Tab. 2. Overshooting of yw and yz changes.

Changes of parameters	Overshooting from „w"			Overshooting from „z"		
	a	d	u	a	d	u
$0.95 \cdot n$, $1.05 \cdot m_p$	42,94	78,48	49,17	77,30	47,77	46,20
n, m_p	43,27	79,05	49,69	76,97	47,71	45,62
$1.05 \cdot n$, $0.95 \cdot m_p$	43,58	79,58	50,18	76,65	47,65	45,07

Finally, it is stated that the tests carried out during the ground tests of the engine could provide full information about its properties from the signal „*w*" (follow-up test) and the „*z*" signal (test of resistance to disturbance) in flight. Characteristics from the „*z*" signal allow to unequivocally evaluate its properties during the flight of an airplane without performing an expensive (often dangerous) aircraft flight after its new adjustment.

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ON STABILIZATION OF NONLINEAR CONTROL SYSTEMS WITH GRÜNWALD-LETNIKOV *h*-DIFFERENCE FRACTIONAL OPERATOR[1]

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Preliminaries. In system theory, in theoretical issues such as, for example, stability and stabilization, representation and identification of nonlinear models, disturbance rejection, a nonlinear dynamic in many cases is represented explicitly as a sum of its Taylor linearization and residual around the equilibrium or working point. Then, results follows from using the known Implicit Function Theorem. Although the concept of the described procedure is simple, but finding the reverse of the Jacobian is not so simple and obvious, it is known to an involved process. In [1] it was proposed another approach based on higher order functions that simplifies the procedure of applying Implicit Function Theorem. This approach was used successfully to examine such properties as controllability and observability of nonlinear discrete-time control systems with fractional difference operators. Now, our goal is to briefly

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sketch how one can answer for the question if the local properties connected with stability and stabilization can be described via global properties of linearized systems.

Let us start from introducing necessary notation and facts. For any real positive h and real number a let $(hN)_a = \{a, a+h, a+2h, \ldots\}$. The *Grünwald-Letnikov h-difference fractional operator* of order $\alpha > 0$ for a function $x: (hN)_a \to R$ is defined as follows (see [2])

$$({}_a\Delta_h^\alpha x)(t) = \sum_{s=0}^{\frac{t-a}{h}} c^{(\alpha)}(s) x(t - sh),$$

where $c^{(\alpha)}(s) = \frac{(-1)^s}{h^\alpha}\binom{\alpha}{s}$ and $\binom{\alpha}{s}$ is the classical binomial coefficient, i.e. $\binom{\alpha}{s} = \frac{\alpha(\alpha-1)\ldots(\alpha-s+1)}{s!}$ for $s \in N$ and $\binom{\alpha}{s} = 1$ if $s = 0$.

Let A be a square $p \times p$ matrix. Recall also that the *discrete Mittag-Leffler function* is the function defined as

$$E_{(\alpha,\beta)}(A, n) = \sum_{k=0}^{\infty} A^k \binom{n - k + k\alpha + \beta - 1}{n - k}.$$

Later on we will denote$(E^{\rho}_{(\alpha,\alpha)}(\lambda, n) = E_{(\alpha,\alpha)}(\lambda, n-1)$.

Besides definitions associated with fractional calculus, we need also functions from particular family. Namely, a continuously differentiable function $F: R^p \to R^m$is called a *higher order function* if $F(0) = 0$ and $\frac{\partial \mathrm{F}}{\partial \mathrm{x_i}}|_{\mathrm{x}=0} = 0$ where $x = (x_1, \ldots, x_p) \in R^p$. The class of higher order functions is denotes by $\mathcal{H}(\mathrm{p}, \mathrm{m})$.

Proposition 1. [3] If $Ax + By + f(x, y) = z, f \in \mathcal{H}(\mathrm{p}, \mathrm{m})$ and A is nonsingular matrix, then $x = A^{-1}(z - By) + g(y, z), g \in \mathcal{H}(\mathrm{p}, \mathrm{m})$.

Linear control system with Grunewald-Letnikov-type h-difference fractional-order operator. Let us consider the following control system

$$({}_0\Delta_h{}^\alpha x)(t + h) = F(x(t), u(t)), \qquad x(0) = x_0 \tag{1}$$

where $t \in (hN)_0$, $x: (hN)_0 \to R^p$ denotes the state vector, the values of control function u are elements of an arbitrary subset Ω of R^m, function F is smooth. Additionally we assume that $F(0,0) = 0$ and that $0 \in int\Omega$. System (1) can be rewritten into the following equivalent form:

$$({}_a\Delta_h{}^\alpha x)(t + h) = Ax(t) + Bu(t) + f(x(t), u(t), \qquad x(0) = x_0 \tag{2}$$

where $t \in (hN)_0$, $A = \frac{\partial F}{\partial x}(0,0)$, $B = \frac{\partial F}{\partial u}(0,0)$ and $f: R^p \times \Omega \to R^p$ is a nonlinear function with the property $f \in \mathcal{H}(\mathrm{p}+\mathrm{m}, \mathrm{p})$. If $f(x(t), u(t)) = 0$, then system (2) is called the *linear approximation* of system (1).

Proposition 2. Let $0 < \alpha \leq 1$ and $u: (hN)_0 \to \Omega$ be a fixed. Then system (2) has the unique solution given by

$$x(t) = E_{(\alpha,\alpha)}\left(Ah^\alpha, \frac{t}{h}\right) x_0 + (E^{\rho}_{(\alpha,\alpha)}(Ah^\alpha, \cdot) * B\bar{u})(\tfrac{t}{h}) + F_t(x_0, U),$$

where $\bar{u}\left(\frac{t}{h}\right) = h^{\alpha}u(t)$, $F_n(x_0, U) \in \mathcal{H}(\mathrm{p}, \mathrm{p})$ and $F_t(x_0, U) = (E^{\rho}_{(\alpha,\alpha)}(Ah^{\alpha}, \cdot) * \bar{f})(\frac{t}{h})$ with $\bar{f}\left(\frac{t}{h}\right) = h^{\alpha}f(x(t+a), u(t))$.

Proof is same as the proof of similar result given in [3] for the case $({}_a\Delta_h{}^{\alpha}x)(t) = Ax(t+a) + Bu(t)) + f(x(t+a), u(t),\ x(a) = x_0$.

Local Controllability of nonlinear system. Let $J_0(m)$ be a set of all sequences $U = (u(0), u(h), u(2h) \dots)$, where $u(nh) = u(t) \in \Omega$. Let $\gamma(t, x_0, U) = x(t)$, $t \in (hN)_a$, denote the *state forward trajectory* of system (2), i.e. the uniquely defined solution of (2) defined by initial state x_0 and control sequence $U \in J_0(m)$. The *reachable set* $\mathcal{R}^q(x_0)$ from the given initial state x_0 in q, $q = 1,2,3, \dots,$ steps is the set of all states to which the given system can be steered from the initial state x_0 in q steps by sequence control $U \in J_0(m)$.

Definition 1. System (1) is *locally controllable in q steps* from initial state x_0 if there exists a neighborhood V of x_0 such that $V \subset \mathcal{R}^q(x_0)$. System (1) is *glabally controllable in q steps* from initial state x_0 if $\mathcal{R}^q(x_0) = R^p$.

Matrix $Q_q = [B \quad E_{(\alpha,\alpha)}(Ah^{\alpha}, 1)B \quad \dots \quad E_{(\alpha,\alpha)}(Ah^{\alpha}, q-1)B]$ is called the *controllability matrix*. Based on Proposition 2, directly from [3] it follows that the linear approximation

$$({}_a\Delta_h{}^{\alpha}x)(t+h) = Ax(t) + Bu(t),\ \ x(0) = x_0, \tag{3}$$

of system (1) is globally controllable if and only if the rank of matrix Q_q is full.

Theorem 1. [3] System (1) is locally controllable in q steps from the origin if its linear approximation (3) is globally controllable in q steps from initial state x_0.

Stabilization. Recall that $x^e = (x_1^e, \dots, x_p^e)$ is an *equilibrium* of system

$$({}_0\Delta_h{}^{\alpha}x)(t+h) = Ax(t) \tag{4}$$

if and only if $({}_0\Delta_h{}^{\alpha}x^e)(t) = Ax^e$ for $t \in (hN)_0$.

Definition 2. Equilibrium $x^e = 0$ of system (3) is said to be *asymptotically stable* if (i) for each $\epsilon > 0$ there exists $\delta > 0$ such that $\|x_0\| < \delta$ implies $\|x(t)\| < \epsilon$ for all $t \in (hN)_0$ and *(ii)* if there exists $\delta > 0$ such that $\|x_0\| < \delta$ implies $\lim_{t\to\infty} x(t) = 0$.

Linear approximation (3) of system (1) is called *stabilizable* if the exists the linear state-feedback controller with gain $F \in R^{m\times p}$, i.e. $u(t) = Fx(t)$, such that the closed loop system

$$({}_a\Delta_h{}^{\alpha}x)(t+h) = h^{\alpha}(A+BF)x(t), \qquad x(0) = x_0$$

is asymptotically stable (for condition of existing the stabilization feedback controller for system (3) see [3]).

Theorem 2. If the linear approximation of system (1) is globally controllable then system (1) can be locally stabilizable in q steps by state-feedback law $u(t) = Fx(t) + \gamma(x(t))$ with $\gamma(\cdot) \in \mathcal{H}(2p, qm)$.

Sketch of the proof: Let x_0 be a fixed initial state. Then control sequence $U_q = Q_q^{-1} E_{(\alpha,\alpha)}(Ah^{\alpha}, q) + \eta(x_0, 0)$, where $\eta(x_0, 0) \in \mathcal{H}(2p, qm)$, transfers in q steps system (1) to the origin, see [3]. Since this sequence is uniquely determined in a neighborhood of the origin and origin is the equilibrium of (3) then using reasoning based on higher order functions (see [1]) we obtain thesis.

Remark: The approach to stabilization of nonlinear control systems with Grünwald-Letnikov h-difference fractional operator given in Theorem 2 is a consequence of controllability results presented in [3]. Its idea is different from the idea of approach to the similar problem presented in [5].

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THE METHOD OF THE PIPELINE DAMAGE DETECTION USING THE ADDITIONAL INSTRUMENTATION - CORRECTORS

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Pipelines are difficult to service due to their large size and complex construction. Comprehensive diagnosis of leakages from long pipelines consists of several activities: leak detection, estimation of its size, searching for the location where the leak occurred, deduct the intentional leak (gas collection) from the damage. There are a number of methods to detect leaks (from monitoring pipelines using trained dogs, monitoring pressure and flows, to methods using neural networks), but each of these methods has its weak and strong sides. Due to the possible catastrophic consequences of misdiagnosis, several methods of detecting and locating leakages from pipelines are often used in parallel.

The article presents the basics of the method that can be used as a supplementary method. It is based on standard signals taken from the pipeline (it can be pressure,