GENERALIZATION OF RUSPINI'S LIKENESS RELATION IN CASES OF SUBNORMAL RELATIONS

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Abstract

The note deals in a preliminary way with the problem of a generalization of a likeness relation. Preliminary results are considered. Definition of generalized likeness relation is proposed. Some preliminary conclusions are made.

Keywords: clustering, fuzzy relation, projection.

1. Introduction

A fuzzy approach to clustering is very effectual, because majority relationships between real objects are fuzzy and initial structure of the set of objects is fuzzy also.

One of the fundamental problems in fuzzy cluster analysis is a relationship between the initial structure of the set of objects and its representation in clustering techniques. As a result of the consideration of this problem is a likeness relation, which was proposed by Ruspini (1982). The main purpose of this theoretical note is generalization of the likeness relation for feeble similarity relations cases.

In part 2 of this paper fuzzy similarity relations and some their properties are considered. In part 3 of the paper the likeness relation is described and its generalization are made. In part 4 some concluding remarks and the outlook for the investigations are discussed.

2. Fuzzy similarity relations

The likeness relation was proposed by Ruspini on a basic of fuzzy similarity relation. Let's consider a few kinds of fuzzy similarity relations.

In general, Kaufmann (1975) defined a fuzzy similarity relation S as a binary fuzzy intransitive relation on X, which possesses the symmetric property

$$\mu_S(x,y) = \mu_S(y,x), \forall x, y \in X, \quad (1)$$

and the reflexivity property

$$\mu_S(x,x) = 1, \forall x \in X, \quad (2)$$

where μ_S is a membership function of the fuzzy relation and X is an initial set of elements. We will call this kind of fuzzy similarity relations the usual similarity relation and indicate S_2.

Viattchenin (1997) proposed a feeble similarity relation which we will indicate S_1, and a powerful similarity relation which we will indicate S_3. A feeble similarity relation was defined as a binary fuzzy intransitive relation on X, which possesses the symmetric property (1) and the feeble reflexivity property:

$$\mu_S(x,y) \le \mu_S(x,x), \forall x, y \in X. \quad (3)$$

The feeble similarity relation which possesses the strict inequality condition in (3):

$$\mu_S(x,y) < \mu_S(x,x), \forall x, y \in X \quad (4)$$

is the strict feeble similarity relation. We will indicate this kind of fuzzy similarity relations S_0.

The feeble similarity relation is a subnormal relation, that is $\operatorname{Pr}oj(S_1) < 1$, where $\operatorname{Pr}oj(S_1)$ is the global projection of feeble similarity relation, if strictly restriction condition is met, that was demonstrated by Viattchenin (1998). A condition

$$\mu_{S_1}(x,x) < 1, \forall x \in X \quad (5)$$

is strictly restriction condition for a feeble similarity relation.

A powerful similarity relation was defined as a binary fuzzy intransitive relation on X, which possesses the symmetric property (1) and a powerful reflexivity property. A fuzzy relation possesses the powerful reflexivity property, if the condition

$$\mu_S(x,y) < 1, \forall x, y \in X, x \ne y \quad (6)$$

is met together with condition (2).

From definitions of fuzzy similarity relations a next condition follows: if S_0, S_1, S_2, S_3 are fuzzy similarity relations on X, where X is an initial set of elements, then for one and the same pairs $(x, y) \in X \times X$ a next relationship

$$S_0 \subseteq S_1 \subseteq S_3 \subseteq S_2 \qquad (7)$$

is met. This condition is obvious and follows from conditions (2), (3), (4), (6) and a definition of an inclusion relationship.

From this condition follows, that the usual similarity relation is most general relation for all fuzzy similarity relations on initial set X.

3. The likeness relation and its generalization

Let's consider the likeness relation now. Ruspini defined the likeness relation as follow.

Let S be some fuzzy similarity relation on X. If an ineqality

$$|\mu_S(x,y) - \mu_S(x,z)| \le 1 - \mu_S(y,z), \qquad (8)$$

is met for all $x, y, z \in X$, then S is a likeness relation on X. We will indicate this relation by L symbol.

One in the right component of the inequality (8) is a significance of maximum of membership function of a fuzzy similarity relation, because we can not limit a connection between different elements of X.

We are considering preliminary results as very important properties of fuzzy similarity relations. In particular, properties of projections of fuzzy similarity relations are a basic for generalization of the likeness relation, because clustering relation may be a subnormal relation. In a case of S_0 or S_1 a significance of maximum of membership function of a fuzzy similarity relation is the global projection of feeble similarity relation.

We can generalize the likeness relation from this point of view. This generalization is met for S_0 and S_1 only, if strictly restriction condition (5) is met.

Let S is S_0 or S_1 be some fuzzy similarity relation on X. If an ineqality

$$|\mu_S(x,y) - \mu_S(x,z)| \le \mathrm{Proj}(S) - \mu_S(y,z), \qquad (9)$$

is met for all $x, y, z \in X$, then S is generalized likeness relation on X. We will indicate this relation by L_g symbol.

If strictly restriction condition (5) isn't met for S and S is a normal fuzzy similarity relation, then $\mathrm{Proj}(S) = 1$. In this case we will consider the likeness relation (8).

Proposition 1. If L_g and L are determined for one and the same $x, y, z \in X$ then a next condition

$$L_g \subseteq L \qquad (10)$$

is met.

Proof. The proof is obvious and follows from (8), (9) and a definition of an inclusion relationship.

Q.E.D.

A next property of L_g is very important for fuzzy clustering.

Proposition 2. If L_g is a generalized likeness relation on X then its addition $D_g = 1 - L_g$ is limited pseudometrics on X.

Proof. The proof follows from a proposition 1 and analogous property of the likeness relation L.

Q.E.D.

4. Summary

We can make some conclusions. Results of investigations are show that feeble similarity relations and strict feeble similarity relations are very interesting and perspective for future investigations, because these relations are soft tools for data representation in fuzzy clustering. We can generalize the likeness relation on a basic of these types of fuzzy similarity relations. However, these results originate some problems, which be required consider for further elaboration of the new methods of fuzzy clustering.

In the first place, a next problem arises. We can normalize fuzzy similarity relation, but this operation can violate the nature of initial data structure sometimes. That's why a problem of normalization of subnormal similarity relation requires special investigation also.

Secondly, the results are show that we must consider is a relationship between generalized likeness relation and its formal representation in fuzzy logic. This problem is require of special investigation, because the problem is very important from theoretical point of view.

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