UDC 621.923.7

AXI-SYMMETRICAL THERMOELASTICITY PROBLEM FOR GRADIENT COATING

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In the present paper the axi-symmetrical thermoelastic problem for a functionally graded coated half-space which surface is heated in circular area is considered. Two approaches to the solution of the problem are investigated: 1°) the analytic method solution of partial differential equations; 2°) inhomogeneous layer modelled by a package of homogeneous isotropic layers. The difference between the obtained solutions were analysed.

Considered nonhomogeneous half-space is composed of homogeneous half-space ($-\infty < z \le 0$) and the gradient coating ($0 \le z \le h$), $h = H/a$, H is the thickness of coating, a – radius of the heat area (Fig. 1). Material properties of the base and the coating are describe by the shear modulus μ, Poisson ratio ν, coefficient of thermal expansion α and thermal conductivity coefficient K (Fig. 1). The surface of the half-space is free from the mechanical loading. Displacements and stresses was arise from heat the surface in the circular area by the heat flux (Fig. 1). Outside the heat area the surface is thermally insulated. We assume the perfect mechanical and thermal contact conditions between coating and substrate.

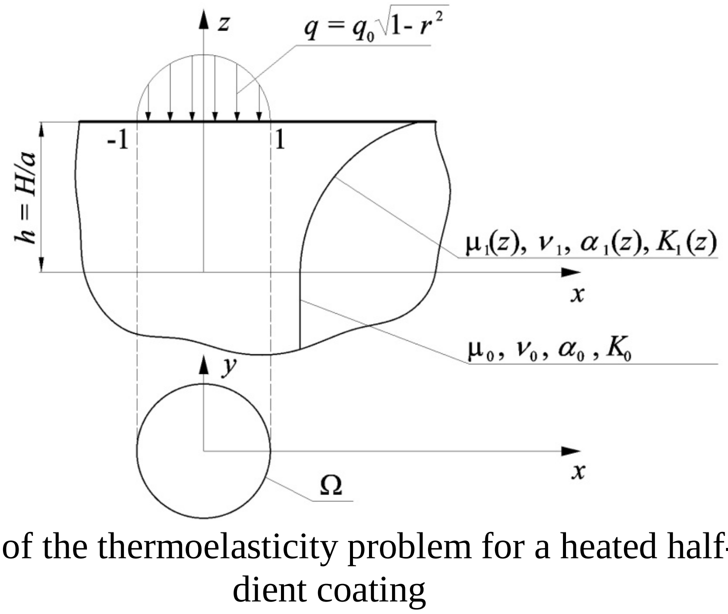

Fig. 1. Scheme of the thermoelasticity problem for a heated half-space with gradient coating

We assume that

$$\begin{bmatrix} \mu_1(z) \\ \alpha_1(z) \\ K_1(z) \end{bmatrix} = \begin{bmatrix} \mu_{\text{int}} \exp(\beta z) \\ \alpha_{\text{int}} \exp(\delta z) \\ K_{\text{int}} \exp(\gamma z) \end{bmatrix}, \quad \begin{bmatrix} \beta h \\ \delta h \\ \gamma h \end{bmatrix} = \begin{bmatrix} \ln(\mu_{\text{sur}}/\mu_{\text{int}}) \\ \ln(\alpha_{\text{sur}}/\alpha_{\text{int}}) \\ \ln(K_{\text{sur}}/K_{\text{int}}) \end{bmatrix},$$

μ_{sur}, α_{sur}, K_{sur} are shear modulus, thermal expansion coefficient and thermal conductivity coefficient, respectively on the surface of the inhomogeneous half space, μ_{int}, α_{int}, K_{int} are shear modulus, thermal expansion coefficient and thermal conductivity coefficient, respectively on the coating at interface between coating and base. The parameters μ_0, α_0, K_0, ν_0 and ν_1 are constant.

We obtain the following boundary value problem:

- the equations of the thermoelasticity and the heat conduction in the coating (differential equations with variable coefficient) and the base;
- two mechanical and one thermal boundary conditions on the surface of the inhomogeneous half space;
- boundary conditions of perfect mechanical (four conditions) and thermal (two conditions) contact between the coating and the base;
- imposed of the stresses at infinity.

In analytic-numerical method we split the coating into *n* layers [1]. It is assumed that all layers are homogeneous. Thermoelastic properties of each layer was obtain by average the properties relative to thickness of layer.

The equations of the thermoelasticity and the heat conduction may be solved by use of the Hankel transform technique [2]. We obtain ordinary differential equations, which we solve analytically. Obtained solution contain nine (approach 1°) or $6n+3$ (approach 2°) unknown functions of the integral transform parameter *s*. These functions are obtained satisfying boundary conditions which may be written in Hankel transform domain. We obtain two systems of linear equations. The first contain 3 (approach 1°) or $2n+1$ (approach 2°) equations and is used to calculate transform of the temperature, second contain 6 (approach 1°) or $4n+2$ (approach 2°) equations. Base on it we can obtain transform of the displacement vector and stress tensor.

The integrals inside the inhomogeneous half space are found with the help of the Gaussian quadratures. The accuracy of calculation of integrals is supported by the continuity of stresses for $z \to h$. Calculating the integrals for z = h, we use the asymptotic behavior of the integrands as $s \to \infty$. The integrals for calculating the stresses in which the integrands are replaced by its asymptotes describe the stresses distributions in homogeneous half-space.

The inhomogeneous layer is modelling by a package of homogeneous layers. After analysis of basic relations which describe the considered problem it can be seen that solution depend on on ten dimensionless parameters: μ_{sur}/μ_0, μ_{int}/μ_0, α_{sur}/α_0, α_{int}/α_0, K_{sur}/K_0, K_{int}/K_0, ν_0, ν_1, h and n. The analytical solution depends on the first nine indicated parameters. To simplify of the calculations we decrease the number of input parameters. The following assumption was taken: the thermoelastic properties on the coating at interface between coating and base is related to the mechanical and thermal properties of the substrate ($\mu_{int}/\mu_0 = \alpha_{int}/\alpha_0 = K_{int}/K_0 = 1$); $\nu_0 = \nu_1 = 0.3$; $\mu_{sur}/\mu_0 = 4$. Also it was assumed that: $\alpha_{sur}/\alpha_0 = 0.25$ or 4; $K_{sur}/K_0 = 0.25$ or 4; $h = 0.2$, 0.4 or 0.8; $n = 10$, 20, 40 or 80.

In Figs. 2 and 3, the rhombus mark the numerical results obtained for the multilayered coating, whereas the solid lines correspond to the coating with continuous variation of mechanical and thermal properties.

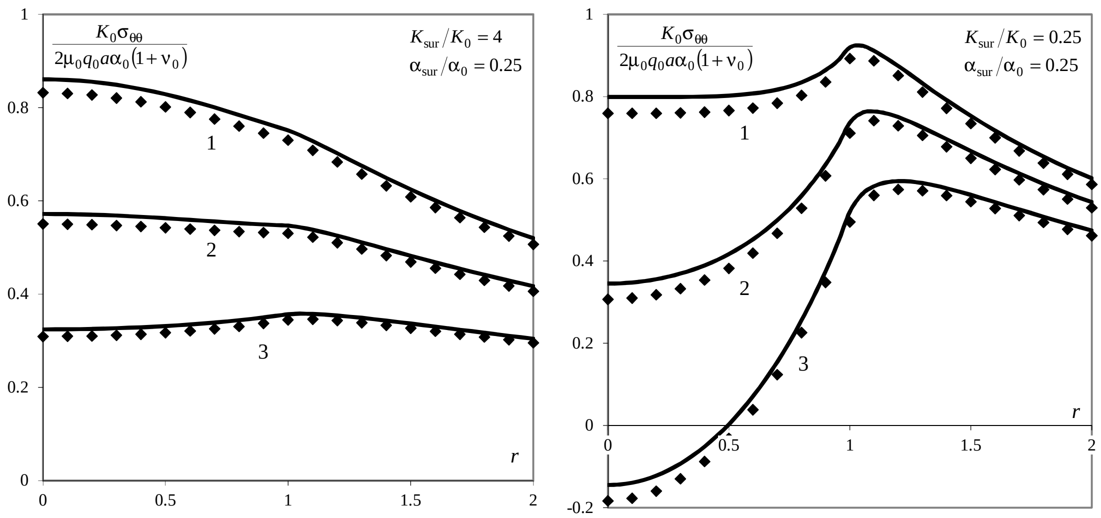

Fig. 2. The hoop stresses over the surface of the non-homogeneous half space: 1 – *h* = 0.2; 2 – *h* = 0.4; 3 – *h* = 0.8; *n* = 40

For the purpose of modelling non-homogeneous coating by package of homogeneous layer the comparative analysis was made.

We can observe that maximum difference between obtained solutions of the considered two approaches was observed on the surface of the nonhomogeneous half-space in the centre of the heat area (Fig. 2.). This difference depend on character of the function which describe the change of the properties in the coating (Fig. 2.).

In Fig. 3. the distribution of the stress depending on the depth in considered body was presented. Stresses obtained for package of homogeneous layers have step changes on the interface between layers. The results in the middle of the layer is accurate (Fig. 3.).

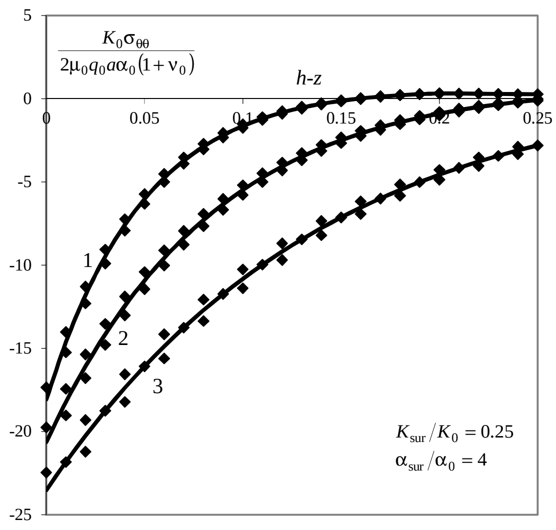

Fig. 3**.** Distributions of the radial (or hoop) stresses along the z-axis: 1 – *h* = 0.2; 2 – *h* = 0.4; 3 – *h* = 0.8; *n* = 40.

The results of the calculations obtained by modelling the gradient coating by package of homogeneous layers is characterized a good agreement with results obtained by analytical calculations. This compliance confirm the thesis that the gradient coating may be modelled by package of the homogeneous layers.

References

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UDC 621.923.7

STRESSES IN COATING WITH INTERLAYER

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Coatings are used for improvement the tribological properties of surface, particularly in cases of frictionally cooperative elements. Appropriate selection of the coatings may cause decrease of the friction and reduction of thermal or chemical adverse influence of environment.

The subject matter of the considered problem is homogeneous half-space with inhomogeneous coating which contain gradient interlayer. Proper evaluation of the strength characteristics of coating is impossible without the calculation of the stress field caused by the contact pressure [1,2].

Considered half-space (Fig. 1.) is composed of homogeneous isotropic linear-elastic half-space and double-layer coatings which contains homogeneous top coat and gradient interlayer. The mechanical properties of the base and top coat were described respectively by Young's moduli E_0 and E_2, and constant Poisson's ratios ν_0 and ν_2. Interlayer's mechanical properties are described by Poisson's ratio $\nu_1(z)$ and Young's modulus $E_1(z)$, which are changing with distance to surface according to determined dependence.

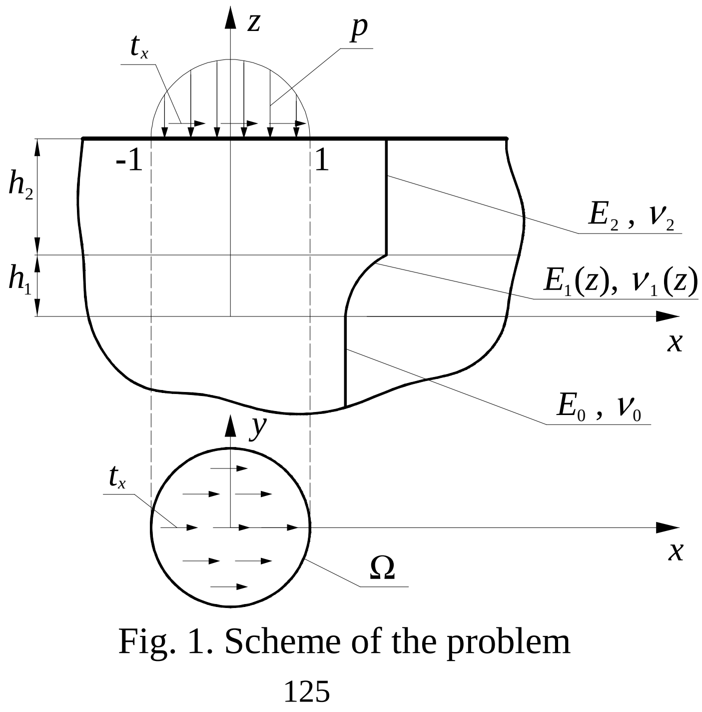

Fig. 1. Scheme of the problem