed;

– When the position of the load on the support, the load is fully accepted by this support (reaction will be equal to the value of the load), and all other forces across multi-span beam will be absent (equal to zero);

– During the movement of the load on the beams, which transmit this load to the underlying beams, influence lines in those (underlying) beams will change linearly (this is due to the fact that the transfer of the action of the load is carried out through reactions in the joints, which are the support reactions for the overlying beams, and those during the movement of the load over them change linearly (Fig. 3.36, *b*, *c*), (Fig. 3.37, *b*, *c*);

– during the movement of load on simple beams located on the floor-by-floor diagram below the beam to which the considered load is related, the load is not transmitted to the beam and considered force will be zero.

Thus, to plot the influence lines in multi-span beam we must first plot the influence line in a simple beam to which considered load is related. Then, in accordance with the specified conditions of interaction and work of the simple beams in the multi-span system, plot the influence lines of load for the rest (remain) simple beams of multi-span beams.

Let's consider the statically determinable multispan beam shown in Fig. 3.38, *a*. Floor-by-floor diagrams of the beam shown in Fig. 3.38, *b*.

The support reaction R_C relates to the beam *BCD*, so we start plotting of the influence lines from the movement of load by this beam. And we plot influence line of R_C just as in the simple beam in according with the dependencies (3.1), (3.2) (see i.l. R_B in Fig. 3.37). Then we consider the movement of the unit load, for example, on the beam DST. When passing through the hinge joint *D* the ordinate of the influence line $1+l_{\text{K}2}/l_2$ remains unchanged.

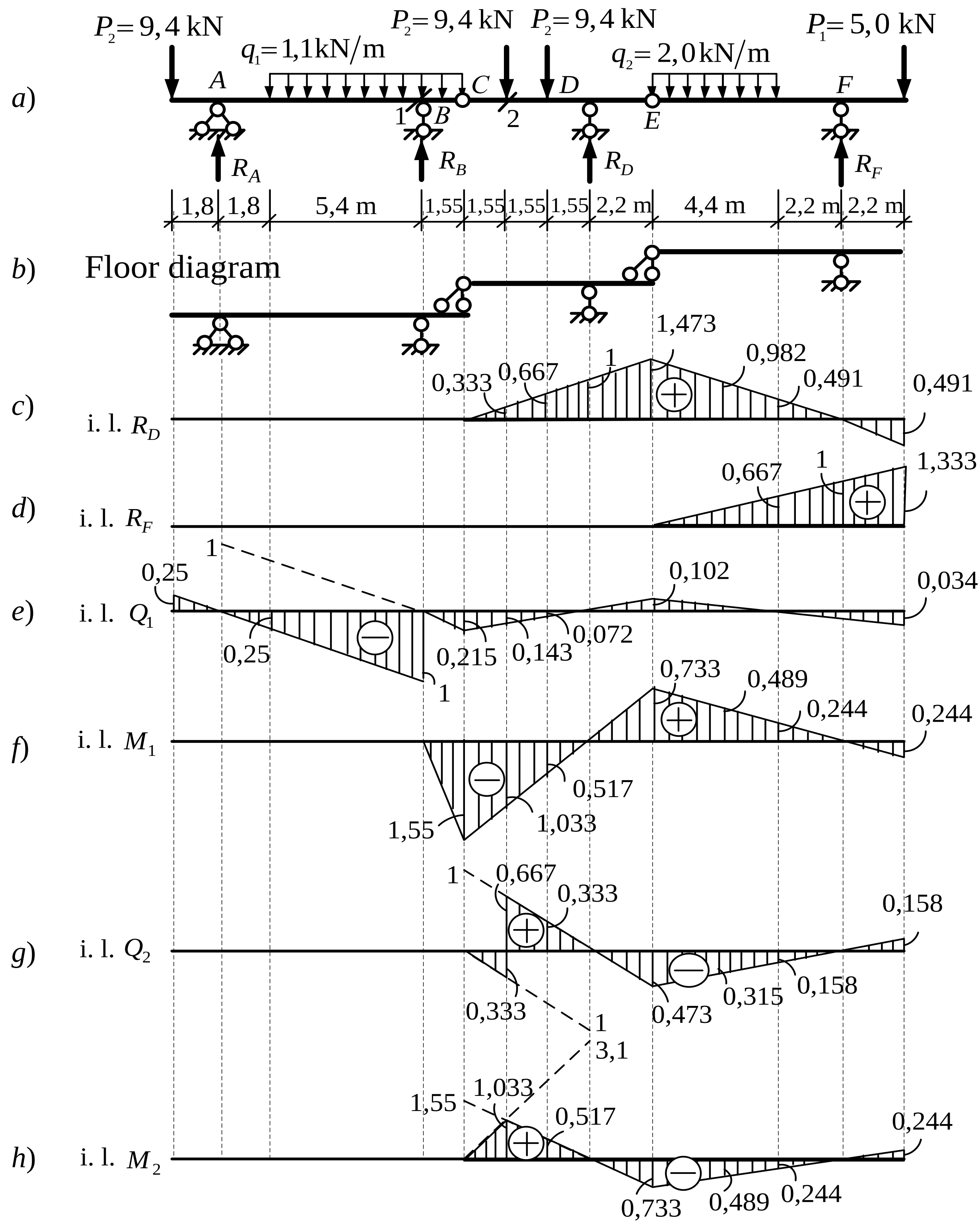


Fig. 3.38

When the load is located on top of the support *S* it is fully accepted by this support and all other forces across the whole multi-span beam including the reaction R_C is equal to zero (the zero ordinate under the base *B*). Let's consider now during the motion of the load on the beam *DST* the influence line of reaction R_C. This part refers to underlying at a floor-by-floor diagram of the beam. That should vary linearly. Connecting the points obtained by depicting the ordinates under the supports of *D* and S of that beam, a straight line and continue that line to the console beams ST (Fig. 3.38, *c*). During the movement of the load on the beam TU the procedure for plotting of the influence lines R_C is the same as well as when the load moves on the beam DST. The last thing to do is to consider the movement of the load on the beam AB, which is the main and located at the diagram below the beam BCD that refers to the required force. As the force from the load on to the underlying beams is not transmitted upward (on the upper beams), so the line of influence of R_C on this site will be zero (Fig. 3.38, *c*).

Influence lines of bending moment and shear force in sections K_1 and K_2 of the beam BCD are constructed similarly (Fig. 3.38, *d*, *e*, *f*, *g*). When plotting the influence lines for shear force and moment in the cross-section K_2 (Fig. 3.38, *f*, *g*) we need to be aware that this section is located on the console beam BCD and during the motion of the load on this beam the influence lines are plotted the same as for console beam (Fig. 3.36).

Plotting of the influence lines for shear force and moment in the section K_3 must begin with consideration of the movement of the load on the beam DST, and since the cross-section is in the span of the beam, the procedure for plotting the influence lines of forces in the section K_3 is the same as for the section K_1 in the simple beam in Fig. 3.37. During the movement of load on the topside beam TU the influence lines in the section K_3 are constructed on the basis of conditions of interaction and work of simple beams in multi-span beam that was discussed above. During the movement of the load on the bottom beams AB and BCD the internal forces in the section K_3 will be missed. Fig. 3.38, *h* shows the influence line for shear force in the section K_3.

Similarly the influence lines are plotted for two support reactions and shear-(moment)-influence lines in two cross-sections for beams with specific dimensions that are represented in Fig. 3.39.

3.7.4. Determination of the Reactions and Internal Forces - Shear, Moment along the Influence Lines From External Loads

For the static determinate systems, that have influence lines having partly-linear type of changing, the general expression for the determination of forces on their influence lines (i. l.) by the action of concentrated forces, uniformly distributed loads and concentrated moments have the form:

$$S = \sum_{i=1}^{n} F_i\, y_i + \sum_{j=1}^{s} q_j\, \Omega_j + \sum_{k=1}^{t} m_k\, \mathrm{tg}\,\alpha_k\,, \tag{3.5}$$

where: n, s, t – accordingly, the number of concentrated forces F_i, uniformly distributed loads q_j and concentrated moments m_k; y_i – the ordinate of the influence line under concentrated force; Ω_j – area of influence line under a uniformly distributed load; $tg\ \alpha_k$ – the tangent of the slope of the plot influence lines under the concentrated moments relative to the reference axis.

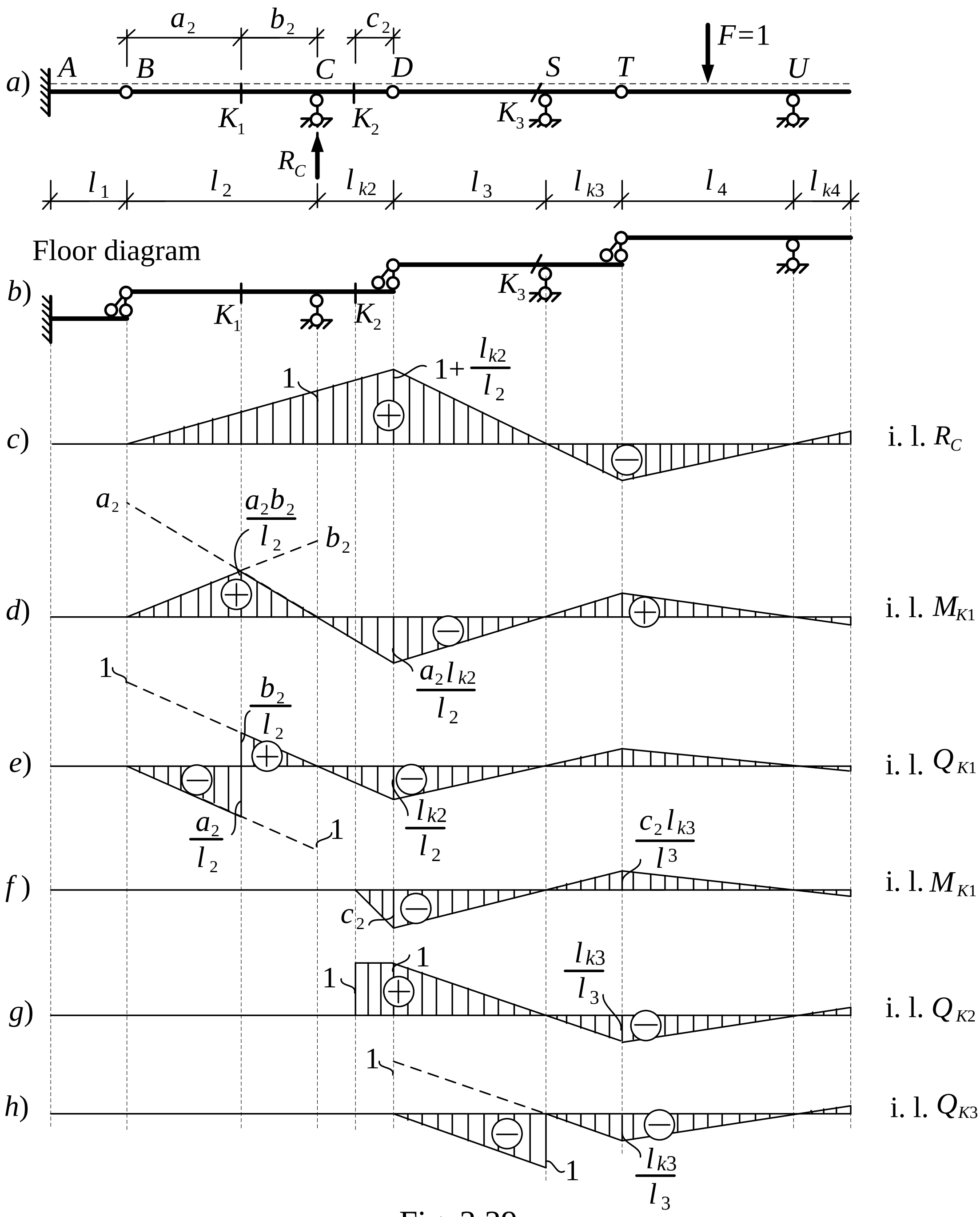


Fig. 3.39

Concentrated forces F_i and uniformly distributed load q_j are taken positive if they act downwards, and concentrated moments m_k are positive if they act in clockwise direction; The ordinate y_i and the area Ω_j of the influence lines are taken with the signs of influence lines respectively under the forces and uniformly distributed loads, and $\operatorname{tg}\alpha_k$ is positive for an increasing function of influence lines (see, for example, plots of *BCD* and *TU* for the influence line of R_C, shown in Fig. 3.38, *c*) and negative for decreasing functions (part *DST* on the same i. l.).

The important factor in determining the forces from external loads is the following property of the straight part of the line of influence:

- at the straight part of the influence line the force from the system of loading can be determined by the product of the resultant of this system of loading on the ordinate of influence line under the resultant

$$S = R\,y_R. \tag{3.6}$$

The considered property simplifies the determination of forces from the action of any loads acting on straight parts of the lines of influence, as long as you can easily find the resultant of these loads and their points of application. Thus, in case of a uniformly distributed load its resultant force is equal to the product of the intensity of the load on the length of the part and is applied to the middle of this area. It is enough to determine the forces on the linear parts of the lines of influence and the loads distributed in a triangular and trapezoidal dependencies. The load that is distributed over the trapezoidal dependencies should be divided into uniformly distributed and triangular, or on two triangular loads.

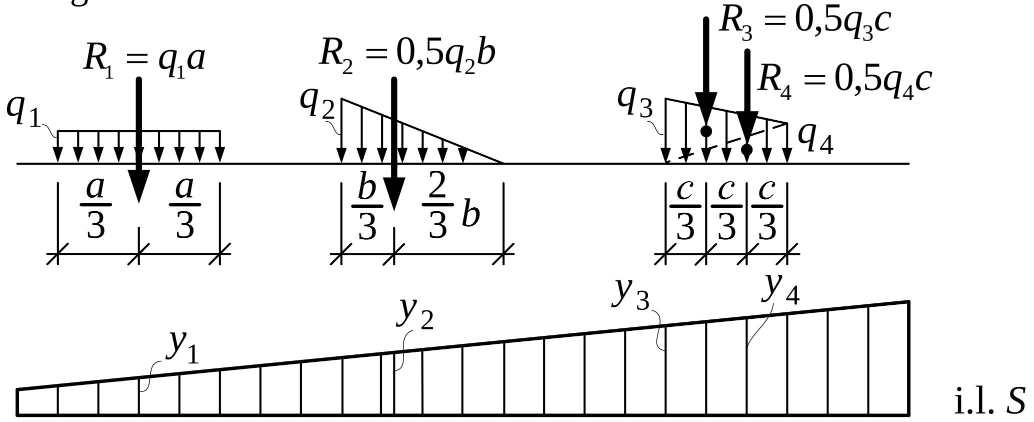


Fig. 3.40

For example, the force ***S*** from the loads shown in Fig. 3.40, can be calculated according to the expression:

$$S = (q_1 a)\,y_1 + (0{,}5 q_2 b)\,y_2 + (0{,}5 q_3 c)\,y_3 + (0{,}5 q_4 c)\,y_4\ .$$

Let’s compute the internal forces, for which the line of influence have been plotted in the beam in Fig. 3.38, from external load, that is presented at the same figure. And then we will compare them with the values that were taken from diagrams of the forces that were plotted for the same beams from the action of the same load in Fig. 3.35:

$$R_D = 9{,}4\cdot 0{,}333 + 9{,}4\cdot 0{,}667 + 2\cdot 4{,}4\cdot 0{,}982 - 5\cdot 0{,}491 = 15{,}587 \text{ kN};$$

$$R_F = 2\cdot 4{,}4\cdot 0{,}333 + 5\cdot 1{,}333 = 9{,}595 \text{ kN};$$

$$M_1 = -1{,}1\cdot 1{,}55\cdot 1{,}55/2 - 9{,}4\cdot 0{,}517 - 9{,}4\cdot 1{,}033 + 2\cdot 4{,}4\cdot 0{,}489 - 5\cdot 0{,}244 = -12{,}808 \text{ kN}\cdot\text{m};$$

$$Q_1 = 9{,}4\cdot 0{,}25 - 5{,}4\cdot 1{,}1\cdot 0{,}625 - 1{,}1\cdot 1{,}55\cdot 0{,}108 - 9{,}4\cdot 0{,}072 - 9{,}4\cdot 0{,}143 + 2\cdot 4{,}4\cdot 0{,}068 - 5\cdot 0{,}034 = -3{,}139 \text{ kN};$$

$$M_2 = 9{,}4\cdot 1{,}033 + 9{,}4\cdot 0{,}517 - 2\cdot 4{,}4\cdot 0{,}489 + 5\cdot 0{,}244 = 11{,}487 \text{ kN}\cdot\text{m};$$

$$Q_2 = -9{,}4\cdot 0{,}333 - 2{,}0\cdot 4{,}4\cdot 0{,}315 + 5\cdot 0{,}158 = 7{,}427 \text{ kN}.$$

A comparison of the values of forces, obtained along the influence lines and taken from the figures:

Designation of the forces	Values of forces obtained according to		The discrepancy between the results	
	the diagrams	the i.l.	absolute	relative, in %
R_D	15,587	15,587	0	0
R_F	9,595	9,6	0,005	0,052
M_1	12,812	12,808	0,004	0,030
M_2	11,49	11,487	0,003	0,026
Q_1	3,142	3,139	0,003	0,088
Q_2	7,413	7,427	0,014	0,194

3.8. Features of the Calculation of Three-Hinged Arches on Vertical Loads

Three-hinged system that has two disks that are connected to each other with hinge joint is represented by the curved rods, is called a three-hinged arch. In three-hinged arches, there are four components of reactions at the supports, just as in the three-hinged frames, that are determined by the four equilibrium equations of the arch as a whole and its separate parts (Fig. 3.41, *a*):

$$\sum M_A = 0; \qquad \sum M_B = 0; \qquad \sum M_C^{left} = 0; \qquad \sum X = 0.$$

In this case only vertical loads act on the three-hinged arch, horizontal reactions (arch thrust) to the left and right are equal to each other, and vertical reactions are determined similarly as the reactions at the support in a simple beam, loaded with the same load (Fig. 3.41, *b*).

Internal forces in the cross-sections of the arches are determined on the basis of the same approaches that are used in the frames and which are reviewed in sections 3.1–3.4.

Forces in sections of three-hinged arches can be determined by the formula:

$$M_k = M_k^{o} - H \cdot y_k; \qquad Q_k = Q_k^{o} \cos\varphi_k - H \cdot \sin\varphi_k;$$

$$N_k = -(Q_k^{o} \sin\varphi_k + H \cdot \cos\varphi_k), \tag{3.7}$$

where M_k^{o}, Q_k^{o} – bending moment and shear force at the cross-section *k* of the simple beam (see Fig. 3.41, *b*), having the same span and loaded with the same load, just as the arch; H – the magnitude of the horizontal reactions of the arch; φ_k – the tangent angle to the axis of the arch in cross-section *k* w.r.t. horizontal axis *x*; y_k – the y coordinate of the center of cross-section *k* w.r.t. horizontal axis *x*, passing through a support (see Fig. 3.41, *a*).

Note that for a given coordinate system with the origin at the left support of the arch (Fig. 3.41, *a*) $\sin\varphi_k$ fot the semi-arch left is positive and for the right is negative; $\cos\varphi_k$ for the both semi-arches is positive.

As an example, let's consider the arch of parabolic shape, shown in Fig. 3.41, *a*. Let's compute the internal forces in the cross-sections K_1 and K_2 of the arch.

First, let's determine the reactions at the supports from a given (set) external load:

$\sum M_A = 0; \quad 12 \cdot 2 + (3 \cdot 8) \cdot 6 + 10 \cdot 12 + 8 \cdot 17 + (4 \cdot 3) \cdot 18{,}5 - R_B \cdot 20 = 0; \quad R_B = 32{,}3$ kN;

$\sum M_B = 0; \quad -12 \cdot 18 - (3 \cdot 8) \cdot 14 - 10 \cdot 8 - 8 \cdot 3 - (4 \cdot 3) \cdot 1{,}5 + R_A \cdot 20 = 0; \quad R_A = 33{,}7$ kN;

$\sum M_C^{left} = 0; \quad 33{,}7 \cdot 10 - 12 \cdot 8 - (3 \cdot 8) \cdot 4 + H \cdot 4 = 0; \quad H = 36{,}25$ kN.

Check the correctness of finding the reactions at the supports:

$\sum Y = 0; \quad 33{,}7 + 32{,}3 - 12 - 3 \cdot 8 - 10 - 8 - 3 \cdot 4 = 0; \quad 66 - 66 = 0;$

$\sum M_C^{right} = 0; \quad 10 \cdot 2 + 8 \cdot 7 + (4 \cdot 3) \cdot 8{,}5 + 36{,}25 \cdot 4 - 32{,}3 \cdot 10 = 0; \quad 323 - 323 = 0.$

Determine the ordinates for the cross-sections and the parameters of the slope angles of the tangent lines to the horizontal line for the cross-sections:

Section K_1: $x_{K1} = 4$ m; $\quad y_{K1} = \dfrac{4f}{l^2} x_{K1}(l - x_{K1}) = \dfrac{4 \cdot 4}{20^2} \cdot 4 \cdot (20 - 4) = 2{,}56$ m;

$\text{tg}\,\varphi_{K1} = \dfrac{4f}{l^2}(l - 2x_{K1}) = \dfrac{4 \cdot 4}{20^2}(20 - 2 \cdot 4) = 0{,}48; \quad \sin\varphi_{K1} = 0{,}4327; \quad \cos\varphi_{K1} = 0{,}9015;$

Section K_2: $x_{K1} = 15$ м; $\quad y_{K2} = \dfrac{4f}{l^2} x_{K2}(l - x_{K2}) = \dfrac{4 \cdot 4}{20^2} \cdot 15 \cdot (20 - 15) = 3$ m;

$\text{tg}\,\varphi_{K2} = \dfrac{4f}{l^2}(l - 2x_{K2}) = \dfrac{4 \cdot 4}{20^2}(20 - 2 \cdot 15) = -0{,}4; \quad \sin\varphi_{K2} = -0{,}3714; \quad \cos\varphi_{K2} = 0{,}9285.$

3) Determine the internal forces in the cross-sections by the formulas (3.7):

Section. К$_1$: $\quad M_{K1}^0 = 33{,}7 \cdot 4 - 12 \cdot 2 - 3 \cdot 2 \cdot 1 = 104{,}8$ kNm;

$Q_{K1}^0 = 33{,}7 - 12 - 3 \cdot 2 = 15{,}7$ kN;

$M_{K1} = 104{,}8 - 36{,}25 \cdot 2{,}56 = 12$ kN·m; – the stretched fibres are at the bottom;

$Q_{K1} = 15{,}7 \cdot 0{,}9015 - 36{,}25 \cdot 0{,}4327 = -1{,}532$ kN;

$N_{K1} = -(15{,}7 \cdot 0{,}4327 + 36{,}25 \cdot 0{,}9015) = -39{,}473$ kN;

Section К$_2$:

$M_{K2} = (32{,}3 \cdot 5 - 8 \cdot 2 - 4 \cdot 3 \cdot 3{,}5) - 36{,}25 \cdot 3 = -5{,}25$ kN·m; – the stretched fibres are at the top;

$Q_{K2} = (-32{,}3 + 8 + 4 \cdot 3) \cdot 0{,}9285 - 36{,}25 \cdot (-0{,}3714) = 2{,}043$ kN;

$N_{K2} = -[(-32{,}3 + 8 + 4 \cdot 3) \cdot (-0{,}3714) + 36{,}25 \cdot 0{,}9285] = -38{,}226$ kN.

Considering the fact that the axes of the arches are curvilinear, the diagrams of forces in the arches also vary in curvilinear relationships, and an accurate representation of their shape is quite difficult. Diagrams of the internal forces in arches are usually ploted up by several ordinates, dividing the spans of the arches into several parts and calculating the ordinates of the diagrams in the extremumes, connecting them with the smooth curves.

It is necessary to calculate the ordinates of the diagrams of internal forces and in the typical cross-sections – under concentrated forces and moments. The more the will be calculated ordinates the more accurate we can depict the diagram of the forces.

Fig. 3.41 presents the results of the calculation of the arch by dividing the span into ten equal parts. Presented values of the forces in the calculated and characteristic cross-sections can be used for self-study of calculation procedure of the arches, considering that the results for showed arche are already known.

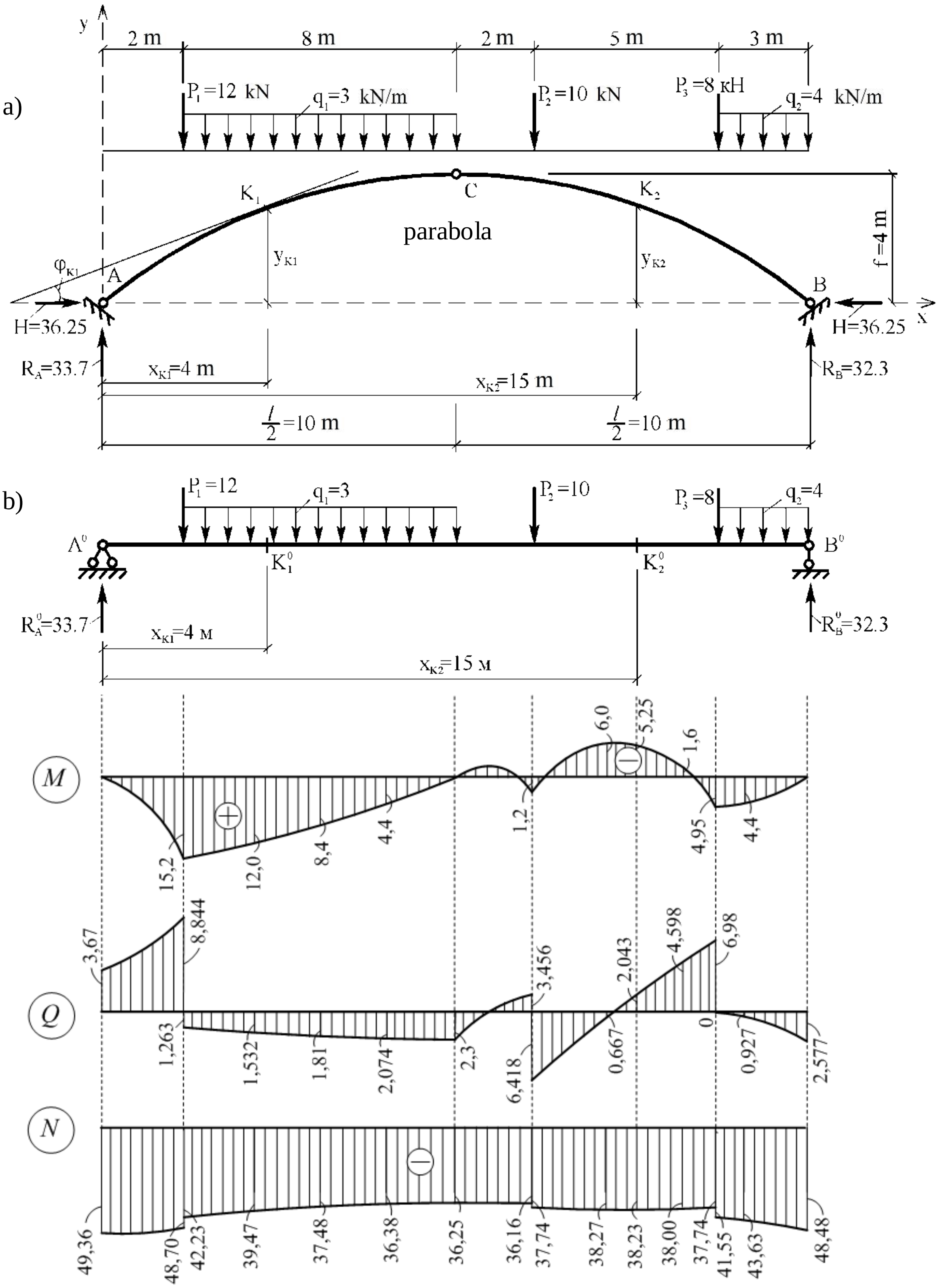


Fig. 3.41

3.9. Determination of the Forces in Truss

Statically determinable *hinged truss* is a geometrically invariable system, calculating scheme of which consists of straight rods connected at the joints of the hinge (Fig. 3.42). The number of degrees of freedom are defined by the formula $W = 2J - R - C_0$, and have to be equal zero. As the joint transfer of the loads the rods of hinged trusses work only in tension-compression mode. We can provide *roof truss rafters* with rigid connection elements at the joints to the design diagram, where during the joint transmission of the load values of the bending moments and shear forces in the rods are unimportant (and accordingly can be neglected). Rigid joints on the design diagram of these trusses are replaced by hinged joint.

Let's determine the basic concepts for trusses. The distance between supports is called the *span* of the truss (*l*), the vertical dimension – the *height* of the truss (*h*).

A set of elements (rods) of the truss that make up its upper and lower contours, called respectively the *upper* (top) and *lower* (bottom) *chords* of the truss (Fig. 3.42). The rods located between the *panels* and connecting them is called the *lattice* of the truss. Rods of the lattice are divided into *bracing* and *vertical posts*. The distance between adjacent joints of chords of truss (in horizon) is called panels. Chords are divided into the *truss top chord* and *bottom (lower) chord* of a truss.

The transfer of loads to the joints of trusses is carried out through so-called *transfer (transmission) beams*, in the real structures it may be overlaps on *girders*, beams; *roof slabs*, etc. On the disigne schemes the transmission beams are represented as a simply beams with a spans equal to the length of the panels of the loaded zones of the trusse.

The general method of determining the internal forces in the rods of statically determinable trusses is *method of sections*. Truss (Fig. 3.43, *a*) is cut by *through* or a closed cross-section into two or more parts so that the rod is splited, in which we define the force. After this we consider the balance of one of the parts, the action on which by the dropped part(s) of the truss is replaced by unknown longitudinal forces. We usually direct these longitudinal forces from the joints (sections), which corresponds the stretching of the rods (Fig. 3.43, *b*).

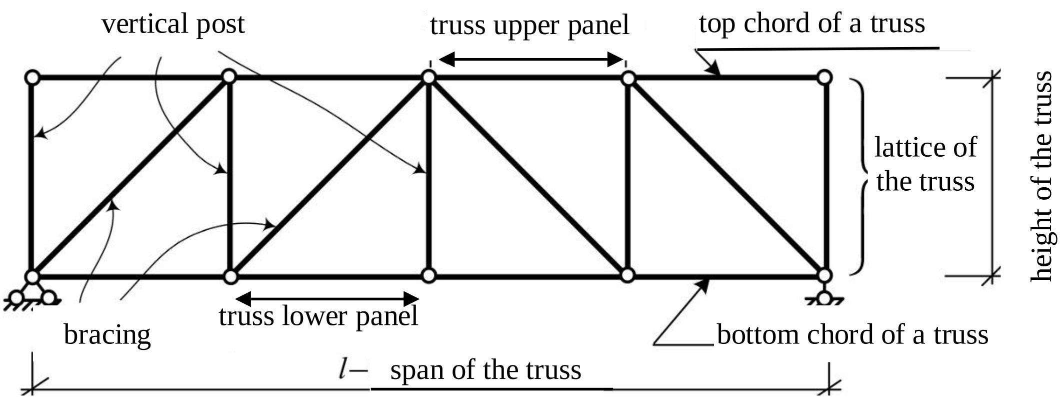


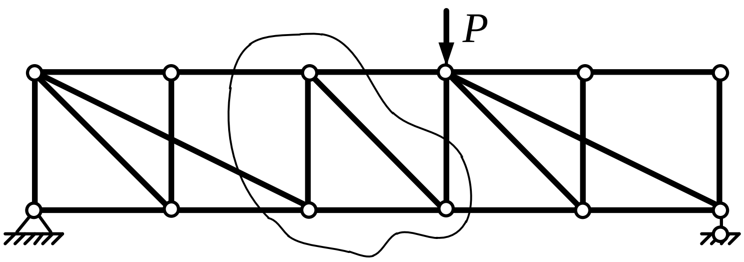
Fig. 3.42 Scheme of the truss and its elements

From the equations of equilibrium we determine the longitudinal force and establishe the true sign of the forces (if the force turned out negative, then it will be directed in the opposite direction and the rod will have a compression). Method of cross-sections for the trusses is implemented by methods*: of cutting of joints, moment points* and *projections*.

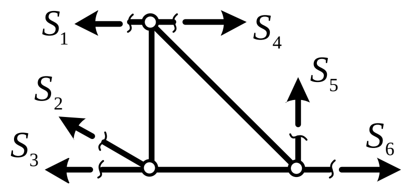
Method of cutting of joints. A truss joint is cut out by the closed cross-section cut. Forces in the splited rods, which connect in the joint, unite into the system of forces converging at one point, for the balance of which it is possible to make two independent equations in the form of sums of projections of forces on two axes:

$$\Sigma X = 0; \quad \Sigma Y = 0; \qquad \text{or} \qquad \Sigma Z_1 = 0; \quad \Sigma Z_2 = 0. \tag{3.8}$$

Direction of these axes can be chosen arbitrarily, with exception of their parallelism. From the rational point of view of calculation we should select the direction of the axis so that, each of the equations (3.8) could consist of only one unknown force. We can cut such joints whose number of unknown forces doesn't exceed two and these forces are not directed along the same straight line. In some cases there is necessity of cutting the joints with a large number of unknowns forces – for example, if it is helps to find the force in at least one of the rods (if in the three rods hinged joint two of rods are directed along the same straight line, we can find the force in the third rod, (see joints 2 and 7 in Fig. 3.44), or allows us to find the dependence between some of the forces that will be used in further calculations.



a) the scheme of the truss and possible variant of cross section



b) cut out considered part of the truss

Fig. 3.43 Usage of the method of sections

For example, for a truss on the Fig. 3.44, *a*, we can cut the joint 1 (Fig. 3.44, *b*), from consideration of equilibrium of which we can find:

$$\Sigma Y = 0; \qquad S_{1-3}\sin\alpha - P = 0; \qquad S_{1-3} = \frac{P}{\sin\alpha};$$

$$\Sigma X = 0; \quad S_{1-2} + S_{1-3}\cos\alpha = 0; \quad S_{1-2} = -S_{1-3}\cos\alpha = -\frac{P}{\sin\alpha}\cos\alpha = -Pctg\alpha.$$

The angle α and the trigonometric functions $\sin\alpha$ and $\cos\alpha$ are found from geometric considerations.

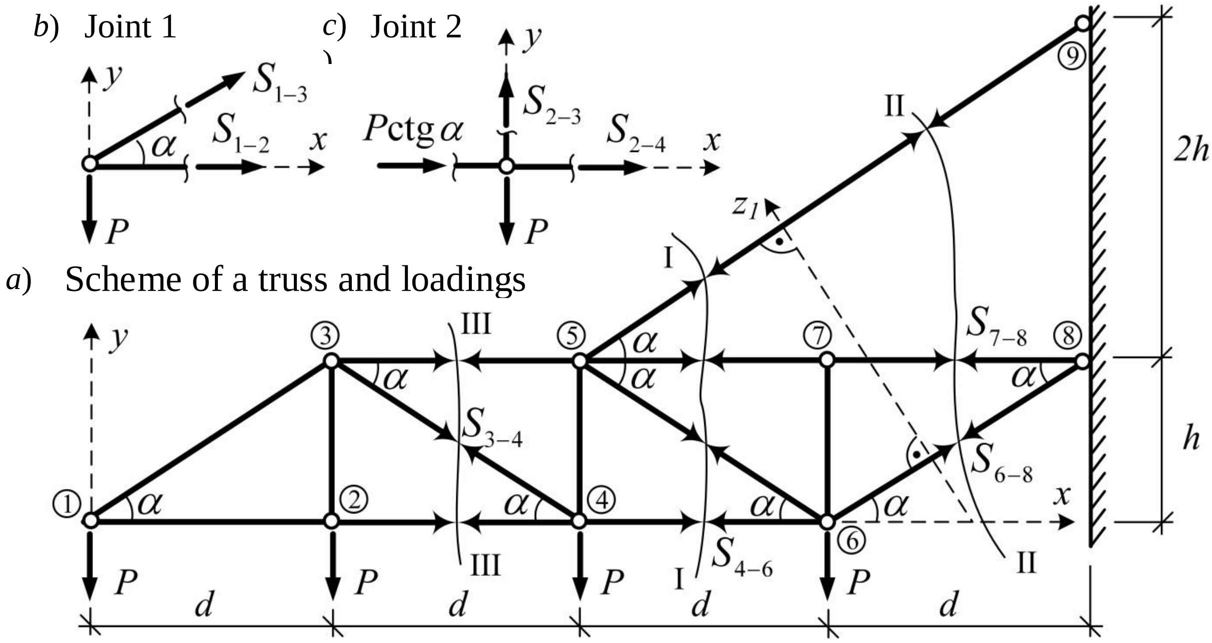


Fig. 3.44 Design scheme of the truss

Then we cut out the joint 2 (Fig. 3.44). In the rod 1–2 we apply the already known force $S_{1-2} = -Pctg\alpha$. From consideration of equilibrium of the joint we will find:

$$\Sigma X = 0; \qquad Pctg\alpha + S_{2-4} = 0;\ S_{2-4} = -Pctg\alpha;$$
$$\Sigma Y = 0; \qquad S_{2-3} - P = 0; \qquad S_{2-3} = P.$$

The further procedure of the calculation of the truss involves the cutting of joints 3 and 4, from the equations of equilibrium of which we will find the forces in the rods 3–4, 3–5, 4–5 and 4–6. Cutting out the joint 7, from the equation $\Sigma Y = 0$ we get result in finding the forces in the rod 7–6 (in this case it will be zero). To determine the forces in the remaining rods a method of cutting joints is not applicable, as in each of the remaining joints converge more than two rods with unknown forces.

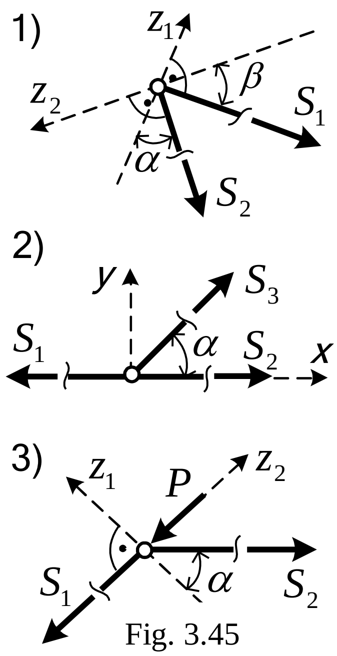


Fig. 3.45

Method of cutting of joints allows us to formulate the ***signs (characteristics) of "zero" rods***, which make it easy to find the rods in which forces are equal to zero:

1) in two-rod unloaded joint, in which the rods do not lie on the same line (Fig. 3.45, position 1), forces in both rods are equal to zero:

$$\Sigma Z_1 = 0; \quad -S_2\cos\alpha = 0; \quad S_2 = 0;$$
$$\Sigma Z_2 = 0; \quad -S_1\cos\beta = 0; \quad S_1 = 0;$$

2) in three-rod unloaded joint, in which two rods are collinear, and the third at an angle to them (Fig. 3.45, position 2), the third force in the rod is zero, and the forces in the first two rods are equal to each other:

$$\Sigma Y = 0; \quad S_3\sin\alpha = 0; \quad S_3 = 0;$$
$$\Sigma X = 0; \quad -S_1 + S_2 + 0 = 0; \quad S_1 = S_2;$$

3) in two-rod joint, in which the rods do not lie on the same line and external force is applied in the direction of one of the rods (Fig. 3.45, position 3), the internal force in the second rod is equal to zero, and the force in the first is equal to the mentioned external force:

$$\Sigma Z_1 = 0; \qquad -S_2 \cos\alpha = 0; \qquad S_2 = 0;$$
$$\Sigma Z_2 = 0; \qquad -S_1 - P = 0; \qquad S_1 = -P.$$

The advantage of this method is its simplicity. *The disadvantages* are:

1) often we can not immediately (without preliminary sequential calculation of the number of joints and sometimes it is rather large number) find the forces in the rods inside the truss;

2) during the process of sequential cutting of the joints the calculation of descrepancy which transfers from the previous joint to the next, gradually accumulating and increasing.

Method of moment point. The truss is divided into two parts. The parts is cut out in such a way (if that's even possible), that the axis of all cut rods with unknown forces, except one (in which we look for the force), intersect in one point; this point is adopted as moment point and w.r.t. this point the sum of the moments of all forces is calculated for the considered part of the truss; from the resulting equation we determine the required force.

For example, for the truss in Fig. 3.44 for determining the forces in the rod 4–6 it is necessary to make cross-section I–I and consider the equilibrium of the left part of the truss. The moment point for force $S_{4\text{-}6}$ will be the point at joint 5, in which the remaining three rods (cut in section I–I) intersect, this are the rods 5–6, 5–7 and 5–9. The required force we will find from the equation:

$$\sum M_5^{left} = 0; \qquad -P\cdot 2d - P\cdot d - S_{4-6}\cdot h = 0; \qquad S_{4-6} = -\frac{3d}{h}P.$$

The force in the rod 6-8 we will find, after making a cross section II–II and considering the equilibrium of the left part of the truss. Considering the fact that the moment point in this case will also be point of node 5, in which the axises of the rods 5–9 and 7-8 intersect with other, which were cut by the section II–II together with the rod 6–8:

$$\sum M_5^{left} = 0; \qquad -2P\cdot d + P\cdot d - P\cdot d - S_{6-8}\cos\alpha\cdot h - S_{6-8}\sin\alpha\cdot d = 0;$$
$$S_{6-8} = -\frac{2d}{h\cos\alpha + d\sin\alpha}P.$$

Method of projections. The truss is divided into two parts. The parts is cut out such a way (if that's even possible) that all rods with unknown forces, except one (in which the force is looked for), would parallel to each other. For the considered part of the truss we constitute the sum of projections on the axis perpendicular to the mentioned parallel rods. From the final equation we determine the required force.

For example, for a force on the Fig. 3.44 for determining the force in the rod 7–8 we can use the earlier made cross-section II–II; where rods 6–8 and 5–9 (parallel to each other) were cut out together with the rod 7–8. Therefore, if we constitute the equation of the projections of all forces, for example, for the left cut out side of the truss on the axis Z_1, that is perpendicular to the rods 6–8 and 5–9, some unknown forces in these rods will not be included in considering equation of equilibrium (their projections on the axis Z_1 is zero); the equation will have only one unknown force S_{7-8}, which we will find from the solution of the equation:

$$\Sigma Z_1 = 0; \qquad -4P\cos\alpha - S_{7-8}\sin\alpha = 0; \qquad S_{7-8} = -4P\text{ctg}\,\alpha .$$

Similarly, we can find the force in the rod 3-4 (Fig. 3.44, *a*), after drowing the cross-section III–III and forming the sum of projections of all the forces on the vertical Y-axis (rods 3–5 and 2–4 are horizontal) for the left cut out part of the truss:

$$\Sigma Y = 0; \qquad -2P - S_{3-4}\sin\alpha = 0; \qquad S_{3-4} = -\frac{2P}{\sin\alpha}.$$

Advantages of methode of the moment point and the projection methode is that in most cases, with their help the forces in the rods can be expressed only through the external loads and the supporting reactions (without expressing through other forces).

Using this methods of the moment point, the projections and cutting out the joints together we can find the forces in all rods for most trusses.

3.10. Determination of Displacements in Flexible Systems

Determination of displacements in *framework structures* (rod systems) from the action of external loads is convenient to perform according to the *Mohr's formula**. For flexible systems (frames, beams) in this case, usually take into account only the bending moments (due to non-significance influence of the shear and longitudinal forces on the values of displacement in such systems). Mohr's formula in this case takes the form:

$$\Delta_{iP} = \sum_{1}^{n}\int_{0}^{l}\frac{\bar{M}_i\, M_P\, dx}{EJ}; \tag{3.9}$$

Where: $\bar{M}_i$ – bending moments in the system from the actions of a unit force, that is applied in the section (point) for which determines the displacement, in the direction of the desired (***i**-th*) displacement; M_P – bending moments in the system from the action of a given load; *EJ* – the flexural rigidity of rods in the system; n – the number of divided parts of the system to calculate the Mohr's integrals; *l* – the lengths of this parts.

The procedure for determining the displacements by the Mohr's formula:

1. Bending moments are determined in the system from the action of a given load and diagram M_P is plotted.

2. In the cross-section (point) for which determins the displacement, in the direction of the desired displacement a unit force is applied; depending on the form of

* This formula was first derived by J.C.Maxwell in 1864 and applied in design practice by O.Mohr in 1874

determining displacement this "force" may be different:

a) if we determine the linear (horizontal, vertical, in any direction) displacement, then a unit concentrated force $P = 1$ is applied (Fig. 3.46, *a*);

b) if there is a mutual convergence (divergence) of the two points, then two unit forces are applied to these points, directed along the straight line connecting these points towards each other (from each other) (Fig. 3.46, *b)*;

c) If determining the *rotation angle* of the cross-section, then we apply a unit concentrated moment $m = 1$ (Fig. 3.46, *c*);

d) If we determine a mutual angle of rotation of the two cross-sections (change in angle between sections), then two unit moments are applied to these two sections which are directed towards each other (Fig. 3.46, *d*).

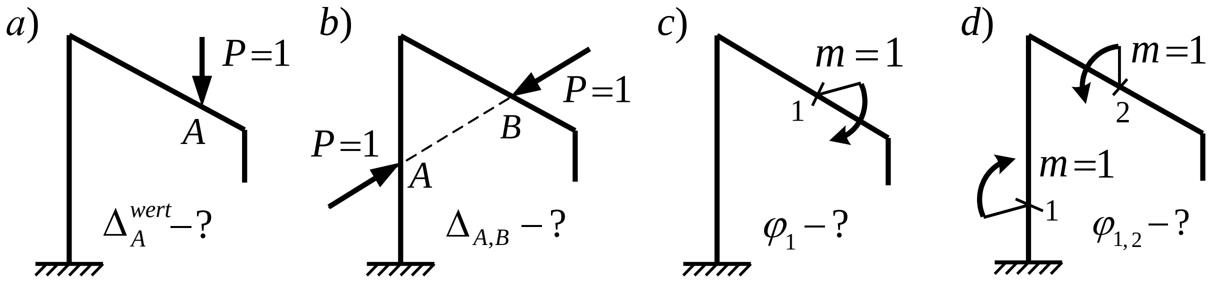


Fig. 3.46

3. From the action of unit force (that is applied according to the paragraph 2) we will find out the bending moments in the borders of each *cheng of load distance* or we plot a diagram $\bar{M}_i$.

4. We calculate the desired displacement according to the Mohr's formula (3.9). The calculation of the Mohr's integrals can be performed by halp of:

a) direct integration (that is not always simple);

b) by the *rule of Vereschagin*;

c) By *trapezoid* formula;

d) By *Simpson's* formula;

e) By *numerical* method.

Note that the computation of Mohr's integrals by the rule of Vereschagin, according to the formulas of trapezoids and Simpson is often called the "*multiplication of diagrams*".

The rule of Vereschagin. To calculate the Mohr's integral $\int_0^l \frac{\bar{M}_i\, M_P\, dx}{EJ}$ on a part of constant rigidity, in which the character of diagrams $\bar{M}_i$ and M_P do not change, we need to multiplicate the area of one of the diagram $\boldsymbol{\Omega}$ by the ordinate, that is taken under the center of gravity of this diagram in front of the other diagram y_c (if one diagram is curvilinear, then we have to compulsory take the area of the curvilinear diagram):

$$\frac{1}{EJ}\int_0^l \bar{M}_i\, M_P\, dx = \frac{1}{EJ}\ \Omega \cdot y_c. \qquad (3.10)$$

Thus, for the diagram, which area is taken, we must be able to calculate this area and be able to find (know) the position of its centre of gravity.

<u>The rule of signs</u>: if the center of gravity of "multiplied" diagram and the corresponding ordinate from the other diagram are located on one side of the axis of the rod (stretched fibers are on one side of the rod), the result of "multiplication" is taken with the "plus" sign.

Here are some examples of application of the rule of Vereschagin.

For diagrams presented on Fig. 3.47, the computation of the Mohr's integral by the rule of Vereschagin can be done in four different ways, which give the same result:

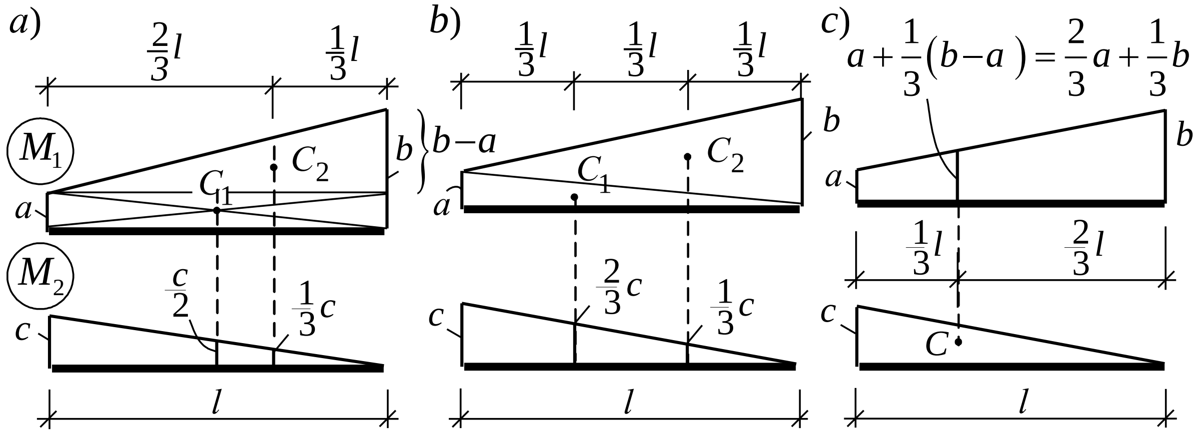


Fig. 3.47

a) during the calculation of the area of this diagram (M_1) and splitting of it (to quick and easy determination of the positions of the centers of gravity) into the rectangle $a\times l$ and the triangle $((b-a)/2)\times l$ (Fig. 3.47, *a*) we obtain:

$$\int_0^l \frac{M_1 M_2\,dx}{EJ} = \frac{1}{EJ}\left[(a\cdot l)\cdot\frac{c}{2} + \frac{1}{2}(b-a)\,l\cdot\frac{1}{3}c\right];$$

b) At the calculation of the area of the diagram M_1 and spliting it into two triangles $(a\times l)/2$ and $(b\times l)/2$ (Fig. 3.47, *b*) we obtain:

$$\int_0^l \frac{M_1 M_2\,dx}{EJ} = \frac{1}{EJ}\left[\left(\frac{a\cdot l}{2}\right)\cdot\frac{2}{3}c + \left(\frac{b\cdot l}{2}\right)\cdot\frac{1}{3}c\right];$$

c) At the calculation of the area of the diagram M_2 (because both diagrams M_1 and M_2 are linear, it does not matter the area which one to take) (Fig 3.47, *c*) we can write:

$$\int_0^l \frac{M_1 M_2\,dx}{EJ} = \underbrace{\frac{1}{EJ}\left(\frac{c\cdot l}{2}\right)\cdot\left[a+\frac{1}{3}(b-a)\right]}_{\text{If diagram } M_1 \text{ can be split into a rectangle and triangle;}}, \quad \text{or} \quad = \underbrace{\frac{1}{EJ}\left(\frac{c\cdot l}{2}\right)\cdot\left(\frac{2}{3}a+\frac{1}{3}b\right)}_{\text{If diagram } M_1 \text{ can be split into two triangles.}}.$$

<u>The trapezoid formula</u> – used for "multiplying", only the linear diagrams (Fig. 3.48):

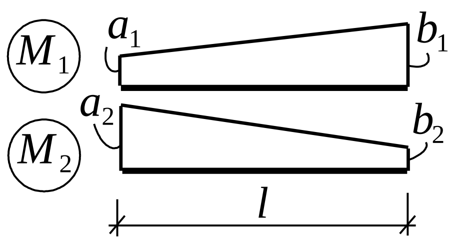


Fig. 3.48

$$\int_0^l \frac{M_1 M_2\,dx}{EJ} = \frac{l}{6EJ}\left(2a_1\cdot a_2 + a_1\cdot b_2 + a_2\cdot b_1 + 2b_1\cdot b_2\right). \quad (3.11)$$

The Simpson's formula can be used to compute Mohr's integrals by the appropriate "multiplying" of linear diagrams, and diagrams, one of which is curved (changes according to the parabolic law) (Fig. 3.49). The Simpson's formula has the form:

$$\int_0^l \frac{M_1 M_2\, dx}{EJ} = \frac{l}{6EJ}\left(a_1 \cdot a_2 + 4\, c_1 \cdot c_2 + b_1 \cdot b_2\right). \quad (3.12)$$

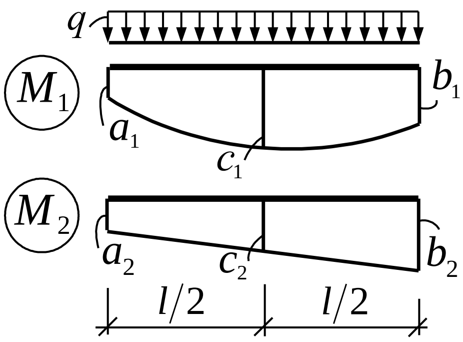


Fig. 3.49

<u>The rule of signs.</u> *In the Simpson's and trapezoid formulas,* multiplications of ordinates are taken with the "plus" sign if the ordinates are on one side of the axis of the rod on both diagrams. The sign "minus", if these ordinates are from different sides of the axis of the rod.

Recommendations to the calculation of Mohr's integrals in the framework and beams systems:

– for diagrams of internal forces (bending moments) with the linear nature of the change and the simple form (rectangular, triangular), it is proposed to use the rule of Vereschagin;

– For diagrams of the internal forces having the linear nature of change, but more complex shape (trapezoidal), it is recommended to use the trapezoid formula;

– For diagrams of internal forces, one of which is curved, we must use the Simpson's formula.

We should note that the Simpson's formula is the most general and can be applied to all cases.

It should be noted that if we use such methods to compute the Mohr's integrals we must satisfy the following requirements:

– the rigidity of the rod in this area must be constant;

– The dependencies of changing of both diagrams (the nature of their changes) must not have change within the area, or, otherwise, within the section of "multiplying" the diagram must not have inflections (discontinuity), "jumps", and transitions to other dependencies of changing the internal forces.

If one of these requirements is not satisfied, then the diagram should be divided into smaller parts, and this must be done so that these requirements are met. The boundaries of the considered parts of framework are the points (sections) of the fracture (break) and branching of rods, applications of concentrated loads (forces, moments), the actions of the reactions, the beginning and the end of distributed loads.

The calculation of displacement from the action of the set loads by the Mohr's formula (3.9) is made by summing up the results of the calculation of the Mohr's integrals with the considered ways in all parts of the system.

Let's consider the finding of the vertical displacement of point C from the action of given loads in the three-hinged frame shown in Fig. 3.50.

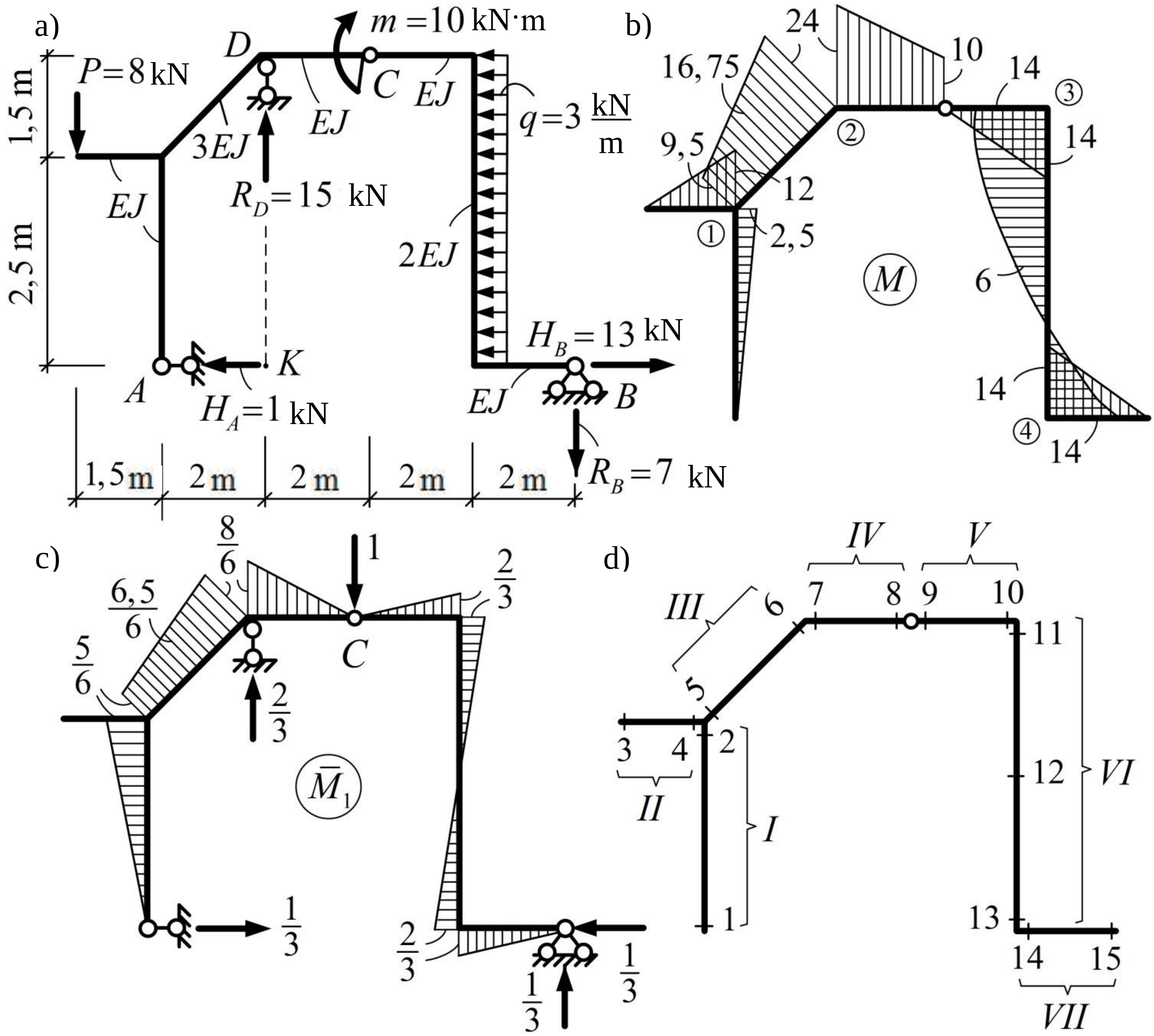


Fig. 3.50

The calculation of the reactions at the supports and plotting of the diagram of bending moments in the frame are performed on the basis of approaches described in sections 3.1–3.3. Diagram of bending moments from the action of a given load is shown in Fig. 3.50, *b*.

As we determine the vertical displacement of point C, so at this point in a vertical direction we apply a single concentrated force and direct it down, assuming that the point C will shift down.

From the action of this force we plot a *unit diagram* of the bending moments $\bar{M}_1$ (Fig. 3.50, *c*). After that, we allocate areas of the frame of continuity of the plot, within which the rigidity of the rods is constant and both of the diagrams change continuously (Fig. 3.50, *d*), and then we calculate the desired displacement according to the Mohr's formula (3.9):

$$\Delta_C^{\text{vert}} = \Delta_{1P} = \sum \int \frac{\overline{M}_1 M_P \, ds}{EJ} = -\frac{1}{EJ}\left(\frac{2,5 \cdot 2,5}{2}\right) \cdot \frac{2}{3} \cdot \frac{5}{6} + 0 +$$

$$+\frac{2,5}{6\cdot 3EJ}\left(9,5\cdot\frac{5}{6}+4\cdot\frac{6,5}{6}\cdot 16,75+24\cdot\frac{8}{6}\right)+\frac{2}{6EJ}\left(2\cdot 24\cdot\frac{8}{6}+\frac{8}{6}\cdot 10+0\cdot 24+2\cdot 10\cdot 0\right)-$$

$$-\frac{1}{EJ}\left(\frac{14\cdot 2}{2}\right)\cdot\frac{2}{3}\cdot\frac{2}{3}+\frac{4}{6\cdot 2EJ}\left(-14\cdot\frac{2}{3}+4\cdot 0\cdot 6-14\cdot\frac{2}{3}\right)-\frac{1}{EJ}\left(\frac{14\cdot 2}{2}\right)\cdot\frac{2}{3}\cdot\frac{2}{3}=\frac{10,333}{EJ}.$$

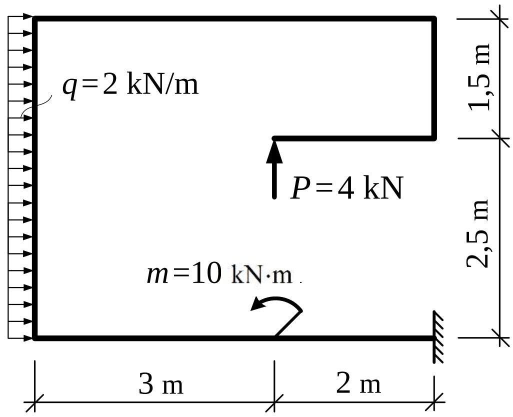
Note that on parts I, V and VII the calculation was carried out using Vereshchagin rule (3.10), for area IV – by trapezoid formula (3.11), and in parts III and VI according to the formula of Simpson (3.12). In the part II a unit diagram of bending moments (Fig. 3.50, *c*) is zero, so the result of calculating the Mohr’s integral – zero.

The value of the displacement is positive, therefore, the point really moves down.

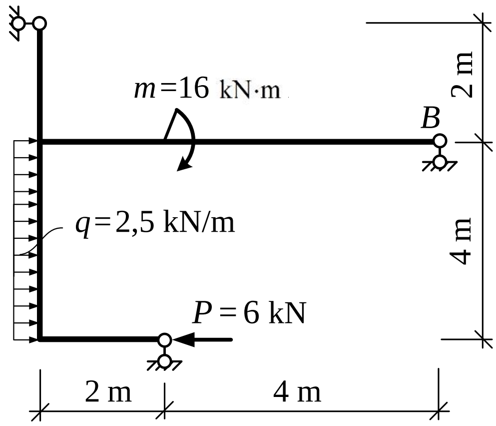
3.11. Tasks for Independent Solutions

To plot the diagrams *M*, *Q* and *N* in frames (tasks 3.1–3.10) and in beam (task 3.11).

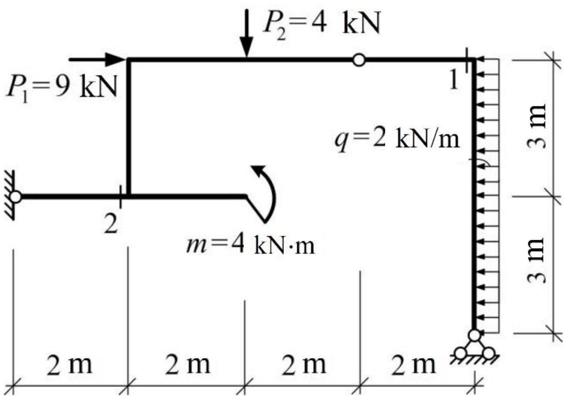
3.1



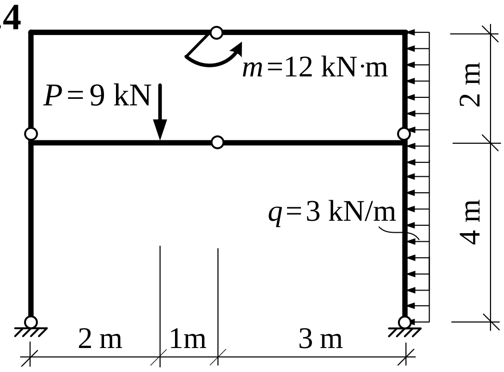
3.2



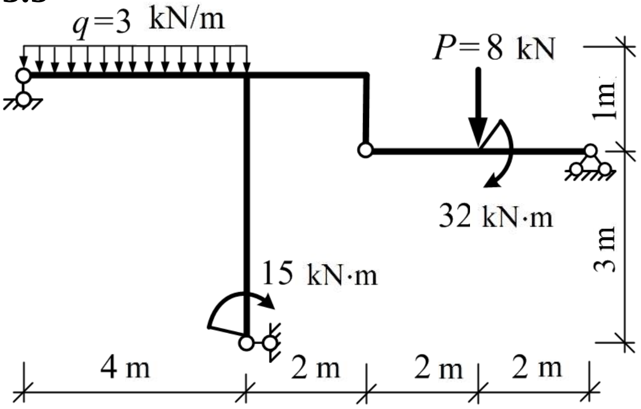
3.3



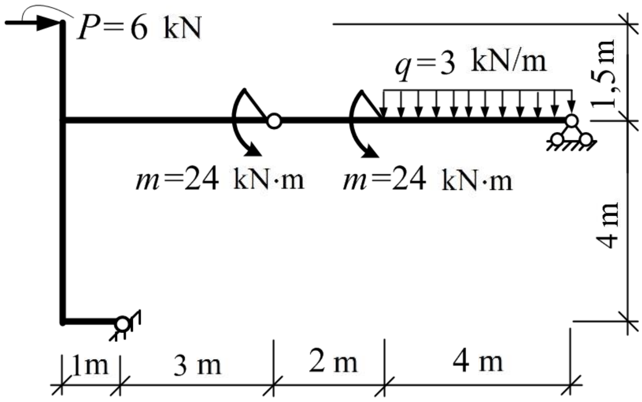
3.4



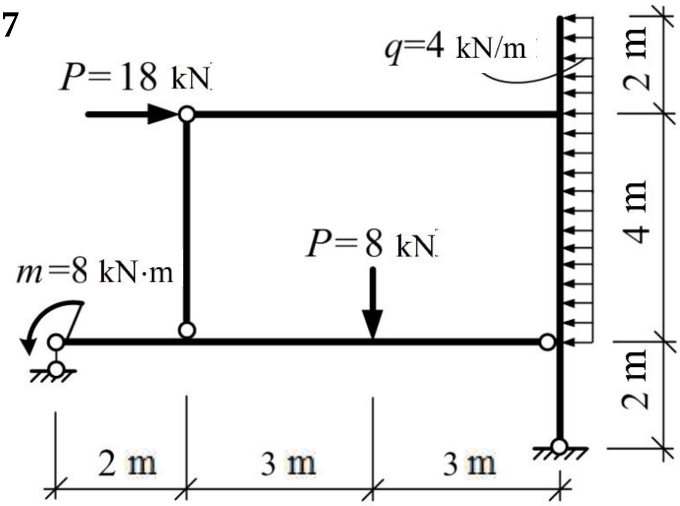
3.5



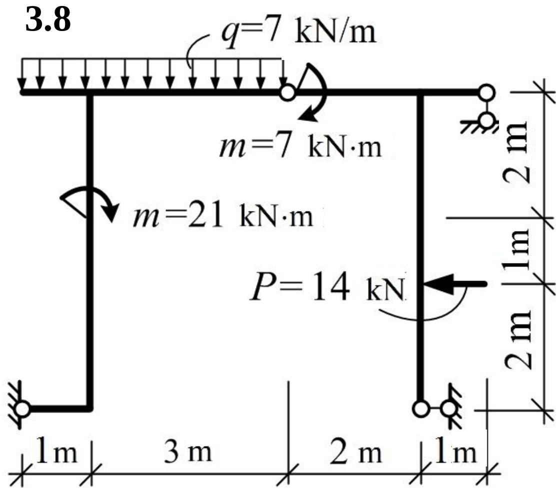
3.6



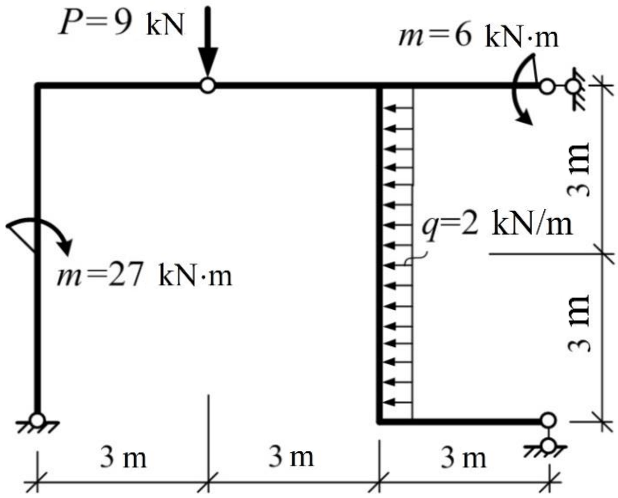
3.7



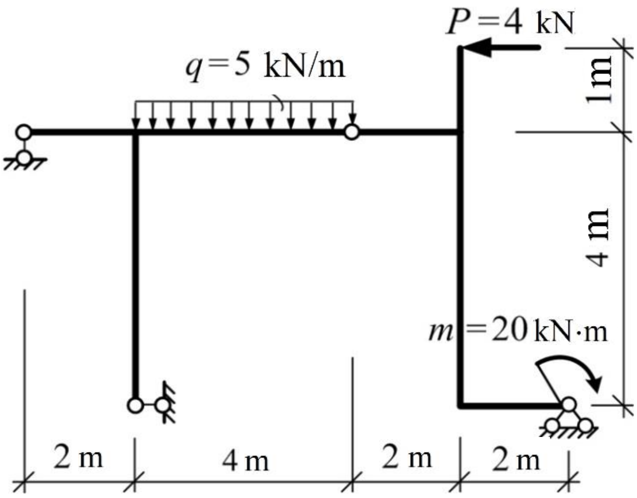
3.8



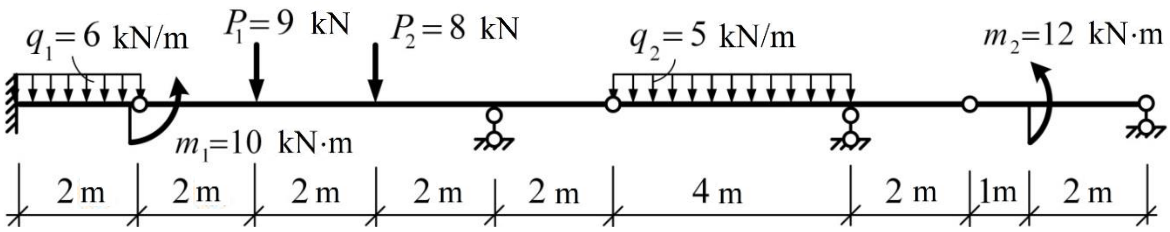
3.9



3.10



3.11



3.11.2 To determine the horizontal displacement of the point *B* in the frame presented in task 3.2 from the action of applied load there, taking the rigidity of all rods of the frame as constant (*EJ* = Const).

3.11.3 To determine the mutual angle of rotation of sections 1 and 2 of the frame presented in problem 3.3 from the action of applied load there, taking the rigidity of all rods of the frame as constant (*EJ* = Const).

The answers to these tasks according to the results of their solution are presented in the section "Answers to tasks for independent solutions" (p. 67–70).

The List of Recommended Literature
For Study of the Discipline "Structural Mechanics»

1. Борисевич, А. А. Строительная механика : учебное пособие / А. А. Борисевич, Е. М. Сидорович, В. И. Игнатюк. – 1-е изд. – Минск : БНТУ, 2007. – 821 с. – 2-е изд. – Минск : БНТУ, 2009. – 756 с.

2. Игнатюк, В. И. Строительная механика : пособие по дисциплине «Строительная механика» для студентов и слушателей ИПКиП строительных специальностей / В. И. Игнатюк, И. С. Сыроквашко, В. В. Молош / Брест. гос. техн. ун-тет. – Брест : Изд-во БрГТУ, 2018. – 227 с.

3. Дарков, А. В. Строительная механика: учебник для строительных специальностей вузов / А. В. Дарков, Н. Н. Шапошников. – 8-е изд. – Москва : Высшая школа, 1986. – 608 с.

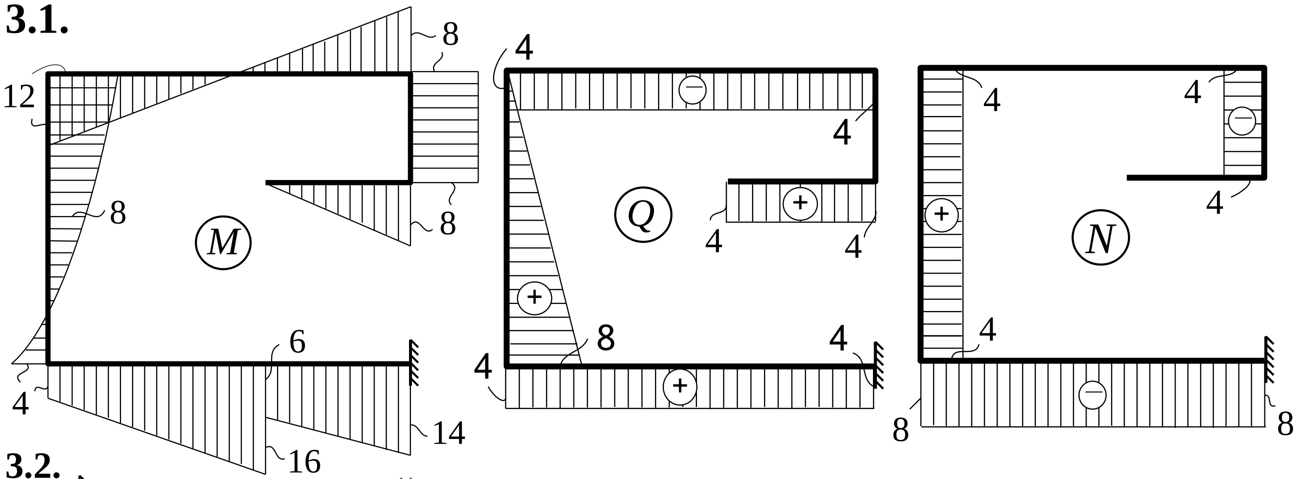
Answers to Tasks for Independent Solutions

Section 2. Kinematic analysis of structures

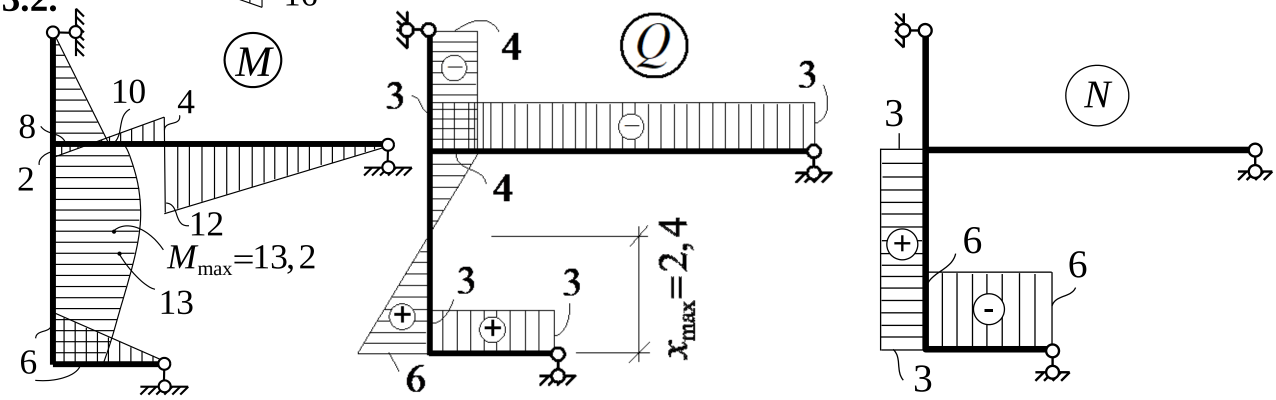
2.1. W = 0; the system is statically determinable and geometrically invariable.
2.2. W = 0; the system is instantaneously variable.
2.3. W = 0; the system is statically determinable and geometrically invariable.
2.4. W = 0; the system is instantaneously variable.
2.5. W = 0; the system is statically determinable and geometrically unchangeable.
2.6. W = 0; the system is invariable.
2.7. W = 0; the system is unstable (variable).
2.8. W = 0; the system is statically determinable and geometrically invariable.
2.9. W = –1; the system is statically indeterminat and geometrically invariable.
2.10. W = 0 the system is statically determinable and geometrically invariable.
2.11. W = 0; the system is instantaneously variable.
2.12. W = 0; the system is statically determinable and geometrically invariable.

Section 3. Calculation of statically determinable systems

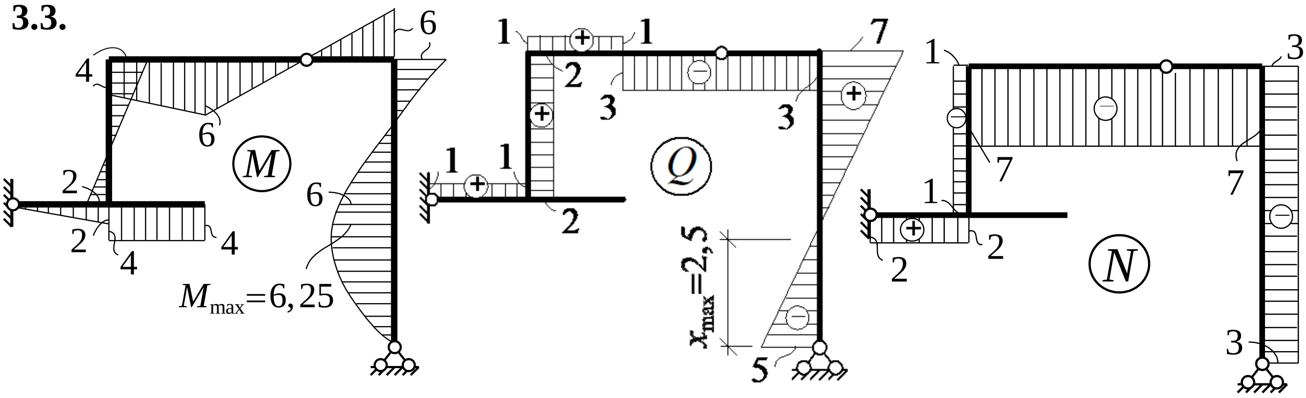
3.1.



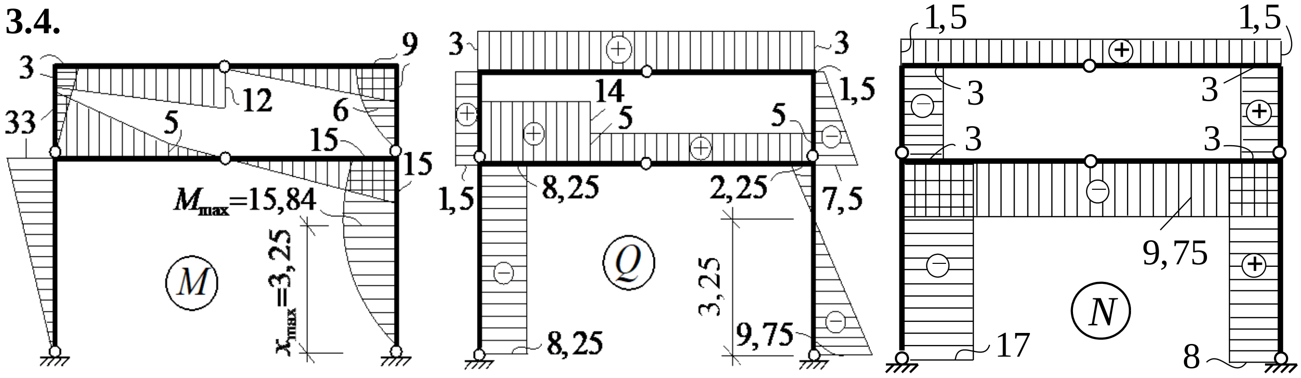
3.2.



3.3.

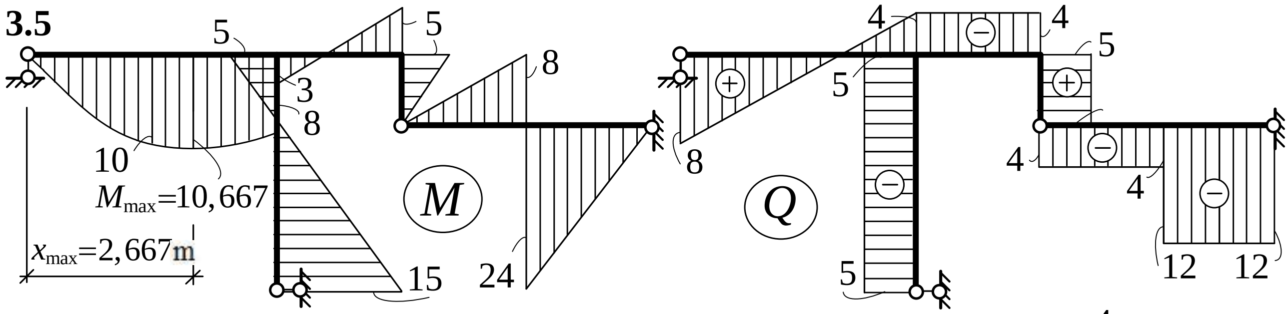


3.4.



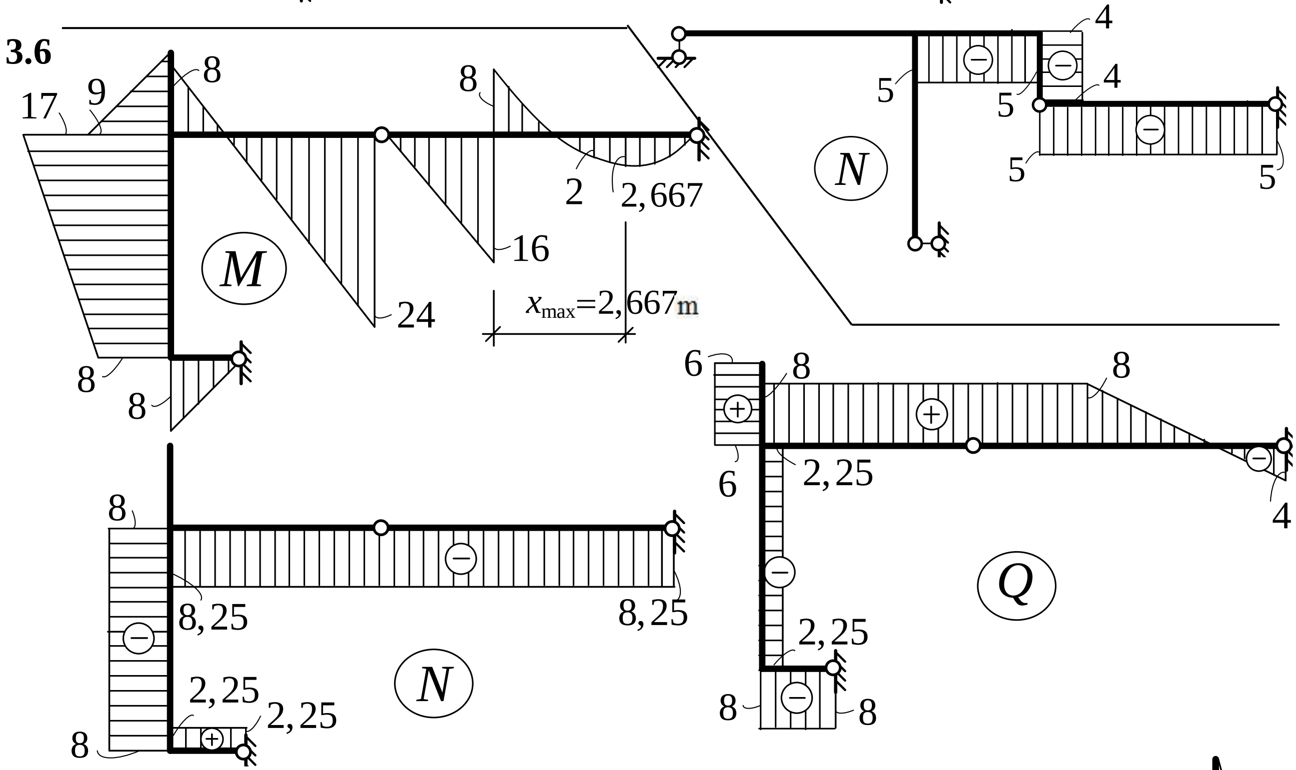
M
Q
N
3
9
12
6
33
5
15
M_{max}=15,84
x_{max}=3,25
14
1,5
8,25
2,25
7,5
3,25
9,75
17
8

3.5



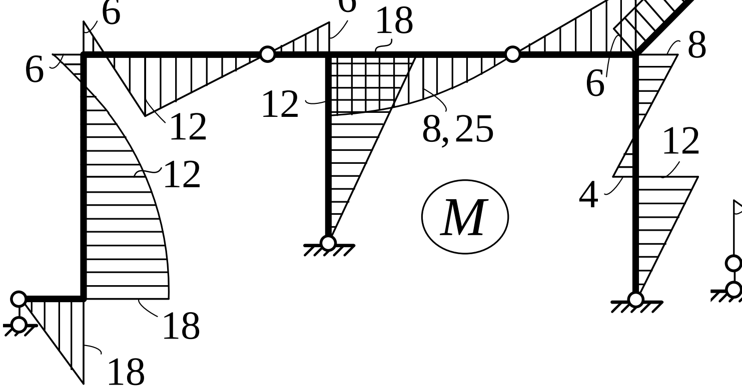
M
Q
5
3
8
10
M_{max}=10,667
x_{max}=2,667m
15
24
4
12

3.6



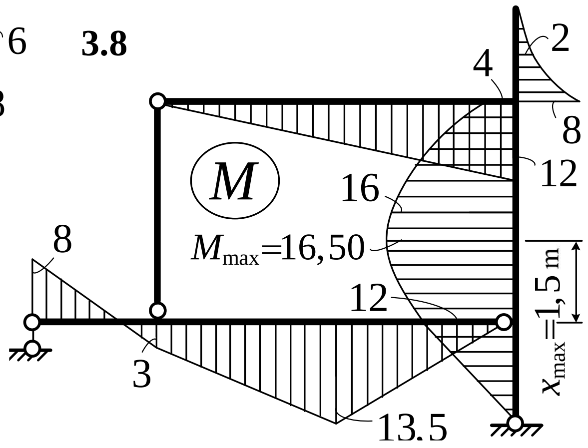
M
N
Q
17
9
8
2
2,667
16
24
x_{max}=2,667m
4
5
6
2,25
8,25

3.7



M
6
12
18
14
8,25
8
4

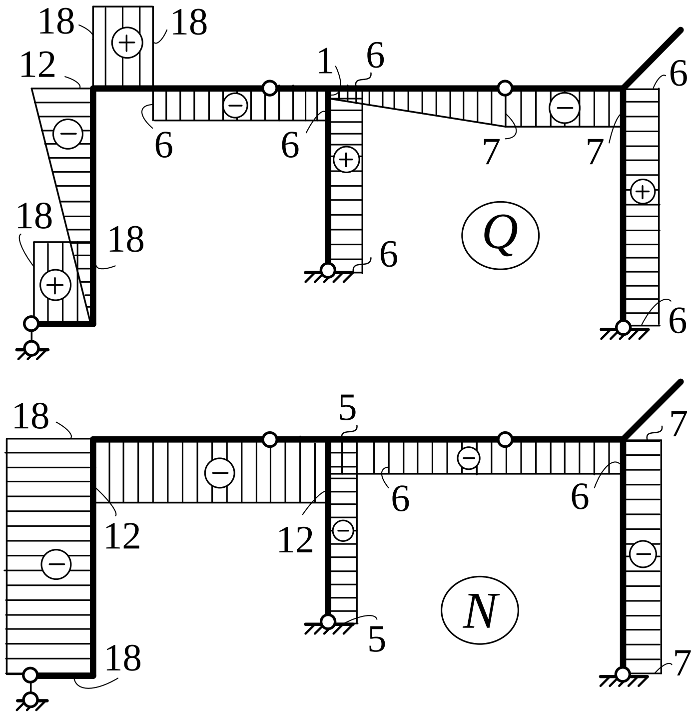
3.8

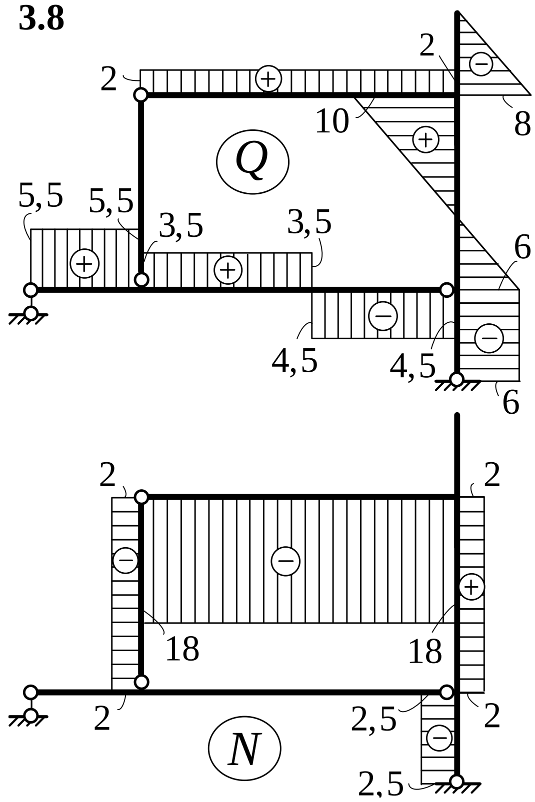


M
2
4
8
12
16
M_{max}=16,50
x_{max}=1,5 m
3
13.5

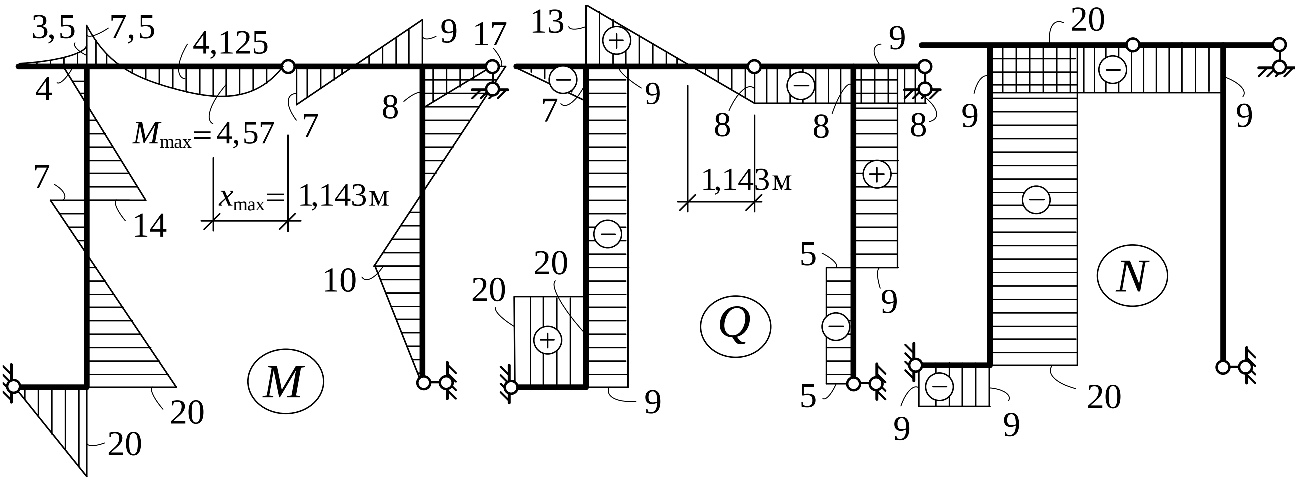
Continuation of the answers to the tasks

3.7 and **3.8**

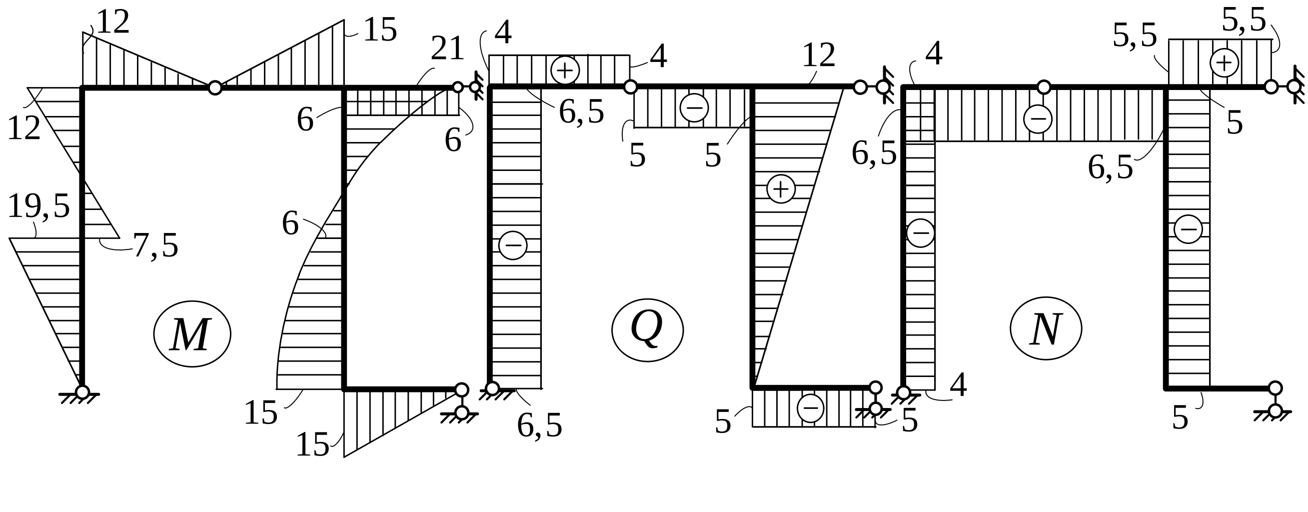




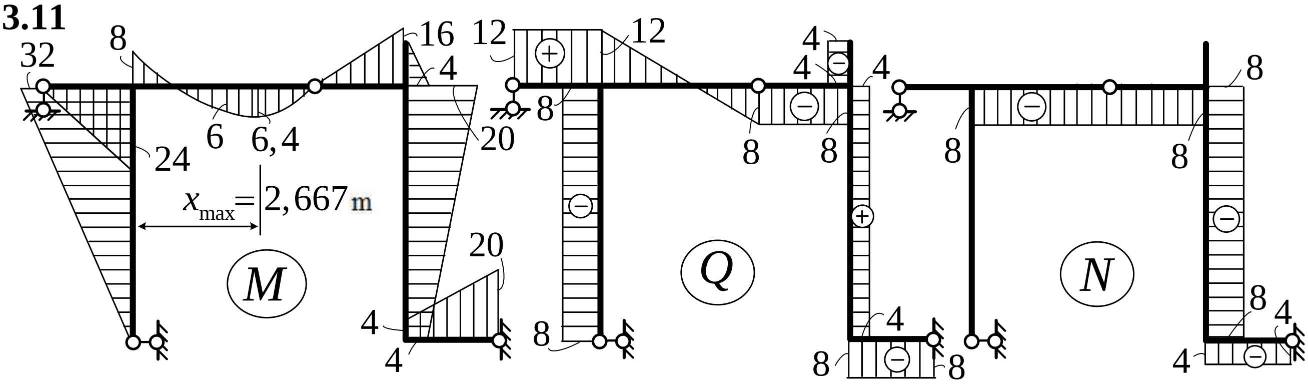
3.9



3.10



3.11



3.12.

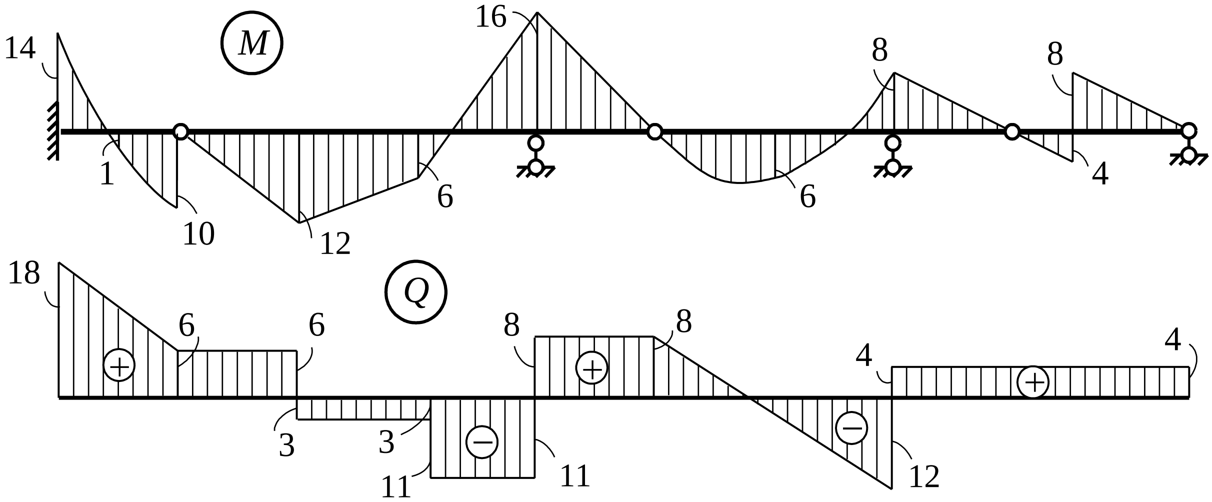
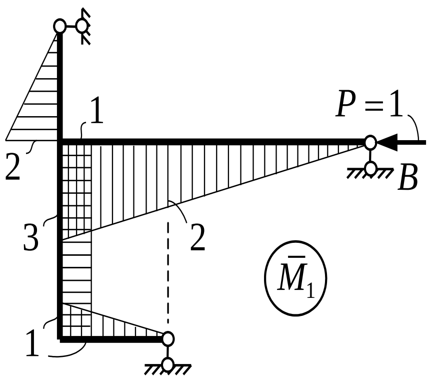
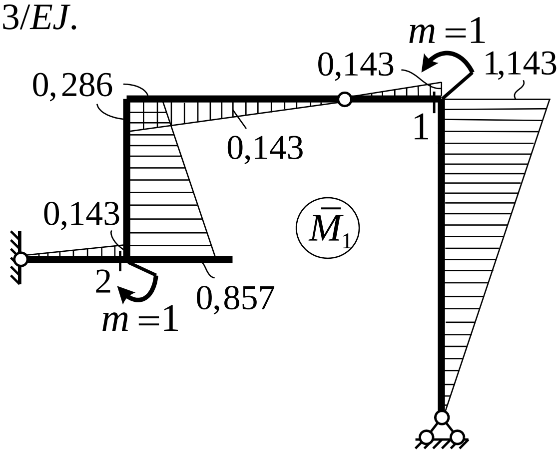


Diagram of longitudinal forces in a beam under the action of the given loads is zero.



3.13. Diagram of bending moments from external loads is presented in the answer to the task 3.2, the unit diagram of the bending moments is shown on the right. The horizontal displacement of point B equals 66,667/EJ (left).

3.14. Diagram of bending moments is given in the answer to problem 3.3, a unit diagram of the bending moments is shown below to the left. Mutual angle of rotation of the angle of cross-sections 1 and 2 is equal to 2,953/*EJ*.



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Редактор: Боровикова Е.А.
Компьютерный набор и верстка: Желткович А.Е.
Корректоры: Гайворонский Р.А.

ISBN 978-985-493-484-6

Издательство БрГТУ.
Свидетельство о государственной регистрации издателя, изготовителя, распространителя печатных изданий № 1/235 от 24.03.2014 г., № 3/1569 от 16.10.2017 г.
Подписано в печать 09.02.2020 г. Формат 60x84 $^{1}/_{16}$.
Бумага «Performer». Гарнитура «Times New Roman».
Усл. печ. л. 4,31. Уч. изд. л. 4,63. Заказ № 93. Тираж 80 экз.
Отпечатано на ризографе Учреждения образования «Брестский государственный технический университет».
224017, г. Брест, ул. Московская, 267.