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UTILIZING THE SELF-ADAPTING SHAPE FUNCTIONS IN NONLINEAR ANALYSIS OF PLANE AND SOLID DOMAINS.

1. INTRODUCTION.

The Finite Element Method FEM [1,2] opens possibility to analyse plain and solid systems. However, numerical integration of the finite element stiffness matrices, especially, when physical and geometrical parameters are changing, requires using complicated and time-consuming computational algorithms. In the case, when stiffness matrices have to be determine repeatedly, utilizing explicite form of stiffness matrices can be more effective. In order to attain right accuracy of calculations, these elements require changing of nonlinear parameters in the whole domain of element and thus mesh refinement.

The artifical mesh refinement (without nodes addition) by using elements which are integrable in subspaces is suggested by authors.

2. MODEL CONCEPTION.

The fact, that element can be divided into parts, which are integrated separately and then put together into one stiffness matrix of element, has been used in this paper.
Hence we have

$$\mathbf{K}_e = \sum_{i=1}^{n} \mathbf{K}_e^i, \tag{1}$$

where:

$\mathbf{K}_e$ - element stiffness matrix,

$\mathbf{K}'_e$ - stiffness matrix of the part of element (subdomain),

n - number of submatrices.

The solutions, which use the method of the summation subdomains energy [3,4], described in (1), don't take into account the effect of influence of changing of stiffness function and subdomain geometry on the strain field distribution inside element. It impairs the solution quality.

The idea of the determination of stiffness matrices of plane and solid elements, with possibility of shape function distribution control during the computational process, has been shown in the paper.

Utilizing the Finite Element Method, after the division elements on subdomains, Fig.1,

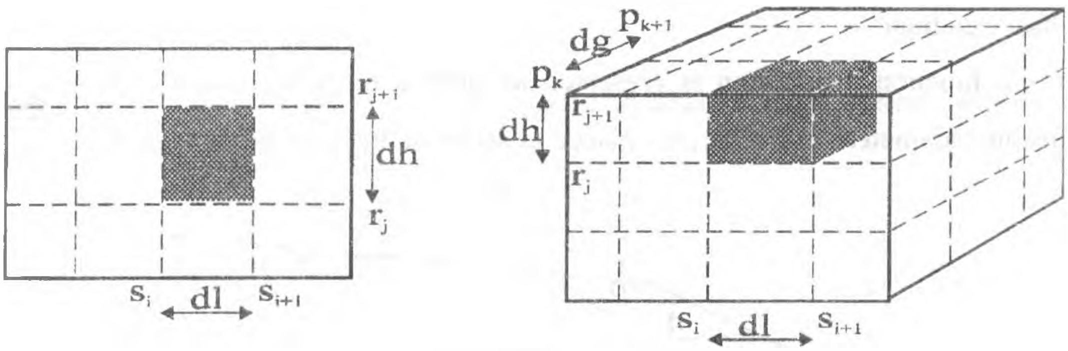


Fig. 1.

subdomain stiffness matrix can be obtained from

$$\mathbf{K}_e^{i\,SUB} = \int_{s_i}^{s_{i+1}} \int_{r_j}^{r_{j+1}} \int_{p_k}^{p_{k+1}} \mathbf{B}'^T_e \mathbf{D}'_e \mathbf{B}'_e \, dp\,dr\,ds, \qquad (2)$$

where $\mathbf{D}'_e$, $\mathbf{B}'_e$ are tensor of elasticity coefficients and strain tensor, which can be modified independently for every subdomain.

Strain matrix (tensor) is given by

$$\mathbf{B}'_e = \mathbf{L}_e \mathbf{N}'_e, \qquad (3)$$

where:

$\mathbf{L}_e$ - differential operator matrix for suitable type of elements,

$\mathbf{N}'_e$ - shape function matrix in subdomain.

Linear functions and parameters on the subdomains boundaries, which control functions distribution, have been used to approximation strain field.

On fig. 2 shown exemplary function running in one direction, where controllable parameters, denoted as m_i, are placed in nodes of inner mesh.

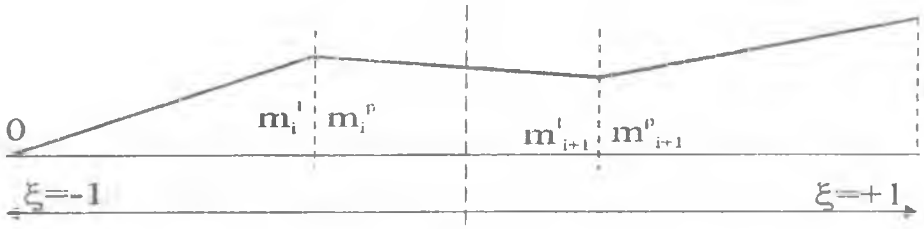


Fig. 2

3. PLANE ELEMENT.

The submatrix of plane shield element, which is integrated in range $\langle s_i, s_{i+1}\rangle, \langle t_j, t_{j+1}\rangle$, fig.1, has been built.

The shape function distribution is presented as surface functions which depend on values of controllable parameters m_1, m_2, m_3, m_4, placed in nodes of analysed subdomain, fig.3.

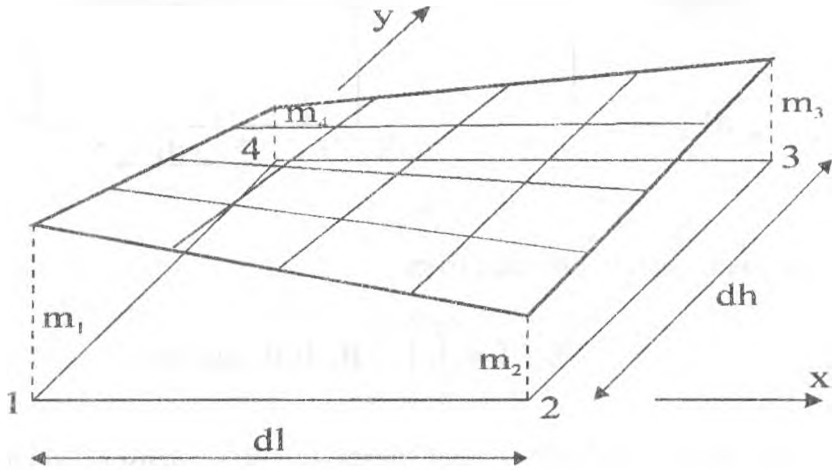


Fig. 3

Subdomain shape function for node *k*, has form

$$f_k = a_1^k + a_2^k x + a_3^k y + a_4^k xy, \quad k = 1,2,3,4. \tag{4}$$

Parameters a^k determined from the boundary conditions are given by

$$\begin{aligned} a_1 &= m_1 - \frac{m_2 - m_1}{dl} x - \frac{m_4 - m_1}{dh} y + \frac{m_1 - m_2 + m_3 - m_4}{dldh} xy, \\ a_2 &= \frac{m_2 - m_1}{dl} - \frac{m_1 - m_2 + m_3 - m_4}{dldh} y, \\ a_3 &= \frac{m_4 - m_1}{dh} - \frac{m_1 - m_2 + m_3 - m_4}{dldh} x, \\ a_4 &= \frac{m_1 - m_2 + m_3 - m_4}{dldh} \end{aligned} \tag{5}$$

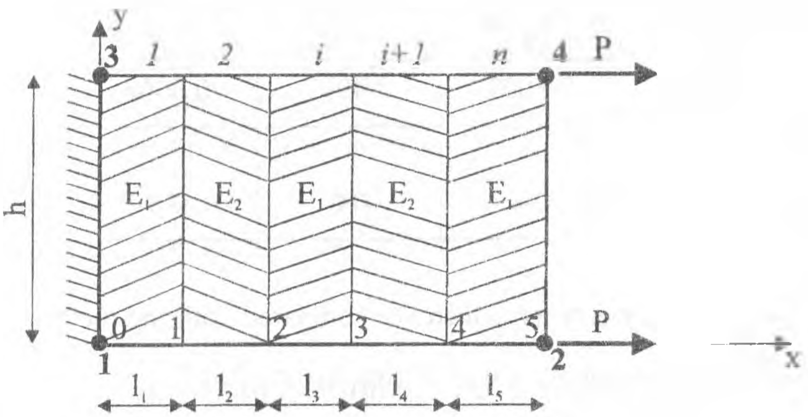
If origin of coordinates is placed in the subdomain node, than the shape function coefficients can be reduced to the form

$$
\begin{aligned}
a_1 &= m_1, \\
a_2 &= \frac{m_2 - m_1}{dl}, \\
a_3 &= \frac{m_4 - m_1}{dh}, \\
a_4 &= \frac{m_1 - m_2 + m_3 - m_4}{dldh}
\end{aligned}
\qquad (6)
$$

The stiffness matrix of the part of element, given in (2), has been determined in explicite form by utilizing computer programme for symbolical processing of algebraical expressions [5].

4. NUMERICAL TESTS.

Controllable shape functions have been used to analyse strips system, shown in fig 4, which is discretized by one four-node element.



$E_1 = 10*10^6$ kPa
$E_2 = 10*10^3$ kPa
$P = 100$ kN
$t = 0.1$ m
l_1-l_5 = 0.2 m
$h = 1.0$ m

Fig. 4

System has been built from strips which have the same thickness and different Young's modules. Big stiffness jumps inside element, have been used because of the possibility of the modelling plasticization zone. The shape function distribution in the direction of the *x*-coordinate axis can be obtained from

$$m_i = \frac{k}{k_i}, \qquad (7)$$

where:

- *k* - stiffness of the system in the direction of the *x*-coordinate axis, which can be determined from

$$k = \frac{1}{\sum_{j=1}^{n} \frac{1}{k_j}} = \frac{1}{\sum_{j=1}^{n} \frac{1}{\frac{E_j A_j}{l_j}}}, \tag{8}$$

- k_i - stiffness of the part of the system, which is given by

$$k_i = \frac{1}{\sum_{j=1}^{i} \frac{1}{k_j}} = \frac{1}{\sum_{j=1}^{i} \frac{1}{\frac{E_j A_j}{l_j}}} \tag{9}$$

The shape function distribution for analysed system, in x-coordinate axis direction, for 2 and 4 nodes, is shown in the Table 1. Function distribution in 1 and 3 nodes can be obtained by one's complement, what is required for the rigid motion criterion.

Table 1

Point	m0	m1	m2	m3	m4	m5
Nodes 2 & 4	0.0	0.0005	0.4997	0.5003	0.9995	1.0
Nodes 1 & 3	1.0	0.9995	0.5003	0.4997	0.0005	0.0
Linear distribution	0.0	0.2	0.4	0.6	0.8	1.0

The result of the analysis of system displacements, which are compared with the method of linear distribution shape function and accurate solution, are presented in the table 2.

Table 2

	Displacements in x-coordinate axis direction	
	Node 2	Node 4
Presented method	0.078 $\nu = 0.16$ 0.080 $\nu = 0.00$	0.078 $\nu = 0.16$ 0.080 $\nu = 0.00$
Linear distribution	0.00033	0.00033
Accurate solution	0.080	0.080

5. CONCLUSIONS.

Results presented in the part 4 allow to note that in some cases utilizing self-adapting (controllable) shape functions can lead to increase the solution accuracy without necessity of increase the number of unknowns (mesh refinement).

Presented method can be useful in such cases as:

- nonlinear analysis of homogeneous and complex systems,
- analysis of cracking effect of the structure,
- analysis of contact problems and geometrical nonlinearity,
- automation of plane and space mesh generation.

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применниение програмированного управления к нелинейному анализу плоских и пространственных тел.

Резюме

Численное интегрирование матриц жесткости элементов при переменных физических и геометрических параметрах требует применения сложных вычислительных алгоритмов. Если требуется матрицы жесткости вычислять многакротно, более эффективным будет представление матриц элементов в явном виде. Применение такого вида элементов требует изменения нелинейных параметров в области элемента и уплотнения сетки деления.