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ESTABLISHMENT OF EDUCATION
BREST STATE TECHNICAL UNIVERSITY

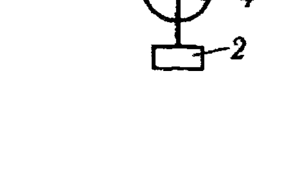
DEPARTMENT OF APPLIED MECHANICS

RESEARCH THE MOVEMENT OF THE MECHANICAL SYSTEM BY MEANS OF THE PRINCIPLE OF D'ALEMBERT LAGRANGE

Tasks and methodical instructions

for performing calculated graphic works on a course
«Theoretical mechanic»

for students of a specialty 1 - 70 02 01 Industrial and civil engineering



Brest 2017

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The theoretical mechanics is one of the main all-technical disciplines which are the base for studying of special disciplines and training of the qualified engineers of technical specialties. For acquirement of skills of engineering calculations students perform calculated graphic works on the main sections of a course.

The present methodical instructions contain brief theoretical material on the Chapter «The general equation of dynamics», section «Dynamics», and condition of tasks for performance of calculated graphic works.

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CONTENTS

INTRODUCTION

Tasks and methodical instructions correspond to basic curriculum (academic plan) of technical specialties and include short theoretical data, conditions of a task for performance of calculated graphic work and examples of calculations. At defense of calculated graphic work it is necessary to answer the questions connected with its performance and to solve control problems of its subject.

INSTRUCTIONS ON REGISTRATION OF CALCULATED GRAPHIC WORKS

1. Calculated graphic works are performed on standard sheets of the A4 format (210 x 297 mm) with a stamp of 15 mm and the indication of numbering of pages.

2. Registration order: the title page with the indication of option; a task with the indication of basic data and schemes of designs; the text of the decision with necessary explanations and schemes; conclusions; list of literature.

3. Drawings and schemes are carried out with observance of rules of graphics and scales of the standard of university.

4. A text part is carried out according to execution requirements of text documents. Calculations are carried out in a general view, in the received expressions values of the sizes, the numerical result with the indication of dimension are substituted (entering them). The corresponding dimensions of the received values are specified in the answer. All calculations are made in decimal fractions to within the third sign after a comma.

5. All drawings (schemes, schedules, etc.) have to be numbered, designated, mentioned in the text.

1. SHORT THEORETICAL DATA

The general equation of dynamics is applied to a research of the movement of not free mechanical systems, bodies or points of which move with some accelerations. According to Dalamber's principle the set of the active forces applied to mechanical system, forces of constraint reaction and forces of inertia of all points of system forms the balanced system of forces.

If to apply the principle of virtual displacement (Lagrange's principle) to such system, then we will receive the integrated principle of Lagrange-Dalambera or the general equation of dynamics: At the displacement of not free mechanical system with bilateral, ideal, stationary and holonomic of constraint reaction the sum of elementary works of all active forces and forces of inertia applied to points of system on any possible displacement of system is equal to zero:

$$\sum_{k=1}^{n}\delta A_k^a+\sum_{k=1}^{n}\delta A_k^{\Phi}=0\ .$$

Expression can give other equivalent forms: a) in the form of a scalar product of vectors

$$\sum_{k=1}^{n}(\overline{F_k}+\overline{\Phi_k})\cdot\delta\overline{r_k}=0;$$

b) in an analytical look

$$\sum_{k=1}^{n}\left[\left(F_{kx}+\Phi_{ky}\right)\delta x_k+(F_{ky}+\Phi_{ky})\delta y_k+(F_{kz}+\Phi_{kz})\delta z_k\right]=0.$$

In these equations force of inertia of a material point $\overline{\Phi_k}=-m_k\overline{a_k}$, and its projection to axes of coordinates $\Phi_{kx}=-m_k\ddot{x}_k$, $\Phi_{ky}=-m_k\ddot{y}_k$, $\Phi_{kz}=-m_k\ddot{z}_k$, then the equations can be presented in the following form:

$$\sum_{k=1}^{n}(\overline{F_k}-m_k\overline{a_k})\cdot\delta\overline{r_k}=0,$$

$$\sum_{k=1}^{n}\left[(F_{kx}-m_k\ddot{x}_k)\delta x_k+(F_{ky}-m_k\ddot{y}_k)\delta y_k+(F_{kz}-m_k z_k)\delta\ddot{z}_k\right]=0.$$

2. TASKS TO CALCULATED GRAPHIC WORK

On the schemes given below, options of mechanical systems are given. Bodies of systems can move in the vertical plane under the influence of forces of weight, elastic forces of springs, friction forces (sliding and rolling friction) and the set active forces. Threads are considered as weightless and inextensible, their inclination is identical with an inclination of the corresponding basic planes. Rolling of bodies happens without slipping. All schemes need to be added with a spring of the set rigidity ***c*** which one end is fixed on a body ***1***, and the second fastens to the motionless surface located at some distance before this body.

The condition of system at $t<0$ is a condition of static balance. It is provided with action of forces of weight, friction and spring elastic force. At $t\geq 0$ the active force of F^a is applied to a body ***1*** and its direction of action matches the direction of displacement of this body specified on schemes $S_1 \leq S \leq S_2$, and the values of force depends on the reached displacement. (***S*** - intermediate position of a trajectory of the displacement of a body ***1***; S_1 – initial position of a body ***1*** at $t=0$; S_2 – final position).

By means of Dalamber-Lagrange's principle to define the law of the displacement of a load 1. To neglect the mass of threads and elastic elements. To consider flexible threads inextensible. Swing of the skating rink representing the uniform cylinder happens without sliding. Numbers of schemes get out of the figure 1. To accept numerical data on table 1.

Explanations to designations and numerical data:

m_1, m_2, m_3, m_4 – the mass of bodies *1-4* expressed through a certain weight m,

R, r – radiuses of circles of wheels (indexes indicate the corresponding body),

i_2, i_3 – radiuses of gyration of the bodies with respect to (w.r.t) axis passing through their centers of masses (if radiuses of inertia of a body aren't set, then it is considered a uniform disk);

α and β – angles of the planes inclination,

f and δ – friction coefficients of sliding and rolling (respectively).

Masses is set in kilograms, the linear sizes – in meters, angles – in radians.

The mass of bodies are accepted on formulas

$$m_1=K_{m1}\cdot m,\quad m_2=K_{m2}\cdot m,\quad m_3=K_{m3}\cdot m,\quad m_4=K_{m4}\cdot m.$$

Radiuses of wheels $R_2=0.30$ ***m***, $R_3=0.10$ ***m*** (if there is no instruction on the scheme),

Radiuses of gyration $i_2=0.20$ ***m***, $i_3=0.15$ ***m***.

Friction coefficient of sliding $f=0.2$, rolling friction coefficient $\delta=0.25\cdot10^{-2}$ ***m***.

Angles $\alpha=K_\alpha\cdot\pi/12$ and $\beta=K_\beta\cdot\pi/12$.

To accept spring constant on a formula $c=K_c\cdot m_1 g/L$, where $g=9.81\ m/s^2$, $L=1.0$ ***m***.

All numerical coefficients ($K_{m1},\ldots, K_C$) and dependence of $F^u(S)$ are specified by the teacher at delivery of a task, for example, as it is specified in the table 1.

Table 1 - Table of basic data

№ one by one	2	3	4
K_{m1}	1.0	1.0	1.0
K_{m2}	2.0	3.0	1.0
K_{m3}	2.0	1.0	2.0
K_{m4}	1.0	1.0	1.0
K_α	3.0	2.0	4.0
K_β	3.0	4.0	2.0
K_c	2.0	3.0	1.0
$F^u(S)$	$Mg\cdot Sin(\pi S/2S_2)$	$Mg\cdot S/S_2$	$Mg\cdot(S/S_2)^2$
M	$3.0\cdot m$	$3.0\cdot m$	$3.0\cdot m$
S_2	0.5	0.5	0.5

Schemes of mechanisms are shown (figure 1).

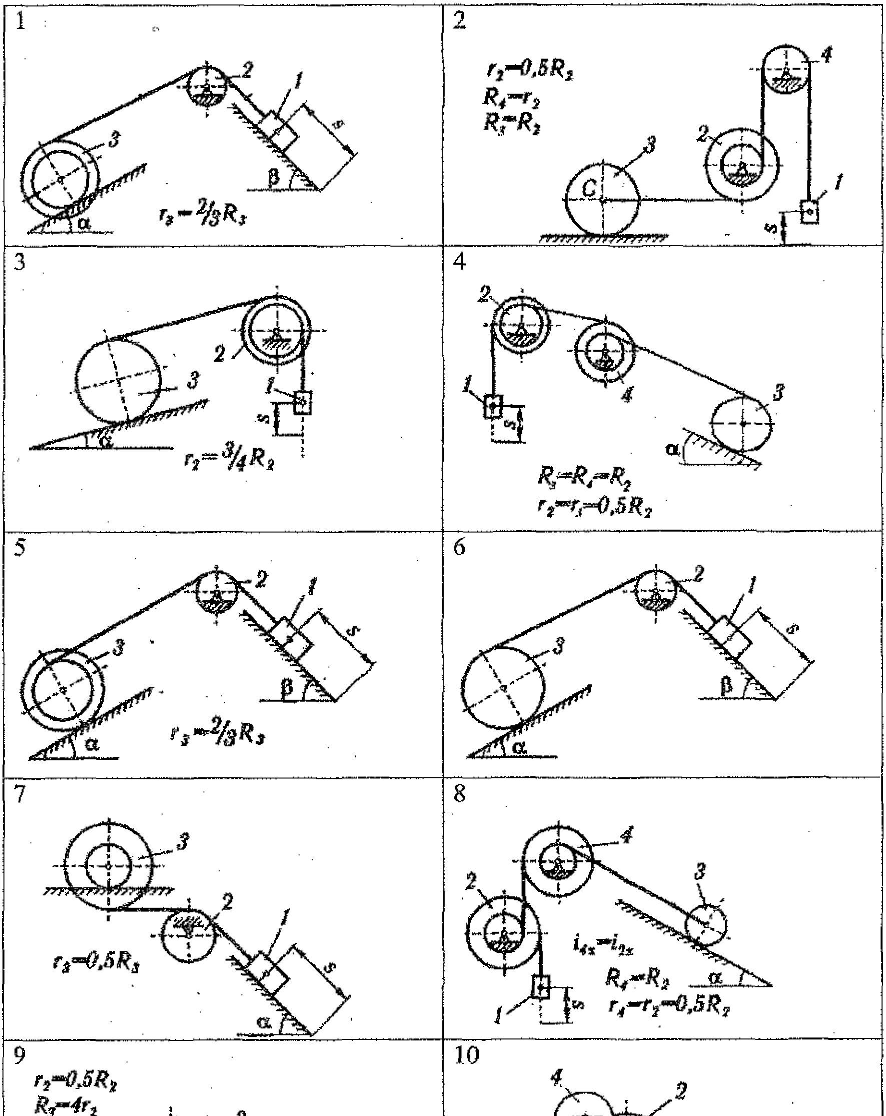

Figure 1 – Schemes of tasks by options

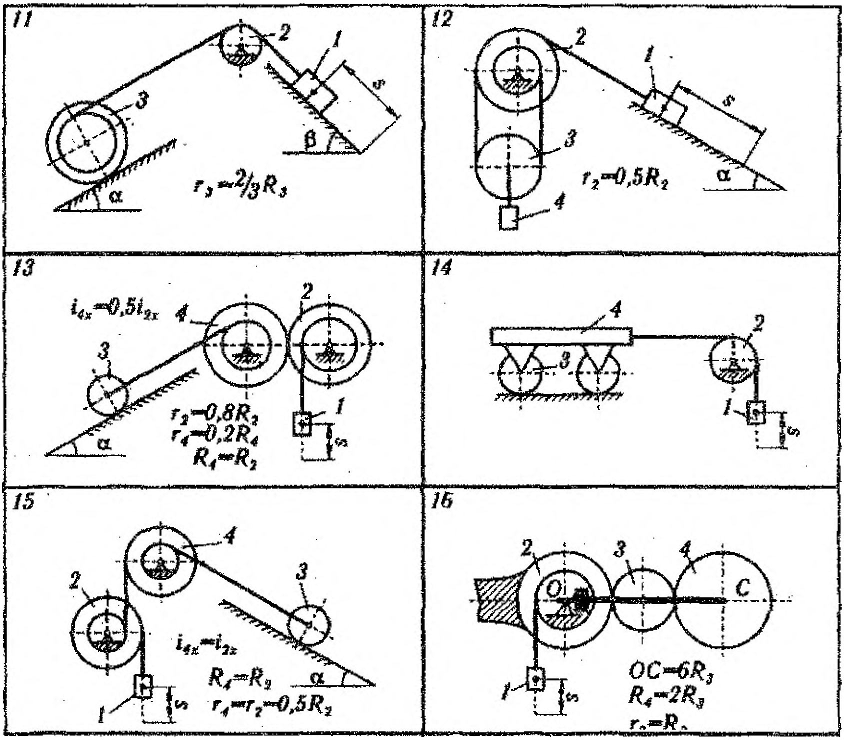

Figure 1 – Schemes of tasks by options

Work is performed in the sequence:

- to show the scheme of the mechanism, supplemented by a spring;
- show all existing in the system of external forces and moments;
- to make calculated schemes and to define static deformation of a spring ***;
- to show system in any position (at $S_1 < S < S_2$);
- to find virtual work of the applied forces;
- to find acceleration of the first body, to carry out calculations for $S = S_2$.

*** the static deformation of a spring is allowed to define by the principle of virtual displacement (Lagrange's principle).

Example:

Basic data: $K_{m1} = 3$, $K_{m2} = 1$, $K_{m3} = 1$, $K_{m4} = 2$, $K_\alpha = 4$, $K_c = 1$,

$M = 3 \cdot m$, $S_2 = 0{,}5$. $F^n(S) = M \cdot g \cdot e^{(\frac{S}{S_2})} = 3 \cdot m \cdot g \cdot e^{(\frac{S}{0.5})} = 3 \cdot g \cdot m \cdot e^{2,0 \cdot S}$.

The mass of bodies are accepted on formulas:

$m_1 = K_{m1} \cdot m = 3m$ [kg], $m_2 = K_{m2} \cdot m = m$ [kg],

$m_3 = K_{m3} \cdot m = m$ [kg], $m_4 = K_{m4} \cdot m = 2m$ [kg].

Radiuses of wheels $R_2 = 0{,}3$ m, ($R_3 = R_2 = 0{,}3\ m$).

Inertia gyration $i_2 = 0{,}2\ m$

Rolling friction coefficient $\delta = 0{,}5 \cdot 10^{-2} m$.

Angle $\alpha = K_\alpha \cdot \frac{\pi}{12} = 4 \frac{\pi}{12} = \frac{\pi}{3}$.

To accept rigidity of spring (constant) on a formula:

$c = K_c \cdot m_1 \cdot \frac{g}{L} = 3 \cdot m \cdot \frac{g}{1} = 3mg$.

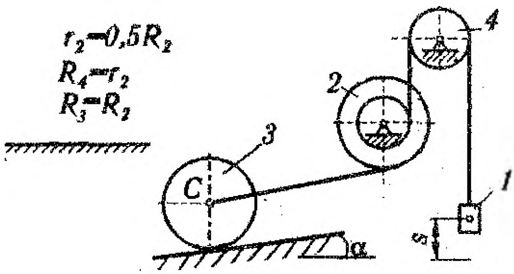

Figure 2 – The original scheme

Decision:

We will represent the scheme of the mechanism added with a spring, and we will show all external forth factors existing in system.

We will consider the movement of the unchangeable mechanical system consisting of the bodies 1, 2, 3 connected by threads. The mechanical system has one degree of freedom. The constraint reaction imposed on this system – ideal. For definition of the law of the displacement of a load 1 it is applicable the general equation of dynamics:

$$\sum \delta A_k^a + \sum \delta A_k^\Phi = 0, \qquad (1)$$

where $\sum \delta A_k{}^a$ – the sum of possible (virtual) works of active forces on virtual displacement of system, $\sum \delta A_k{}^\Phi$ – the sum of virtual works of forces of inertia.

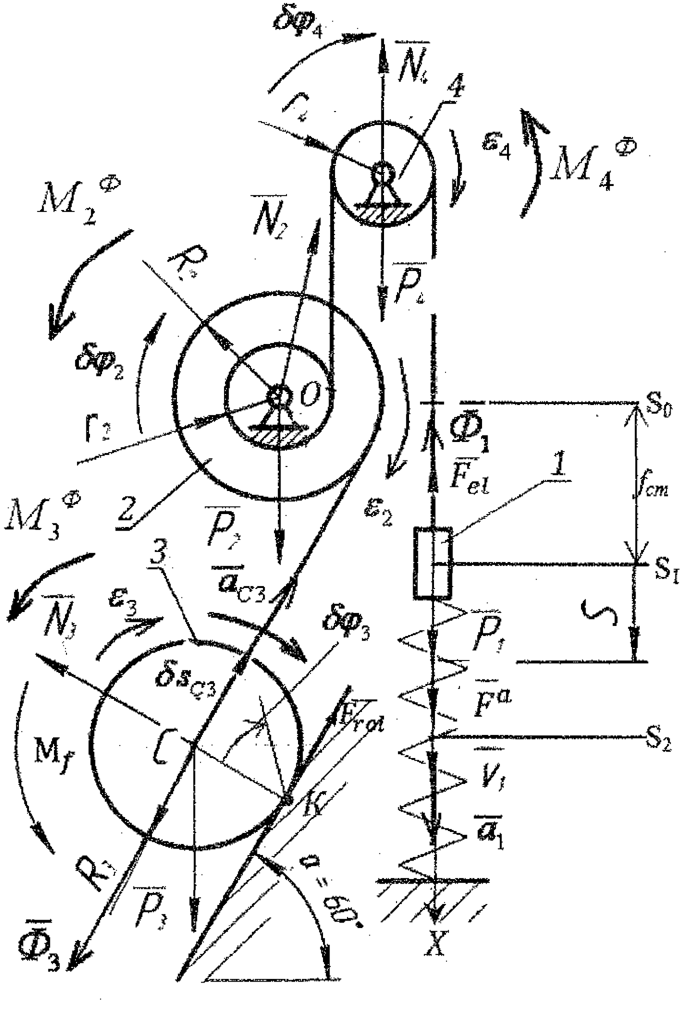

Figure 3 – Rated scheme of the mechanism

Where F_{rol} – rolling friction; M_f – the moment from rolling friction; F_{el} – elastic force; S_0 – position of a body 1 at not deformed spring, i.e. a body 1 is kept by some external force.

Having set by the direction of acceleration $\bar{a}_1$, we represent on the drawing of force of inertia $\bar{\Phi}_1, \bar{\Phi}_3$, and pairs of forces of inertia with the moments M_2^{Φ}, M_3^{Φ} и M_4^{Φ}, and which sizes are equal:

$$\Phi_1 = m_1 a_1, \qquad \Phi_3 = m_3 a_{C3},$$

$$M_2^{\Phi} = I_2 \varepsilon_2 = m_2 i_2^2 \varepsilon_2, \quad M_3^{\Phi} = I_{C3}\varepsilon_3 = \frac{m_3 R_3^2}{2}\varepsilon_3, \quad M_4^{\Phi} = I_4 \varepsilon_4 = \frac{m_4 r_4^2}{2}\varepsilon_4. \tag{2}$$

We will report to system virtual displacement and we will find virtual work of active forces as the sum of possible works:

$$\sum \delta A_k^a = \delta A(\overline{P}_1) + \delta A(\overline{P}_3) + \delta A(\overline{F}_{el}) + \delta A(\overline{F}^a) + \delta A(M_f),$$

where: $\delta A(\overline{P}_1) = P_1 \cdot \delta s_1$,

$\delta A(\overline{F}_{el}) = -F_{el} \cdot \delta s_1$,

$\delta A(\overline{F}_a) = F_a \cdot \delta s_1$,

$\delta A(M_f) = -M_f \cdot \delta\varphi_3, \quad M_f = N_3 \cdot \delta = P_3 \cos\alpha \cdot \delta = mg\cos\alpha \cdot \delta$.

We will express all movements through δs_1, and accelerations through a_1:

$$\begin{aligned}
&\delta\varphi_4 = \frac{\delta s_1}{r_4}, && \varepsilon_4 = \frac{a_1}{r_4},\\
&\delta\varphi_2 = \frac{\delta s_1}{r_2}, && \varepsilon_2 = \frac{a_1}{r_2},\\
&\delta s_{C3} = \delta\varphi_2 \cdot R_2 = \frac{\delta s_1}{r_2}\cdot R_2, && a_{C3} = \varepsilon_2 \cdot R_2 = \frac{a_1}{r_2}\cdot R_2\\
&\delta\varphi_3 = \frac{\delta s_{C3}}{R_3} = \frac{R_2}{R_3}\frac{\delta s_1}{r_2}. && \varepsilon_3 = \frac{a_{C3}}{R_3} = \frac{R_2}{R_3}\frac{a_1}{r_2}.
\end{aligned} \tag{4}$$

$$\begin{aligned}
\sum \delta A_k^a &= m_1 g \cdot \delta s_1 - F_{el}\cdot \delta s_1 + F^a \cdot \delta s_1 - m_3 g \cdot \delta s_{C3}\cdot \sin\alpha - M_f \cdot \delta\varphi_3 =\\
&= m_1 g \cdot \delta s_1 - F_{el}\cdot\delta s_1 + F^a \cdot \delta s_1 - m_3 g \cdot \frac{\delta s_1}{r_2}\cdot R_2 \cdot \sin\alpha - M_f \cdot \frac{R_2}{R_3}\frac{\delta s_1}{r_2} =\\
&= \left(m_1 g - F_{el} + F^a - m_3 g \cdot \frac{R_2}{r_2}\cdot \sin\alpha - M_f \cdot \frac{R_2}{r_2 \cdot R_3}\right)\cdot \delta s_1.
\end{aligned} \tag{5}$$

Elastic force of a spring is proportional to its deformation:

$$F_{el} = c\cdot(f_{st} + s_{C3}) = c\cdot(0,42 + s_1), \tag{6}$$

where $f_{st} = 0,42\,m$ – it is defined in the previous calculated and graphic work [2].

Then expression (5) taking into account basic data will take a form:

$$\sum \delta A_k^a = \left(3mg - 3mg \cdot (0,42 + s_1) + 3mg \cdot e^{2s_1} - mg \cdot \frac{R_2}{r_2} \cdot \sin\alpha - \right.$$

$$\left. -mg\cos\alpha \cdot \delta \cdot \frac{R_2}{r_2 \cdot R_3} \right) \cdot \delta s_1 = mg \cdot \left(3 - 3 \cdot (0,42 + s_1) + 3 \cdot e^{2s_1} - \frac{R_2}{r_2} \cdot \sin\alpha - \right. \tag{7}$$

$$\left. -\cos\alpha \cdot \delta \cdot \frac{R_2}{r_2 \cdot R_3} \right) \cdot \delta s_1.$$

We will find the sum of virtual works of forces of inertia:

$$\sum \delta A_k^{\Phi} = -\Phi_1 \cdot \delta s_1 - M_2^{\Phi} \cdot \delta\varphi_2 - M_4^{\Phi} \cdot \delta\varphi_4 - M_3^{\Phi} \cdot \delta\varphi_3 - \Phi_3 \cdot \delta s_{C3} =$$

$$= -m_1 a_1 \cdot \delta s_1 - m_2 i_2^{\,2} \varepsilon_2 \cdot \delta\varphi_2 - \frac{m_3 R_3^{\,2}}{2} \varepsilon_3 \cdot \delta\varphi_3 - \frac{m_4 r_4^{\,2}}{2} \varepsilon_4 \cdot \delta\varphi_4 - m_3 a_{C3} \cdot \delta s_{C3} =$$

$$= -m_1 a_1 \cdot \delta s_1 - m_2 i_2^{\,2} \varepsilon_2 \cdot \frac{\delta s_1}{r_2} - \frac{m_3 R_3^{\,2}}{2} \varepsilon_3 \cdot \frac{R_2}{R_3} \frac{\delta s_1}{r_2} - \frac{m_4 r_4^{\,2}}{2} \varepsilon_4 \cdot \frac{\delta s_1}{r_4} -$$

$$-m_3 a_{C3} \cdot \frac{\delta s_1}{r_2} \cdot R_2 = -m_1 a_1 \cdot \delta s_1 - m_2 i_2^{\,2} \cdot \frac{a_1}{r_2} \cdot \frac{\delta s_1}{r_2} - \frac{m_3 R_3^{\,2}}{2} \frac{R_2}{R_3} \frac{a_1}{r_2} \cdot \frac{R_2}{R_3} \frac{\delta s_1}{r_2} -$$

$$-\frac{m_4 r_4^{\,2}}{2} \cdot \frac{a_1}{r_4} \cdot \frac{\delta s_1}{r_4} - m_3 \cdot \frac{a_1}{r_2} \cdot R_2 \cdot \frac{\delta s_1}{r_2} \cdot R_2 =$$

$$= \left(-m_1 - m_2 \frac{i_2^{\,2}}{r_2^2} - \frac{m_3}{2} \frac{R_2^2}{r_2^2} - \frac{m_4}{2} - m_3 \frac{R_2^2}{r_2^2} \right) a_1 \cdot \delta s_1 =$$

$$= \left(-3 - \frac{i_2^{\,2}}{r_2^2} - \frac{3}{2} \frac{R_2^2}{r_2^2} - 1 \right) m a_1 \cdot \delta s_1. \tag{8}$$

We work out the equation (1):

$$mg \cdot \left(3 - 3 \cdot (0,42 + s_1) + 3 \cdot e^{2s_1} - \frac{R_2}{r_2} \cdot \sin\alpha - \cos\alpha \cdot \delta \cdot \frac{R_2}{r_2 \cdot R_3} \right) \cdot \delta s_1 +$$

$$+ \left(-4 - \frac{i_2^{\,2}}{r_2^2} - \frac{3}{2} \frac{R_2^2}{r_2^2} \right) m a_1 \cdot \delta s_1 = 0$$

Having separated on $\delta s \neq 0$, we will receive:

$$g \cdot \left(3 - 3 \cdot (0,42 + s_1) + 3 \cdot e^{2s_1} - \frac{R_2}{r_2} \cdot \sin\alpha - \cos\alpha \cdot \delta \cdot \frac{R_2}{r_2 \cdot R_3} \right) +$$

$$+ \left(-4 - \frac{i_2^{\,2}}{r_2^2} - \frac{3}{2} \frac{R_2^2}{r_2^2} \right) a_1 = 0$$

from where:

$$a_1 = \frac{g \cdot \left(3 - 3 \cdot (0{,}42 + s_1) + 3 \cdot e^{2s_1} - \frac{R_2}{r_2} \cdot \sin\alpha - \cos\alpha \cdot \delta \cdot \frac{R_2}{r_2 \cdot R_3}\right)}{4 + \frac{i_2^{\ 2}}{r_2^2} + \frac{3}{2}\frac{R_2^2}{r_2^2}}.$$

We substitute numerical values:

$$a_1 = \frac{g \cdot \left(3 - 3 \cdot (0{,}42 + s_1) + 3 \cdot e^{2s_1} - \frac{0{,}3}{0{,}15} \cdot \sin 60^0 - \cos 60^0 \cdot 0{,}005 \cdot \frac{0{,}3}{0{,}15 \cdot 0{,}3}\right)}{4 + \frac{0{,}2^2}{0{,}15^2} + \frac{3}{2}\frac{0{,}3^2}{0{,}15^2}} =$$

$$= \frac{g \cdot (1{,}74 - 3 \cdot s_1 + 3 \cdot e^{2s_1} - 1{,}732 - 0{,}0165)}{4 + 1{,}78 + 6} =$$

$$= \frac{g \cdot (0{,}0085 - 3 \cdot s_1 + 3 \cdot e^{2s_1})}{11{,}78} = -0{,}007 - 2{,}49 \cdot s_1 + 2{,}49 \cdot e^{2s_1}.$$

We build the schedule of dependence of ***$a_1(s)$***.

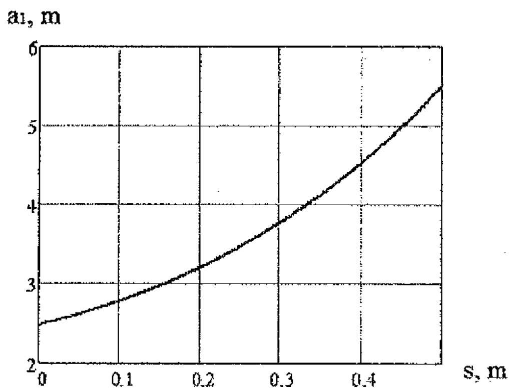

Figure 4 – The schedule of dependence of acceleration of a load 1 from coordinate

LITERATURE

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