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STRESSES IN COATING WITH INTERLAYER

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Coatings are used for improvement the tribological properties of surface, particularly in cases of frictionally cooperative elements. Appropriate selection of the coatings may cause decrease of the friction and reduction of thermal or chemical adverse influence of environment.

The subject matter of the considered problem is homogeneous half-space with inhomogeneous coating which contain gradient interlayer. Proper evaluation of the strength characteristics of coating is impossible without the calculation of the stress field caused by the contact pressure [1,2].

Considered half-space (Fig. 1.) is composed of homogeneous isotropic linear-elastic half-space and double-layer coatings which contains homogeneous top coat and gradient interlayer. The mechanical properties of the base and top coat were described respectively by Young's moduli E_0 and E_2, and constant Poisson's ratios ν_0 and ν_2. Interlayer's mechanical properties are described by Poisson's ratio $\nu_1(z)$ and Young's modulus $E_1(z)$, which are changing with distance to surface according to determined dependence.

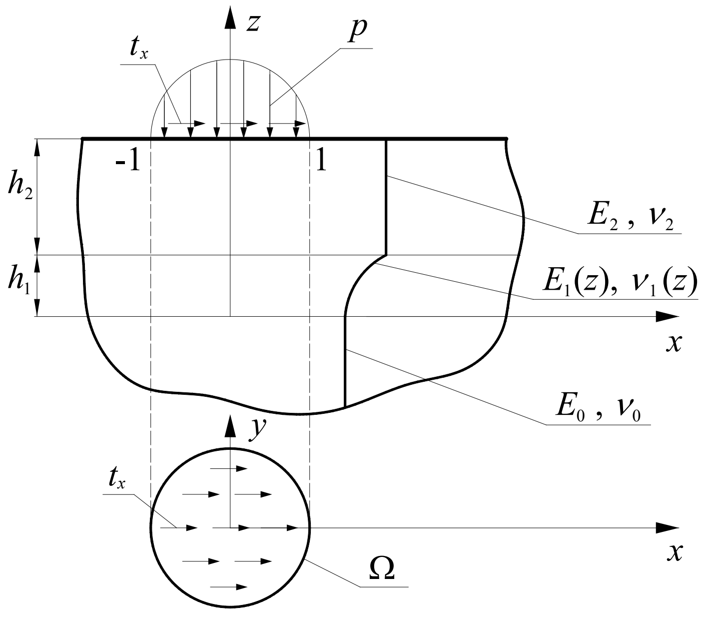

Fig. 1. Scheme of the problem

The surface is under normal p and tangential ($\tau = fp$, f – coefficient of friction) loading applied in circular area Ω. We assume that the distribution of the loading is elliptical.

Solving the problem analytically we should solve the elasticity equations which in case inhomogeneous interlayer is linear partial differential equations with variable coefficients. Only in a few cases the analytical solutions of these equations can be construct.

The problem is described by equations of the theory of elasticity [3], in terms of displacements for top layer, interlayer and base. In gradient interlayer linear partial differential equation with variable coefficients

$$(1-2\nu_i)\Delta\mathbf{u}^{(i)} + grad\,\mathrm{div}\,\mathbf{u}^{(i)} = 0,\ i = 0,\ 1,\ 2; \tag{1}$$

which can be solved fulfil the boundary conditions:

- loading the surface of non-homogeneous body

$$\sigma_{xz}^{(n)}(x, y, z = h) = t_x(x, y)H(x, y), \tag{2}$$

$$\sigma_{yz}^{(n)}(x, y, z = h) = 0, \tag{3}$$

$$\sigma_{zz}^{(n)}(x, y, z = h) = -p(x, y)H(x, y); \tag{4}$$

- perfect contact between components of considered half-space

$$\mathbf{u}^{(i+1)}(x, y, z = z_i) = \mathbf{u}^{(i)}(x, y, z = z_i),\ i = 0,1; \tag{5}$$

$$\boldsymbol{\sigma}^{(i+1)}(x, y, z = z_i)\cdot\mathbf{n} = \boldsymbol{\sigma}^{(i)}(x, y, z = z_i)\cdot\mathbf{n},\ i = 0,\ 1; \tag{6}$$

- conditions in infinity

$$\mathbf{u}^{(i)}(x, y, z) \to 0,\ \ x^2 + y^2 + z^2 \to \infty,\ i = 0,\ 1,\ 2. \tag{7}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\mathbf{u}^{(i)}$ – dimensionless vector of displacement related to a parameter; $\boldsymbol{\sigma}^{(i)}$ – stress tensor; indexes i = 0, 1, 2, characterize parameters and functions respectively in base, in the interlayer and in the top layer; $\mathbf{n}$ = (0, 0, 1); $H(x,y)$ – Heaviside step function ($H(x,y) = 1$, when $(x,y) \in \Omega$ and $H(x,y) = 0$, when $(x,y) \notin \Omega$).

Solving the problem analytically we should solve the elasticity equations which in case inhomogeneous interlayer is linear partial differential equations with variable coefficients [4]. Only in a few cases the analytical solutions of these equations can be construct.

The solution of the boundary problem was constructed using the two-dimensional integral Fourier transform, which is defined by the following equation:

$$\tilde{f}(\xi, \eta, z) = \mathcal{F}(f(x, y, z), x \to \xi, y \to \eta) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} f(x, y, z)\exp(-ix\xi - iy\eta)\,dxdy. \tag{8}$$

Using the integral Fourier transform partial differential equations was written as ordinary differential equations. We can write the general solution which contain 15 unknown functions of integral Fourier transform parameters. These functions were obtained computing the results of the system of the linear equations formed through fulfilment of the boundary conditions in a transform space. Using the inverse integral Fourier transform the relationships between components of stress tensor were obtained in two-dimensional integrals form.

We consider coatings (Fig. 2.) with homogeneous outer layer and:

- a) without interlayer
- b) interlayer Young's modulus is constant
- c) interlayer Young's modulus is changing by the power function

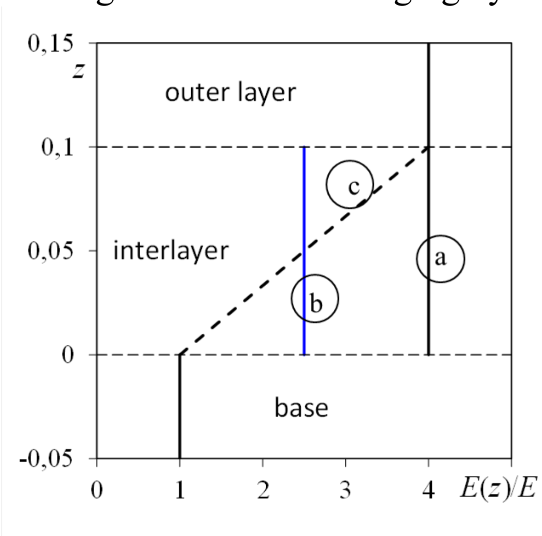

Fig. 2. Change of the properties (Young modulus)

For analysis the following values were taken

- $\nu_0 = \nu_1(z) = \nu_2 = 1/3$,
- $E_2/E_0 = 4$.
- $f = 0$ and $f = 0{,}25$;
- $h_1/h = 1/4$,
- $h = h_1 + h_2 = 0{,}4$ or $h = 0{,}8$.

The stress distribution for various parameter was presented in Fig. 3.

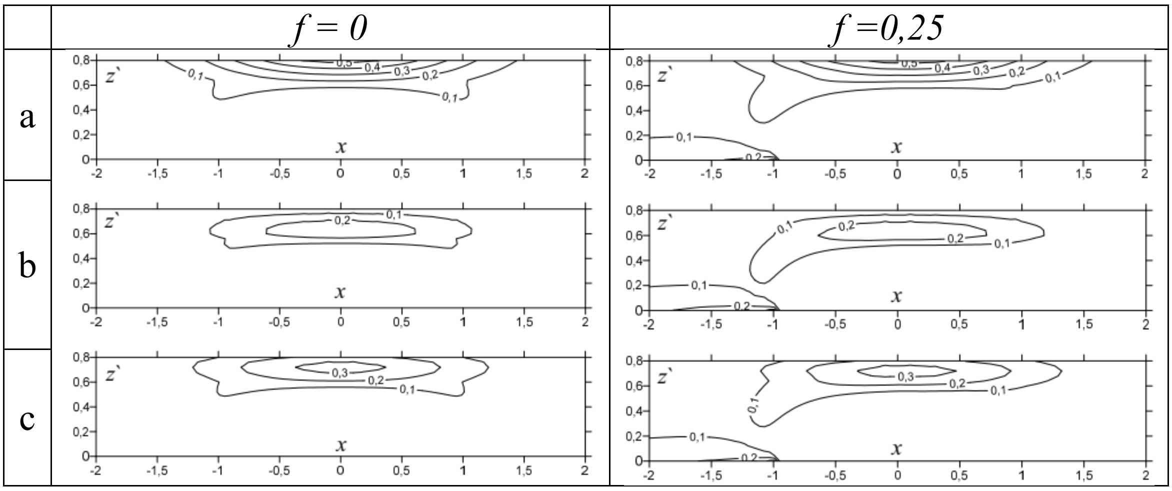

Fig. 3. Distribution of first principal stress $\sigma_1/p_{\max}$ in the xz plane $h = 0{,}8$;
a) without interlayer, b) interlayer Young's modulus is constant,
c) interlayer Young's modulus is changing by the power function

In considered coatings, like in homogeneous coatings, tensile stresses may mostly occur in two areas: in the unbiased surface of considered inhomogeneous half-space and in the surroundings of boundary between coatings and base.At some mechanical properties the area of the tensile stresses was displaced from interface to internal zone of the coating. The tangential loading causes appreciable increase of level of the tensile stresses in the surface $z = h$. In surroundings of boundary between coatings and base the influence of friction is small. Occurrence of the interlayer and its properties have significant influence on tensile stress distribution in considered medium. The highest tensile stress at the interface between a coating and a base occurs in cases without the interlayer.

For all of analysing coatings the tensile stress distribution in the external surface was similar. For coatings of greater thickness stresses in the external surface have smaller values. Occurrence of the gradient interlayer causes insignificant increase of tensile stress in this surface.

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ОПРЕДЕЛЕНИЕ ОСТАТОЧНЫХ НАПРЯЖЕНИЙ В ПОКРЫТИЯХ ПРИ ДВУХСТОРОННЕМ УПРОЧНЕНИИ ПРЯМОУГОЛЬНОГО СТЕРЖНЯ

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В процессе магнитно-электрического упрочнения (МЭУ) композиционными ферромагнитными порошками (ФМП) плоских поверхностей рабочих органов почвообрабатывающих машин (например, дисков сошников) в системе покрытие-основа имеют место все три вида напряжений (I, II и III рода) [1, 2]. Однако причиной нарушения прочности покрытий, появления микротрещин в них являются напряжения I рода [3]. В связи с этим рассмотрим напряжения I рода σ_H, возникающие в покрытиях после МЭУ.

Так как в процессе МЭУ длительность воздействия электрических разрядов составляет около 10^{-3}с [1], то температурное поле по ширине стержня (оси *X*) и в направлении оси *Z* (рисунок 1) можно считать постоянным. Изменение температуры будем рассматривать только по высоте стержня в направлении оси *Y*.