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LABORATORY WORKS
STRENGTH OF MATERIALS

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(Part 2)

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When studying resistance of materials the experiment plays extremely important role. It gives the chance to receive the mechanical characteristics of materials necessary for creation of the theory of calculations on strength. With the help of the experiment the check of theoretical conclusions and formulas of materials resistance is made. Usually these conclusions and formulas turn out on the basis of assumptions (hypotheses) and therefore demand check on experience.

The main objective of methodical instructions is to help students with their independent preparation for laboratory works.

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CONTENTS

Part II. Determination of deformations and stresses

LABORATORY WORK № 9

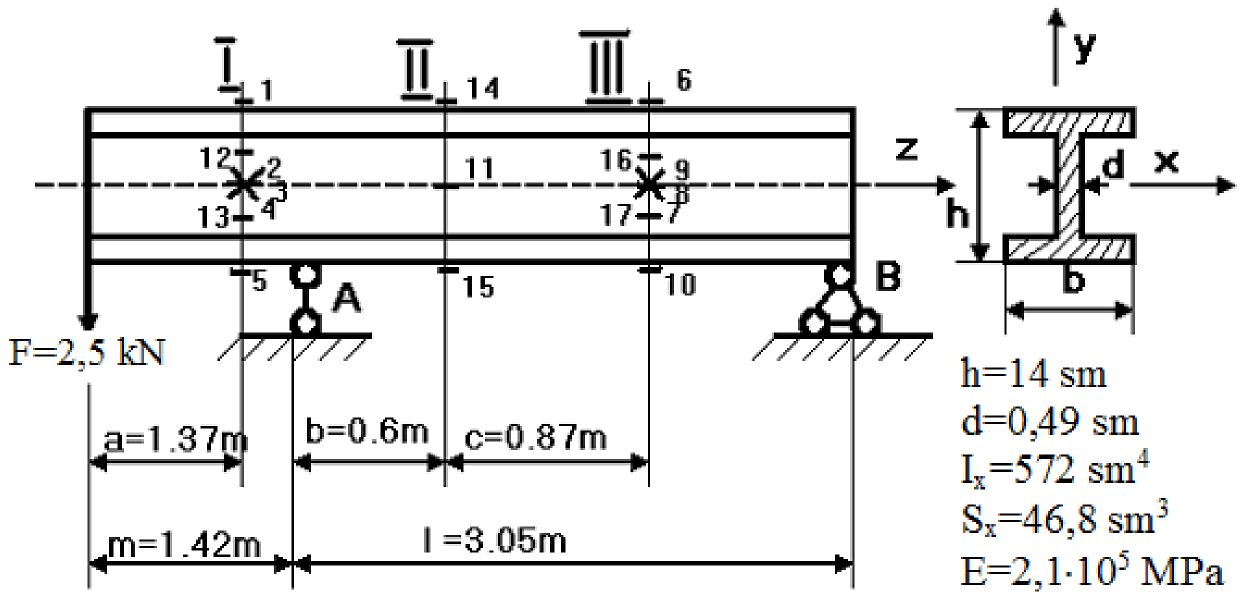
Determination of stresses in a metal beam at transverse (lateral) bending (uniplanar bend)

I. Work purpose: Theoretically and experimentally to determine the stress at the given points of the beam cross sections. Investigate the distribution of normal stresses over the beam cross section and determine the magnitude and direction of the main stresses in the neutral layer of the beam.

II. Content of work

The metal beam of double-T section is loaded with F force applied on the console. Tension is defined in three sections (I, II, III) and in points of sections, as shown in fig. 9.1.

a)



b)

Fig. 9.1. – Beam appearance (a) and scheme (b)

a) theoretical determination of stresses

At transverse bending normal stresses at any point in the cross section of the beam are determined by the formula:

$$\sigma = \pm \frac{M}{I_x} y, \qquad (9.1)$$

where M – bending moment in the considered section; I_x – axial moment of inertia of section; y – ordinate of a point in which stress is defined.

It is easy to determine the sign of tension by the M diagram (the diagram of M should be plotted on the stretched fibers).

From a formula 9.1 it is visible that at $\boldsymbol{y} = 0$, $\sigma = 0$, and at $y = \frac{h}{2}$: $\sigma = \sigma_{\min}^{\max}$.

The stress-strain condition research in beams shows that on a neutral axis ($y=0$) deformation of pure shift takes place (fig. 9.2) i.e. $\sigma_{\max} = |\sigma_{\min}| = \tau$, where σ_{max}; σ_{min} – principal stresses acting (are directed) at an angle 45° toward beam axes; τ – shear stress.

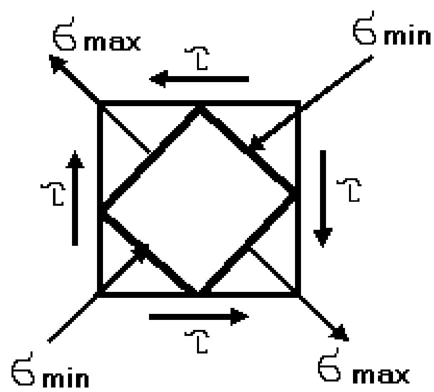


Fig. 9.2. – Stresses in a point at Q>0

Shearing stress is determined by Zhuravsky's formula:

$$\tau = \frac{Q \cdot S_x}{I_x \cdot d}, \qquad (9.2)$$

where Q – shear force in the considered section; I_x, S_x, d – geometrical characteristics of section (accepted from range for rolling profiles cross-sections (for the double-T section)).

b) ***experimental determination of stresses***

Electric tensiometers (wire sensors of ohmic resistance) are widely used. The sensor is pasted by special glue on the studied surface in the set points (fig. 9.1).

The results of measurements of stresses are processed by a computer. As a result, we obtain experimental stress values at 13 points of the beam.

III. Order of carrying out tests

1. To get acquainted with the resistance strain gauge and tensiometer.
2. To study the device and work of a laboratory unit, a technique of measurement of stress by means of electrotensometry.
3. To sketch the scheme of a beam, to measure the sizes with an accuracy of 1 mm *(a; b; c; m; l; h)* specified in the fig. 9.1, b. To switch on a computer and in no-load condition of a beam to take consistently indications (of a strain gauge) for all sensors pasted in sections I II, III.
4. To load a beam loading of ***F*** and again to take indications for the corresponding sensors.
5. Determine the experimental values of the stress.

To enter data of calculation in table 9.1.

IV. Processing of results of an experiment

1. Using the statics equations, basic reactions of the supports are determined.
2. Plot diagrams of shearing forces and bending moments.
3. From a range of rolling profiles (for the I-beam No.14) geometrical characteristics are written out.
4. Stresses in the studied points (of the considered sections) is determined by formulas (9.1–9.2). In one of sections plot σ diagram. Results of calculation are entered in table 9.1.
5. The results received analytically and experimentally are compared. The errors % is determined by a formula:

$$\delta = \left| \frac{\sigma_i^{theor} - \sigma_i^{\exp}}{\sigma_i^{theor}} \right| \cdot 100\%. \qquad (9.4)$$

V. Conclusions

The conclusions should answer the questions posed by the purpose of laboratory work.

After analyzing the table of experimental data, you can make sure that the experimental plot of normal stresses is almost a straight line. So the hypothesis of flat sections (Bernoulli hypothesis) should be confirmed.

Comparing σ^{theor} and $\sigma^{\exp}$ showing that the results are the same or slightly different from each other. This allows us to draw a conclusion about the permissible application of those hypotheses and simplifications that are accepted in the theory of transverse bending.

Table 9.1

Sections	Measurement point	Stresses		% discrepancy
		$\sigma^{\exp}$, MPa	σ^{theor}, MPa	
I-I	1			
	2			
	3			
	4			
	5			
	12			
	13			
II-II	11			
	14			
	15			
III-III	6			
	7			
	8			
	9			
	10			
	16			
	17			

Control questions

1. What are the hypotheses and assumptions taken in the theory of bending?
2. What is the hypothesis of flat sections?
3. How are the normal stresses distributed over the height of the beam section?
4. What is the stress state of the material at the studied points on the beam surface?
5. What is the position of neutral layer of a beam?
6. Formulate a common goal of laboratory work.
7. What is the formula determined by the normal bending stress at any point in the cross section of the beam?
8. Why formula for the shear stresses in bending the beam is used?
9. What is the direction of the main stresses at the level of the neutral layer of the beam and by what formula they are determined?
10. What measuring instruments are used in laboratory work?
11. What is measured by means of the sensors resistance?
12. How are located (in relation to the longitudinal axis of the beam) sensors used to measure the deformation of fibers?

13. Show where the cross section of the beam has a pure shear?
14. Describe the construction and principle of operation of the sensor.
15. What kind of condition of strength for the normal and shear stresses?

LABORATORY WORK № 10

Determination of deformations in a metal beam at transverse (uniplanar) bending

I. Work purpose: Theoretically and experimentally to define deflections and angles of rotation of the specified beam sections.

II. Content of work

The metal I-beam is loaded with a force *F* applied to the console. Deflections should be defined in sections ***0, 1***, and rotation angles in sections ***0, A*** (fig. 10.1).

Moving the center of gravity of the beam section in a direction perpendicular to the axis of the beam is called the deflection of the beam in this section or deflection of the beam section. The angle at which each section rotates relative to its original position is called the section rotation angle.

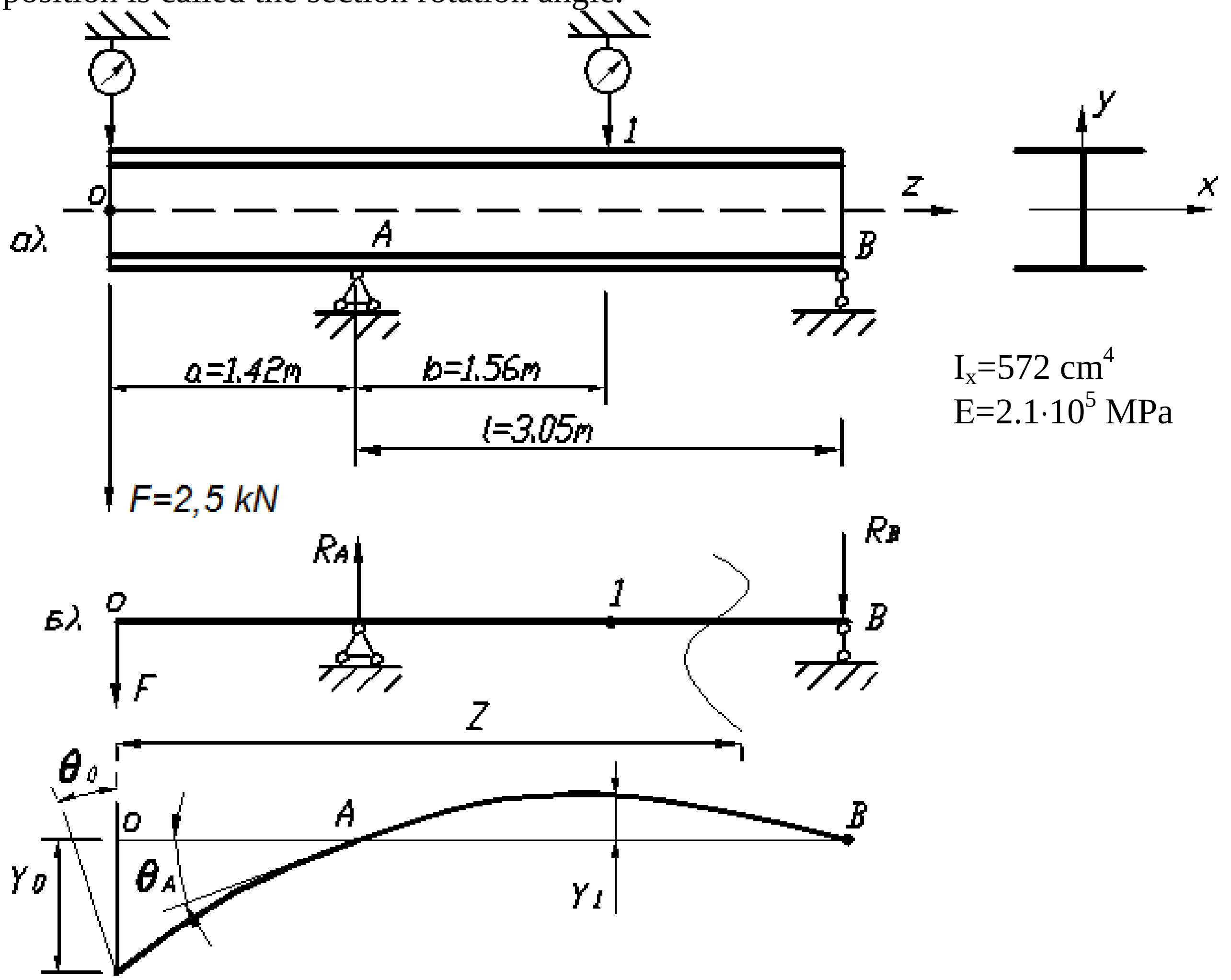


Fig. 10.1. – Beam patterns

a) theoretical determination of deflections and angles of rotation

Deflections and angles of rotation of the set sections are defined by method of initial parameters.

For any section of "z" on the site of AB the universal equation of deflections will have an appearance:

$$EI_x y_z = EI_x y_0 + EJ_x \theta_0 \cdot z - \frac{F \cdot z^3}{6} + \frac{R_A (z-a)^3}{6}; \qquad (10.1)$$

where: EI_x – rigidity of a beam at a bend, θ_0; y_0– initial parameters, i.e. angle of rotation and deflection, respectively, at the beginning of coordinates (section «0»).

For determination θ_0 and y_0 we use a condition of fixing of a beam.

$$\begin{aligned} &as\ z = a; \\ &as\ z = a + l; \end{aligned} \left\{ \begin{aligned} EI_x y_A &= EI_x y_0 + EI_x \theta_0 \cdot a - \frac{F \cdot a^3}{6} = 0, \\ EI_x y_B &= EI_x y_0 + EI_x \theta_0 (a+l) - \frac{F(a+l)^3}{6} + \frac{R_A \cdot l^3}{6} = 0. \end{aligned} \right. \qquad (10.2)$$

Having solved the system (10.2) equations, we define $EI_x \theta_0$ and $EI_x y_0$, and then $\theta_0 ; y_0$.

We define a deflection of section "1" from the equation (10.1) under a condition $z = a + b$, i.e.

$$EI_x y_1 = EI_x y_0 + EI_x \theta_0 (a+b) - \frac{F(a+b)^3}{6} + \frac{R_A \cdot b^3}{6}. \qquad (10.3)$$

We define an angle of rotation of basic section "A" from the universal equation of angles of rotation:

$$EI_x \theta_A = EI_x \theta_0 - \frac{F \cdot a^2}{2}. \qquad (10.4)$$

b) experimental determination of deflections and angles of rotation

For measurement of deflections of sections "1", "0" of a beam dial indicators (indicators of hour type, needle indicators) with an accuracy of 0,01 mm are used (fig. 10.1). The device and the principle of work are given in the section "Probing devices").

We determine the size of deflections by a formula:

$$y = n \cdot c; \qquad (10.5)$$

where n – indications of indicators (number of divisions); c – the division value of the indicator.

For determination of an angle of rotation of section "0" the device (fig. 10.2) is used.

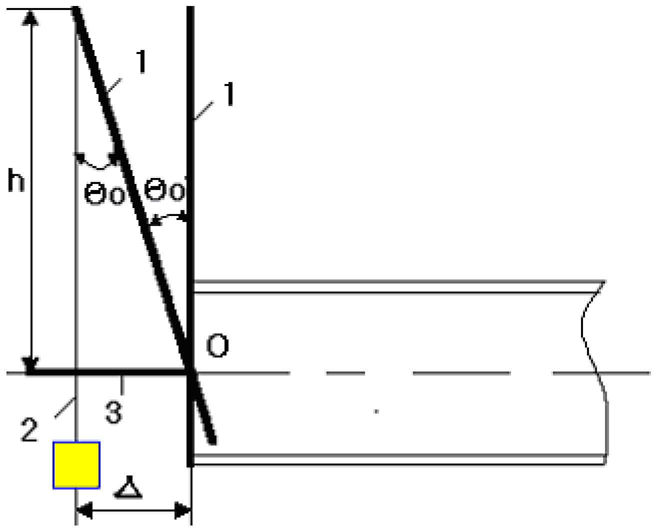


Fig. 10.2. – The device for determination of an angle of rotation (θ_0)
1 – a bar (h = 1500 mm), 2 – a plumb, 3 – a ruler

Owing to the smallnesses of deformations we can write down:

$$tg\theta_0 \approx \theta_0 = \frac{\Delta}{h}, rad. \qquad (10.6)$$

Approximately the angle of rotation of basic section "A" (fig. 10.2) is:

$$tg\theta_A \approx \theta_A = \frac{y_0}{a}. \qquad (10.7)$$

where y_0 it is determined by a formula (10.5).

III. Order of carrying out tests

The device of needle indicators, their installation and technique of definition of displacement with their help is studied.

1. The scheme of a metal beam (fig. 10.1) is sketched, the sizes are measured (a; b; l), specified on the scheme.
2. Tests are performed:

a) prior to the loading of the beam in all indicators shall be set at zero,
b) smoothly without jerking (jumping) the beam is loaded with a load ***F***,
c) the indicator readings are taken, as well as the horizontal offset (shift from initial position) of the plumb (Δ).

IV. Processing of results of an experiment

1. On formulas (10.5; 10.6; 10.7) experimental values are defined: y_0; y_1; θ_0; θ_A and results are entered in table 1.
2. Basic reactions from the statics equations are defined.
3. Analytically on formulas (10.1; 10.2; 10.3; 10.4) are defined y_0; y_1; θ_0; θ_A and results are entered in table 1.
4. Experimental and theoretical results are compared. The discrepancy % is determined by formulas:

$$\delta_y = \left| \frac{y_i^{theor} - y_i^{\exp}}{y_i^{theor}} \right| \cdot 100\,\%; \quad \delta_\theta = \left| \frac{\theta_i^{theor} - \theta_i^{\exp}}{\theta_i^{theor}} \right| \cdot 100\,\%.$$

Table 10.1

No.	Section	Indications indicators	Division scale of indicator	Counting on to plumb Δ (mm)	Deflections of sections, ***y*** (mm)		% discrepancy (y)	The angles of rotation of sections, θ (rad)		% discrepancy (θ)
					Exp.	Theor.		Exp.	Theor.	
1	0									
2	A									
3	1									

V. Conclusions

To give the analysis of experimental and theoretical results.

Control questions

1. What parameters characterize deformation at a uniplanar bend?
2. What methods of determination of these parameters do you know?
3. What differential dependence between a deflection and the angle of rotation of section of a beam exists?
4. Formulate the purpose of laboratory work.
5. Describe the type of installation and devices used to measurement of deflections and angles of rotation of sections of a beam.
6. What method is applied to theoretical determination of deflections and angles of rotation of sections?
7. What is called rigidity of a rod at a bend?
8. What is initial parameters and from what conditions they are defined?
9. How according to the indication of the indicator the measured deflection is determined?
10. Explain why after unloading of a beam indicators showed initial counting?

LABORATORY WORK № 11
Research of statically indeterminate beam

I. Work purpose: To confirm the possibility of theoretical calculations of statically indeterminate beams using displacement equations, i.e. to compare the results of experimental determination of the moment of pinching of the beam with the theoretical one. On the basis of experimental data to establish a proportional dependence of the beam deflection on the load.

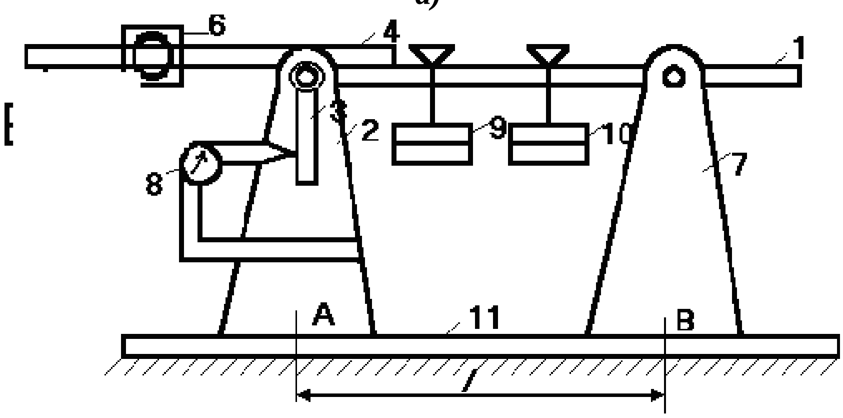
II. Content of work

Statically indeterminate beams are widely used in engineering practice, because they are more economical, allowing to perceive large loads, to cover large spans. Such beams are produced by the introduction of additional support pins. In these cases, the number of support reactions exceeds the number of possible statics equations. This leads to the compilation of additional equations related to the consideration of deformations in beams. Additional equations are generalized displacement equations and can be solved in various ways.

For carrying out a research on this work desktop installation (fig. 11.1) which represents the beam (1) made of strip steel of rectangular lateral section is used. The beam lies on two support (2) ***A*** and (7) ***B***. Support ***A*** – isn't mobile, the support ***B*** can allow the beam to move (11). On a surface of a beam (1) there is a centimetric marking from a support A to a support B that allows to set position of suspenders (9) and (10). Except the hinge support the console G - a figurative form of rigide fixed support (pinching). A horizontal part of the console is executed in the form of a rail with a millimetric marking from a support A towards an end.

The beam deflects when loading suspenders (9), (10). All lateral sections (including and sections at the supports) undergoes turn. Together with basic section A the console (3) on a corner θ_A turns (fig. 11.2). The deviation is fixed by the indicator (8). Return of the console to initial situation with the purpose of an exception of turn of basic section A (embedded imitation) is made by means of load (6). Knowing the size and the location of these loads, it is possible to define the moment at the support A.

a)



b)

Fig. 11.1. – Appearance (a) and scheme of installation (b)

3 – beam; 2, 7 – supports; 3 – vertical part of the console; 4 – horizontal part of the console; 9, 10 – replaceable loads; 6 – mobile load (F_o = 9,6 N); 8 – indicator; 11 – bed

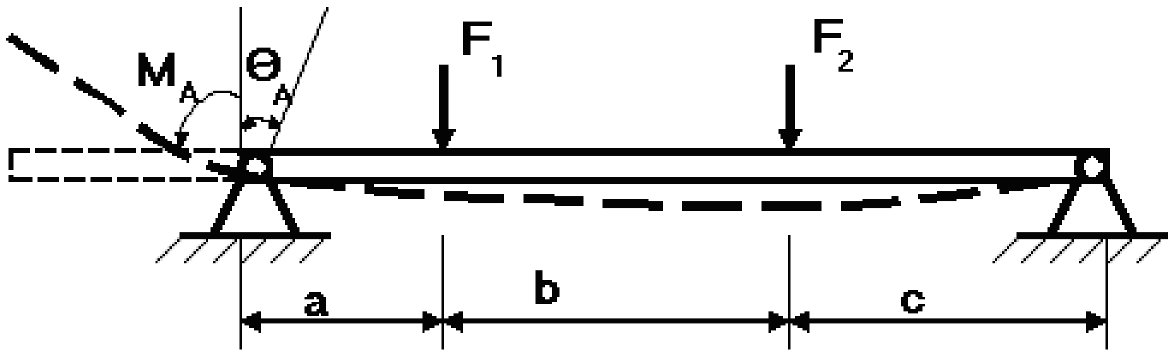


Fig. 11.2. – Beam design diagram

a) theoretical determination of the moment at the support

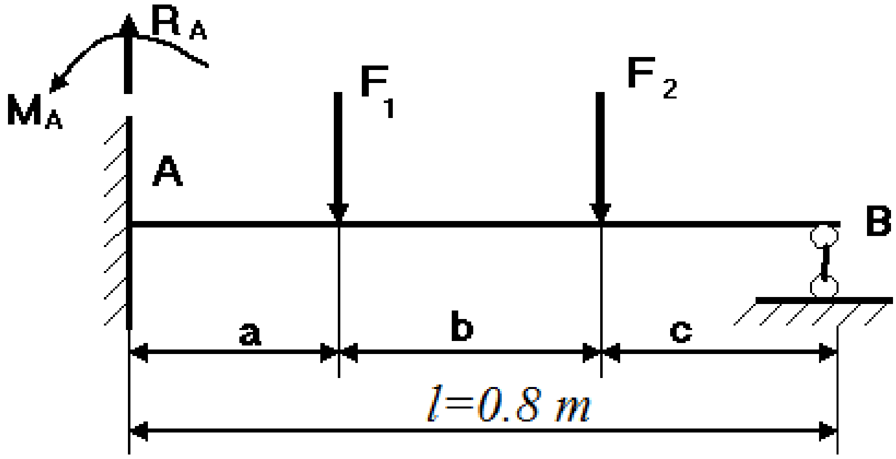


Fig. 11.3. – The calculated scheme of a beam

For determination of reaction of the M_A and R_A we write down the system of two equations:

$$\sum M_A = 0; \quad M_A - R_A \cdot l + F_1 \cdot (b+c) + F_2 \cdot c = 0, \tag{11.1}$$

$$y_B = 0; \quad \frac{-M_A \cdot l^2}{2} + \frac{R_A \cdot l^3}{6} - \frac{F_1 \cdot (b+c)^3}{6} - \frac{F_2 \cdot c^3}{6} = 0. \tag{11.2}$$

The equation (11.1) represents the static equation, and the equation (11.2) – geometrical.

Excepting R_A reaction, we come to the following expression for the moment at the support A:

$$M_A = \frac{F_1(b+c)\cdot\left[l^2 - (b+c)^2\right] + F_2 c \cdot \left(l^2 - c^2\right)}{2l^2}. \tag{11.3}$$

Being set by values (sizes) of forces of F_1 and F_2 and also having chosen sizes a, b, c, we define the moment at the support A.

b) experimental determination of the moment at the support A

We load a beam by forces of F_I and F_2. For each case of loading we fix counting (u) on the indicator (8).

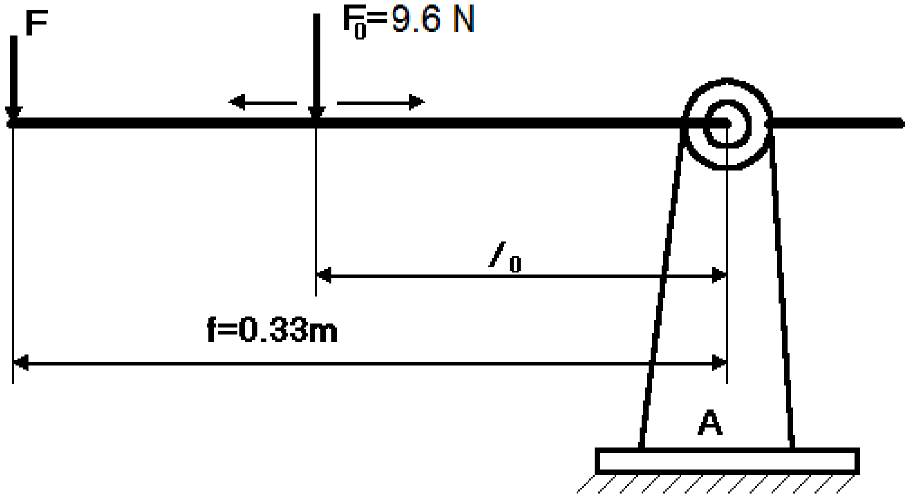


Fig. 11.4. – Scheme for experimental determination of the moment at the support A

By means of loads (6) we return indications of the indicator in home (initial) position that there corresponds equality to zero turn of section A. We fix counting $\boldsymbol{l_o}$ on a scale of the console (fig. 11.4). The experimental value of the moment of the M_A is calculated on a formula:

$$M_A^T = F_0 \cdot l_0. \qquad (11.4)$$

III. Order of carrying out tests

1. The indication of the indicator (8) is set to zero and loads are prepared (F_O, F_1, F_2).
2. The beam loads (with F_1 and F_2) and for each loading fix counting of the indicator (U).
3. By means of loads (F_0) return the indication of the indicator in home position (zero).
4. Define distances from loads (F_0) to support A.

IV. Processing of results of an experiment

1. For each case of loading is determined by a formula (11.4) support moment.
2. The scheme of dependence "F – u" is elaborated
3. The moment at the support M_A^{theor} theoretically is determined by a formula (11.3).
4. Results of measurements and calculations are entered in table 11.1.
5. The % of discrepancy of experimental and theoretical determinations on a formula is defined:

$$\delta = \frac{M_A^{theor} - M_A^{exp}}{M_A^{theor}} \cdot 100\,\%.$$

Table 11.1

№	Dimensions, m						Replacement loads, N			Indicator reading U, mm	Moment, N·m		δ, %
	l	l_0	*a*	*b*	*c*	*f*	F_0	F_1	F_2		M_A^{theor}	M_A^{exp}	
1	0,8					0,33							
2													
3													

Notes:

1. It is possible to determine M_A^{theor} by instructions of the teacher (by a comparison method deformations).

2. To plot diagrams of lateral forces (Q) and bending moments (M) in statically indeterminable beam.

V. Conclusions

To give the answer to the questions posed at statement of a research objective.

Control questions

1. Which beams are called statically indeterminate?
2. What is the basic system?
3. Which restrictings imposes on the beam support with a pinching?
4. How the experimental value of the moment M_A was determined?
5. How to determine the theoretical value of M_A?
6. What supports are superfluous (redundant)?
7. What kind of movement corresponds to the moment of pinching?
8. What measuring device was used in the experiment?
9. What is the role of load acting on the console?
10. List the methods for determining displacement.
11. What analytical method was used in this work to determine the moment of pinching?

LABORATORY WORK № 12
Research of oblique (unsymmetrical) beam bending

1. Work purpose: Familiarization with the oblique (unsymmetrical) bending of the cantilever beam and comparison of experimental values of stresses, deflections with theoretical. Comparison of the results of oblique and transverse **(**uniplanar) bends.

II. Content of work

Installation consists of two identical console beams (console). Section of beams - an equilateral angles'. Beams are loaded with ***F*** force. In the set sections and points of sections stresses are defined, and also are defined deflections of the end of the console (fig. 12.1).

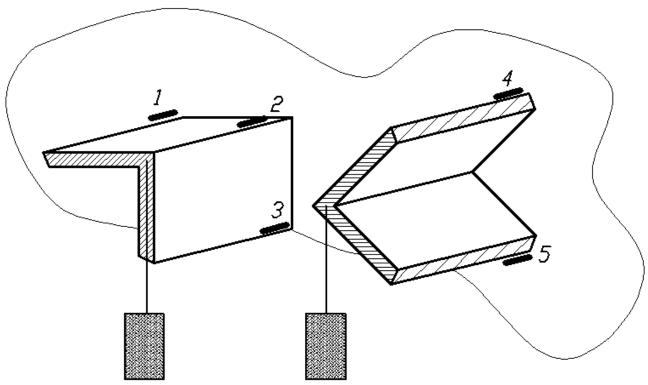


Fig. 12.1. – Scheme of installation

a) theoretical determination of stress and deflections

A. **Unsymmetrical (oblique) bend.** Unsymmetrical bend is called such type of a bend when the plane of action of bending moment doesn't match one of the main central axes of inertia of lateral section of a rod. The unsymmetrical bend can be provided as a combination of two uni-planar bends.

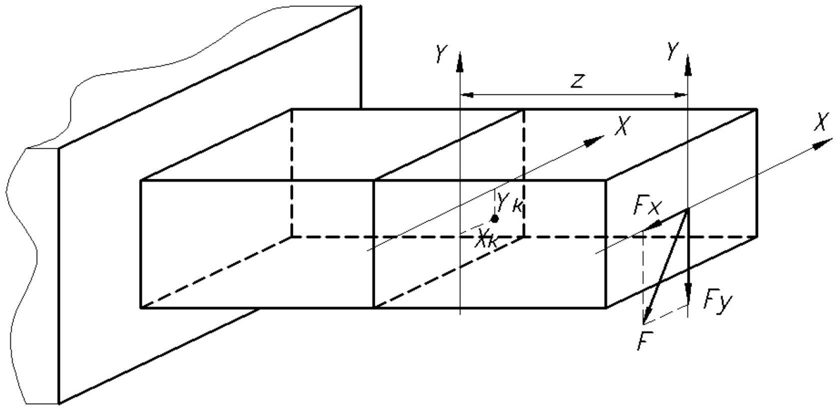


Fig.12.2. – Unsymmetrical bend

Normal stresses in any point of section Z can be determined by a formula (12.1):

$$\sigma = \pm\frac{M_x}{I_x}y \pm \frac{M_y}{I_y}x, \tag{12.1}$$

where $F_X = F \cdot \sin\alpha,\ F_y = F \cdot \cos\alpha,$ (12.2)

$$M_y = F_x \cdot z = F\sin\alpha \cdot z = M \cdot \sin\alpha,$$
$$M_x = F_y \cdot z = F\cos\alpha \cdot z = M \cdot \cos\alpha, \tag{12.3}$$

x, y — point coordinates where stresses is defined.

We will accept the sign "+" or "–" in a formula (12.1) on deformation of a beam, i.e. without connecting it with signs of coordinates of a point and bending moments.

In different sections of a beam at an unsymmetrical bend we will apply a method of superposition of forces to determination of deflections. We find (by different methods) a deflection from forces F_y and F_x, and we find a full deflection on a formula:

$$f = \sqrt{f_x^2 + f_y^2}\ . \tag{12.4}$$

The design diagram of a beam has an appearance (fig. 12.3).

Stresses in points 1, 2, 3 respectively will be:

$$\sigma_1 = \frac{M_x}{I_x}y_1 - \frac{M_y}{I_y}x_1,$$

$$\sigma_2 = \frac{M_x}{I_x}y_2 + \frac{M_y}{I_y}x_2, \quad \text{where} \tag{12.5}$$

$$\sigma_3 = -\frac{M_x}{I_x}y_3 - \frac{M_y}{I_y}x_3,$$

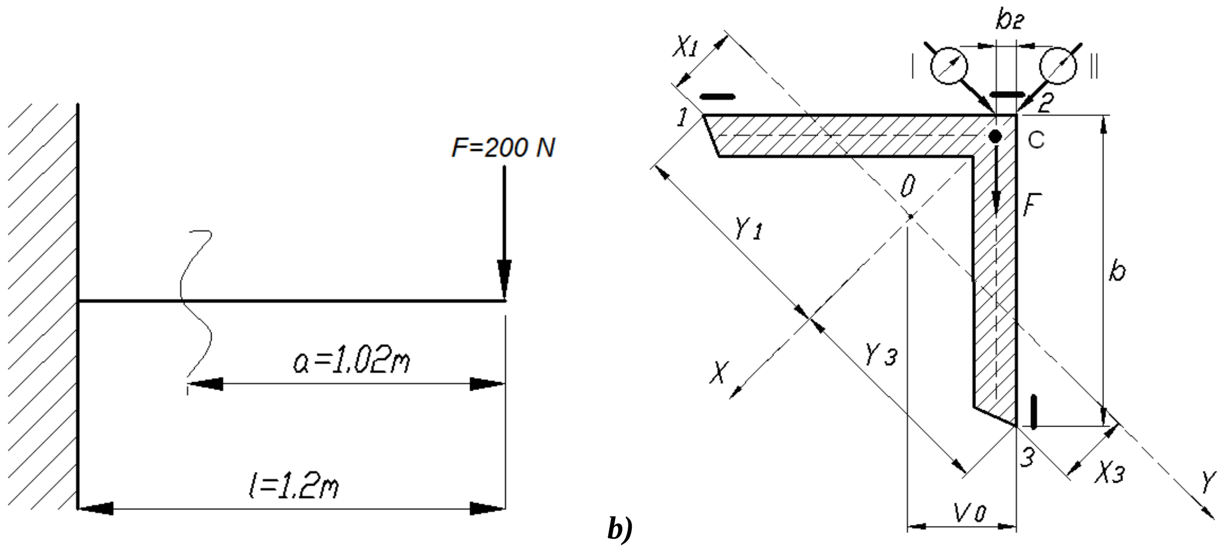
$$M_x = M_y = F_x \cdot a = F_y \cdot b = F\cos\alpha \cdot a,$$

$$x_1 = x_3 = b\cos 45° - \frac{v_0}{\cos 45°},$$

$$y_1 = |y_3| = b\sin 45°, \qquad (12.6)$$

$$x_2 = \frac{v_0}{\cos 45°} - b_2\cos 45°,$$

$$y_2 = b_2\cos 45°, \quad b_2 = \frac{d}{2}.$$



a) ***b)***

Fig. 12.3. – Scheme of a beam

Geometrical characteristics of section:
$\angle 75\times 75\times 8$, $I_x = 94{,}8$ cm^4, $I_y = 24{,}8$ cm^4, $v_0 = 2{,}15$ cm.

We determine deflections of a free end of the console by the known formula:

$$f_x = \frac{F_x \cdot l^3}{3EI_y}, \quad f_y = \frac{F_y \cdot l^3}{3EI_x}. \qquad (12.7)$$

Note:

To avoid torsion of a beam force of *F* is applied in a point C (the center of a bend), which is on crossing of average lines of flanges of an angle.

B. Uni-planar bend. The design diagram of a beam has an appearance (fig.12.4).

We determine stresses in points 4 and 5 by a formula:

$$\sigma_{4,5} = \pm\frac{M}{I_x} y_{4,5}. \qquad (12.8)$$

We determine a deflection of a free end by a formula:

$$f = f_y = \frac{F \cdot l^3}{3EI_x}. \qquad (12.9)$$

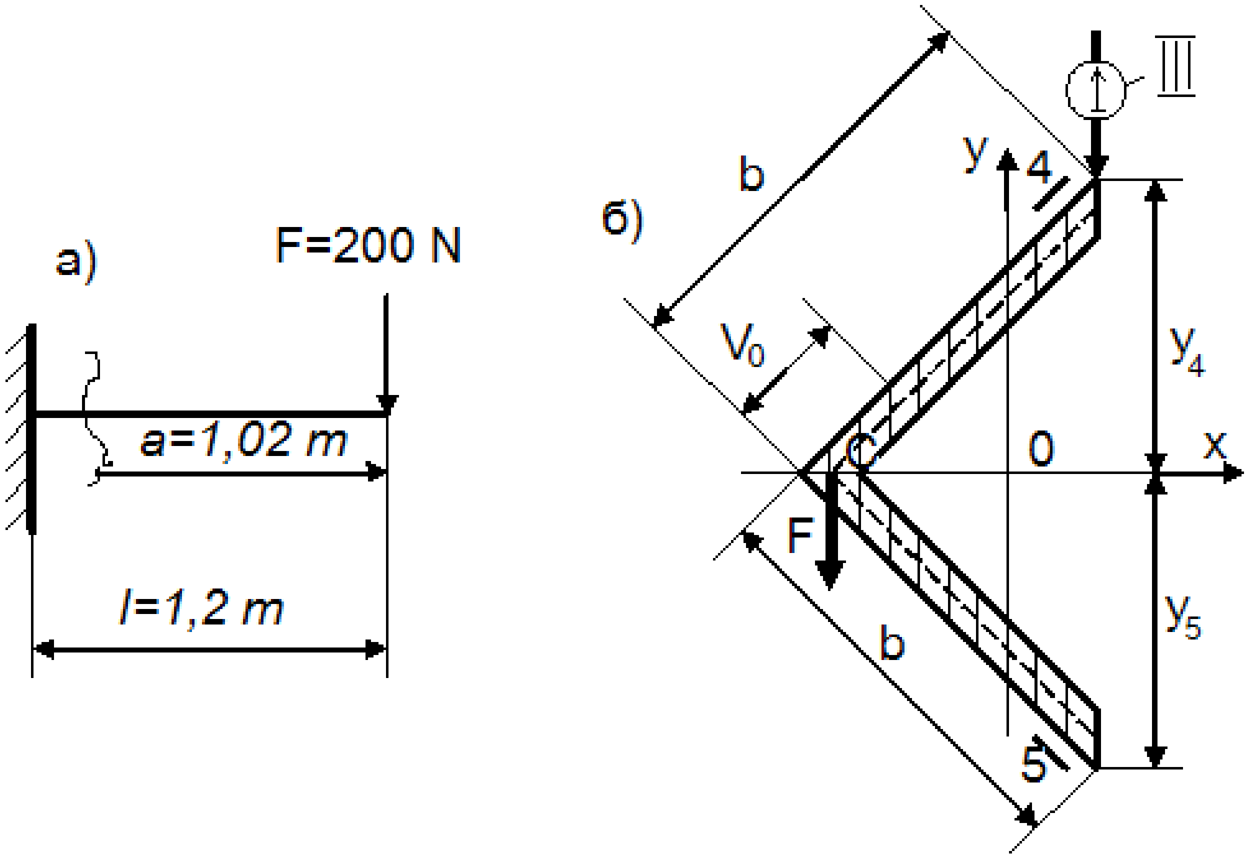


Fig. 12.4. –Scheme of a beam

b) experimental determination of stresses and deflections

The stresses at the given points of the cross sections are found by the method of tensometry. For each point readings (counts) of load by means of electrotensometry method before and after the load are taken. The experimental stress values are calculated using a computer.

We take deflections on indicators I, II, III (fig. 12.3–12.4).

III. Order of carrying out tests

1. With the help of a ruler with an accuracy of 1 mm we measure the dimensions of the beam.
2. Set the indicator readings I, II, III to zero.
3. For beams in the unloaded condition we write down of the tensometry readings for each point of the cross-sections.
4. Load the beams with force *F* and take tensometry readings (of indicators I, II, III) using a computer.
5. The results are recorded in table 12.1.

IV. Processing of results of an experiment

1. By formulas (12.5) and (12.8) we determine stress, and by formulas (12.4), (12.7) and (12.9) deflections at an unsymmetrical and uniplanar bend.
3. We determine deflection value at an unsymmetrical bend by a formula (12.4), and we take f_y on the indicator I, f_x – on the indicator II.

Deflection value at uniplanar bend is determined by the indicator III.

Table 12.1

<table>
<tr><th rowspan="3">l, m</th><th rowspan="3">a, m</th><th rowspan="3">F, N</th><th rowspan="3">Type of a bend</th><th rowspan="3">No. of points</th><th colspan="2">Stress, MPa</th><th colspan="6">Deflections, mm</th><th colspan="2">% discrepancy</th></tr>
<tr><th rowspan="2">σ^{exp}</th><th rowspan="2">σ^{teor}</th><th colspan="3">exp.</th><th colspan="3">theor.</th><th rowspan="2">σ</th><th rowspan="2">F</th></tr>
<tr><th>f_x</th><th>f_y</th><th>f</th><th>f_x</th><th>f_y</th><th>f</th></tr>
<tr><td rowspan="3">1,2</td><td rowspan="3">1,02</td><td rowspan="3">200</td><td rowspan="3">unsymmetric</td><td>1</td><td></td><td></td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td><td></td></tr>
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<tr><td rowspan="2">1,2</td><td rowspan="2">1,02</td><td rowspan="2">200</td><td rowspan="2">uni-planar</td><td>4</td><td></td><td></td><td rowspan="2">–</td><td rowspan="2">–</td><td rowspan="2"></td><td rowspan="2">–</td><td rowspan="2">–</td><td rowspan="2"></td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td></td><td></td></tr>
</table>

V. Conclusions

1. To give the analysis of results of experimental and theoretical researches.
2. To compare stress and deflections an unsymmetrical and uniplanar bends.

Control questions

1. What bend is called unsymmetrical? Where fundamental difference between an unsymmetrical and uniplanar bends?
2. What is the principle of independence of action of forces?
3. What purpose of work?
4. How to define theoretically normal stresses at an unsymmetrical bending?
5. What conclusions can be drawn on the basis of comparison of normal stress at an unsymmetrical and uniplanar bends?
6. What condition of strengths at an unsymmetrical bend?

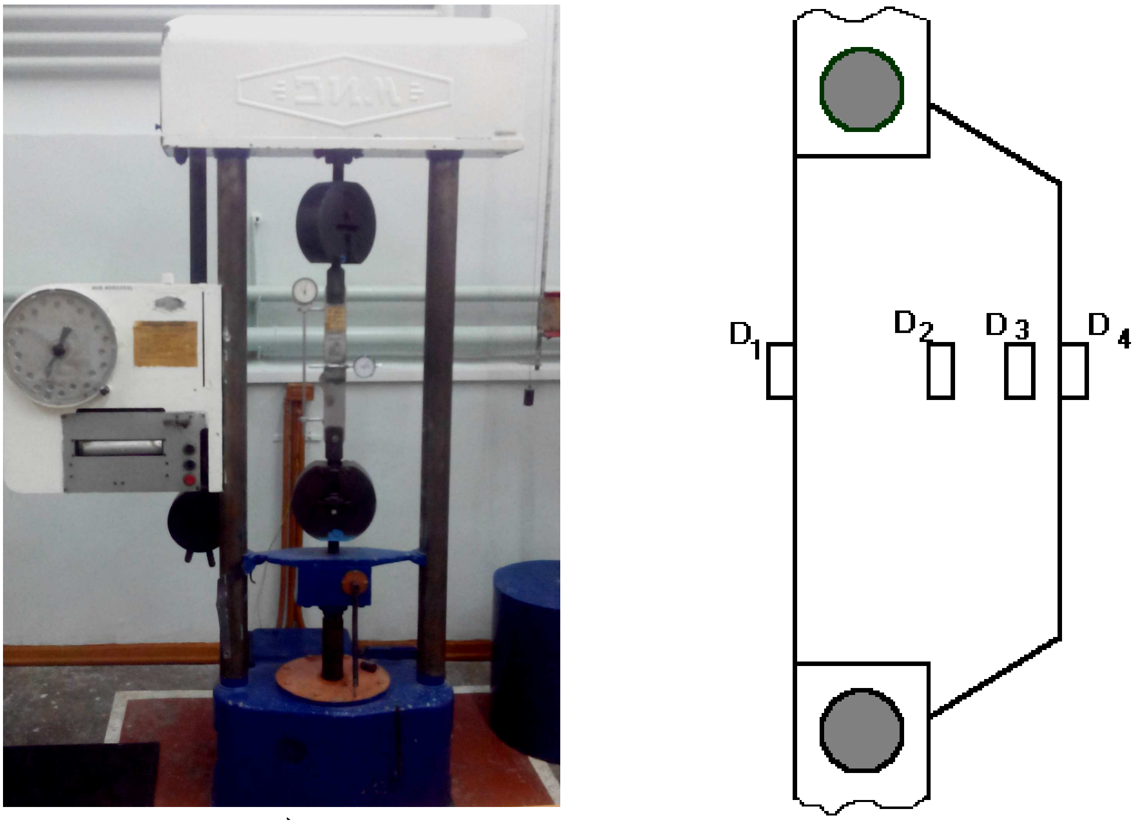
LABORATORY WORK № 13

Research of the unsymmetrical stretching of a straight-axis bar

I. Work purpose: Theoretically and experimentally determine the normal stresses at the designated points of the cross section. Determine the position of the zero line. To confirm Hooke's law at off-center tension-compression and the law of distribution of normal stresses on the cross section of a bar (to plot their diagrams).

II. Content of work

The installation is a rectangular strip with sensors glued to its side surface. Tests are carried out on the machine UMM-5 (fig. 13.1).



a) *b)*

Fig. 13.1. – Scheme of the machine UMM-5 (a) and the layout of strain sensors (b)

a) theoretical determination of tension and position of the zero line (n.l.)

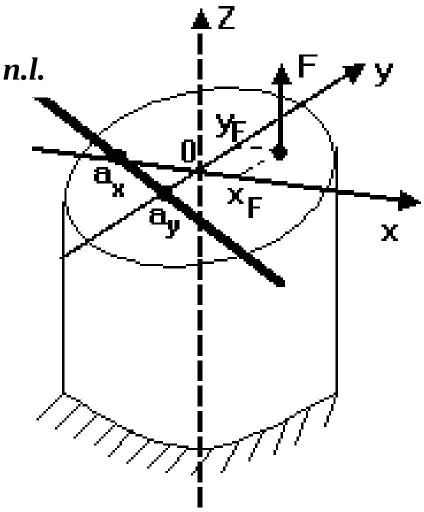


Fig. 13.2. – Scheme of off-center tension-compression

The off-center (unsymmetrical) tension-compression is compound resistance. At the same time in its lateral section work: N, M_x, M_y, i.e. $N = F$,

$$M_x = F \cdot y_F; \quad M_y = F \cdot x_F, \tag{13.1}$$

where y_F, x_F – coordinates of a point of application of force of F.

Normal stresses in any point of lateral section of a bar are determined by a formula:

$$\sigma = \frac{F}{A} + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x \qquad (13.2)$$

where F – external force, A – the cross-section area, I_x, I_y – the principal moments of inertia of section, x, y – the current coordinates (coordinate of points where stress is defined).

Taking into account (13.1) formula for stress will take a form:

$$\sigma = \frac{F}{A}\left(1 + \frac{y_F y}{i_x^2} + \frac{x_F x}{i_y^2}\right); \qquad (13.3)$$

$i_x^2 = \frac{I_x}{A}$; $i_y^2 = \frac{I_y}{A}$ – radiuses of gyration.

From (13.4) we will receive segments which are cut by the zero line on coordinate axes (fig. 13.2):

$$a_x = -\frac{i_y^2}{x_F}; \quad a_y = -\frac{i_x^2}{y_F}; \qquad (13.4)$$

In our case the line of action of force passes through axis x, then (13.3) and (13.4) will take a form:

$$\sigma = \frac{F}{A}\left(1 + \frac{x_F \cdot x}{i_y^2}\right); \qquad (13.5)$$

$$a_x = -\frac{i_y^2}{x_F}; \; a_y = \infty . \qquad (13.6)$$

The analysis (13.5) shows that stresses changes under the linear law. The zero line is parallel to y axis and its position doesn't depend on the size of force F (13.6).

<u>b) experimental determination of stresses and position of the zero line</u>

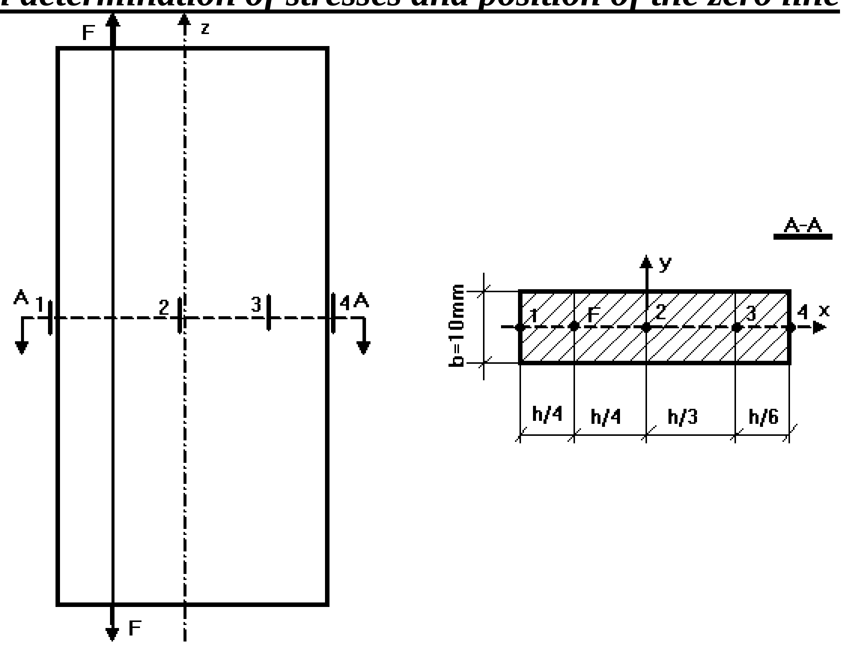


Fig. 13.3. – The geometric dimensions of the section and scheme of loading

We determine stresses in the set points of the section (fig. 13.3) by a tensometry method. For each point indications of a sensors are taken and we determine stresses by computer.

We take F_1, F_2, F_3 loading with any step of ΔF. A maximum load ($F_{\max}$) on a experimental bar, according requirement of testing $(\sigma_{\max} \le \sigma_{pr})$, shouldn't exceed 80 kN, (UMM–5 opportunities – 50 kN).

III. Order of carrying out tests

1. To get acquainted with the device of the UMM–5 machine.
2. By means of a ruler with an accuracy of 1 mm we measure the sizes of section of a bar of *h, b* and we define positions of sensors in section.
3. We take counting's for each sensor in not loaded state by computer.
4. We load a bar with F_i forces (ΔF – any) not exceeding F_{max} and we take counting on sensors.
5. We enter results in table 13.1.

IV. Processing of results of an experiment

1. We determine stresses in points: 1, 2, 3, 4 by a formula (13.5) for all loadings of F_i and also we plot diagrams of this stresses σ_i^{theor}.
2. With the help of a computer, we take readings of sensors before and after loading and determine the experimental stress values. $\sigma_i^{\exp}$.
4. We compare stresses σ_i^{theor} and $\sigma_i^{\exp}$, i.e. we define discrepancy percent.
5. We enter results in table 13.1.

Table 13.1

Geometrical characteristics					№ points	F_i, kN	Stresses, MPa		% discrepancy
b, *cm*	*h*, *cm*	*A*, cm^2	i_y^2, cm^2	x_i, *cm*			σ_i^{theor}	$\sigma_i^{\exp}$	
					1				
					2				
					3				
					4				

V. Conclusions

1. To specify whether Hooke's law at the off-center tension-compression is carried out.
2. To confirm a theoretical conclusion about position of the neutral line at the off-center (unsymmetrical) tension-compression and the distribution law of normal stress.

Control questions

1. What does mean the principle of independence of action of forces?
2. Formulate the work purpose.

3. What type of deformation is called the off-center tension-compression?
4. By what formula normal stresses in any point of lateral section of a bar at the off-center tension-compression are determined?
5. How normal stresses on lateral section of a bar at the off-center tension-compression are distributed?
6. What position is occupied by the neutral line (in the plane of lateral section of a bar) at the off-center tension-compression?
7. What experimental devices are used in experience and what directly were measured by them?
8. Why when testing in bar section the neutral line is perpendicular one of the principal axes of inertia?
9. What mutual positioning of points of application of force, center of gravity of section and neutral line?
10. Whether the distribution law of normal stresses on lateral section confirms experience (at off-center tension-compression bar)?
11. How experimental values of stress were received?
12. What internal forces arise in a bar at the off-center tension-compression?
13. What is called the core section?
14. Why do you need to know the shape of the core section?

LABORATORY WORK № 14
Research of stresses in a curved bar

I. Work purpose: Determination of stresses in a curved bar with plotting of their diagrams on section height.

II. Content of work

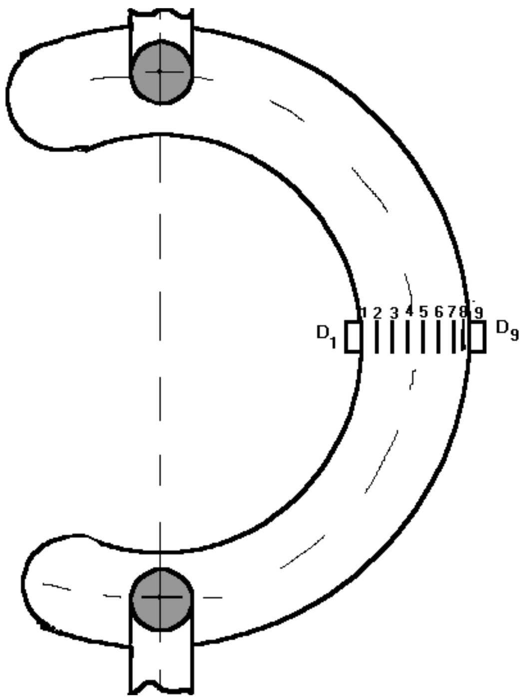


Fig. 14.1. – Scheme of a curved bar

Curved bars are widely used in construction and the equipment. Treat them: hooks, eyes, links of chains, arches, rim of pulleys, wheels, etc.

Studies show that the bending distribution of normal stresses in the cross section, as well as the value of the maximum stresses in the curved bars, other than in the bars with a straight axis.

Installation represents a circular bar of radius of *R* which is subject to stretching by *F* forces by the UMM–5 machine (fig. 14.1).

In horizontal section of a bar (A-A) in 9 points sensors with the help of which the stresses are defined.

a) theoretical determination of stresses

The bar and its sizes are specified on fig. 14.2.

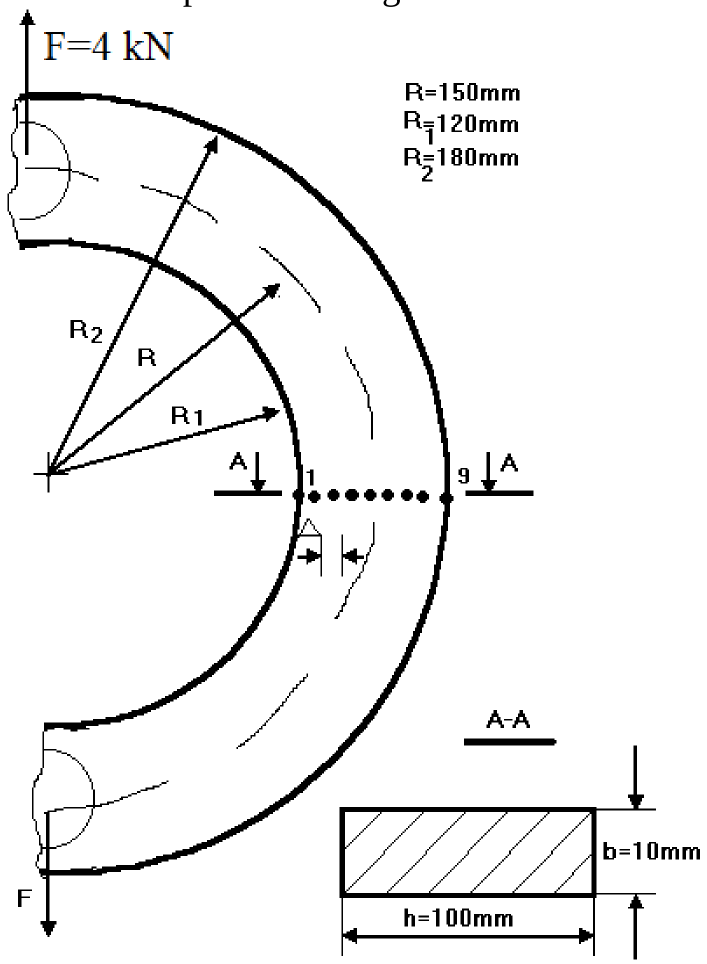


Fig. 14.2. – Scheme of loading of a curved bar and its geometrical sizes

In lateral section A–A 9 sensors with Δ step are pasted.

In a curved bar there are at the same time normal stresses from the longitudinal (N) force and bending moment (M). Tension is determined by a formula:

$$\sigma = \frac{N}{A} \pm \frac{M}{S_x}\frac{y}{\rho}; \qquad (14.1)$$

where $N = F$ – longitudinal force; $A = b \times h$ – bar cross-sectional area; $M = -F \cdot R$ – bending moment in section A–A (the sign "–" means the curvature of a bar decreases); $S_x = A{\cdot}y_0$ – static moment of section w.r.t. neutral line at a poor bend; y_0 – distance from the neutral line to the center of gravity of the section; ($y_0 = R - r$); y – coordinate point where stress is defined; $\rho = r + y$ – the distance from the center of the curvature of the bar to the point where the stress is determined (current radius); r – the distance from the center of curvature to the neutral line, depending on the shape of the cross section of the bar.

For rectangular section:

$$r = \frac{h}{\ln\frac{R_2}{R_1}}; \qquad (14.2)$$

Distance from the neutral line to a point in which stress is defined, it can be found from expression:

$$y_n = -\frac{h}{2} + y_0 + \Delta(n-1); \qquad (14.3)$$

where n – number of a point in which stress is defined; Δ – step with which sensors are located.

<u>Note:</u> when determining stresses, it is necessary to consider signs of bending moment (M) and coordinate of points (y) where stresses is defined.

The sizes and distances mentioned above are given in fig. 14.3.

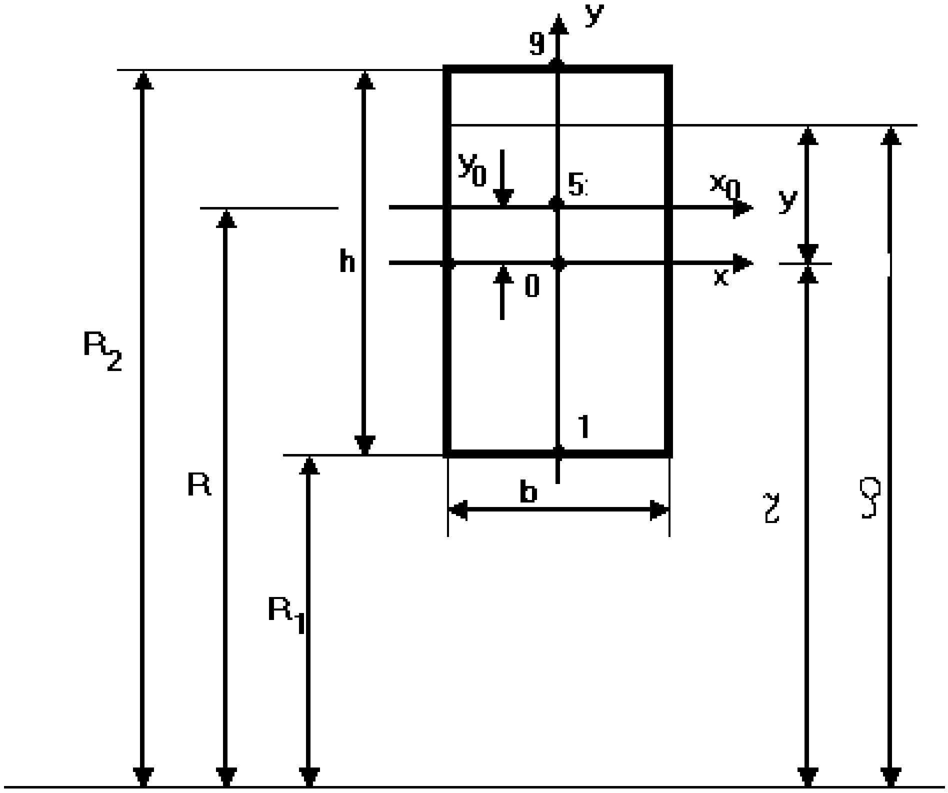


Fig. 14.3. – Geometrical sizes

<u>b. experimental determination of stresses</u>

Experimentally stress is defined by an electrotensometry method (by help of computer).

II. Order of carrying out tests.

1. To get acquainted with the machine device UMM–5.
2. By means of a ruler and a caliper with an accuracy of *0,1* mm we measure *h* bar section sizes, and we define places of sensors in section.
3. We take reading for each sensor in not loaded state.
4. We load a bar with force of *F* and we take counting on sensors.
5. We enter results in table 14.1.

IV. Processing of results of an experiment

1. We determine stresses in points by a formula (14.1): 1, 2, 3, … 9 also we plot diagrams σ_N; σ_M; σ.
2. We determinate by a formula (14.1) stresses in the set points experimentally.
3. We compare stresses received experimentally and theoretically, we define discrepancy percent.
4. We enter results in table 14.1.

Table 14.1

№	y_n, *mm*	ρ_n, *mm*	σ_N, MPa	σ_M, MPa	σ^{theor}, MPa	σ^{exp}, MPa	% discrepancy
1							
2							
3							
4							
5							
6							
7							
8							
9							

V. Conclusions

1. To assess the theoretical and experimental data.
2. Compare the law of stress changes in a curved bar with the law of stress changes for a straight bar.

Control questions:

1. On what signs curved bars subdivide into curved bars of big and small curvature?
2. What formula for theoretical determination of stresses in the studied points was used?
3. How to determine stress in the set points experimentally?
4. To what law of distribution of stress does the curved bar submit?
5. Where the zero line is in curved bar at poor bend?
6. How position of the zero line for the set curved bar is defined?
7. Whether position of the zero line depends on form of lateral section of curved bar? Prove that.

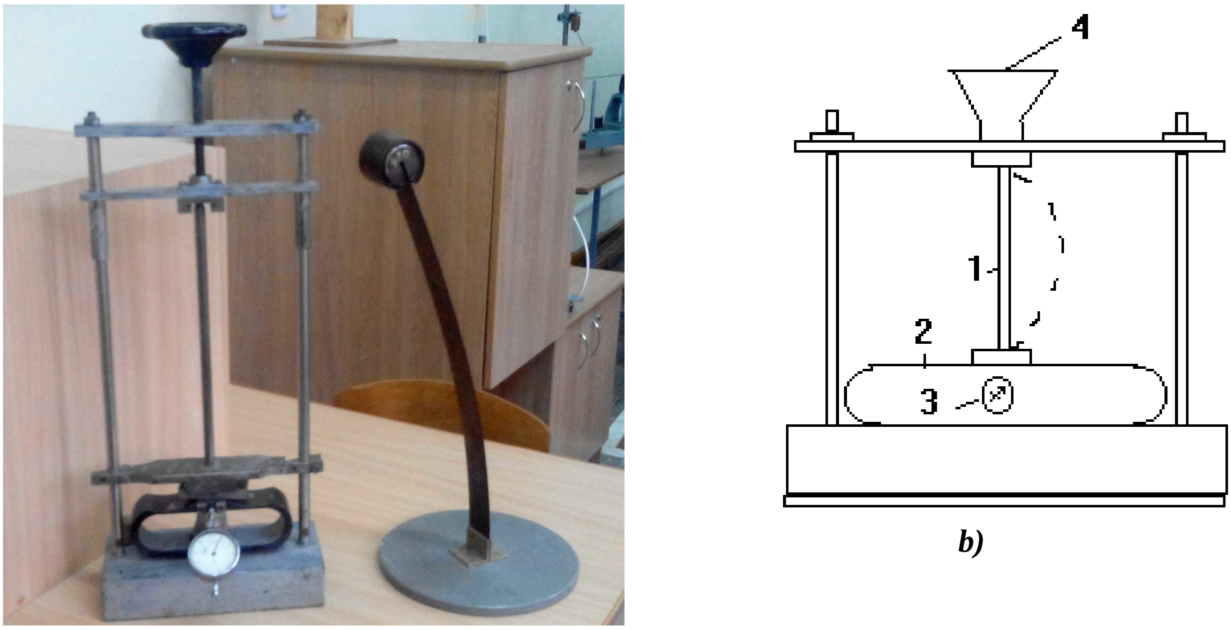
LABORATORY WORK № 15
Research of a longitudinal bend of a rod in an elastic stage

I. Work purpose: To make observation over the phenomenon of loss of stability of a steel rod. To determine by practical consideration the value of critical force and to compare its value with rated one. To calculate critical stress and to compare it with a yielding limit (σ_y).

II. Content of work

The compressed rod of big flexibility at a certain value of the press force called by critical loses a stable equilibrium. At the same time the rod with a straight axis is a little bent. The type of a curve depends on a way of fixing of its ends.

For performance of laboratory work the installation shown in the fig. 15.1 is used.



a)

Fig. 15.1. – Installation scheme
1 – the examine a sample, 2 – the dynamometer, 3 – the indicator,
4 – the loading screw

a) Theoretical determination of critical force and critical stress

At calculation of the critical force F_{cr} it is necessary to know flexibility of a rod which is determined by a formula:

$$\lambda = \frac{\mu l}{i_{\min}}, \qquad (15.1)$$

where: l – rod length; μ – coefficient of reduction of length of a rod to rated (to etalon); i_{min} – the minimum radius of inertia of section of a rod;

$$i_{\min} = \sqrt{\frac{I_{\min}}{A}}; \qquad (15.2)$$

I_{min} – the minimum central moment of inertia of section; A – rod cross-sectional area.

If $\lambda \geq \lambda_{\lim}$, then the value of critical force is determined by Euler's formula:

$$F_{cr} = \frac{\pi^2 E I_{\min}}{(\mu \ell)^2}. \qquad (15.3)$$

If $\lambda < \lambda_{\lim}$, then it is necessary to use Yasinsky-Tetmayer's formula:

$$F_{cr} = (a - b\lambda) \cdot A; \qquad (15.4)$$

where *a* and *b* – the coefficients defined from the reference book depending on rod material (for construction steel: *a = 314 MPa; b = 1,14 MPa*).

The extreme value of flexibility is determined by a formula:

$$\lambda_{\lim} = \sqrt{\frac{\pi^2 E}{\sigma_{pr}}}. \qquad (15.5)$$

Critical stress is determined by a formula:

$$\sigma_{cr} = \frac{F_{cr}}{A}. \qquad (15.6)$$

b) experimental determination of critical force and critical stress

Having fixed the studied sample (1), gradually we load a rod by means of the screw (4) and we monitor indications of the indicator (3).

We determine experimental value of critical force by a formula:

$$F_{cr}^{\exp} = n \cdot c; \qquad (15.7)$$

where *n* – number of divisions; *c* – the scale division interval (tick spacing) of the indicator of a dynamometer.

III. Order of carrying out tests

1. By means of a caliper to measure the sizes of lateral section of a rods with an accuracy of 0,1 mm, and a ruler – rod length, with an accuracy of 1 mm, we enter results of measurements in tab. 15.1.
2. The sample is fixed in the device for tests (fig. 15.1).
3. Gradually increasing loading we fix the maximum deviation of an indicator needle of a dynamometer.
4. We determine by a formula (15.7) experimentally the value of critical force.
5. Results are entered in table 15.1.

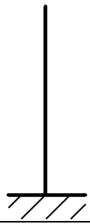
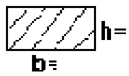
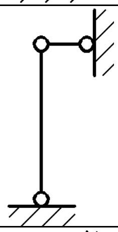
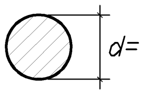
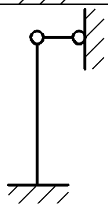
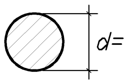
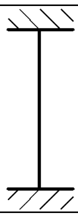
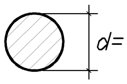
IV. Processing of results of an experiment

1. By practical consideration we determine critical force (15.7).
2. We determine critical stress $\sigma_{cr}^{\exp} = \frac{F_{cr}^{\exp}}{A}$.
3. We determine theoretically critical force and critical stress (15.3); (15.6).
4. We compare experimental and theoretical values.

V. Conclusions

To specify in conclusions whether Euler's formula for a compressed rod of big flexibility is confirmed. To offer an explanation why theoretical critical force is more than experimental. Whether the theoretical character of a curvature of an axis of the rod which lost stability is confirmed.

Table 15.1

№	Length mm	Way of fixing	μ	Form of lateral section and its sizes	$\lambda = \frac{\mu l}{i_{\min}}$	Critical force, N		F_{cr} discrepancy %	Critical stress, MPa	
						F_{cr}^{theor}	$F_{cr}^{\exp}$		σ_{cr}^{theor}	$\sigma_{cr}^{\exp}$
1	500		2							
2	350		1							
3	350		0,7							
4	350		0,5							

Control questions

1. Formulate the purpose of laboratory work.
2. What force is called critical and how behaves the compressed rod under this force?
3. How the way of fixing of a rod influence on the value of critical force?
4. How the form of lateral section of a compressed rod influence on the value of critical force (other things being equal)?
5. How critical force is defined? Write down Euler's formula for compressed rods.
6. By what formula the flexibility of a rod is determined?
7. How the extreme flexibility is determined and where it is used?
8. What experimental devices are used in laboratory work and what they measure?
9. Represent the scheme of testing of a rod.
10. Was the critical stress (in a rod) exceeding of a limit of proportionality of a material?
11. Is Euler's formula for practical purposes applicable?

LABORATORY WORK № 16

Determination of dynamic coefficient at impact loading on beam

I. Work purpose: Determination of dynamic coefficient at impact loading on beam.

II. Content of work

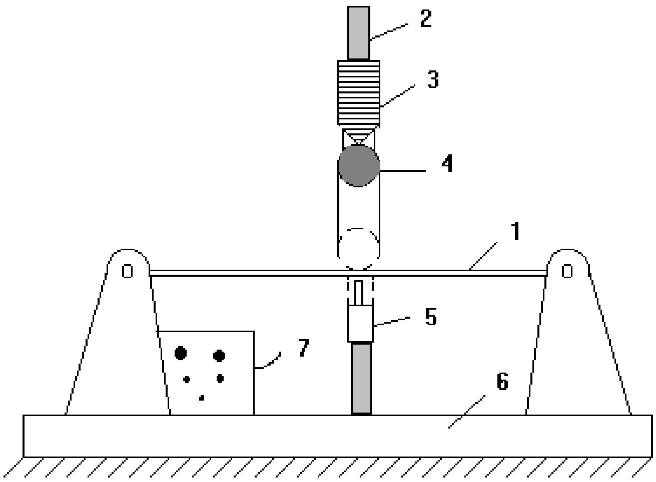
In "Strength of materials" the approximate theory at impact loading based on two assumptions is considered, i.e. the blow (hit) is considered inelastic and the struck system is accepted with one degree of dynamic freedom.

The formulas received on the basis of these assumptions approximate and, actually, demand check.

In work the case at which the blow is directed perpendicular to rod axil is investigated. Such impact loading is called uni-planar.

Installation of CM-21M represents the steel beam, rectangular section lying on two pivoted supports (fig. 16.1).

a)



b)

Fig. 16.1. – Installation scheme

1 – beam, 2 – support, 3 – temporary magnet, 4 – load (ball), 5 – micrometer, 6 – the bad, 7 – the control panel

Load (weight of G) is kept over beam by means of temporary magnet.

Static $f_{st}^{\exp}$ and dynamic $f_{din}^{\exp}$ deflections are measured by the micrometric screw installed under beam 1.

Experimentally dynamic coefficient $K_{din}^{\exp}$ is determined by formula:

$$K_{din}^{\exp} = \frac{f_{din}^{\exp}}{f_{st}^{\exp}}. \qquad (16.1)$$

Theoretically dynamic coefficient K_{din}^{teor} at blow is determined by formula (fig. 16.2). The load falls on beam from some height of H:

$$K_{din}^{theor} = 1 + \sqrt{1 + \frac{2H}{f_{st}^{theor} + \left(1 + \eta \frac{Q}{G}\right)}}, \qquad (16.2)$$

where H – height of fall of the striking body; G – the weight of the striking body (ball); Q – the weight of the struck system (beam); η – the coefficient depending on mode of fixing of beam and the place of impact point of load (ball) (in our case $\eta = 17/35$); f_{st} – static deflection in the direction of blow.

For two-support bar the greatest static deflection in the middle of span, is equal:

$$f_{st}^{theor} = \frac{G \cdot l^3}{48 E J_x}. \qquad (16.3)$$

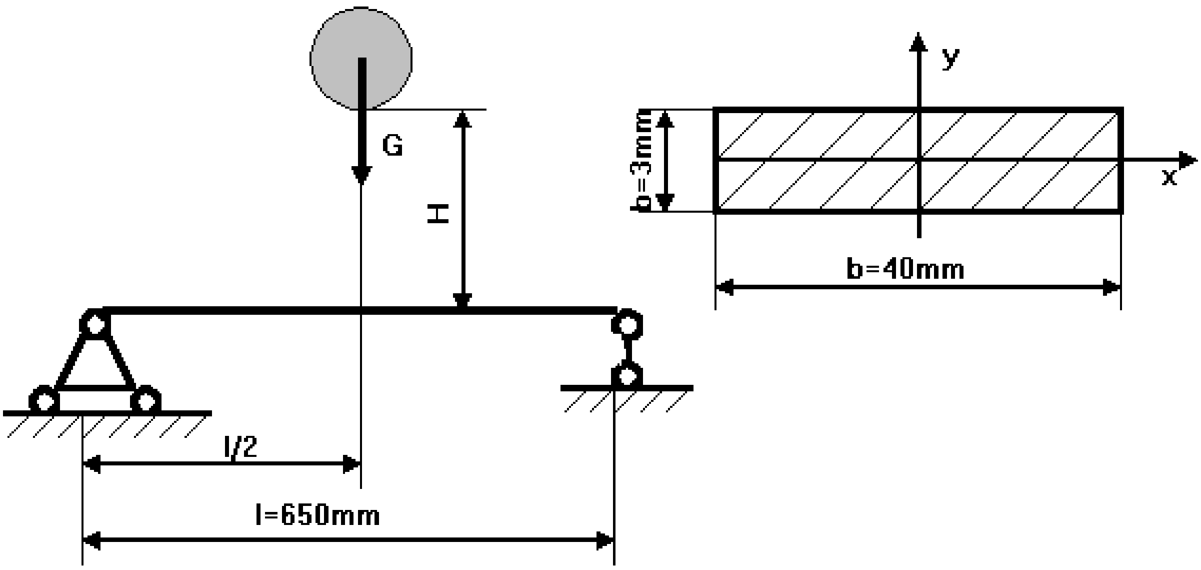


Fig. 16.2. – The scheme of the beam and its geometric dimensions

<u>III. Order of carrying out tests</u>

1. Rotating the micrometric screw of micrometer (5), the space between (5) and beam cleans up (1). So that the get signal from alarm lamp on (7) panel. The electric contour becomes isolated. Is fixed (h_0) on micrometer (5) with accuracy up to 0.01 mm.
2. Load by the weight of G (ball) is put on the midpoint of beam. Rotating the micrometric screw (5) is fixed (h_{st}).
3. Load (ball) on H height is established, previously having switched on the (7) panel temporary magnet (3).

4. Having disconnected temporary magnet (3) the ball falls to beam (1).
5. By means of micrometer (5) is fixed (h_{din}) at which there will be contact between the (5) screw and (1) beam (the lamp shines).

IV. Processing of results of an experiment

1. Are determined static $f_{st}^{\exp}$ and dynamic $f_{din}^{\exp}$ deflection of beam:

$$f_{st}^{\exp} = h_{st} - h_0; \qquad f_{din}^{\exp} = h_{din} - h_{0.}$$

2. The dynamic coefficient K_{din}^{theor} on formula (16.2) is calculated, where

$$G = mg; \qquad \left(m = 70\,gr, \qquad g = 9{,}81\,m/sek^2\right),$$

$$Q = \gamma V = \gamma \cdot A \cdot l = \gamma b \cdot h \cdot l; \qquad \left(\gamma = 78{,}5\,kN/m^2\right),$$

b, h, l – beam sizes, f_{st}^{theor} – is determined by formula (16.3) which can be transformed to look:

$$f_{st}^{theor} = \frac{mgl^3}{4Ebh^3}. \qquad (16.4)$$

Results of tests are entered in table 16.1.

Table 16.1

№	Height N, mm	Statich deflections, mm		% discre-pancy	Dynamic deflections, mm		% discre-pancy	Dynamic coefficients		% F_{cr}^{theor}
		$f_{st}^{\exp}$	f_{st}^{theor}		$f_{din}^{\exp}$	f_{din}^{theor}		$K_{din}^{\exp}$	K_{din}^{theor}	
1										
2										
3										
4										

Control questions

1. Why coefficient is called “dynamic” and what it characterizes?
2. How the experimental value of dynamic coefficient has been found in work?
3. Write formula for dynamic coefficient. Explain the parameters entering it.
4. How the ratio of weight of the striking body and the struck system influence the value of dynamic coefficient?
5. What will be the simplified formula for dynamic coefficient?

EDUCATIONAL EDITION

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LABORATORY WORKS
STRENGTH OF MATERIALS

For students full-day studies
Faculty of civil and industrial engineering

(Part 2)

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