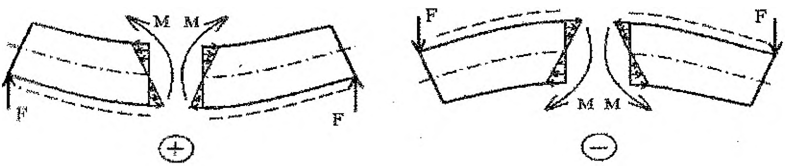
MINISTRY OF EDUCATION OF THE REPUBLIC OF BELARUS
ESTABLISHMENT OF EDUCATION
BREST STATE TECHNICAL UNIVERSITY
DEPARTMENT OF APPLIED MECHANICS

# TASKS AND METHODICAL INSTRUCTIONS

for performing calculated graphic works on a course

**«Resistance of materials»**

for students of a specialty 1 – 70 02 01 «Industrial and civil engineering»




Brest 2017

УДК 539.3/.6(07)

Methodical instructions contain individual tasks, initial data for performing a calculated graphic work (CGW) and examples of solving problems.

The main purpose of the methodical instructions is to provide assistance to students of construction specialties when studying the main sections of resistance of materials and to activate individual work.

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# CONTENTS



## GENERALREGULATIONS

When designing buildings and constructions of different purposes specialists have to possess fundamental knowledge of basic technical disciplines. Resistance of materials also belongs to such disciplines. The ability to create calculated schemes (models) of construction elements, to define reactions of construction supporting devices and also to evaluate their strength and rigidity characteristics is gained by students after studying the main sections of resistance of materials.

The standard plan for training students provides a small amount of school hours that makes possible to examine only elementary sections of resistance of materials. Each student performs the calculated graphic works (CGW) on the main sections of the discipline.

Methodical instructions allow to study, taking into account the reference list, the main sections of a course and to apply theoretical material when performing a CGW. It is necessary to answer questions on the CGW's subject and to be able to solve test problems on its subject when defending the work. An exam in a course is held after the CGW are defended.

## 1.REQUIREMENTS FOR WRITING CALCULATED GRAPHIC WORKS

1. CGW are carried out on separate sheets of A4 format.

2. The order of writing the CGW is: a title page; a task with the indication of initial data and the schemes of structures; the text of calculations with necessary explanations and calculated schemes; conclusions; references list.

3. Drawings and schemes are carried out following the rules of graphics and scales according to the standard of «BrSTU».

4. A text part is carried out in accordance with the requirements for text documents presentation. Pages are numbered. The calculations are carried out in a general way, size values are substituted. Numerical results with the indication of obtained values dimensions are written down. All the calculations are made with an accuracy of one-hundredths of a unit.

5. Diagrams should be built on the same sheet of paper with the rated scheme, numerical values of ordinates and units of the calculated values should be indicated on the diagrams.

## 2. BRIEF THEORETICAL INFORMATION

### 2.1. Short theoretical data

#### Internal forces at axial stretching-compression. Stress. Strength calculation

At stretching (compression) a direct bar (rod) in its cross-sections there is only one internal force factor - the longitudinal force which is defined by the method of sections. This force is equal to the algebraic sum of projections to a longitudinal axle of all external loadings applied to one of the cut parts of the bar:

$$\sum Z = 0;\ F - N = 0;\ N = F.$$

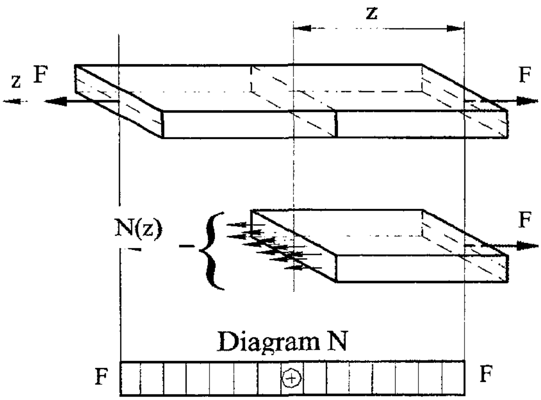



***Figure 2.1*** – Determination of longitudinal force of N

In case of action of several loads, internal force is calculated: $N = \Sigma F_i$. Stretching (i.e. acting from a section) force is considered positive, compressing – negative.

The law of longitudinal force change along the bar length is convenient to be presented graphically in a form of longitudinal forces $N$ diagram. When distributed axial loads with the intensity $q$ acton a bar it is possible to use differential dependence $q = \frac{dN}{dz}$ for checking the correct construction of $N$ diagram. The diagram allows to find out the greatest value of longitudinal force $N$ and the location of section in which it arises in cases when longitudinal forces in different lateral sections of a bar are not identical.

At stretching (compression) a bar in its cross-sections there are only normal stresses. To define them (when the value of longitudinal load is known) it is necessary to know the distribution law of normal stresses in a bar cross-section. The problem is solved using a hypothesis of plain sections (Ja. Bernoulli's hypothesis): bar sections, plain and normal to an axis before deformation, remain plain and normal to an axis during the deformation too. This hypothesis suggests that all fibers in the longitudinal direction are deformed equally. Therefore we consider that at stretching (compression) a bar normal stresses are distributed on its cross-sections evenly. Considering that $\sigma$ on all cross-sectional area $A$ are constant, we obtain

$$N = \int_A \sigma dA = \sigma \int_A dA = \sigma \cdot A, \sigma = \frac{N}{A}. \tag{2.1}$$

At stretching stress is considered positive, under compression - negative.

When normal stresses in different cross-sections of a bar are not identical, it is reasonable to show the law of their change along the bar length graphically in a form of a diagram of normal stresses.

Strength condition must be respected for all points of the calculated (rated) element:

$$\sigma \le [\sigma], \tag{2.2}$$

where: $\sigma$ is calculated stress which arises in a constructional element under the influence of applied loads; $[\sigma]$ is allowable stress that ensures safe, reliable and long-lasting work of a construction.

Strength condition at stretching (compression) looks like:

$$\sigma = \frac{N}{A} \le [\sigma], \tag{2.3}$$

where: $A$ is cross-sectional area; $N$ – longitudinal force in the specified section.

**Deformations and displacements. Stiffness calculation**

The ability to calculate deformations and displacements is necessary for stifness calculations and also for forces (reactions) determination in statically indeterminate systems.

Let's consider longitudinal deformation of a bar.

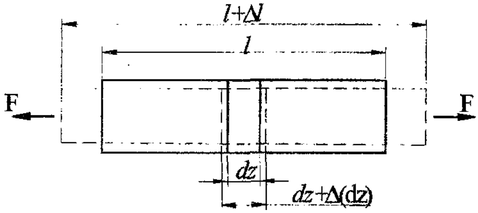



***Figure 2.2*** – Longitudinal deformation of a bar

We will allocate from a bar (figure 2.2) an infinitesimal element with $dz$ length. We will designate an element length increment as a result of deformation $\Delta(dz)$. The element length increment ratio to its initial length is called relative elongation or longitudinal deformation:

$$\varepsilon = \frac{\Delta(dz)}{dz}. \tag{2.4}$$

Experiments proved that there exists directly proportional dependence between longitudinal deformation and the normal stress acting in its direction for the majority of materials within elastic work. This situation carries the name of Hooke's law and is written down like this: $\sigma = E\varepsilon$, where $E$ is the module of longitudinal elasticity (or Jung's module) - the physical constant of material characterizing its rigidity (it is measured in *Pa* or *MPa*).

For stretching (compression) of an element of infinitesimal length Hooke's law looks like:

$$\Delta(dz) = \frac{Ndz}{EA},$$

where $EA$ is the magnitude called rigidity of a bar at stretching (compression).

Change of length of a bar:

$$\Delta l = \int_l \frac{Ndz}{EA}. \tag{2.5}$$

If the bar rigidity and longitudinal force are constant along the bar length, from (2.5) we obtain:

$$\Delta l = \frac{Nl}{EA}. \tag{2.6}$$

Generally, if laws of $N$, $E$ or $A$ change are different for certain sites of a bar, integration of expression (2.5) is made within every site and the results are algebraically summarized:

$$\Delta l = \sum_{i=1}^{n} \int_{l_i} \frac{Ndz}{EA}. \tag{2.7}$$

Displacement of any bar section is equal to length change of the site concluded between this section and rigidly fixed support. Mutual displacement of two sections is equal to length change of the bar part concluded between these sections.

The function $\delta = f(z)$ that shows displacement $\delta$ of cross sections as their distance $z$ from the motionless bar end (or the section which is conditionally taken for motionless) is graphically represented by a displacement diagram which is checked by differential dependence $\delta = \frac{d\sigma}{dz}$.

Bar rigidity calculation must implement a rigidity condition:

$$\delta \le [\delta], \tag{2.8}$$

where $\delta = \sum_{i=1}^{n} \Delta l_i$ is length change of a bar (absolute deformation), $[\delta]$ is allowable value of displacement(it is usually set as some part of full bar length).

**Internal force factors determination under a direct cross (transvers) bending. Strength calculations**

The sections method allows to find shearing forces and the bending moments in any beam section under any load action. In strength calculation sit is required to know the location of dangerous sections, i.e. sections where the internal forces or their adverse combinations, maximum in values, work. Therefore it is convenient to present graphically the distribution law of force factors along the bar length using the diagrams.

Shearing force of Q and bending moment of M are calculated as the algebraic sum of external forces projections or the moments of the external forces acting on one of the bar parts (left or right).

Rule of signs:

a) Shearing force of Q is positive if it is directed clockwise concerning section and is negative if it acts counterclockwise (figure 2.3).

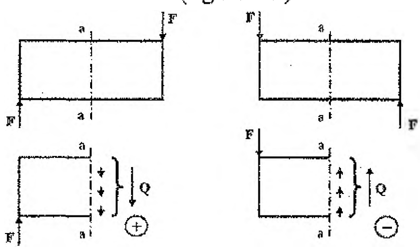



***Figure 2.3*** – Rule of signs for determination of shearing force

b) Bending moment of M is considered positive if the bar element is bent by convex down, i.e. the stretched fibers are below. Negative bending moment bends an element convex (figure 2.4) up.

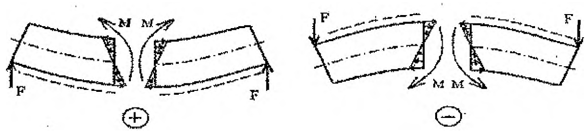



*Figure 2.4* – Rule of signs for determination of bending moment

Positive values of shearing forces lay above (postpone) from the basic line. The diagram of bending moments is drawn on the stretched fibers (positive values lay below the basic line).

In most cases beams strength calculation is carried out on the greatest normal stresses that arise in dangerous cross section. Strength condition for beams which material equally resists stretching and compression $[\sigma_p] = [\sigma_c] = [\sigma]$, looks like:

$$\sigma_{max} = \frac{|M_{max}|}{W_x} \le [\sigma], \tag{2.9}$$

where: $M_{max}$ is the bending moment, maximum on an absolute value, in dangerous section; $W_x$– axial section module with respect to (w.r.t) neutral axis of a beam; $[\sigma]$– the allowable normal stress.

The necessary value of axial section module is determined to select a beam cross section from a strength condition (2.9).

$$W_x = \frac{|M_{max}|}{[\sigma]}. \tag{2.10}$$

According to the calculated $W_x$, a form of cross section is chosen (a rectangle, a square, a channel, a I-section) and its sizes are found.

For the beams which are strongly loaded close to the supporting structures and thin-walled sections where shearing stresses have big value, the calculation should be done not only on the greatest normal, but also on the largest shearing stresses. Strength condition on shearing stresses looks like (D.I. Zhuravsky's formula):

$$\tau_{max} = \frac{Q_{max} \cdot S_x}{I_x \cdot b} \le [\tau], \tag{2.11}$$

where: $|Q_{max}|$ is the maximum shearing force (is accepted from a diagram of shearing forces); $S_x$ is static moment with respect to neutral axis of the cut part of cross section located on one side from the level at which shearing stresses are defined; $I_x$ is the inertia moment of the entire cross section with respect to axis; $b$ is beam section width at the level where shearing stresses are defined $\tau$; $[\tau]$– the allowable shearing stress. It is usually accepted $[\tau] = (0{,}5 \div 0{,}6)[\sigma]$ for steel beams.

## 3. EXAMPLES OF THE TASKS SOLUTION

**Example 1**

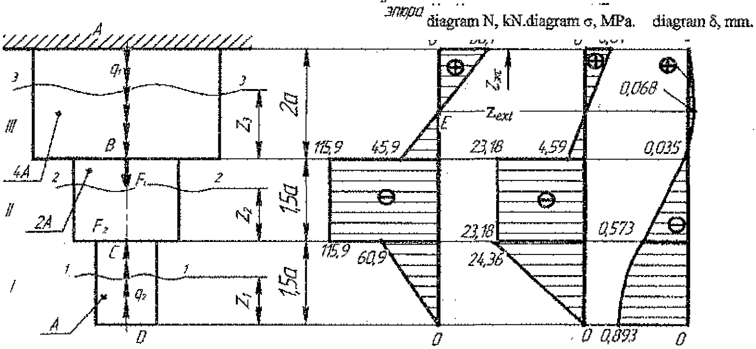
For the step bar loaded by longitudinal axial loads as shown in the figure 3.1, a) it is required:

1) to construct a diagram of longitudinal forces N;
2) to construct a diagram of normal stresses σ;
3) to construct a diagram of displacement δ;
4) to do a check of bar strength and rigidity.

It is given: $a$ = 1,4 m; $F_1$ = 70 кN; $F_2$ = 55 кN; $q_1$ = 40 кN/m; $q_2$ = 29 кN/m;
$[\sigma_s]$ = 130 MPa; $[\sigma_c]$ = 160 MPa; $E = 0{,}8\cdot10^5$MPa; $A$ = 2500 mm$^2$; $k = \dfrac{1}{1000}$.

Solution:

We draw a bar in scale with the necessary loads and sizes indication.




a) b) c) d)

***Figure 3.1*** – The scheme of a step bar (a) and N (b), σ (c), δ (d) diagrams

1. We divide a bar into 3 sectors, beginning from the free end.

We designate sections in the chosen sectors.

We compose(constitute) expressions for longitudinal forces and stresses in the respective sections.

*Sector 1*; $0 \le z_1 \le 1{,}5a$.

$$N_1 = -q_2 \cdot z_1, \quad \sigma_1 = \frac{N_1}{A_1},$$

where $A_1 = A$ – area of section 1

With $z_1 = 0, \quad N_1 = -q_2 \cdot z_1 = -29 \cdot 0 = 0\ kN;$

$$\sigma_1 = \frac{N_1}{A} = 0 \text{ MPa}.$$

With $z_1 = 1{,}5a, \quad N'_1 = -q_2 \cdot z_1 = -29 \cdot (1{,}5 \cdot 1{,}4) = -60{,}9\,kN;$

$$\sigma'_1 = \frac{N'_1}{A} = \frac{-60{,}9 \cdot 10^3}{2500} = -24{,}36 \text{ MPa}.$$

*Sector 2*; $0 \le z_2 \le 1{,}5a$.

$$N_2 = -q_2 \cdot 1{,}5a - F_2 = -29 \cdot (1{,}5 \cdot 1{,}4) - 55 = -115{,}9 \text{ kN};$$

$\sigma_2 = \dfrac{N_2}{A_2}$, where $A_2 = 2A$ – area of section 2.

$$\sigma_2=\frac{N_2}{2A}=\frac{-115{,}9\cdot 10^3}{2\cdot 2500}=-23{,}18\text{ MPa.}$$

*Sector 3*; $0\le z_3\le 2a$.

$$N_3=-q_2\cdot 1{,}5a-F_2+F_1+q_1\cdot z_3,\quad \sigma_3=\frac{N_3}{A_3};$$

where $A_3=4A$ – area of section 3.

With $z_3=0$,

$$N_3=-q_2\cdot 1{,}5a-F_2+F_1+q_1\cdot z_3=-29\cdot(1{,}5\cdot 1{,}4)-55+70+40\cdot 0=-45{,}9\text{kN};$$

$$\sigma_3=\frac{N_3}{4A}=\frac{-45{,}9\cdot 10^3}{4\cdot 2500}=-4{,}59\text{ MPa};$$

With $z_3=2a$,

$$N'_3=-q_2\cdot 1{,}5a-F_2+F_1+q_1\cdot z_3=-29\cdot(1{,}5\cdot 1{,}4)-55+70+40\cdot(2\cdot 1{,}4)=66{,}1\text{kN};$$

$$\sigma'_3=\frac{N'_3}{4A}=\frac{66{,}1\cdot 10^3}{4\cdot 2500}=6{,}61\text{ MPa.}$$

According to the calculations of longitudinal forces and normal stresses the diagrams are constructed (figure 3.1, b and 3.1, c).

2. We define absolute change of rod length.

*Sector1*:

$$\Delta l_1=\int_0^{l_1}\frac{N_1}{E\cdot A_1}dz_1=\int_0^{1.5a}\frac{-q_2\cdot z_1}{E\cdot A}dz_1=\frac{1}{2}\frac{-q_2\cdot z_1^2}{E\cdot A}\Bigg|_{z_1=0}^{z_1=1.5a}=$$

$$=\frac{1}{2}\frac{-29\cdot 10^3\cdot(1{,}5\cdot 1{,}4)^2}{0{,}8\cdot 10^{11}\cdot 2500\cdot 10^{-6}}=-0{,}32\cdot 10^{-3}\text{ m (compression).}$$

*Sector2*:

$$\Delta l_2=\frac{N_2\cdot 1{,}5a}{E\cdot A_2}=\frac{N_2\cdot 1{,}5a}{E\cdot 2A}=\frac{-115{,}9\cdot 10^3\cdot(1{,}5\cdot 1{,}4)}{0{,}8\cdot 10^{11}\cdot(2\cdot 2500\cdot 10^{-6})}=$$

$$=-0{,}608\cdot 10^{-3}\text{ m (compression).}$$

*Sector 3*:

$$\Delta l_3=\int_0^{l_3}\frac{N_3}{E\cdot A_3}dz_3=\int_0^{2a}\frac{-q_2\cdot 1{,}5a-F_2+F_1+q_1\cdot z_3}{E\cdot 4A}dz_3=$$

$$=\frac{(-q_2\cdot 1{,}5a-F_2+F_1)\cdot z_3+\frac{q_1\cdot z_3^2}{2}}{E\cdot 4A}\Bigg|_{z_3=0}^{z_3=2a}=$$

$$=\frac{(-29\cdot 10^3\cdot(1{,}5\cdot 1{,}4)-55\cdot 10^3+70\cdot 10^3)\cdot(2\cdot 1.4)+\frac{40\cdot 10^3\cdot(2\cdot 1{,}4)^2}{2}}{0{,}8\cdot 10^{11}\cdot 2500\cdot 10^{-6}}=$$

$$=0{,}035\cdot 10^{-3}\text{ m (stretching).}$$

Absolute length change:

$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 = (-0,32) + (-0,608) + 0,035 = -0,893$ mm.

Extreme value of deformation in sector III:

$$\Delta l_{3extr} = \frac{\omega_N}{E \cdot A_3},$$

where $\omega_N$ area of a diagram of $N$.

We find the extremum location in sector III from rigidly fixed support:

$$z_{extr} = \frac{N'_3}{q_1} = \frac{66,1 \cdot 10^3}{40 \cdot 10^3} = 1,653 \text{ m},$$

$$\omega_N = \frac{1}{2} \cdot N'_3 \cdot z_{extr} = \frac{1}{2} \cdot 66,1 \cdot 10^3 \cdot 1,653 = 54,62 \cdot 10^3 \text{ N} \cdot m.$$

$$\Delta l_{3extr} = \frac{\omega_N}{E \cdot 4A} = \frac{54,62 \cdot 10^3}{0,8 \cdot 10^{11} \cdot \left(4 \cdot 2500 \cdot 10^{-6}\right)} = 0,068 \cdot 10^{-3} \text{ m (stretching).}$$

We define displacement.

Section A displacement:

$\delta_A = 0$ because the bar is rigidly fixed;

Section E displacement:

$\delta_E = \Delta l_{3\,extr} = 0,068$ mm;

$\delta_B = \Delta l_3 = 0,035$ mm;

$\delta_C = \Delta l_3 + \Delta l_2 = 0,035 + (-0,608) = -0,573$ mm;

$\delta_D = \Delta l_3 + \Delta l_2 + \Delta l_1 = 0,035 + (-0,608) + (-0,32) = -0,893$ mm.

According to the calculation the diagram of cross sections displacements is constructed (figure 3.1, d).

3. Checking the bar strength.

The diagram σ analysis shows that dangerous sections are section in p. C (in compressed bar area) and section in p. A (in the stretched bar area):

$\sigma^C = |\sigma'_1| = 24,36$ MPa $< [\sigma_C] = 160$ MPa,

$\sigma^A = \sigma'_3 = 6,61$ MPa $< [\sigma_P] = 130$ MPa.

Both conditions of strength are implemented.

4. Checking bar rigidity.

Rigidity condition: $\delta \le [\delta]$,

$$\frac{|\delta_D|}{1,5a + 1,5a + 2a} = \frac{|-0,893 \cdot 10^{-3}|}{1,5 \cdot 1,4 + 1,5 \cdot 1,4 + 2 \cdot 1,4} = 0,128 \cdot 10^{-3} < k = 1 \cdot 10^{-3}.$$

**Example 2**

Absolutely rigid bar that is suspended on two steel rods and has not movably hinged support is loaded by the concentrated force of $F = 610$ kN. The linear bar dimensions $a$, $b$, and height $h$ are respectively 1,2 m; 1,8 m; 0,6 m. Areas ratio $\frac{A_1}{A_2}$ of rods cross sections n=2, allowable stress is $[\sigma] = 160$ MPa, material yielding limit is $\sigma_T = 240$ MPa.

It is required to choose rods sections from two equal lag angles and also to determine value of coefficient of safety factor by the value of a rupture load.

Solution:

1. We construct the rated scheme of rods system (figure 3.2) in scale:

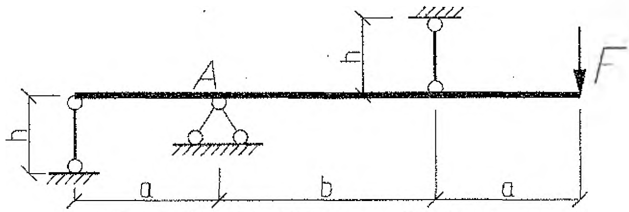



***Figure 3.2*** – Initial scheme of rod system

2. We establish the static indefinability degree. We consider bar balance. The bar is in balance under the influence of force $F$ and four unknown reactions: $N_1$, $N_2$, $X_A$, $Y_A$. But for the arbitrary forces plane system we can work out only three statics equations. It means that the static indefinability degree is $S=4-3=1$. The system is once statically undefinable.

a) In our case it is required to define only $N_1$ and $N_2$, therefore we use one of the three statics equations (of moments):

$$\Sigma M_A=0;\ N_1 \cdot a+N_2 \cdot b-F(a+b)=0,$$

$$1,2N_1+1,8N_2=1830.$$

b) We work out the deformation scheme (figure 3.3).

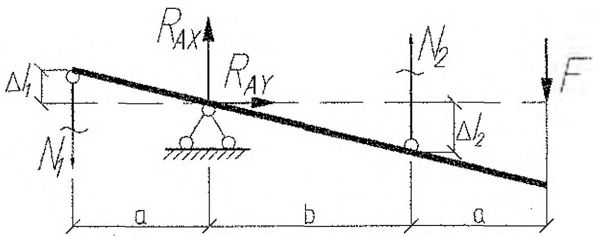



***Figure 3.3*** – Deformation scheme of rod system

From the deformation scheme we work out the additional deformation equation using the triangles similarity:

$$\frac{\Delta l_1}{\Delta l_2}=\frac{a}{b},$$

$$1,8\Delta l_1=1,2\Delta l_2.$$

c) We express $\Delta l_1$ and $\Delta l_2$ using Hooke's law through efforts in rods, their length and rigidity:

$$b\frac{N_1 l_1}{EA_1}=a\frac{N_2 l_2}{EA_2}.$$

Taking into account $l_1 = l_2 = h$ and $\frac{A_1}{A_2} = n$ the deformations equation takes the form of:

$$N_1 = anMN_2,$$
$$N_1 = 1{,}33N_2.$$

d) We compose the equations system which includes the static equation and the deformation equation:

$$\begin{cases} N_1 + 1{,}52N_2 = 1525 \\ N_1 - 1{,}33N_2 = 0 \end{cases}$$

From here:

$$N_1 = 718 \ kN,$$
$$N_2 = 540 \ kN.$$

3. We define the most stressed rod. For this purpose we compare stresses $\sigma_1$ and $\sigma_2$:

$$\sigma_1 = \frac{N_1}{A_1} = \frac{N_1}{n \cdot A_2},$$
$$\sigma_2 = \frac{N_2}{A_2}.$$

We compose the ratio:

$$\frac{\sigma_1}{\sigma_2} = \frac{N_1}{n \cdot N_2} = \frac{718 \cdot 10^3}{2 \cdot 540 \cdot 10^3} = 0{,}7 \Rightarrow \sigma_1 < \sigma_2.$$

The second rod is more stressed.

4. We determine rod cross-section area.

As $\sigma_2 > \sigma_1$, we determine cross-section area $A_2$:

$$A_2 \geq \frac{N_2}{[\sigma]} = \frac{540 \cdot 10^3}{160 \cdot 10^6} = 0{,}00338 \ m^2 = 33{,}8 \ cm^2.$$

Where $[\sigma] = 160$ MPa.

5. In accordance with GOST 8509-72 we select the rod section that consists of two equal lag angles. We use a condition:

$$A_2^* \geq \frac{A_2}{2} = \frac{33{,}8}{2} = 16{,}9 \ cm^2.$$

In our case $A_2^* \geq 16{,}9\,\text{cm}^2$.

The area of an equal lag angle No 110×110×8, i.e. $A_2^* = 17{,}2\,\text{cm}^2$ is close. We define underload percent $\delta$:

$$\delta = \left| \frac{\sigma_2 - [\sigma]}{[\sigma]} \right| \cdot 100\,\%.$$

For this purpose we determine stress $\sigma_2$:

$$\sigma_2 = \frac{N_2}{2A_2^*}. = \frac{540 \cdot 10^3}{2 \cdot 17{,}2 \cdot 10^3} \approx 157 MPa.$$

Then

$$\delta = \left|\frac{157-160}{160}\right| \approx 2\,\%,$$

what is admissible as $|\delta| = 2\% < [\delta] = 5\%$, where $[\delta] = 5\%$ is the allowable percent of rod overload (underload). We accept an equal lag angle № 110×110×8.

6. We find the cross-sectional area of the first equal lag angle $A_1^*$,

$$A_1 = n \cdot A_2,$$

$$A_1^* \geq \frac{A_1}{2} = \frac{67,6}{2} = 33,8\ \ cm^2.$$

In accordance with GOST 8509-72 we select the section of an equal lag angle№160×160×11 for which $A_1^* = 34,4\ \ cm^2$.

7. We determine the value of a rupture load

$$F_p = \frac{N_1^{max} \cdot a + N_2^{max} \cdot b}{a+b}.$$

We also calculate limit efforts in rods $N_1^{max}$ and $N_2^{max}$:

$$N_1^{max} = 2 \cdot A_1^* \cdot \sigma_T = 2 \cdot 34,4 \cdot 10^{-4} \cdot 240 \cdot 10^6 = 1651,2\ \ kN,$$

$$N_2^{max} = 2 \cdot A_2^* \cdot \sigma_T = 2 \cdot 17,2 \cdot 10^{-4} \cdot 240 \cdot 10^6 = 825,6\ \ kN.$$

We substitute values $N_1^{max}$ and $N_2^{max}$ in a calculation formula of a rupture load andwe obtain:

$$F_p = \frac{1651,2 \cdot 1,2 + 820,8 \cdot 1,8}{1,2 + 1,8} \approx 1153\ kN.$$

We find the safety factor:

$$m = \frac{F_p}{F} = \frac{1153 \cdot 10^3}{610 \cdot 10^3} \approx 1,89.$$

**Example 3**

It is given: Compound section.

To define:

1) gravity center position of a section respectively to any axes $x$, $y$;

2) inertia sections moment $I_{Xc}$ and $I_{Yc}$ respectively to central axes $x_c$ and $y_c$;

3) principal central axes disposition $U$ and $V$;

4) principal central inertia moments;

5) to do checks $I_{Xc} + I_{Yc} = I_u + I_v$; $I_{uv} = 0$;

6) to construct an inertia ellipse.

To accept: channel No. 24; sheet: $b = 2\ cm$; $h = 18\ cm$.

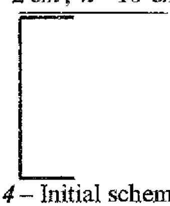
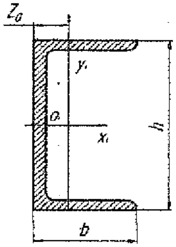


***Figure 3.4*** – Initial scheme

Solution:

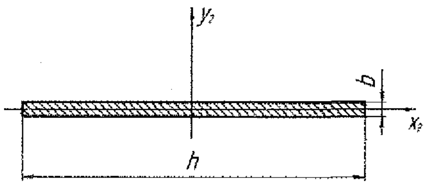
1. We write out all data required in further calculations from an assortment of rolling profiles tables:

Channel:




$h = 24\ cm;$
$b = 9\ cm;$
$A_{ch} = 30{,}6\ cm^2;$
$I_X = 2900\ cm^4;$
$I_Y = 208\ cm^4;$
$z_0 = 2{,}42\ cm;$
$I_{XY} = 0.$

Sheet:




$b = 2\ cm;$
$h = 18\ cm;$
$A_{sh} = b \cdot h = 2 \cdot 18 = 36\ cm^2;$
$$I_X = \frac{h \cdot b^3}{12} = \frac{18 \cdot 2^3}{12} = 12\ cm^4;$$
$$I_Y = \frac{b \cdot h^3}{12} = \frac{2 \cdot 18^3}{12} = 972\ cm^4;$$
$I_{XY} = 0.$

We draw section in scale 1:2. We denote the random (accidental) axes, and we define the gravity center of the set section.

a) total area of section:
$$A = A_{ch} + A_{sh} = 30{,}6 + 36 = 66{,}6\ cm^2.$$

b) coordinates of the gravity center of each section element in axes $x$ and $y$:
$$x_1 = z_0 = 2{,}42\ cm;$$
$$y_1 = \frac{h_{шв}}{2} = \frac{24}{2} = 12\ cm;$$
$$x_2 = \frac{h_л}{2} = \frac{18}{2} = 9\ cm;$$
$$y_2 = h_{шв} + \frac{b_л}{2} = 24 + \frac{2}{2} = 25\ cm.$$

c) the section static moments relating to axes $x$ and $y$:
$$S_X = A_{ch} \cdot y_1 + A_{sh} \cdot y_2 = 30{,}6 \cdot 12 + 36 \cdot 25 = 1267{,}2\ cm^3;$$
$$S_Y = A_{ch} \cdot x_1 + A_{sh} \cdot x_2 = 30{,}6 \cdot 2{,}42 + 36 \cdot 9 = 398\ cm^3.$$

d) gravity center section:
$$x_C = \frac{S_Y}{A} = \frac{398}{66{,}6} = 5{,}98\ cm;$$

$$y_C = \frac{S_X}{A} = \frac{1267,2}{66,6} = 19\ cm.$$

Through the gravity center we draw axes $x_C$ and $y_C$ parallel to axes $x_1$, $y_1$ and $x_2$, $y_2$.

2. We calculate inertia section moments about axes $x_C, y_C$.

a) The position of the gravity center (GC) of each section element with respect to central axes:

$m_1 = x_1 - x_c = 2,42 - 5,98 = -3,56\ cm$;

$n_1 = y_1 - y_c = 12 - 19 = -7\ cm$;

$m_2 = x_2 - x_c = 9 - 5,98 = 3,02\ cm$;

$n_2 = y_2 - y_c = 25 - 19 = 6\ cm$.

b) using the rule of parallel axes translation, we define axial moments and centrifugal inertia moment:

$I_{Xc} = I_{X_{ch}} + n_1^2 \cdot A_{ch} + I_{X_{sh}} + n_2^2 \cdot A_{sh} = 2900 + (-7)^2 \cdot 30,6 + 12 + 6^2 \cdot 36 = 5707,4\ cm^4$;

$I_{Yc} = I_{Y_{ch}} + m_1^2 \cdot A_{ch} + I_{Y_{sh}} + m_2^2 \cdot A_{sh} = 208 + (-3,56)^2 \cdot 30,6 + 972 + 3,02^2 \cdot 36 = 1896\ cm^4$;

$I_{XcYc} = I_{XY_{ch}} + m_1 \cdot n_1 \cdot A_{ch} + I_{XY_{sh}} + m_2 \cdot n_2 \cdot A_{sh} = 0 + (-3,56) \cdot (-7) \cdot 30,6 + 0 + 3,02 \times$

$\times 6 \cdot 36 = 1414,87\ cm^4$

3. We determine the principal axes location using a formula:

$$tg\,2\alpha = -\frac{2I_{XcYc}}{I_{Xc} - I_{Yc}} = -\frac{2 \cdot 1414,87}{5707,4 - 1896} = -0,742;$$

$$\alpha = \frac{1}{2} arctg(-0,742) = -18,3^\circ.$$

We draw the principal axes of $U_{(max)}$ and $V_{(min)}$.

4. We calculate the principal inertia moments:

$$\left.\begin{matrix} I_u \\ I_v \end{matrix}\right\} = \frac{5707,4 + 1896}{2} \pm \frac{1}{2}\sqrt{(5707,4 - 1896)^2 + 4 \cdot (1414,87)^2} = 3801,7 \pm 2373,5$$

$I_u = 6175,2\ cm^4$ – *max* value;

$I_v = 1428,2\ cm^4$ – *min* value.

5. We check the calculations:

a) $I_{Xc} + I_{Yc} = I_u + I_v$:

$5707,4 + 1896 = 6175,2 + 1428,2$;

$7603,4 = 7603,4$.

б) $I_{uv} = 0$:

$$I_{uv} = \frac{I_{Xc} - I_{Yc}}{2} \cdot \sin 2\alpha + I_{XcYc} \cdot \cos 2\alpha = \frac{5707,4 - 1896}{2} \cdot (-0,596) + 1414,87 \cdot 0,803 = 0,34 \approx 0.$$

6. We build inertia ellipse.

We calculate inertia radiuses:

$$i_u = \sqrt{\frac{I_u}{A}} = \sqrt{\frac{6175,2}{66,6}} = 9,63\ cm;$$

$$i_v = \sqrt{\frac{I_v}{A}} = \sqrt{\frac{1428,2}{66,6}} = 4,63\ cm.$$

We draw obtained values on the principal axes and we build a momental ellipse (figure 3.5).

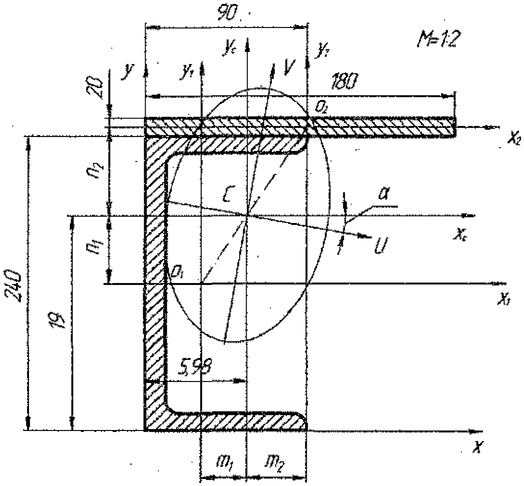



***Figure 3.5*** – The momental ellipse

**Example 4**

The double-support beam is loaded by external loadings. It is required:

1. To construct diagrams of shearing forces $Q$ and bending moments $M$;

2. To specify the position of dangerous section of a beam.

3. For a I-shaped beam choose number of a rolling profile from strength condition and also to make check of strength on shearing stresses. When calculating to accept for steel: $[\sigma] = 160$ MPa, $[\tau] = 100$ MPa.

4. To define geometrical characteristics of rectangular section, on condition of a ratio of the sides: $h=2b$, where $h$ – height, $b$ – section width.

5. To calculate the weight of both beams and to compare results. To reflect the reason of the choice of a beam with this or that section in a conclusion (criterion – a material consumption).

It is given: $a = 2$ m, $b = 2$ m, $c = 2$ m, $F = 26$ N, $q = 30$ kN/m, $M = 38$ kN·m.

Solution:

We work out (constitute) the equation of the moments with respect to a support A:

$$\sum m_A(\overline{F_k}) = 0, \quad -q \cdot b \cdot \left(a + \frac{b}{2}\right) - M + R_B \cdot (a+b+c) - F \cdot a = 0;$$

also we find reaction of the $R_B$ support:

$$R_B = \frac{M + F \cdot a + q \cdot b \cdot \left(a + \frac{b}{2}\right)}{a+b+c} = \frac{38 + 26 \cdot 2 + 30 \cdot 2 \cdot \left(2 + \frac{2}{2}\right)}{2+2+2} = 45\ \text{kN}.$$

We work out the equation of the moments about a support B:

$$\sum m_B(\overline{F_k})=0, \quad q\cdot b\cdot\left(c+\frac{b}{2}\right)-M-R_A\cdot(a+b+c)+F\cdot(b+c)=0;$$

also we find reaction of the $R_A$ support:

$$R_A=\frac{F\cdot(b+c)-M+q\cdot b\cdot\left(c+\frac{b}{2}\right)}{a+b+c}=\frac{26\cdot(2+2)-38+30\cdot 2\cdot\left(2+\frac{2}{2}\right)}{2+2+2}=41 \text{ kN}.$$

Verification:

$$\sum F_{ky}=0, \quad -q\cdot b-F+R_A+R_B=-30\cdot 2-26+41+45=0, \quad 0=0.$$

We break a beam into 3 forces sites (sectors).

We draw any section on each of sites at distance of z and we consider a condition of balance of the cut part:

Sector I, $0\le z_1\le a$.

$Q_1=R_A=41\ kN,$ $\quad M_1=R_A\cdot z_1;$

With $z_1=0,$ $\quad M_1=R_A\cdot z_1=41\cdot 0=0.$

With $z_1=a=2$ m, $\quad M'_1=R_A\cdot z_1=41\cdot 2=82\ kN\cdot m.$

Sector II, $0\le z_2\le b$.

$$Q_2=R_A-F-q\cdot z_2,\ M_2=R_A\cdot(a+z_2)-F\cdot z_2-q\cdot\frac{z_2^2}{2};$$

With $z_2=0,$ $\quad Q_2=R_A-F-q\cdot z_2=41-26-30\cdot 0=15\ kN,$

$$M_2=R_A\cdot(a+z_2)-F\cdot z_2-q\cdot\frac{z_2^2}{2}=41\cdot(2+0)-26\cdot 0-30\cdot\frac{0^2}{2}=82\ kN\cdot m;$$

With $z_2=b=2$ m, $\quad Q'_2=R_A-F-q\cdot z_2=41-26-30\cdot 2=-45\ kN\cdot m,$

$$M'_2=R_A\cdot(a+z_2)-F\cdot z_2-q\cdot\frac{z_2^2}{2}=41\cdot(2+2)-26\cdot 2-30\cdot\frac{2^2}{2}=52\ kN\cdot m.$$

$Q_2>0$, $Q'_2<0$, therefore site 2 is an extremum. We find its location:

$$z_э=\frac{Q_2}{q}=\frac{15}{30}=0{,}5 \text{ m}.$$

We find value of the extreme moment:

$$M_{max}=R_A\cdot(a+z_{extr})-F\cdot z_{extr}-q\cdot\frac{z_{extr}^2}{2}=41\cdot(2+0{,}5)-26\cdot 0{,}5-30\cdot\frac{0{,}5^2}{2}=$$

$$=85{,}75\ kN\cdot m.$$

Sector III, $0\le z_3\le c$.

$Q_3=R_B=-45\ kN,$ $\quad M_3=R_B\cdot z_3;$

With $z_3=0,$ $\quad M_3=R_B\cdot z_3=45\cdot 0=0.$

With $z_3=c=2\ m,$ $\quad M'_3=R_B\cdot z_3=45\cdot 2=90\ kN\cdot m.$

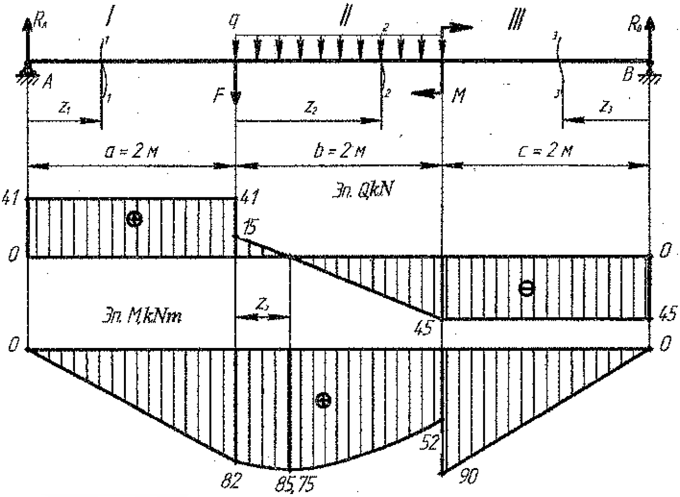



*Figure 3.6* – Scheme beam on two supports and diagram of internal force factors

Dangerous section: sector 3 at $z_3 = 2$ m.

$M_{\max} = |M'_3| = 90\ kN \cdot m.$

Find required moment of resistance::

$$W_0 = \frac{M_{\max}}{[\sigma]} = \frac{90 \cdot 10^3}{160 \cdot 10^6} = 562{,}5 \cdot 10^{-6}\ \text{m}^3 = 562{,}5\ \text{cm}^3.$$

We select a I-section with the closest section module of $W_x = 597$ cm$^3$ for a range (assortment) of rolling steel (No. 33).

Since the accepted $W_x = 597$ cm$^3$ is more than required 562,3 cm$^3$, we don't carry out check of strength on normal stresses.

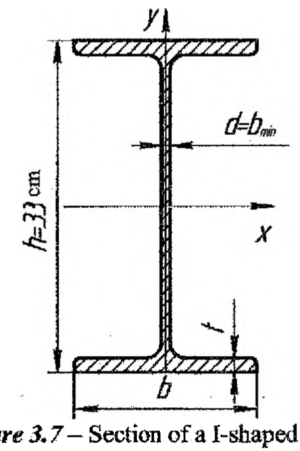



*Figure 3.7* – Section of a I-shaped beam

Tangent bending stresses are determined according to the Zhuravsky's formula:

$$\tau = \frac{Q \cdot S_x}{I_x \cdot b(y)},$$

where: Q – shearing force in the considered section; $S_x$ – static moment of the cut part of cross section; $I_x$ – moment of inertia of all section about neutral axis; $b$ – width of lateral section of a beam at that level at which tangent (shearing) stresses is defined.

We will find $\tau_{max}$:

$$\tau_{max} = \frac{Q_{max} \cdot S_{x\,max}}{I_x \cdot b_{min}}.$$

The static moment of the cut part of cross section is maximum for semi-section and on a range for a I-shaped section No. 33 is equal $S_{x\,max} = 339$ cm$^3$. At the same time section width at this level is minimum and equal to $b_{min}=d=0,7$ *cm*. Moment of inertia of I-shaped section w.r.t. neutral axis of $I_x = 9840$ $cm^4$. The maximum shearing force acts on sector3:

$$Q_{max} = |Q_3| = 45\ kN.$$

Then:

$$\tau_{max} = \frac{Q_{max} \cdot S_{x\,max}}{I_x \cdot d} = \frac{45 \cdot 10^3 \cdot 339 \cdot 10^{-6}}{9840 \cdot 10^{-8} \cdot 0,7 \cdot 10^{-2}} = 22,147 \cdot 10^6\ \text{Pa} < [\tau] = 100\ \text{MPa}.$$

Strength condition on shearing stresses is satisfied.

5. We select a beam of rectangular section with $h=2b$ ratio. We consider that axial section module of rectangular section

$$W_x = \frac{bh^2}{6}.$$

From strength condition on normal stresses:

$$W_x \geq \frac{M_{max}}{[\sigma]}.$$

From here:

$$b \geq \sqrt[3]{\frac{3M_{max}}{2[\sigma]}} = \sqrt[3]{\frac{3 \cdot 63,3 \cdot 10^3}{2 \cdot 160 \cdot 10^6}} = 0,084\ m = 8,4\ cm.$$

Then $h = 2b = 16,8\ sm.$

We calculate the weight of a beam of standard (I-shaped) section and a beam of rectangular section ($\rho = 7800 \frac{kg}{m^3}$):

$$P_1 = M_0 \cdot 4a = 33,9 \cdot 4 \cdot 1 = 135,6\ kg,$$

where $m_0$ – weight (specific weight) is 1 m of a profile;

$$P_1 = hb \cdot \rho \cdot 4a = 0,084 \cdot 0,168 \cdot 7800 \cdot 4 \cdot 1 = 440,3\ kg,$$

$$\frac{P_1}{P_2} = \frac{440,3}{135,6} \approx 3,25.$$

It is obvious that use of rolling I-shapid section allows to save considerably material when ensuring necessary strength.

**Example 5**

To construct diagrams $Q$, $M$ and $N$ for the flat frame represented in the figure 3.8.

Solution:

Frames are the systems consisting of rigidly connected rectilinear rods. A frame axis – the broken line. It is convenient to consider each straight section as a beam, however, in a frame, except bending moments $M$ and shearing forces $Q$, also longitudinal forces $N$ acts. Rules of signs for $N$ and $Q$ also remain earlier accepted. For bending moments $M$ the rule of signs is usually not established and at creation of diagrams $M$ of ordinate draw on that side where the stretched fiber from a bend. (*Note.* Some authors consider convenient to build diagrams $M$ from compressed fiber). For convenience any moment can be taken for positive.

Diagrams $N$, $Q$, $M$ for frames build by a method of sections, applying, brought earlier for beams, rules.

Analytical expressions of functions $N$, $Q$, $M$ write down seldom (for example, for determination of extreme values on curvilinear sites of diagrams). Usually diagrams $N$, $Q$, $M$ build on points. calculating values in characteristic sections.

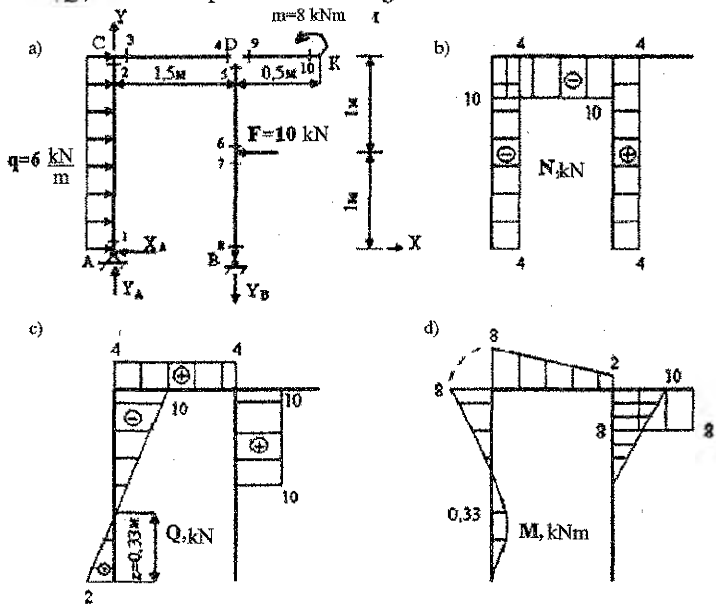



*Figure 3.8* – Diagrams of Q, M, N

We define reactions of support:

$$\sum M_A = 0; \quad q \cdot 2 \cdot \frac{1}{2} \cdot 2 - m - F \cdot 1 + Y_B \cdot 1{,}5 = 0 \ ;$$

$$Y_B = \frac{-q \cdot 2 \cdot 1 + m + F \cdot 1}{1{,}5} = \frac{-6 \cdot 2 \cdot 1 + 8 + 10 \cdot 1}{1{,}5} = 4 \ kN .$$

$\sum M_B = 0; \quad q \cdot 2 \cdot \frac{1}{2} \cdot 2 - m - F \cdot 1 + Y_A \cdot 1{,}5 = 0$ ;

$Y_A = \frac{-q \cdot 2 \cdot 1 + m + F \cdot 1}{1{,}5} = \frac{-6 \cdot 2 \cdot 1 + 8 + 10 \cdot 1}{1{,}5} = 4 \ kN$.

$\sum X = 0; \qquad -X_A - F + q \cdot 2 = 0;$

$X_A = -F + q \cdot 2 = -10 + 6 \cdot 2 = 2 \ kN$.

Check: $\sum M_K = 0; \ Y_A \cdot 2 - Y_B \cdot 0{,}5 - q \cdot 2 \cdot \frac{1}{2} \cdot 2 - m + X_A \cdot 2 + F \cdot 1 = 0$ ;

$4 \cdot 2 - 4 \cdot 0{,}5 - 6 \cdot 2 \cdot 1 - 8 + 2 \cdot 2 + 10 \cdot 1 = 0$.

Reactions are found truly.

We choose characteristic sections at borders of sites and we determine in them values $N$, $Q$, $M$ :

$N_1 = -Y_A = -4 \ кN$; $\ N_2 = -Y_A = -4 \ кN$; $\ N_3 = N_4 = X_A - q \cdot 2 = -10 \ кN$;

$N_5 = N_6 = N_7 = N_8 = Y_B = 4 \ кN$; $\ N_9 = N_{10} = 0$.

Building diagram (figure 3.8, b).

$Q_1 = X_A = 2 \ кN$; $Q_2 = X_A - q \cdot 2 = 2 - 6 \cdot 2 = -10 \ кN$;

$Q_3 = Q_4 = Y_A = 4 \ кN; Q_5 = Q_6 = F = 10 \ кN$; $Q_7 = Q_8 = Q_9 = Q_{10} = 0$.

Building diagram $Q$ (figure 3.8, c). On the site $1-2$ the diagram crosses an axis. We will determine the coordinate of a point of intersection:

$z_0 = \frac{Q}{q} = \frac{2}{6} = 0{,}33 \ m$.

$M_1 = 0; \ \ M_2 = X_A \cdot 2 - q \cdot 2 \cdot \frac{1}{2} \cdot 2 = 2 \cdot 2 - 6 \cdot 2 = -8 \ кN \cdot m$;

$M_{extr} = X_A \cdot z_0 - q \cdot \frac{z_0^2}{2} = 2 \cdot 0{,}33 - 6 \cdot \frac{0{,}33^2}{2} = 0{,}33 \ kN \cdot m$; $\ M_3 = M_2 = -8 \ kN \cdot m$;

$M_5 = -F \cdot 1 = -10 \ \ kN \cdot m; M_6 = M_7 = M_8 = 0$;

$M_9 = M_{10} = m = 8 \ \ kN \cdot m$..

We build a diagram $M$ (figure 3.8, d).

At the correct creation of diagrams static balance of each node has to be observed. We check balance of nodes $C$ and $D$ (figure 3.9).

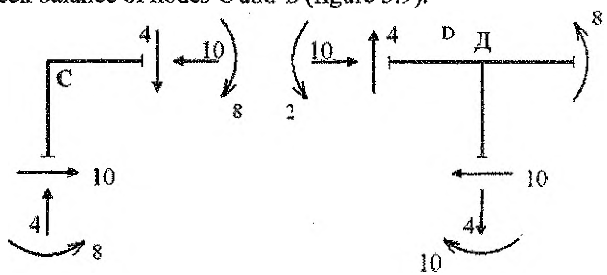



***Figure 3.9*** – Check of nodes

The balance of the nodes is observed.

## 4. THE TASKS FOR PERFORMANCE IS CALCULATED GRAPHIC WORKS

CGW include tasks 1-5. Room diagrams and numerical data are selected in accordance with the instructions of the lecturer.

### TASK 1.
### CALCULATION OF STATICALLY DETERMINATE STEP BAR

It is necessary for the vertical or horizontal rod having rigidly fixed support on one of the ends:
1) to draw the scheme in any scale;
2) to define values of normal force on each sector of a rod;
3) to construct a diagram of normal force;
4) to construct a diagram of displacement;
5) to check bar strength;
6) to check rigidity of a bar.

Schemes of rods are provided on the figure 6. Lengths of sites of a rod and loading attached to it are specified in table 1, the cross-sectional area of narrow site $A = 0,2\ m^2$, wide site $2A$. When calculating to accept: permissible stresses on stretching $[\sigma_p] = 20\ МПа$; on compression $[\sigma_c] = 80\ МПа$; the allowed (permissible) deformation $[\delta] = \frac{l}{500}$, the module of elasticity of $E = 2\cdot10^5$ MPa.

***Table 1*** - Numerical data to calculation of step bars

| ***Number lines*** | ***a, m*** | ***$q_1$=$q_3$, kN/m*** | ***$q_2$, kN/m*** | ***$F_1$, kN*** | ***$F_2$, kN*** | ***$F_3$, kN*** |
|---|---|---|---|---|---|---|
| 0 | 1,2 | 20 | 15 | 20 | 25 | 20 |
| 1 | 0,8 | 5 | 30 | 10 | 35 | 10 |
| 2 | 1 | 10 | 25 | 15 | 30 | 20 |
| 3 | 1,2 | 15 | 20 | 20 | 25 | 30 |
| 4 | 1,4 | 20 | 15 | 25 | 20 | 40 |
| 5 | 1,6 | 25 | 10 | 30 | 15 | 10 |
| 6 | 1,8 | 30 | 5 | 35 | 10 | 20 |
| 7 | 2 | 5 | 30 | 40 | 5 | 30 |
| 8 | 0,8 | 10 | 25 | 10 | 35 | 40 |
| 9 | 1 | 15 | 20 | 15 | 30 | 10 |

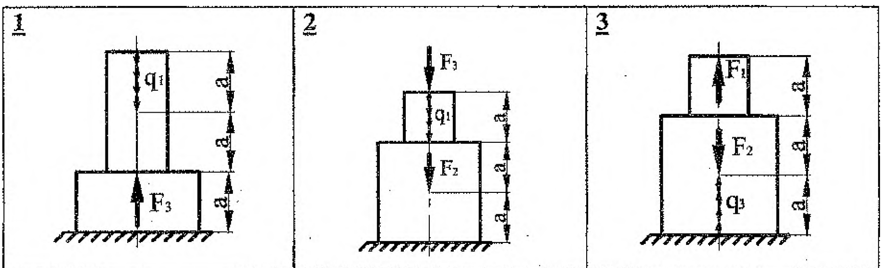
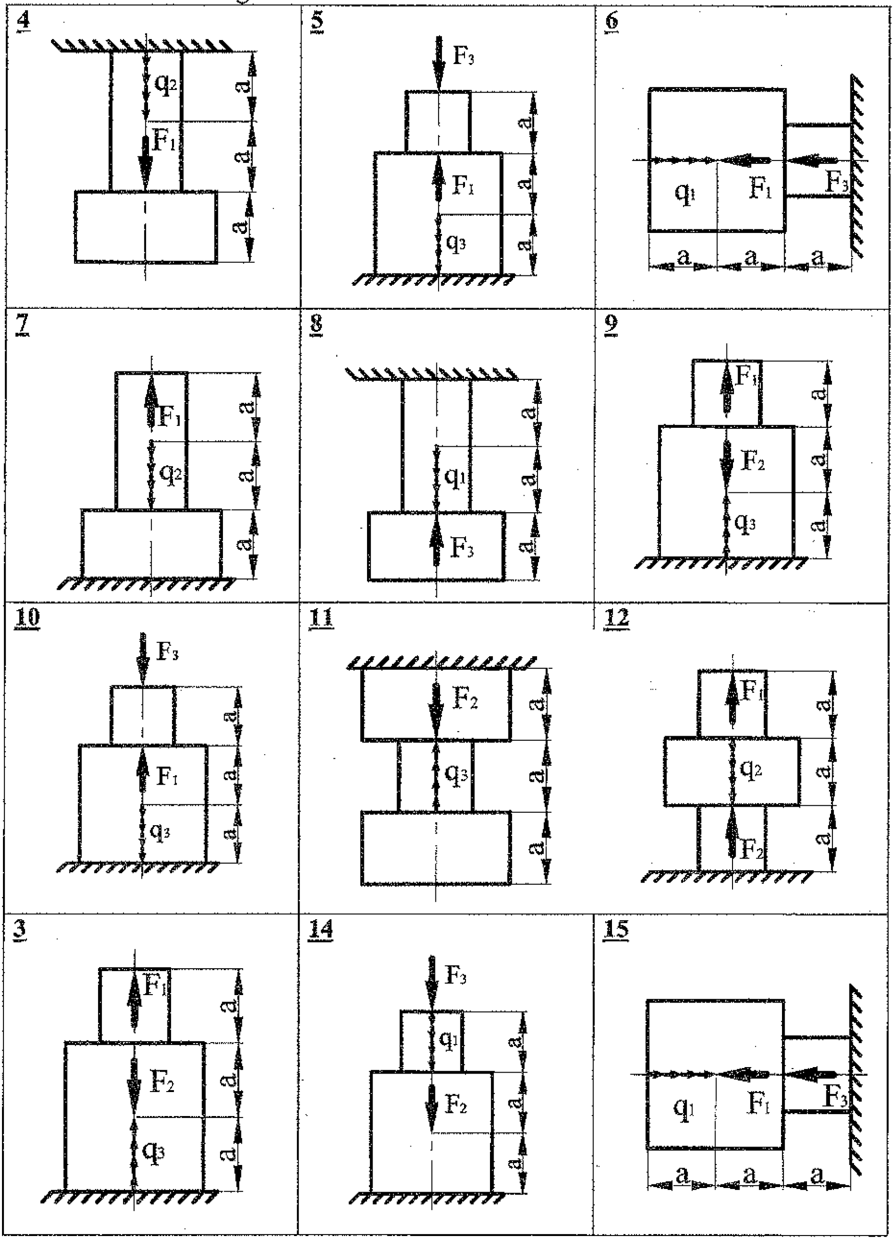



***Figure 4.1*** - Schemes of step bars

Continuation of the figure 4.1



## TASK 2.
## CALCULATION OF STATICALLY INDETERMINATE ROD SYSTEM

Absolutely rigid beam suspended on 2 steel rods fixed hinged fixed bearing. Beam loaded with a concentrated force $F$.

It is required:

1. To disclose static indefinability of system for what:

a) to establish degree of static indefinability;

b) to write down the necessary equations of static balance;

c) to compose the plan of deformations;

d) from the plan of deformations to work out the additional equation of deformations;

e) to solve jointly the statics equation with the equation of deformations and to define efforts in rods $N_1$ and $N_2$.

2. In accordance with GOST 8509-72 to choose sections of rods from two equilateral lag angles for what:

a) to determine stresses in rods and to establish the most stressed rod;

b) from a condition of strength for more stressed rod to determine necessary cross-sectional area and to choose in accordance with GOST number of a profile;

c) to check percent of underload or an overload of more stressed rod;

d) from a ratio $\frac{A_1}{A_2} = n$ to find the cross-sectional area of less loaded rod and to choose a profile in accordance with GOST 8509-72.

3. To determine the value of a rupture load and to compare it to the set loading

To accept basic data according to schemes (figure 4.2) and table 2.

***Table 2*** - Numerical data to calculation of rod systems

| *№ lines* | *a, m* | *b, m* | *h, m* | *α, deg* | $A_1/A_2$ | *F, kN* |
|---|---|---|---|---|---|---|
| 0 | 2,9 | 2,0 | 1,5 | 70 | 4 | 600 |
| 1 | 2 | 1,2 | 1,5 | 20 | 2 | 200 |
| 2 | 2,1 | 1,4 | 1 | 40 | 4 | 300 |
| 3 | 2,2 | 1,6 | 2 | 50 | 1,5 | 400 |
| 4 | 2,3 | 1,8 | 1,5 | 60 | 3 | 500 |
| 5 | 2,4 | 2,0 | 1 | 70 | 2 | 600 |
| 6 | 2,5 | 1,2 | 2 | 20 | 4 | 200 |
| 7 | 2,6 | 1,4 | 1,5 | 40 | 1,5 | 300 |
| 8 | 2,7 | 1,6 | 1 | 50 | 3 | 400 |
| 9 | 2,8 | 1,8 | 2 | 60 | 2 | 500 |

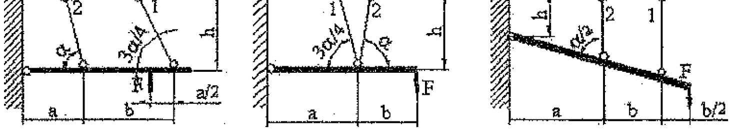
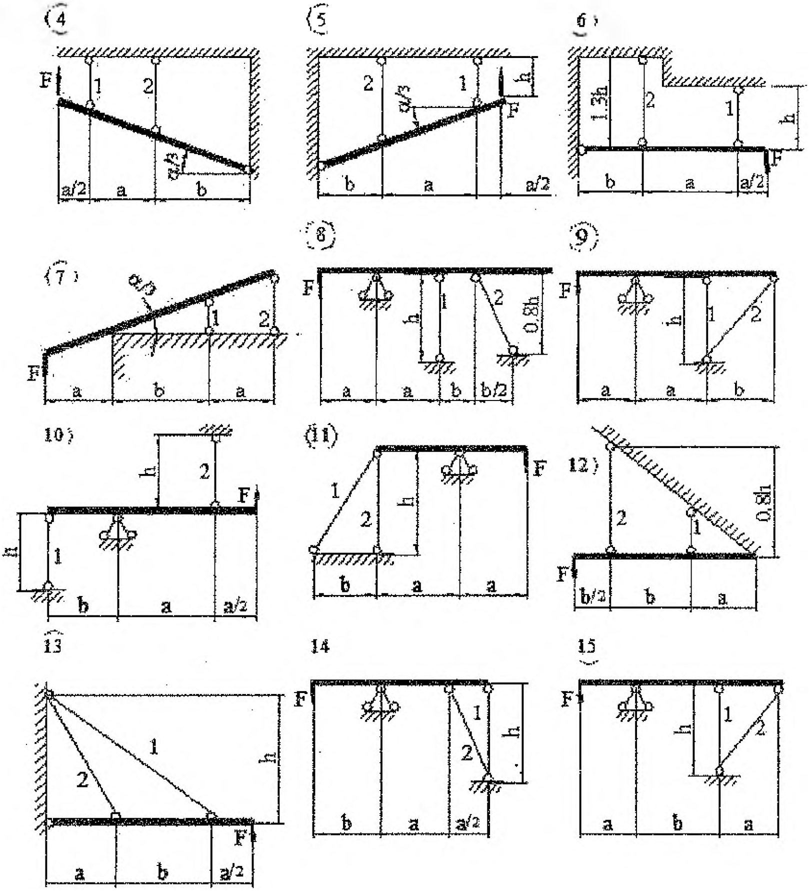



***Figure 4.2*** – Schemes of statically indeterminate rod systems

Continuation of the figure 4.2




## TASK 3.
## GEOMETRICAL CHARACTERISTICS OF PLAIN FIGURES

It is given: the compound section consisting of three simple elements of certain geometrical sizes. It is required to define:

1) center of gravity position of compound section;
2) moments of inertia of section about central axes;
3) disposition of the principal central axes of inertia;
4) values of the principal central moments of inertia;
5) values of the principal radiuses of inertia;
6) to construct an momental ellipse.

Schemes of compound sections are accepted on the figure 4.3, numerical data – table 3.

***Table 3*** - Numerical parameters to compound sections

| ***№ lines*** | ***I-section*** | ***Channel*** | ***Equal lag angle, mm*** | ***Unequal lag angle, mm*** | ***Sheet*** | |
|---|---|---|---|---|---|---|
| | | | | | *h, cm* | *b, cm* |
| 0 | 36 | 20 | 100x100x16 | | 20 | 1.8 |
| 1 | 24 | 14a | | 125x80x12 | 20 | 2.2 |
| 2 | 27 | 24a | 80x80x8 | | 26 | 1.8 |
| 3 | 30 | 16a | | 100x63x64 | 22 | 1.8 |
| 4 | 16 | 20a | 100x100x16 | | 24 | 2.4 |
| 5 | 22 | 18a | | 110x70x7 | 18 | 2.0 |
| 6 | 20 | 16 | 90x90x9 | | 19 | 2.6 |
| 7 | 30 | 22a | | 160x100x10 | 20 | 1.8 |
| 8 | 27a | 18 | 110x110x8 | | 22 | 2.0 |
| 9 | 33 | 24 | | 100x63x10 | 24 | 2.6 |

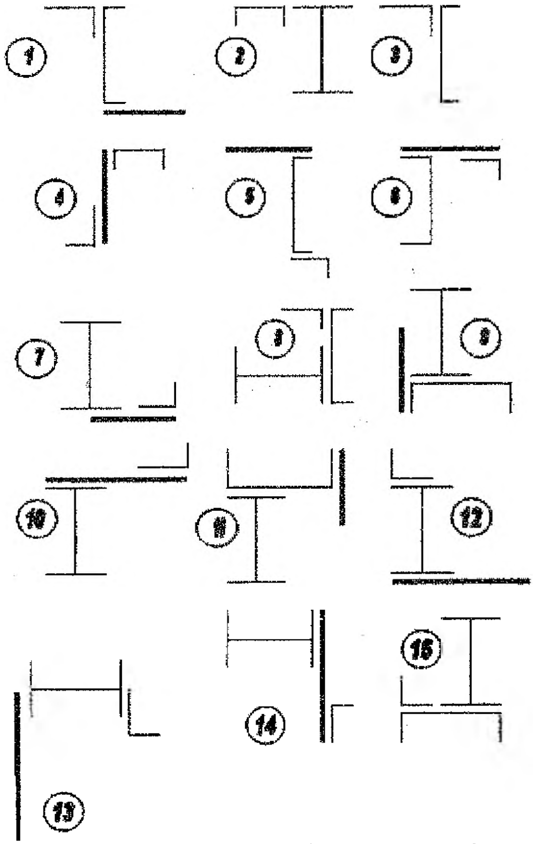



***Figure 4.3*** – Schemes of compound sections

## TASK 4.
## DIRECT TRANSVERSE BENDING

The beam is fixed in a different way, loaded with external loads (concentrated force, couple of forces, distributed load).

Required:

1. To construct diagrams of shearing forces of $Q$ and bending moments of $M$ for what follows, which requires:

a) to write down in a general view analytical expressions for shearing forces of $Q(z)$ and bending moments of $M(z)$:

$$Q = \sum_{i=1}^{n} F_i,\ M = \sum_{i=1}^{n} M_i.$$

b) to calculate values of shearing force $Q$ and bending moment $M$ for characteristic sections of a bar (on borders of force sites);

c) on the received values to construct on the scale of a diagram (graphics) of shearing forces $Q$ and bending moments $M$ ;

d) to verify the correctness of building of diagrams on differential dependences:

$$q = \frac{dQ}{dZ} = \frac{d^2M}{dZ^2}.$$

2. To specify the location of dangerous section of beams.

3. For a (timber) wooden beam (a) to choose the sizes of square lateral section from strength condition if $[\sigma] = 10$ MPa.

4. For a steel I-shaped beam (b) to choose number of a rolling profile from strength condition and also to make check of strength on shearing stresses.

When calculating to accept for steel: module of elasticity of $E = 2 \cdot 10^5$ MPa, $[\sigma] = 160$ MPa, $[\tau] = 100$ MPa.

To accept basic data according to schemes (figure 9) and table 4.

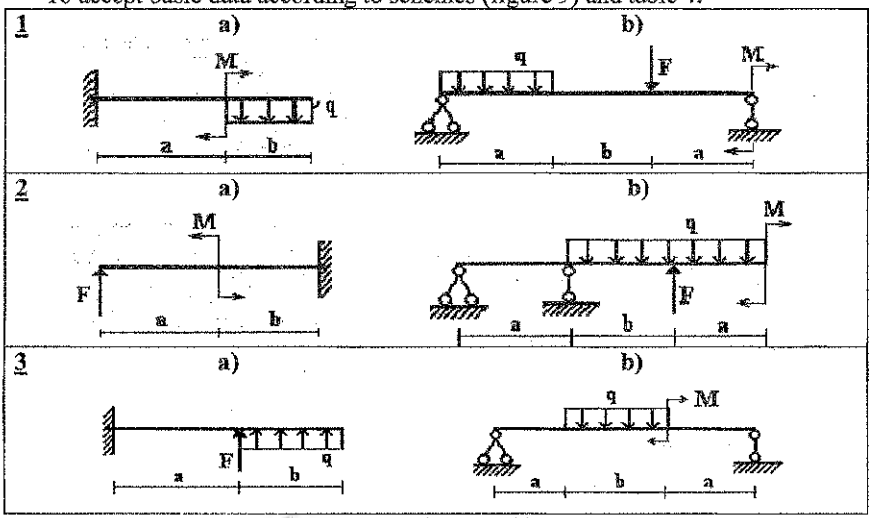
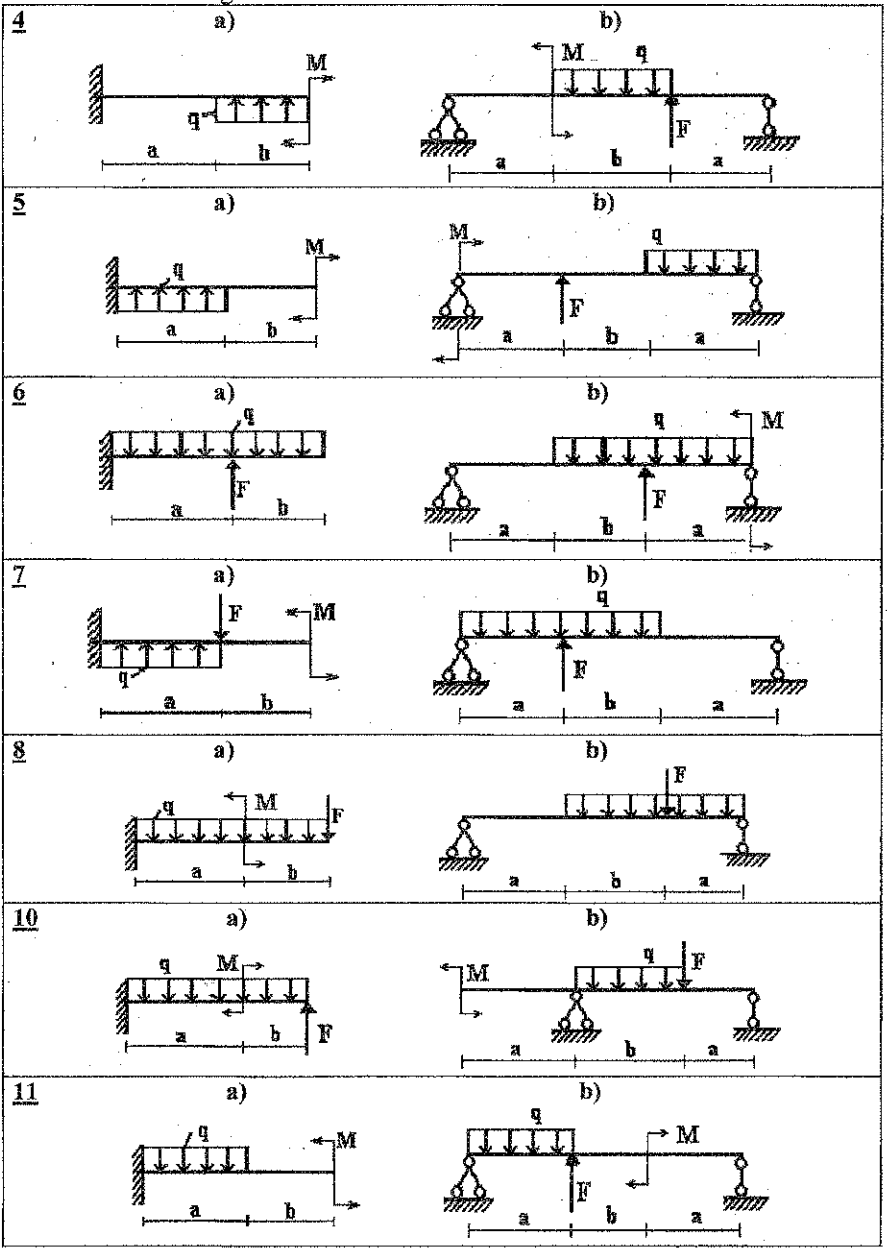



***Figure4.4*** – Schemes of beams

Continuation of the figure 4.2



Continuation of the figure 4.2

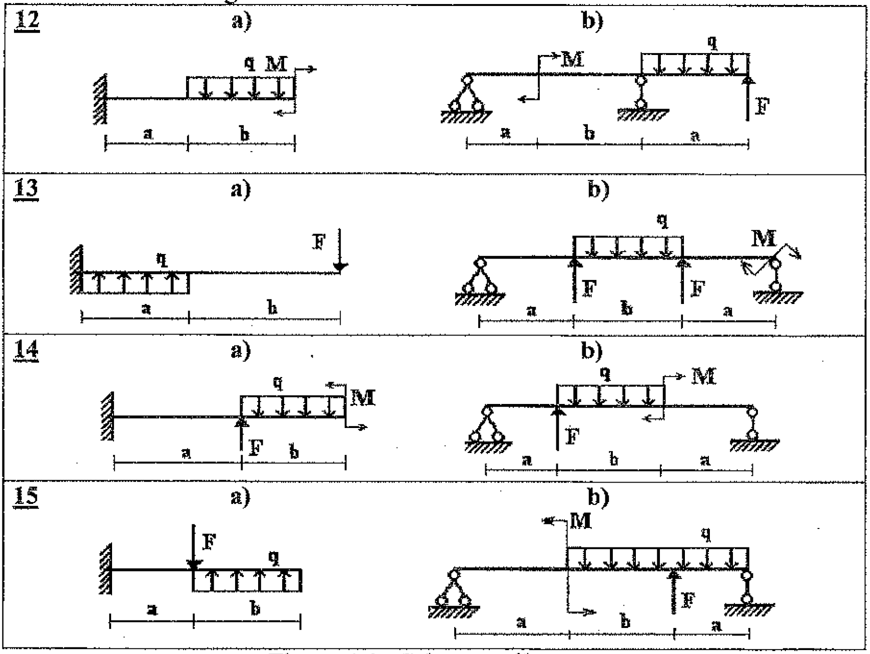



*Figure 4.4* – Schemes of beams

*Table 4* - Numerical data to calculation of beams and frames (tasks 4, 5)

| *№ lines* | *F, kN* | *M, kN·m* | *q, kN/m* | *a, m* | *b, m* |
|---|---|---|---|---|---|
| 0 | 50 | 60 | 15 | 1 | 3 |
| 1 | 40 | 40 | 10 | 1 | 2 |
| 2 | 50 | 60 | 15 | 2 | 2 |
| 3 | 60 | 80 | 20 | 3 | 2 |
| 4 | 70 | 100 | 25 | 2 | 3 |
| 5 | 80 | 40 | 10 | 1 | 3 |
| 6 | 70 | 60 | 15 | 3 | 1 |
| 7 | 60 | 80 | 20 | 2 | 2 |
| 8 | 50 | 100 | 25 | 1 | 2 |
| 9 | 40 | 40 | 10 | 2 | 1 |
| 0 | 50 | 60 | 15 | 1 | 3 |

## TASK 5.
## CREATION OF DIAGRAMS OF Q, M, N IN FRAMES

For the set frame to construct diagrams of internal force factors. To accept numerical data on table 4, schemes are accepted on the figure 4.5.

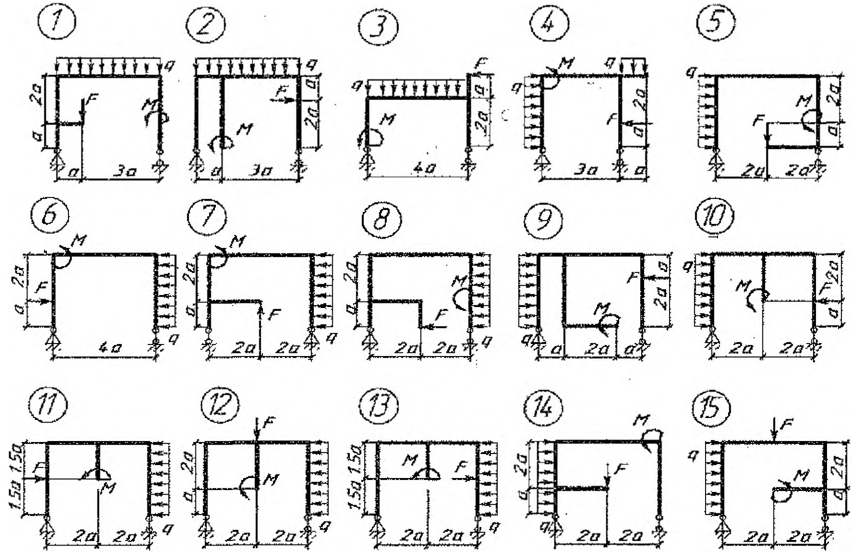


*Figure 4.5* – Schemes of frames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EDUCATIONAL EDITION

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# TASKS AND METHODICAL INSTRUCTIONS

to performance calculated graphic works on a course

## «Resistance of materials»

for students of specialty1 – 70 02 01 «Industrial and civil engineering»

Responsible for release: *A. Zheltkovich*
Editor: *E. Borovikova*
Computer imposition: *K. Kolb*

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It is passed for the press 13.11.2017. Paper writing No. 1.
Font of Times New Roman. Condish. print. page 1,86. Ed. edit. l. 2.
Order No. 1134. Circulation 30 piece.
It is printed on risograph educational institutions Brest state technical university.
224017, Brest, Moskovskaya St., 267.