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DEPARTMENT OF PHYSICS

Methodical instructions to perform laboratory work M4

“DETERMINATION OF BULLET’S SPEED WITH TORSIONAL BALLISTIC PENDULUM“

Brest 2014

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Methodical instructions drawn up in accordance with a model program of physics course for engineering majors. The guidance given necessary for the understanding of the physical processes theoretical information explaining the principle of operation of devices and equipment used, describes a technique experiments, are tasks for independent work. Given control questions and a list of recommended reading.

Methodical instructions to carry out the laboratory work designed for foreign students BrGTU all technology sectors and forms of education.

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LABORATORY WORK M4
"DETERMINATION OF BULLET'S SPEED WITH TORSIONAL BALLISTIC PENDULUM"

1. GOAL

Studying operation principles of a torsional ballistic pendulum and the law of conservation of an angular momentum.

1.1 TASKS

- experimental determination of the inertia moment of a ballistic torsional pendulum and the elastic torsional forces constant;
- experimental determination of a bullet speed by the method of a ballistic torsional pendulum.

2. INSTRUMENTS AND ACCESSORIES

The ballistic pendulum with counter periods, millisecond timer and shooting device.

3. INSTALLATION DESCRIPTION

The installation allows to measure the period of oscillations and the angular amplitude of the ballistic pendulum, as well as the angular amplitude of the pendulum with perfectly inelastic bullet's collision on target with plasticine. The main element of the installation is a torsion pendulum, shown in Figure 1. It is a horizontal rod 12 tightened to the vertical wire 13 stretched between the upper 4 and the lower bracket 5 of the installation. Two cylinders 11 with mass $m_{CYLINDER}$ = 200 grams each can move along the rod. The screws in these cylinders are used for fastening them in a definite position. At the ends of the rod plates 10, with one side coated by plasticine, are fixed. At the end of the plate there is a trait which is determined by the angle of deviation from the equilibrium position of the pendulum. Dividing lines which are on the plates show the distance from the axis of suspension of the pendulum. Cross strokes are applied to the rod at the distance of 1 cm from each other. The first stroke is located at the distance of 30 mm from the axis of the pendulum.

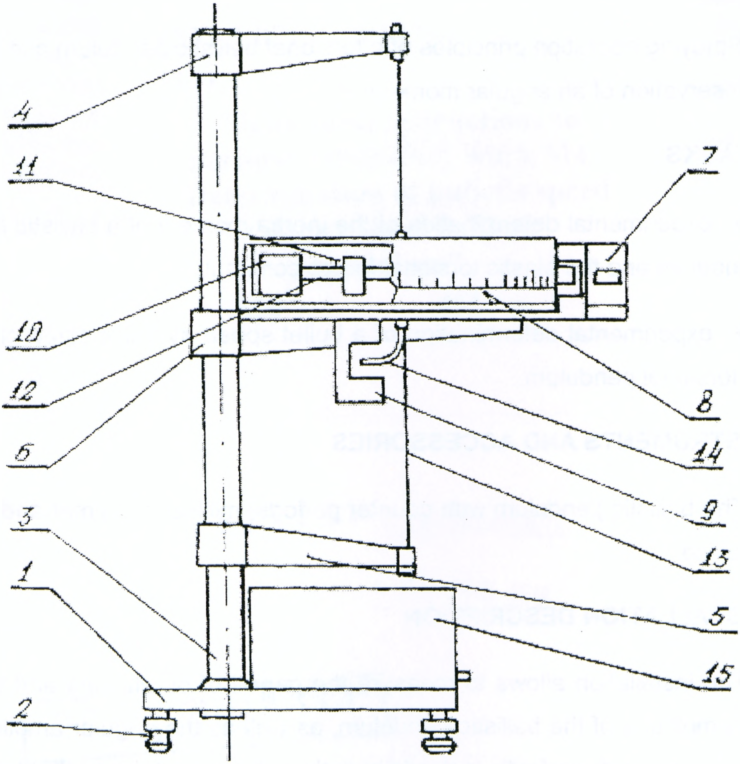

Figure 1. The torsional ballistic pendulum

To the middle bracket 6 is attached shooting device 7, as well as a transparent screen coated with it, an angular scale 8 and photoelectric sensor 9 which show number of oscillations of the pendulum.

The number of oscillations is displayed on the left display. The indicator which is based on the right shows the total time of all oscillations. The oscillations are counted when the carrier 14 passes in front of a photocell in a certain direction. On the basis of the pendulum, there are three buttons: "Power" ("СЕТЬ"), "Stop" ("СТОП") and "Reset" ("СБРОС"), which are used to turn on the instrument, the termination of the account and are re-incorporated in the work.

4. ORDER OF PERFORMANCE

4.1 Determination of the inertia moment of the ballistic torsional pendulum and the torsion constant of the wire

1) Set movable cylinders at the same distance R_1 from the rotation axis. The equilibrium position of the ballistic pendulum is exposed so that it corresponds to zero of the angular scale. Click the "Power", then press the "RESET". Reject the ballistic pendulum from its equilibrium position by an angle of about 20° and release it. At a time when the count of the number of oscillations shows the number "9", press the button "STOP". The clock will stop, the display will show the time t_{11}. The counter will show that the number of oscillations is equal to "10".

Find the period of oscillation T_{11} by the formula

$$T_{11} = \frac{t_{11}}{10}.$$

Repeat the experiment two more times and take the times t_{12} and t_{13}. Determine the oscillation periods T_{12} and T_{13}. The average period T_1 is determined by the formula:

$$T_1 = \frac{T_{11} + T_{12} + T_{13}}{3}.$$

2) Install the movable cylinders at the same distance R_2 for both cylinders from the axis of rotation and repeat the previous experience. Find the average period T_2.

3) Calculate the inertia moment of a ballistic pendulum

$$I_0 = \frac{2m_{CYLINDER}(T_2^2 R_1^2 - T_1^2 R_2^2)}{T_1^2 - T_2^2}$$

and the elastic torsional forces constant

$$C = \frac{8\pi m_{CYLINDER}(R_1^{\,2} - R_2^{\,2})}{T_1^2 - T_2^2}.$$

4.2 Determination of a bullet speed

1) Charge the shooting device with specially prepared "bullet". The moving cylinders are set on the same distances R. Having released the compressed spring, make a shot, and measure the angle φ_{max1} of maximum deviation ballistic pendulum and the distance l_1 from the axis of rotation to the center of the dent from incident bullet. Calculate the bullet velocity from the formula

$$V_1 = \sqrt{C\left(I_0 + 2m_{CYLINDER}R^2 + m_{BULLET}l_1^2\right)} \cdot \frac{\varphi_{max1}}{m_{BULLET}l_1}.$$

2) Repeat the experiment two more times and calculate an average ve- ve-
locity of the bullet

$$\langle V \rangle = \frac{V_1 + V_2 + V_3}{3}.$$

5. THEORY

The ballistic pendulum method for measuring the bullet's speed is based on the fact that during the interaction of the bullet with the pendulum its angular velocity varies considerably, but the angular displacement of the pendulum at the same time can be neglected. This condition will occur if the mass of the pendulum is much greater than that of the bullet. In addition, we assume that the interaction of the bullet with the ballistic pendulum is absolutely inelastic.

Torsional oscillations are believed to be undamped that is we neglect the forces of viscous friction.

The basic equation of the dynamics of rotational motion describing the rotational motion of a ballistic pendulum can be written as:

$$I\varepsilon = M_{ELASTIC},$$

where $I = I_0 + 2m_{CYLINDER}R^2$ – an inertia moment of a ballistic pendulum, $\varepsilon = \dot{\omega} = \ddot{\varphi}$ – an angular acceleration, ω – an angular velocity, φ – the angle of deviation from the equilibrium position of the pendulum. The moment of the elastic forces of a torsional deformation is proportional to the angle of deflection of the ballistic pendulum from its equilibrium position:

$$M_{ELASTIC} = -C\varphi ,$$

where C – the elastic torsional forces constant. Since the angular acceleration is the second time derivative of the rotation angle, the basic equation of rotational motion can be rewritten as:

$$I\ddot{\varphi} = -C\varphi .$$

It is easy to verify that the solution of this equation is a function of the form:

$$\varphi(t) = \varphi_{max}\cos(\omega_0 t + \alpha),$$

which describes the harmonic oscillations, where φ_{max} – an angular amplitude, α – an initial phase, ω_0 – the circular frequency of oscillation. It is associated with the period of oscillation by the following relation: $\omega_0 = \frac{2\pi}{T}$. In the case of a ballistic pendulum the circular frequency is determined from the expression $\omega_0^2 = \frac{C}{I}$. Then the period of oscillation of a torsion pendulum is equal to

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{C}} .$$

Squaring both sides of the equation and dividing by $4\pi^2$, we obtain that

$$\frac{T^2}{4\pi^2} = \frac{I}{C}.$$

If we take into account that $I = I_0 + 2m_{CYLINDER}R^2$ and write the last expression for two values of period T_1 and T_2 then we obtained

$$\begin{cases} \dfrac{T_1^2}{4\pi^2} = \dfrac{I_0 + 2m_{CYLINDER}R_1^2}{C}, \\ \dfrac{T_2^2}{4\pi^2} = \dfrac{I_0 + 2m_{CYLINDER}R_2^2}{C}. \end{cases}$$

Solving the resulting system, we get hold of the inertia moment I_0 of the ballistic pendulum

$$I_0 = \frac{2m_{CYLINDER}(T_2^2R_1^2 - T_1^2R_2^2)}{T_1^2 - T_2^2},$$

and the elastic torsional forces constant

$$C = \frac{8\pi m_{CYLINDER}(R_1^2 - R_2^2)}{T_1^2 - T_2^2}.$$

To determine the speed of a bullet, you can use the laws of conservation of an angular momentum and mechanical energy. The angular momentum of the bullet before and after the shock:

$$m_{BULLET}Vl = (I_0 + 2m_{CYLINDER}R^2 + m_{BULLET}l^2)\omega.$$

The mechanical energy of the bullet before and after the impact can be written as:

$$\frac{(I_0 + 2m_{CYLINDER}R^2 + m_{BULLET}l^2)\omega^2}{2} = \frac{C\varphi_{max}^2}{2}.$$

If we express the value of

$$I_0 + 2m_{\text{CYLINDER}} R^2 + m_{\text{BULLET}} l^2 = \frac{m_{\text{BULLET}} Vl}{\omega}$$

from the law of conservation of angular momentum and then substitute this expression into the law of conservation of mechanical energy, we can obtain the bullet velocity before the collision with the ballistic pendulum

$$V = \sqrt{C\left(I_0 + 2m_{CYLINDER} R^2 + m_{BULLET} l^2\right)} \cdot \frac{\varphi_{\max}}{m_{BULLET} l}.$$

6. CONTROL QUESTIONS:

1) What does the ballistic pendulum method consist in?
2) Formulate the conservation laws that were used in this study.
3) How to determine the inertia moment of the torsion ballistic pendulum?
4) How to determine the elastic torsional forces constant?
5) How to determine a bullet speed by the method of a ballistic pendulum?

7. LITERATURE

1. Чопчиц Н.И., Гладыщук А.А., Янусик И.С. Лабораторный физический практикум "Механика" / Методическое пособие для студентов технических специальностей. – Брест: изд-во БрГТУ, 2011. – 80 с.

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