# ON THE DETECTING OF SYMMETRIES OF SWITCHING FUNCTIONS FOR EFFICIENT DESIGN OF DECISION TREES AND DECISION DIAGRAMS*

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**ABSTRACT**

*This paper presents an approach to detect symmetries of switching functions for efficient design of Reed-Muller decision trees (DTs) and decision diagrams (DDs). The information theory measures of switching functions are used to determine possible symmetric variables and therefore to reduce search space. This approach allows to detect different types of symmetries of any variables coincident. We consider the technique to apply symmetries properties that can significantly improve Reed-Muller DT or DD design. We implement symmetry detection algorithm as a part of program for switching function minimization based on DT design. Experiments have been performed on MCNC benchmarks and the results verify the efficiency of our approach.*

**KEY WORDS**: Switching functions, symmetry, decision trees and diagrams, information theory measures

## 1. INTRODUCTION

Determining symmetries among groups of variables is important in problems of logic synthesis [5], design verification and testing [7], and in problems of technology mapping, i.e. Boolean matching [10, 18]. The effectiveness of matching procedure can be increased if the groups of symmetric variables are known.

The symmetry properties are used in different areas of logic design. There are well known methods of circuit design, decomposition and minimization [4, 6, 15, 16]. In our investigation we focus on the detection of symmetries for Reed-Muller DT and DD design and further application for switching function minimization.

There are several techniques to recognize symmetries based on different principles, namely,

(i) manipulation of a truth table and truth column vector developed in this paper;
(ii) transformation of the given function into spectral domain;
(iii) formal representation of symmetric functions (decision trees and diagrams, Reed-Muller forms, etc.).

The well known algorithms explore properties of symmetries via manipulation of the truth tables. For example, in [17] an effective method to detect different types of symmetries based on numerical methods has been proposed.

The second direction exploits features of spectra to determine the symmetries in variables for given function. There are many results on detecting symmetries in Hadamard, Haar and other transform bases [5]. However, spectral coefficients are very expensive to compute and store for functions with large number of variables.

Formal representation of symmetric switching functions in positive polarity Reed-Muller (PPRM) expressions are studied for recognition symmetries in [1]. Authors use an additional program to obtain PPRM expression.

In recent years binary decision diagrams have been used as an efficient data structure to store

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functions, and symmetry detection has become feasible for functions with large number of variables [2, 8, 9]. Fast detection of symmetric variables is important for DTs and DDs design.

In our approach we consider switching function represented by truth tables or truth column vectors and use symmetric properties for efficient trees and diagrams design.

The paper is organized as follows. Section 2 outlines background of investigation and presents necessary definitions and notations. Section 3 presents an approach to detect different types of symmetries. Section 4 outlines algorithm to detect symmetries *InfoRECSym* and gives principles of Reed-Muller DTs and DDs design for symmetric switching functions. Section 5 presents experimental results and Section 6 concludes the paper.

## 2. BACKGROUND

Let give some essential definitions and prepositions that are important for the understanding the paper.

We use the following notations:

| | |
|---|---|
| $X = \{x_1, x_2, ..., x_n\}$ | a set of variables |
| $f(x_1, x_2, ..., x_n)$ or $f$ | a switching function |
| $f_x$, $f_{\bar{x}}$ | cofactors of $f$ with respect to arbitrary variable $x$. |
| $\{x_i, x_j\}$ | a *symmetric pair* |
| $\Omega$ | a set of expansion types |
| $H(f)$ | *entropy* of switching function $f$ |
| $H(f \mid x)$ | *conditional entropy* of function $f$ with respect to variable $x$ |

Switching function $f$ can be represent as *positive Davio (pD) expansion*

$$f = f_{\bar{x}} \oplus x(f_{\bar{x}} \oplus f_x),$$

and *negative Davio (nD) expansion*

$$f = f_x \oplus \bar{x}(f_{\bar{x}} \oplus f_x).$$

We consider Reed-Muller DT or DD as directed acyclic graph. Each node is labeled with possible expansion $\omega$ with respect to arbitrary variable $x$. A couple $(x, \omega)$ is assigned to a node, where $x \in X$ and $\omega \in \Omega$. For Reed-Muller DTs and DDs: $\Omega = \{pD, nD\}$. $pD$ and $nD$ nodes and cofactors of expansion are shown in Fig. 1.

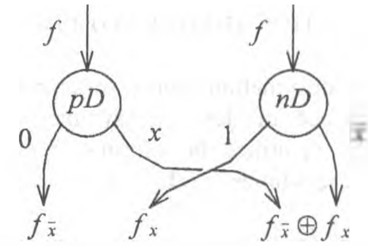


**Fig. 1. Nodes and cofactors of *pD* and *nD* expansion of switching function *f* with respect to variable *x*.**

### 2.1. SYMMETRIC SWITCHING FUNCTIONS

For any pair of variables $x_i$ and $x_j$ there are four cofactors $f_{x_i x_j}$, $f_{\bar{x}_i x_j}$, $f_{x_i \bar{x}_j}$, $f_{\bar{x}_i \bar{x}_j}$.

A function $f$ is *nonequivalent symmetric* in variables $x_i$ and $x_j$, denoted as $\{x_i, x_j\}$ or $\{\bar{x}_i, \bar{x}_j\}$, if $f$ remains invariant when this variables are interchanged: $f_{\bar{x}_i x_j} = f_{x_i \bar{x}_j}$ [3].

A function $f$ is *equivalent symmetric* in variables $x_i$ and $x_j$, if it remains invariant when $x_i$ and $\bar{x}_j$ ($\bar{x}_i$ and $x_j$) are interchanged: $f_{x_i x_j} = f_{\bar{x}_i \bar{x}_j}$ [3]. This type of symmetry is denoted by $\{x_i, \bar{x}_j\}$ or $\{\bar{x}_i, x_j\}$.

If function $f$ is simultaneously nonequivalent and equivalent symmetric in $x_i$ and $x_j$, then $f$ is *multiform symmetric* [3].

**Example 1.**

(i) Function $f = x_2 \oplus x_3 \oplus x_2 x_3 \oplus x_1 \cdot x_2 x_3$ is nonequivalent symmetric in $x_2$ and $x_3$; function $f = x_1 \cdot x_3 + x_1 \cdot x_2$ is nonequivalent symmetric in $x_2$ and $x_3$;

(ii) $f = \bar{x}_1 \oplus x_2 \oplus \bar{x}_1 \cdot x_2 \cdot x_3$ is equivalent symmetric in variables $\{\bar{x}_1, x_2\}$ or $\{x_1, \bar{x}_2\}$;

(iii) $f = x_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_2$ is multiform symmetric in $\{x_1, x_2\}$ ($\{\bar{x}_1, \bar{x}_2\}$) and $\{x_1, \bar{x}_2\}$ ($\{\bar{x}_1, x_2\}$).

A function $f$ is *partially symmetric* with respect to $X_i \subset X$, if any permutation of variables in $X_i$ leaves $f$ unchanged.

A function $f$ is *totally symmetric* if every pair of variables in the function is either nonequivalent or equivalent symmetric.

**Example 2.**

(i) Function $f = \bar{x}_1 \oplus x_2 \oplus \bar{x}_1 \cdot x_2 \cdot x_3$ is partially symmetric in variables $\{\bar{x}_1, x_2\}$ or $\{x_1, \bar{x}_2\}$.

(ii) $f = x_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_2$ and $f = x_1 \cdot x_2 \oplus x_1 \cdot x_3 \oplus x_2 \cdot x_3$ are totally symmetric functions.

### 2.2. INFORMATION THEORY NOTATIONS

We use information theory measures to reduce the search space of detecting symmetric variables pairs. Besides we utilize the information measures for efficient Reed-Muller DTs and DDs design [11, 12, 14].

In order to quantify the information content revealed by the outcome for finite field of events with probabilities distribution, Shannon introduced the concept of entropy. *Entropy* of switching function $f$ is given by [13]:

$$H(f) = -p_{|f=0} \cdot \log_2 p_{|f=0} - p_{|f=1} \cdot \log_2 p_{|f=1}. \quad (1)$$

We calculate probabilities $p_{|f=b} = k_{|f=b} / k$, where $k_{|f=b}$ is the number of assignments of values to variables (patterns) for which $f = b$, and $k$ is the total number of assignments.

**Example 3.** Let us calculate entropy of switching function $f$ given by truth column vector [1100000111000010]: $H(f) = -{}^{6}/_{16} \cdot \log_2 {}^{6}/_{16} - {}^{10}/_{16} \cdot \log_2 {}^{10}/_{16} = 0.95$ bit/pattern.

We consider the process of DT or DD design as recursive decomposition of switching function. A step of this recursive decomposition corresponds to the expansion of switching function $f$ with respect to variable $x$. Assume that variable $x$ of function $f$ carries information that is, in some sense, the rate of influence of the input variable to output, $f$.

For positive Davio and negative Davio expansion we use conditional entropy $H^{\omega}(f|x)$ as information measure [14]:

$$H^{pD}(f|x) = p_{|x=0} \cdot H(f_{\bar{x}}) + p_{|x=1} \cdot H(f_{\bar{x}} \oplus f_x), \quad (2)$$

$$H^{nD}(f|x) = p_{|x=1} \cdot H(f_x) + p_{|x=0} \cdot H(f_{\bar{x}} \oplus f_x). \quad (3)$$

Thus, in proposed approach we consider the following tasks, that will be partially solved by information theoretic measures:

1. *How can we determine different types of symmetries in variables for a given switching function f?*
2. *How can we use symmetric properties for efficient design of Reed-Muller DTs and DDs?*

## 3. DETECTION OF SYMMETRIES

In this section we focus on detecting different types of symmetries (nonequivalent, equivalent, multiform, totally symmetry) by information measures, presented in subsection 2.2.

### 3.1. DETECTION OF NONEQUIVALENT SYMMETRY

**Statement 1.** Switching function $f$ is nonequivalent symmetric in $\{ x_i, x_j \}$ or $\{ \bar{x}_i, \bar{x}_j \}$ if

$$f_{\bar{x}_i} \oplus f_{x_i} = f_{\bar{x}_j} \oplus f_{x_j} \text{ and } f_{\bar{x}_i} = f_{\bar{x}_j}, f_{x_i} = f_{x_j}.$$

**Proof.** It is easy to show that $f_{x_i} = f_{x_i x_j} + f_{x_i \bar{x}_j}$ and $f_{x_j} = f_{x_i x_j} + f_{\bar{x}_i x_j}$, hear '+' means union of cofactors. The nonequivalent symmetry condition $f_{\bar{x}_i x_j} = f_{x_i \bar{x}_j}$ implies $f_{x_i} = f_{x_j}$. Similarly, $f_{\bar{x}_i} = f_{\bar{x}_j}$. Hence, we can write $f_{x_i} \oplus f_{\bar{x}_i} = f_{x_j} \oplus f_{\bar{x}_j}$. ∎

DT or DD nodes with nonequivalent symmetric variables are assigned together as primitives (Fig.2).

**Property 1.** If switching function $f$ is nonequivalent symmetric in $x_i$ and $x_j$ then

$$H^{pD}(f|x_i) = H^{pD}(f|x_j) \text{ and}$$
$$H^{nD}(f|x_i) = H^{nD}(f|x_j).$$

This property of entropy equality is necessary but not enough to detect symmetry.

**Example 4.** Consider switching function $f$ of four variables that correspond to truth column vector [1100000111000010].

The cofactors for this function presented in Table 1. The information measures for $pD$ and $nD$ expansion are given in Table 2. According property 1 we analyze pairs with equal information measures. Nonequivalent symmetries in $\{ x_2, x_3 \}$ and in $\{ x_1, x_4 \}$ are possible.

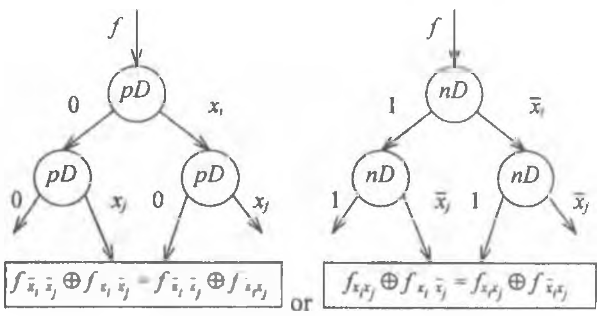


**Fig. 2. Nodes of Reed-Muller DT or DD that correspond to nonequivalent symmetry.**

**Table 1. The cofactors to detect symmetries (Example 4).**

| | $f_{\bar{x}}$ | $f_x$ | $f_{\bar{x}} \oplus f_x$ |
|---|---|---|---|
| $x_1$ | [11000001] | [11000010] | [00000011] |
| $x_2$ | [11001100] | [00010010] | [11011110] |
| $x_3$ | [11001100] | [00010010] | [11011110] |
| $x_4$ | [10001001] | [10011000] | [00010001] |

**Table 2. The information measures (in bit/pattern) for positive and negative Davio expansion (Example 4).**

| | $H^{pD}(f\mid x)$ | $H^{nD}(f\mid x)$ |
|---|---|---|
| $x_1$ | *0.88* | *0.88* |
| $x_2$ | 0.91 | **0.81** |
| $x_3$ | 0.91 | **0.81** |
| $x_4$ | *0.88* | *0.88* |

The function $f$ is nonequivalent symmetric in $\{x_2, x_3\}$: $f_{\bar{x}_2} \oplus f_{x_2} = f_{\bar{x}_3} \oplus f_{x_3}$ and $f_{\bar{x}_2} = f_{\bar{x}_3}$.

This function is nonequivalent symmetric in $\{x_1, x_4\}$: $f_{\bar{x}_1} \oplus f_{x_1} = f_{\bar{x}_4} \oplus f_{x_4}$ and $f_{\bar{x}_1} = f_{\bar{x}_4}$, taking into consideration the necessary permutation of variables assignments.

As a result of DT or DD design we obtain the following fixed polarity Reed-Muller expression: $f = 1 \oplus x_2 \oplus x_3 \oplus x_2 x_3 \oplus x_1 \cdot x_2 \cdot x_3 \oplus x_4 \cdot x_2 x_3$.

### 3.2. DETECTION OF EQUIVALENT SYMMETRY

**Statement 2.** Switching function $f$ is equivalent symmetric in $\{x_i, \bar{x}_j\}$ or $\{\bar{x}_i, x_j\}$ if

$$f_{\bar{x}_i} \oplus f_{x_i} = f_{\bar{x}_j} \oplus f_{x_j} \text{ and } f_{\bar{x}_i} = f_{x_j}, f_{x_i} = f_{\bar{x}_j}.$$

**Proof.** It is easy to show that $f_{x_i} = f_{x_i x_j} + f_{x_i \bar{x}_j}$ and $f_{\bar{x}_j} = f_{\bar{x}_i \bar{x}_j} + f_{x_i \bar{x}_j}$, hear '+' means union of cofactors. The equivalent symmetry condition $f_{x_i x_j} = f_{\bar{x}_i \bar{x}_j}$ implies $f_{x_i} = f_{\bar{x}_j}$. Similarly, $f_{\bar{x}_i} = f_{x_j}$. Hence, we can write $f_{x_i} \oplus f_{\bar{x}_i} = f_{x_j} \oplus f_{\bar{x}_j}$. ■

DT or DD nodes with equivalent symmetric variables are placed together (Fig.3).

**Property 2.** If switching function $f$ is equivalent symmetric in $x_i$ and $x_j$, then

$$H^{pD}(f \mid x_i) = H^{nD}(f \mid x_j) \text{ and}$$
$$H^{nD}(f \mid x_i) = H^{pD}(f \mid x_j).$$

This of entropy equality is necessary but not enough to detect symmetry.

**Example 5.** Consider switching function $f$ of three variables that correspond to truth column vector [11100011].

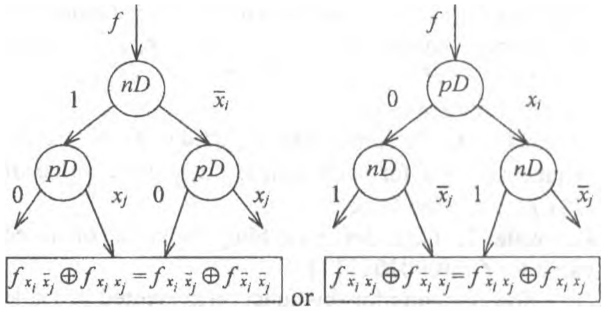


**Fig. 3. Nodes of Reed-Muller DT or DD that correspond to equivalent symmetry.**

The cofactors for this function given in Table 3. The information measures are presented in Table 5. Equivalent symmetries in $\{x_1, \bar{x}_2\}$ and in $\{x_2, \bar{x}_3\}$ are possible.

The function $f$ is equivalent symmetric in $\{x_1, \bar{x}_2\}$: $f_{\bar{x}_1} \oplus f_{x_1} = f_{\bar{x}_2} \oplus f_{x_2}$ and $f_{\bar{x}_1} = f_{x_2}$ and $f_{x_1} = f_{\bar{x}_2}$.

This function is not equivalent symmetric in $\{x_2, \bar{x}_3\}$ cause $f_{\bar{x}_2} \oplus f_{x_2} \neq f_{\bar{x}_3} \oplus f_{x_3}$.

Finally, we obtain the following fixed polarity Reed-Muller expression: $f = \bar{x}_1 \oplus x_2 \oplus \bar{x}_1 \cdot x_2 \cdot x_3$.

### 3.3. DETECTION OF MULTIFORM AND TOTALLY SYMMETRIES

**Property 3.** If switching function $f$ is multiform symmetric in $\{x_i, x_j\}$, then

$$H^{pD}(f \mid x_i) = H^{pD}(f \mid x_j) =$$

**Table 3. The cofactors to detect symmetries (Example 5).**

| | $f_{\bar{x}}$ | $f_x$ | $f_{\bar{x}} \oplus f_x$ |
|---|---|---|---|
| $x_1$ | [1110] | [0011] | [1101] |
| $x_2$ | [1100] | [1011] | [0111] |
| $x_3$ | [1101] | [1001] | [0100] |

**Table 4. The information measures (in bit/pattern) for positive and negative Davio expansion (Example 5).**

| | $H^{pD}(f\mid x)$ | $H^{nD}(f\mid x)$ |
|---|---|---|
| $x_1$ | **0.81** | 0.91 |
| $x_2$ | 0.91 | **0.81** |
| $x_3$ | **0.81** | 0.91 |

$$H^{nD}(f \mid x_i) = H^{nD}(f \mid x_j).$$

**Example 6.** (*Continue of Example 4*) The function $f$ is multiform symmetric in $\{x_1, x_4\}$, cause $H^{pD}(f \mid x_i) = H^{pD}(f \mid x_j) = H^{nD}(f \mid x_i) = H^{nD}(f \mid x_j)$ = 0.88 bit/pattern.

**Property 4.** If switching function $f$ is totally symmetric, then for each couple $(x, \omega)$ the value of $H^{\omega}(f \mid x)$ is the same.

**Example 7.** Consider switching function of three variables $f$ = [00010111].

The cofactors for this function presented in Table 5. The information measures for *pD* and *nD* expansion are given in Table 6. Information measures for all variables are equal. Function $f$ is totally symmetric: $f = x_1 x_2 \oplus x_2 x_3 \oplus x_3 x_1$.

## 4. Algorithm to detect symmetries and principles of Reed-Muller DTs or DDs design

We propose an algorithm to detect symmetries of switching functions for Reed-Muller DT design. The algorithm called *InfoRECSym* (*Infor*mation *REC*ognizer *of Sym*metries) is described in Fig.4.

We incorporate the symmetry detection algorithm *InfoRECSym* to DT or DD design by following stages.

**Stage 1.** For each variable $x$ of function $f$ calculate information measures $H^{\omega}(f \mid x)$. Select subset of couples $(x, \omega)$ for which $H^{\omega}(f \mid x) \to \min$.

**Stage 2.** Check for symmetries according symmetry detection algorithm (Fig. 4).

**Stage 3.** For each symmetry pair construct DTs or DDs primitives (Fig.2 and Fig. 3).

**Table 5. The cofactors to detect symmetries (Example 7).**

| | $f_{\bar{x}}$ | $f_x$ | $f_{\bar{x}} \oplus f_x$ |
|---|---|---|---|
| $x_1$ | [0001] | [0111] | [0110] |
| $x_2$ | [0001] | [0111] | [0110] |
| $x_3$ | [0001] | [0111] | [0110] |

**Table 6. The information measures (in bit/pattern) for positive and negative Davio expansion (Example 7).**

| | $H^{pD}(f \mid x)$ | $H^{nD}(f \mid x)$ |
|---|---|---|
| $x_1$ | **0.91** | **0.91** |
| $x_2$ | **0.91** | **0.91** |
| $x_3$ | **0.91** | **0.91** |

/* (input) $f = f(x_1, x_2, \ldots, x_n)$ */
/*(output) *symmetry pairs* $\{x_i, x_j\}$ */

***InfoRECSym***($f$) {
*for* each variable $x \in X$ {
Determine cofactors $f_0$ and $f_1$ and store into *sub_table*:

| *pD* | *nD* |
|---|---|
| $f_0 = f_{\bar{x}}$ | $f_0 = f_x$ |
| $f_1 = f_{\bar{x}} \oplus f_x$ | $f_1 = f_{\bar{x}} \oplus f_x$ |

Compute information measures $H^{\omega}(f \mid x)$ for *pD* and *nD* expansion, according (1)-(3)
}
*for* each pair $\{x_i, x_j\}$ {
*if* $H^{pD}(f \mid x_i) = H^{pD}(f \mid x_j)$ and $H^{nD}(f \mid x_i) = H^{nD}(f \mid x_j)$
*if* $f_{x_i} \oplus f_{\bar{x}_i} = f_{x_j} \oplus f_{\bar{x}_j}$ and $f_{x_i} = f_{x_j}$
*then* $f$ is nonequivalent symmetric in $x_i$ and $x_j$
*if* $H^{pD}(f \mid x_i) = H^{nD}(f \mid x_j)$ and $H^{nD}(f \mid x_i) = H^{pD}(f \mid x_j)$
*if* $f_{x_i} \oplus f_{\bar{x}_i} = f_{x_j} \oplus f_{\bar{x}_j}$ and $f_{x_i} = f_{\bar{x}_j}$
*then* $f$ is equivalent symmetric in $x_i$ and $x_j$
*if* $f$ is nonequivalent and equivalent symmetric simultaneously
*then* $f$ is multiform symmetric in $x_i$ and $x_j$
}
*if* checked pairs are either nonequivalent or equivalent symmetric then $f$ is totally symmetric function.
}

**Fig. 4. Sketch of the algorithm to detect symmetries of switching function *f*.**

**Example 8.** Let us consider the process of Reed-Muller DT design for switching function $f$ that given by truth column vector [11111001]. The cofactors and information measures are presented in Table 7 and Table 8 respectively.

**Step 1.** According information measures for DT node we assign the couple $(x_1, pD)$ (Fig. 5).

**Table 7. The cofactors (Example 8, Step 1).**

| | $f_{\bar{x}}$ | $f_x$ | $f_{\bar{x}} \oplus f_x$ |
|---|---|---|---|
| $x_1$ | [1111] | [1001] | [0110] |
| $x_2$ | [1110] | [1101] | [0011] |
| $x_3$ | [1110] | [1101] | [0011] |

**Table 8. The information measures (in bit/pattern) for positive and negative Davio expansion (Example 8, Step 1).**

| | $H^{pD}(f \mid x)$ | $H^{nD}(f \mid x)$ |
|---|---|---|
| $x_1$ | 0.5 | 1 |
| $x_2$ | 0.91 | 0.91 |
| $x_3$ | 0.91 | 0.91 |

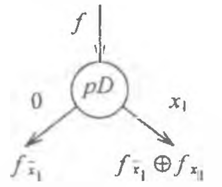


**Fig. 5. Step 1 of DT design (Example 8).**

Let us consider cofactor $f_{\bar{x}_1}$ = [1111]. The cofactor is constant, therefore we assign a leaf (logic value 1) to Reed-Muller DT.

Step 2. Consider cofactor $f_{\bar{x}_1} \oplus f_{x_1}$= [0110]. The cofactors and information measures are given in Table 9 and Table 10 respectively. The information measures are equal. The cofactor $f_{\bar{x}_1} \oplus f_{x_1}$ is multiform symmetric in $x_2$ and $x_3$. Finally, we assign primitives with *pD* nodes for decision tree (Fig. 6).

We obtain the fixed polarity Reed-Muller expression: $f = 1 \oplus x_1 \cdot x_2 \oplus x_1 \cdot x_3$.

**Table 9. The cofactors (Example 8, Step 2).**

| | $f_{\bar{x}}$ | $f_x$ | $f_{\bar{x}} \oplus f_x$ |
|---|---|---|---|
| $x_2$ | [01] | [10] | [11] |
| $x_3$ | [01] | [10] | [11] |

**Table 10. The information measures (in bit/pattern) for positive and negative Davio expansion (Example 8, Step 2).**

| | $H^{pD}(f \mid x)$ | $H^{nD}(f \mid x)$ |
|---|---|---|
| $x_2$ | 0.5 | 0.5 |
| $x_3$ | 0.5 | 0.5 |

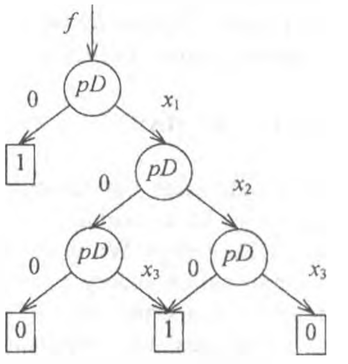


**Fig. 6. Reed-Muller DT (Example 8).**

## 5. Experimental results

We incorporate *InfoRECSym* algorithm in program of minimization of switching function via Reed-Muller DT design *InfoEXOR* [14] - on Pentium 100 MHz (RAM 48 Mb), programming language *C++* under OS Windows 95.

To verify the efficiency of symmetry detection approach, we tested program on MCNC benchmarks (completely specified Boolean functions). Table 7 contains fragments of our results. The column with label **in** shows the number of variables, column with label **out nr.** shows the output number (we consider single output benchmarks). In column *Time* the running times for the algorithms in CPU seconds are given. The column $C_T$ refers the number of products in minimized FPRM expressions.

The running time of *InfoEXOR* program with

**Table 11. Comparison of running times for Sym*pathy* [4], *InfoEXOR* [14] and *InfoEXOR* with algorithm *InfoRECSym*.**

| | | | Sym*pathy* | *InfoEXOR* (FPRM) | *InfoEXOR* (FPRM)+ *InfoRECSym* |
|---|---|---|---|---|---|
| | **in/ out nr.** | $C_T$ | *Time*[◊], s | *Time*[&], s | *Time*[&], s |
| xor5 | 5/1 | 5 | 0.1 | 0.001 | 0.001 |
| rd84 | 8/1 | 28 | 0.1 | 0.010 | 0.001 |
| rd84 | 8/4 | 70 | 0.1 | 0.092 | 0.001 |
| 9sym | 9/1 | 173 | 0.1 | 0.341 | 0.087 |
| sym10 | 10/1 | 266 | 0.2 | 2.620 | 0.152 |
| ***Total*** | | | 0.6 | 3.064 | 0.242 |

-60%

13 times

[◊] **Sparc 1+ workstation, OS UNIX**
[&] **Pentium 100MHz processor, OS Windows 95**

symmetry detection algorithm *InfoRECSym* is better for 60% than program Sym*pathy* [4] and in 13 times better than original program *InfoEXOR*.

## 6. CONCLUDING REMARKS

This paper addresses the detection of different types of symmetries of switching function for Reed-Muller DTs or DDs design. We investigate symmetry detection by information theory point of view that gives us additional properties for switching function minimization techniques. Our program *InfoRECSym* successfully recognize symmetries for efficient Reed-Muller DTs or DDs design.

In future it will be interesting to extend our results to detect symmetries in multivalued functions.

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