

DERIVATIVE π -LENGTH OF π -SOLVABLE GROUPS, THE ORDER OF COFACTORS IS SQUARE-FREE

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Only finite groups are considered. All designations and definitions used correspond to [1, 2].

A number of subgroups

$$1 = G_0 \subseteq G_1 \subseteq G_2 \subseteq \dots \subseteq G_m = G \quad (1)$$

is called subnormal if for any i the subgroup G_i is normal in G_{i+1} . A group G is called π -solvable if it has a subnormal series (1), each factor of which is either a π -group or a π' -group.

Recall that the cofactor of a subgroup H of a group G is the factor group $H/\text{Core}_G H$, where $\text{Core}_G H$ is the core of the subgroup H в группе G , т.е. наибольшая нормальная подгруппа в G , содержащаяся в H . В in the group G , i.e. the largest normal subgroup of G contained in H . In what follows, the cofactor of a subgroup H in a group G will be denoted by $\text{Core}_G H$.

In [3], the structure of a group with cyclic cofactors of primary subgroups was studied. In particular, it has been proven that the p -length of such groups does not exceed 1 for all primes p . The structure of solvable groups with bicyclic cofactors of primary subgroups is given in [4].

A number n is said to be free of m th powers if p^m does not divide n for all primes p . When $m = 2$ they say that n is free from squares, when $m = 3$ – from cubes.

The following theorem has been proven.

Theorem. If the cofactor of an arbitrary π -subgroup X of a π -soluble group G is square-free, then $l_\pi^a(G/\Phi(G)) \leq 4$.

Bibliography

1. Huppert, B. Endliche Gruppen I / B. Huppert // Berlin-Heidelberg-New York: Springer, 1967. – 793 c.
2. Monakhov, V.S. Introduction to the theory of finite groups and their classes / V.S. Monakhov // Minsk: Higher School. – 2006. – 207 p.
3. Yufeng Liu, Yi Xiaolan Finite groups in which primary subgroups have cyclic cofactors // Bull. Malays. Math. Sci. Soc. 2011. Vol. 34. №. 2. P. 337–344.
4. Trofimuk, A.A. Finite solvable groups with bicyclic cofactors of primary subgroups / A.A. Trofimuk, D.D. Daudov // NAS of Belarus. Proceedings of the Institute of Mathematics. – 2016. – Т. 24. – No. 1. – pp. 95–99.