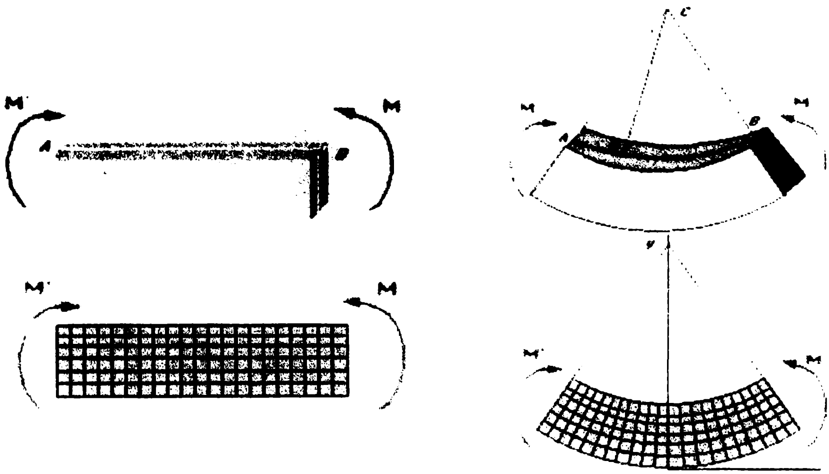
MINISTRY OF EDUCATION OF THE REPUBLIC OF BELARUS

ESTABLISHMENT OF EDUCATION
«BREST STATE TECHNICAL UNIVERSITY»

DEPARTMENT OF APPLIED MECHANICS

TASKS AND METHODICAL INSTRUCTIONS FOR PERFORMING CALCULATION-DESIGN WORKS

«STRENGTH OF MATERIALS» FOR STUDENTS OF A SPECIALTY 1 – 70 02 01 «INDUSTRIAL AND CIVIL ENGINEERING»



Brest – 2022

UDC 531.8

Strength of materials is one of the general professional disciplines in the training of construction engineers.

To consolidate theoretical material and acquire skills in engineering calculations, students perform calculation and design work on the main sections of the course.

This task and methodological guidelines allow individualize and activate the independent work of students when studying the course "strength of materials".

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INTRODUCTION

Tasks and methodological guidelines for calculation-design works (design works) correspond to the curricula of specialties 1–70 02 01 and cover the most important sections of the Strength of Materials course that students study in the spring semester. Methodological guidelines allow students to study and apply theoretical material to solve problems using examples of calculating statically determined beams (that experience direct bending), an eccentrically loaded (short) column and a compressed column (for stability analysis).

The methodical instructions contain the demands for the design works execution and examples of tasks.

INSTRUCTIONS FOR EXECUTION OF CALCULATION-DESIGN WORKS

1. The necessary data for the execution of design works should be accepted according to the variant number and diagram number (diagrams and tables are given).
2. Design work is performed on standard sheets of A4 format (210 x 297 mm) and is executed in the following order: cover sheet, task for design work, calculation text, conclusions, list of literature.
3. Drawings and diagrams should be executed on separate sheets in accordance with the rules of graphics and scales. You must specify on the diagrams the numerical values and unites of measurement of specific dates, used (or has been gotten) in calculations.
4. The calculated values should be rounded to tenths or hundredths with the dimensions.
5. To check the correctness of the calculations, you need to use a certain software package (or programs) at the computing center of the Department of Computer Science and Applied Mathematics or by use of Internet Online Calculators.

1. SYMMETRICAL BENDING

1.1. Abstract concepts

For most building elements, bending is almost the most common type of deformation. A straight bar experiencing a bend is called a beam. Bending of beams causes forces (loads) that are perpendicular to the longitudinal axis of the beam, or couples (pairs of forces) lying in planes passing through this axis. If all loads act in the same plane, called a plane of forces, passing through the geometric axis of the beam and one of the principal central axes of inertia of the cross section, then such a bend is called a symmetrical bending. If only bending moments act in cross sections, then such a bend is called a pure bending. With a symmetrical bending, the longitudinal axis (the geometric location of the centers of gravity of the cross sections) from a straight line turns into a smooth curve line called the curved axis of the beam or the elastic line of the beam. The elastic line shows the vertical movements of the centers of gravity of the cross sections under the applied loads.

1.2. Calculation of beams strength

In general, the calculation of beams for strength is reduced:

a) Calculation of the maximum normal stress. The strength condition is as follows:

$$\sigma_{\max} = \frac{M_{\max}}{W_x} \le [\sigma] = R,$$

where $M_{\max}$ – highest bending moment (set by diagram M); W_x – axial section moduli of beam cross section; R – design strength of beam material.

According to the condition of strength, three types of problems can be solved. Of greatest interest is the design task – determining the required section moduli of the cross section of the beam:

$$W_x = \frac{M_{\max}}{R}$$

b) Calculation on the highest shearing stress, the strength condition is used in the form:

$$\tau_{\max} = \frac{Q_{\max} S_{x,\max}^{cut}}{I_x b} \le [\tau],$$

where $Q_{\max}$ – highest shear force (set by diagram Q); $S_{x,\max}^{cut}$ – static moment of cut-off part of cross section with respect to (w.r.t.) neutral axis X; I_x – axial moment of inertia of beam cross section w.r.t. principal central axis X; b – the width of the beam cross section at the point level at which the τ is defined.

Usually the calculation on shearing maximum strength in order to checking the strength of beams (the strength conditions) for critical (dangerous) points of critical cross sections is limited by $\tau_{\max}$. Critical sections are those sections in which the greatest shearing forces (Q) act, and critical points are points of cross section located on the neutral axis X. In cases where the strength condition is not met (overstressing $> 2 \div 5\%$), the cross section should be increased.

c) Principal stress strength calculation. Checking the strength of beams by principal stresses is reduced to compiling strength conditions using one of the strength theories. For beams made of ductile materials, a third or, most often, fourth theory of strength (as the most economical) is used. Thus, the strength conditions according to the third and fourth strength theories are as follows:

$$\sigma_{r_3} = \sqrt{\sigma^2 + 4\tau^2} \le R;$$

where $\sigma_{r_3}, \sigma_{r_4}$ – reduced stresses (or equivalent – $\sigma_{eqv}^{III}, \sigma_{eqv}^{IV}$) for test section points; σ, τ – normal and shearing stresses for corresponding points.

The principal stresses should be checked for those sections in which the greatest or close to them, shear forces Q and bending moments M are acted. The dangerous points of such sections are usually located where the width of the cross sections changes dramatically. So, for example, in I-beams, channels - these are the points of contact between the flange and web, the width (d) of which is an order of magnitude less than the width of the flange.

1.3. Calculating the Stiffness of Beams

When checking beams for stiffness, the stiffness condition is used:

$$y_{max} \le [f], \qquad [f] = \frac{l}{k};$$

where y_{max} – maximum deflection of beam in span, cantilevers, etc.; $[f]$ – permissible deflection value; l – length of beam sections to be checked; k – coefficient, the value of which is specified by norms (k=100; 200; 400; 500; 1000).

permissible deflection value.

It is better to use the universal method to define beam deflections, i.e.

$$EI_x y = EI_x y_0 + EI_x \theta_0 z + \sum \frac{m_i (z-a_i)^2}{2} + \sum \frac{F_i (z-b_i)^3}{6} + \sum \frac{q_i (z-c_i)^4}{24} - \sum \frac{q_i (z-d_i)^4}{24};$$

$$EI_x \theta = EI_x \theta_0 + \sum m_i \cdot (z-a_i) + \sum \frac{F_i (z-b_i)^2}{2} + \sum \frac{q_i (z-c_i)^3}{6} - \sum \frac{q_i (z-d_i)^3}{6};$$

where $y_0; \theta_0$ – initial parameters, i.e. deflection and angle of rotation the section located at the origin, i.e. point O. The origin is most often located at the left end of the beam at the center of gravity of the cross section. The distance z from the origin point (p. O) to the section, for which y, θ are determined. The distances a_i, b_i, c_i, d_i – from the origin to the sections in which m_i, F_i, q_i, $-q_{comp\,i}$ are applied respectively.

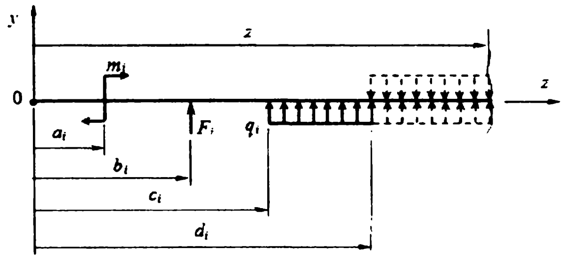


Figure 1.1 – Initial parameters method scheme

1.4. Self-Test Questions

1. What is the essence of the design methods for permissible stresses, collapsible loads and limit states?
2. What is meant by the plastic hinge and the plastic moment of resistance, how is its value determined?
3. Which sections and points in the section when calculating beams for strength (a – by normal stresses, b – by shearing stresses, c – by principal stresses): are considered critical (dangerous)?
4. Which beam cross sections are more rational?
5. What is meant by an elastic beam line (or deflection curve)?
6. What are the known methods for determining beam deflections?
7. What is the approximate differential equation of the elastic beam line?
8. What is the universal equation of the deflection curve?

9. What do you mean by the initial parameters? How are their magnitude determined?

10. How the calculation for beams stiffness is realized?

11. What is the connection between the diagram of deflections and the diagram of moments?

12. How can be defined (analytically) the maximum moments and deflections of beams?

1.5. Task for design work "Calculation of statically defined I-beam for strength and stiffness"

Given: I-beam (to accept by diagram number) is loaded with external load (to accept numeric data taking into account number of variant). Design resistance of beam material: R_c=210 MPa, R_{sh}=120 MPa; longitudinal elastic modulus – E = 200 GPa; allowable deflection – $[f/l]$ = 1/500 and 1/100 for consoles.

Required:

1) Determine the reactions at the supports;

2) Plot diagrams of shearing forces and the bending moments (Q, M);

3) Choose the section of a beam and check strength on shear and principal stresses with use of the third failure theory;

4) Determine principal stresses for one of dangerous points (p. 2);

5) Plot diagrams of normal, shear, principal and extreme shear stresses for dangerous section on principal stresses;

6) Plot diagram of deflections of a beam, having determine deflections of three sections: one section in span and two for a consoles;

7) Beam stiffness test;

8) Check correctness of plotting of diagrams Q, M, y by means of a computer and attach the printout.

1.6. Numeric load values for beams

Table 1.1.

Variant number	Size a, m	Loading			Index of loading		
		q, kN/m	F, kN	M, kN·m	q	F	M
1	0,9	30	110	40	2	1,2	2
2	1,1	40	100	30	2	1	1
3	1,0	32	120	34	1	1	1,2
4	0,8	34	90	36	1,2	2	1
5	1,1	30	70	32	2	1	2
6	1,0	36	80	38	1	2	1,2
7	1,2	38	90	28	1,2	1	1
8	0,9	40	100	30	2	2	2
9	1,0	30	110	32	1,2	1	1,2
10	1,1	40	80	30	1	2	2
11	0,9	34	90	40	2	1	1
12	1,2	36	65	38	1,2	2	1
13	1,1	38	70	32	1	1	2
14	0,9	40	75	30	2	2	1
15	1,2	42	80	28	1	1	1,2
16	1,1	44	60	34	1,2	2	1

1.7. Diagrams of I–beams with loads

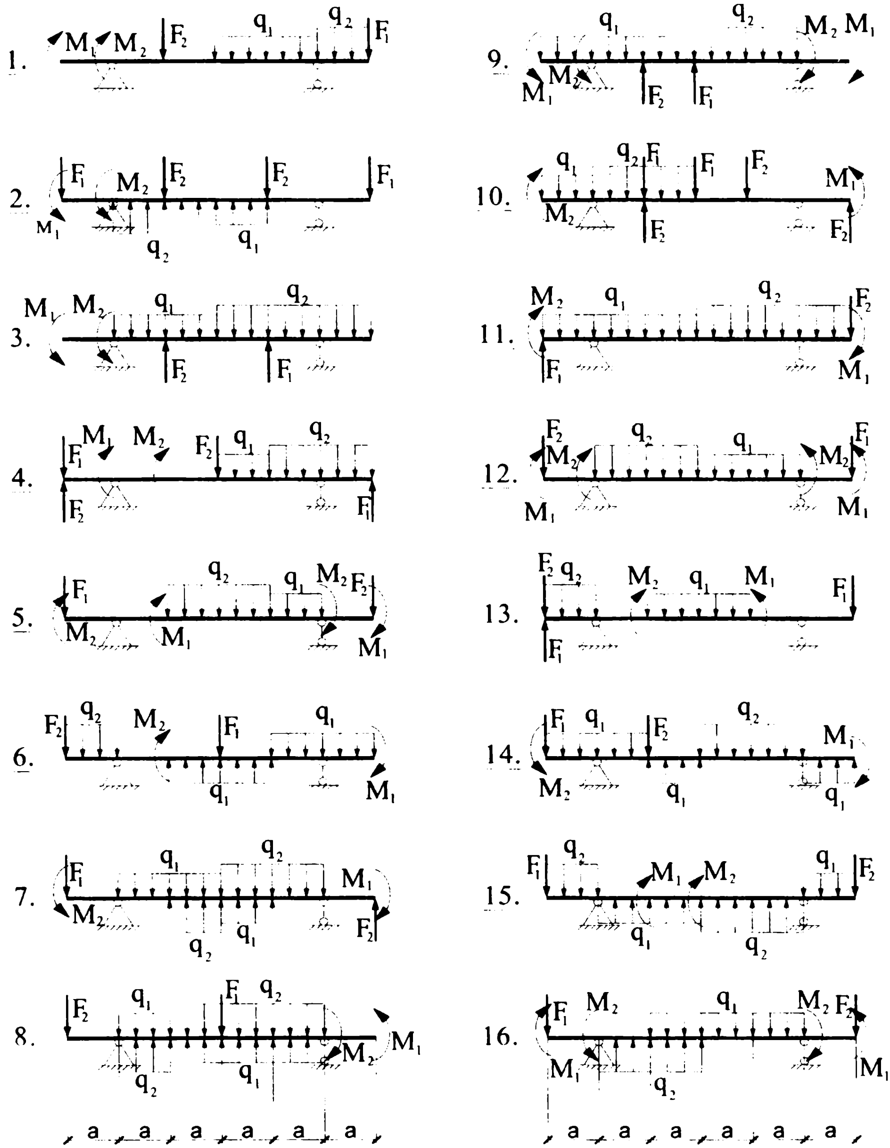


Figure 1.2 – Schemes of beams

1.8. Example of I-beam strength and stiffness calculation

Given: I-beam (Figure 1.3) with length – L=6 m (a=1 m) is loaded with concentrated force – F = 40 kN, moment – M = 50 kN·m and distributed load – q=30 kN/m. Design resistance of beam material: R_c = 210 MPa, R_{sh} = 120 MPa; longitudinal elastic modulus – E = 200 GPa; allowable deflection – [f/l] = 1/500 and 1/100 for consoles.

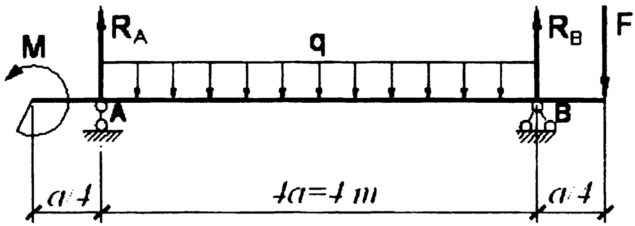


Figure 1.3 – Design diagram of beam and load

Solution:

1. Define support reactions.

Compose the static equilibrium equation:

$\sum m_A$=0; $4{,}25\cdot Fa+8qa^2-m-4R_B\cdot a=0$; $\quad R_B=\dfrac{4{,}25\cdot Fa+8\cdot qa^2-m}{4a}$;

R_B=90 kN.

$\sum m_B$=0; $-m-8qa^2+Fa+4R_A\cdot a=0$; $\quad R_A=\dfrac{m+8\cdot q\cdot a^2-F\cdot a}{4\cdot a}$;

R_A=70 kN.

Check: $\sum Y=0$; $R_A+R_B-4qa-F=0$; $\quad 70+90-120-40=160-160=0$.

2. Plotting of diagrams Q, M.

Determinate Q, M values in characteristic sections of a beam.

$Q_0=0$; $Q_{A(\text{left.})}=0$; $Q_{A(\text{right})}=R_A=70$ kN; $Q_C=F=40$ kN;

$M_0=-m=50$ kN·m; $M_A=-m=-50$ kN·m; $M_C=0$; $M_B=-F\cdot a/4=-40\cdot 0.25=-10$ kN·m.

Define the position of the section (z_0) in which $Q=0$.

$$Q_{z_0}=R_A-qz_0=0;\quad z_0=\frac{R_A}{q}=\frac{70}{30}=2{,}33\text{ m}.$$

Then the value of the maximum moment for the section (z_0) will be equal:

$$M_{\max}=-m+R_Az_0-q\frac{z_0^2}{z}=-50+70\cdot 2{,}33-30\cdot\frac{2{,}33^2}{2}=32\ \text{kN}\cdot\text{m}.$$

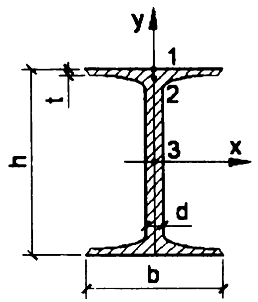
3. Selection of section of beam and check the strength.

We select section from strength condition on design resistance of beam material (permissible strength): $\sigma_{\max}=\dfrac{M_{\max}}{W_x}\le R_C$

The required section module is equal:

$$W_x = \frac{M_{max}}{R_C} = \frac{50 \cdot 10^3}{210 \cdot 10^6} = 238\ \text{cm}^3.$$

According to tables of rolling profile we choose the **I** section of № 22.
Geometrical characteristics of the accepted **I** section № 22 are:



h=22 cm;
b=12 cm;
t=0,89 cm;
d=0,54 cm;
I_x=2790 cm^4;
W_x=254 cm^3;
$S^{cut}_{x,max} = 143\ \text{cm}^3$.

Figure1.4 – Scheme of I section

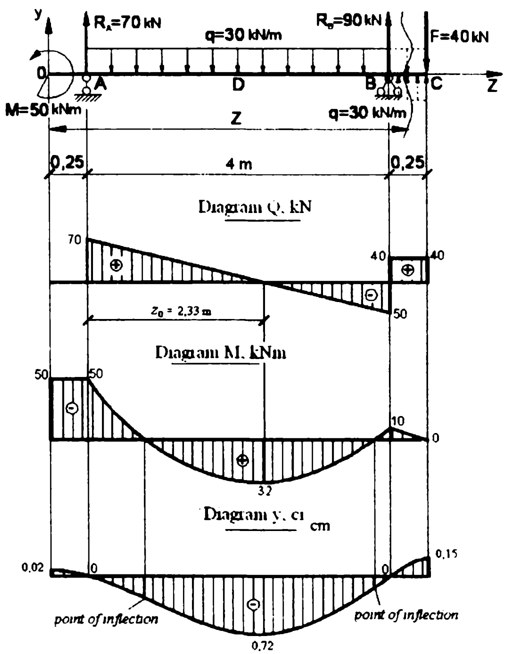


Figure 1.5 – Diagrams of shear forces, the bending moments, deflections

Check of strength of beam on shear stress in point 3 (see Figure 1.5) for section A (on the right), where Q_{max}=70 kN.

$$\tau_{\max} = \frac{70 \cdot 10^3 \cdot 143 \cdot 10^{-6}}{2790 \cdot 10^{-8} \cdot 0{,}54 \cdot 10^{-2}} = 66{,}4 \cdot 10^6 \text{ Pa};$$

$$\tau_{\max} = 66{,}4 \text{ MPa} < R_{Sh} = 120 \text{ MPa}.$$

The strength condition on shear stress is satisfied.

Check of strength of beam on the principal stresses, using the third theory of strength.

We make check for points of adjunction of the web and flange of the double T section (point of 2, see Figure 1.4), in section A on the right where Q_A=70 kN; M_A=50 kN·m.

The strength condition on the third theory of strength has appearance:

$$\sigma_{r_3} = \sqrt{\sigma^2 + 4\tau^2} \le R_C$$

For point 2: $y_2 = \frac{h}{2} - t = \frac{22}{2} - 0{,}89 = 10{,}11$ cm;

$$S_{X.2}^{cut} = b \cdot t \cdot \frac{h-t}{2} = 12 \cdot 0{,}89 \cdot \frac{22 - 0{,}89}{2} = 112{,}7 \text{ cm}^3;$$

$$\sigma_2 = \frac{M_A \cdot y_2}{I_x} = \frac{50 \cdot 10^3 \cdot 10{,}11 \cdot 10^{-2}}{2790 \cdot 10^{-8}} = 181{,}2 \text{ MPa};$$

$$\tau_2 = \frac{Q_A \cdot S_{X.2}^{cut}}{I_x \cdot d} = \frac{50 \cdot 10^3 \cdot 112{,}7 \cdot 10^{-6}}{2790 \cdot 10^{-8} \cdot 0{,}54 \cdot 10^{-2}} = 52{,}4 \text{ MPa};$$

$$\sigma_{r_3} = \sqrt{181{,}2^2 + 4 \cdot 52{,}4^2} = 209{,}5 \text{ MPa} < R = 210 \text{ MPa}.$$

The strength condition on the principal stresses is satisfied.

4. Determination of the principal stresses for one of dangerous points of dangerous section in the graphic way.

Show the stress condition in point 2 of section A (on the right).

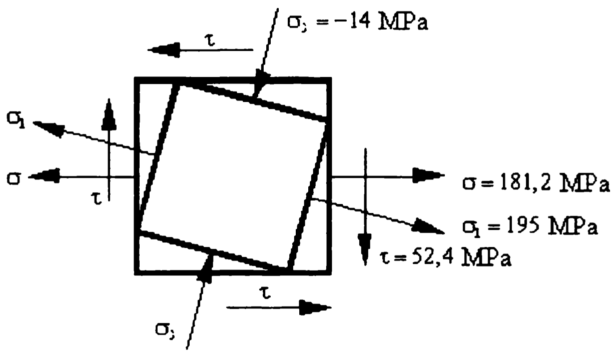


Figure 1.6 – Stress condition in point 2

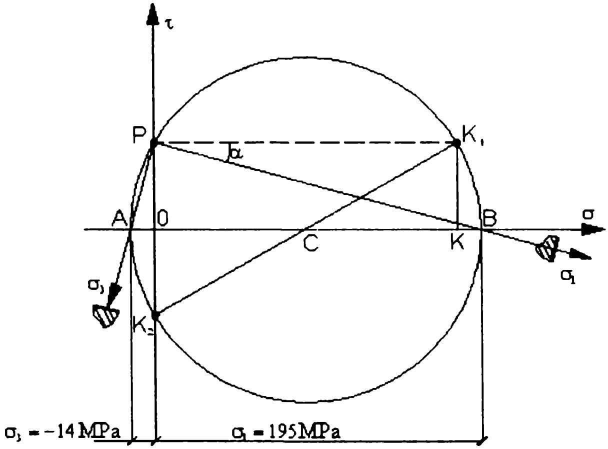


Figure 1.7 – Mohr's circle

In the system of coordinates – «$\tau - \sigma$» (figure 1.7) in scale is laying the straight-line segment: OK $=\sigma=181{,}2$ MPa; KK_1 $=\tau=52{,}4$ MPa, OK_2= –52,4 MPa. Having connected points of K_1 and K_2 we receive diameter of required circle of stresses with the center in point C (on which can be drawn circle). Crossing of circle with axis σ gives two points A and B, which characterize the principal stresses values. So, OB straight-line (in scale) represents the σ_1=195 MPa, and OA – σ_3= –14 MPa. Having drawn straight lines through points of K_1 and K_2 until their crossing, P – the pole can be found. Having connected pole P and point B, we get direction of tension stress σ_1, and points P and A – direction of σ_3. Then we transfer in parallel the direction of action of stresses σ_1 and σ_3 to the element presented in fig. 4 and show position of the principal platforms which are perpendicular to the corresponding stresses σ_1 and σ_3.

5. Plotting of diagrams of normal, shear, principal and extreme shear stresses for critical section.

When determining values of principal stresses and extreme shear stresses have to be used the biaxial stress-strain theory.

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\}=\frac{1}{2}(\sigma\pm\sqrt{\sigma^2+4\tau^2});\ \tau_{max,min}=\pm\frac{1}{2}\sqrt{\sigma^2+4\tau^2}=\pm\frac{\sigma_1-\sigma_3}{2};$$

where σ and τ may be determined by known formulas for the corresponding points (of section): $\sigma=-\frac{M_X}{I_X}\cdot y;\quad \tau=\frac{Q\cdot S_X^{cut}}{I_X\cdot b}$

At the same time parameters Q, M, y it is necessary to substitute in formulas taking into account their signs.

Point 1 : $\sigma = -\frac{M_X}{W_X} = \frac{50 \cdot 10^3}{254 \cdot 10^{-6}} = 196{,}8$ MPa,

$\tau = 0$, because $S_X^{cut} = 0$; $\sigma_1 = 196{,}8$ MPa,

$\sigma_3 = 0$; $\tau_{\max,\min} = \pm\frac{\sigma_1}{2} = \pm 98.4$ MPa.

Point 2 : $\sigma = \frac{50 \cdot 10^3}{2790 \cdot 10^{-8}} \cdot 10{,}11 \cdot 10^{-2} = 181{,}2$ MPa,

$$\tau^{flange} = \frac{70 \cdot 10^3 \cdot 112{,}7 \cdot 10^{-6}}{2790 \cdot 10^{-8} \cdot 12 \cdot 10^{-2}} = 2{,}4 \text{ MPa},$$

$$\tau^{web} = \frac{70 \cdot 10^3 \cdot 112{,}7 \cdot 10^{-6}}{2790 \cdot 10^{-8} \cdot 0{,}54 \cdot 10^{-2}} = 52{,}4 \text{ MPa}.$$

The further course of calculations is reduced in Table 1.2 and by the received results we plot diagrams of stresses (Figure 1.7).

Table 1.2.

The point of section		Y cm	$S_{X,2}^{cut.}$ cm^3	σ MPa	τ MPa	$\sqrt{\sigma^2+4\tau^2}$ MPa	σ_1 MPa	σ_3 MPa	$\tau_{\max,\min}$ MPa
1		11	0	196,8	0	196,8	196,8	0	±98,4
2	flange	10,11	112,7	181,2	2,4	181,5	181,4	-0,15	±90,8
	web	10,11	112,7	181,2	52,4	209,5	195,4	-14,2	±104,8
3		0	143	0	66,4	132,8	66,4	-66,4	±66,4
2'	flange	-10,11	112,7	-181,2	2,4	181,5	0,15	-181,4	±90,8
	web	-10,11	112,7	-181,2	52,4	209,5	14,2	-195,4	±104,8
1'		11	0	-196,8	0	196,8	0	-196,8	±98,4

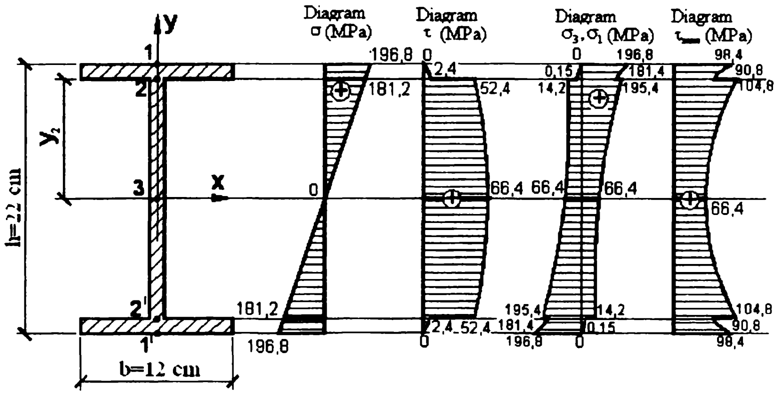


Figure 1.8 – Diagrams of stresses in section A (on the right)

6. Plotting of a deflection diagram of a beam.

We select the origin of coordinate system on the left end of a beam (point O). Next, we prolong **q** – uniformly distributed load up to the right end of a beam and counterbalance it with the compensating distributed load of backward direction (see fig. 2.).

Having worked out universal equation of deflections for the last force section (zone ***BC***) we write.

Where θ_O and y_O initial parameters (a turning angle and a deflection of section in origin of coordinates) which we will determine from conditions of supporting sections:

At z=0,25a, y_A=0

$$E\cdot I_X\cdot y_A = E\cdot I_X\cdot y_O + E\cdot I_X\cdot\theta_O\cdot 0{,}25a - \frac{m\cdot(0{,}25a)^2}{2} = 0;$$

at z=4,25a, $y_B=0$,

$$E\cdot I_X\cdot y_B = E\cdot I_X\cdot y_O + E\cdot I_X\cdot\theta_O\cdot 4{,}25a - \frac{m\cdot(4{,}25a)^2}{2} + \frac{R_A\cdot(4a)^3}{6} - \frac{q\cdot(4a)^4}{24} = 0.$$

We solve the system of equations with boundary condition: y_A=0; $y_B=0$.

$$\begin{cases} E\cdot I_X\cdot y_O + 0{,}25\cdot E\cdot I_X\cdot\theta_O - \dfrac{50\cdot 0{,}25^2}{2} = 0; \\ E\cdot I_X\cdot y_O + 4{,}25\cdot E\cdot I_X\cdot\theta_O - \dfrac{50\cdot 4{,}25^2}{2} - \dfrac{70\cdot 4^2}{6} - \dfrac{30\cdot 4^2}{24} = 0. \end{cases}$$

After simplification we receive:

$$\begin{cases} E\cdot I_X\cdot y_O + 0{,}25\cdot E\cdot I_X\cdot\theta_O = 1{,}5625\ \text{кN}\cdot\text{m}^3; \\ E\cdot I_X\cdot y_O + 4{,}25\cdot E\cdot I_X\cdot\theta_O = 24{,}8965\ \text{кN}\cdot\text{m}^3. \end{cases}$$

From where:

$E\cdot I_X\cdot\theta_O$=5,8335 kN·m^2; $E\cdot I_X\cdot y_O$=0,1041 kN·m^3.

Checking:

$$E\cdot I_X\cdot y_A = E\cdot I_X\cdot y_O + E\cdot I_X\cdot\theta_O\cdot 0{,}25a - \frac{m\cdot(0{,}25a)^2}{2} = 1{,}5625 - 1{,}5625 = 0;$$

$$E\cdot I_X\cdot y_B = E\cdot I_X\cdot y_O + E\cdot I_X\cdot\theta_O\cdot 4{,}25a - \frac{m\cdot(4{,}25a)^2}{2} + \frac{R_A\cdot(4a)^3}{6} - \frac{q\cdot(4a)^4}{24} =$$

$$= 771{,}5631 - 771{,}5625 \approx 0.$$

For plotting of a diagram of deflections of a beam, in our case, we will calculate deflections only of the following sections:

In section O (z=0):

$E\cdot I_X\cdot y_O$=0,1041 kN·m^3; $E\cdot I_X = 2\cdot 10^{11}\cdot 2790\cdot 10^{-8} = 5580$ kN·m^3.

$$y_O = \frac{0{,}1041}{5580} = 0{,}02\ \text{mm}.$$

In section A (z=a/4):

y_A=0.

In section D (midspan, z=2,25a):

$$E\cdot I_x\cdot y_D=E\cdot I_x\cdot y_0+E\cdot I_x\cdot\theta_0\cdot 2{,}25a-\frac{m\cdot(2{,}25a)^2}{2}+\frac{R_A\cdot(2a)^3}{6}-\frac{q\cdot(2a)^4}{24}=$$

$$=-40\ \text{kN}\cdot\text{m}^3;$$

$$y_D=-\frac{40}{5580}=-0{,}72\ \text{cm}.$$

In section B (z=4,25a):

y_B=0.

In section C (z=4,5a):

$$E\cdot I_x\cdot y_C=E\cdot I_x\cdot y_0+E\cdot I_x\cdot\theta_0\cdot 4{,}5a-\frac{m\cdot(4{,}5a)^2}{2}+\frac{R_A\cdot(4{,}25a)^3}{6}-\frac{q\cdot(4{,}25a)^4}{24}+$$

$$+\frac{R_B\cdot(0{,}25a)^3}{6}+\frac{q\cdot(0{,}25a)^4}{24}=0{,}1041+5{,}8335\cdot 4{,}5-\frac{50\cdot 4{,}5^2}{2}+\frac{70\cdot 4{,}25^3}{6}-$$

$$-\frac{30\cdot 4{,}25^4}{24}+\frac{90\cdot 0{,}25^3}{6}+\frac{30\cdot 0{,}25^4}{24}=812\ \text{kN}\cdot\text{m}^3;$$

$$y_C=\frac{812}{5580}=0{,}15\ \text{cm}.$$

According to calculation the diagram of deflections of a beam is plotted (Figure 1.5).

Note. In calculated-design work it is necessary to define deflections of two sections in span of a beam, i.e. at z=2a; z=4a, for more exact plotting of deflection diagram.

7. Check of a beam on stiffness.

In span: l=4 *m*, a=4 *m*;

$$\frac{y_D}{l}=\frac{0{,}72}{400}=\frac{1}{555}<\left[\frac{f}{l}\right]=\frac{1}{500}.$$

On the right console: $l=\frac{a}{4}=0{,}25$ m;

$$\frac{y_C}{l}=\frac{0{,}15}{25}=\frac{1}{167}<\left[\frac{f}{l}\right]=\frac{1}{100}.$$

The condition of stiffness is satisfied. Finally we accept a double T section №22.

8. Determination of safety factors for strength and stiffness.

Safety factor on stresses:

– on normal stresses: $K_\sigma=\frac{R_C}{\sigma_{max}}=\frac{210}{196{,}8}=1{,}07;$

– on shear stresses: $K_\tau=\frac{R_{Sh}}{\tau_{max}}=\frac{120}{66{,}4}=1{,}81;$

– on principale stresses: $K_{red.3}=\frac{R_C}{\sigma_{red.3}}=\frac{210}{209{,}5}=1{,}00;$

Safety factor on stiffness: $K_{Y_D}=\frac{[y_D]}{y_D}=\frac{0,8}{0,72}=1,1;$

$$K_{Y_C}=\frac{[y_C]}{y_C}=\frac{0,25}{0,15}=1,67;$$

where: $[y_D]=\frac{l}{500}=0,8\ cm,\qquad [y_C]=\frac{l}{100}=0,25\ cm.$

2. STRENGTH OF SHORT POLUMNS ON EXCENTRIC COMPRESSION-TENSION

2.1. General information

In real designs, there are often cases where two or more internal force factors act in cross sections, causing two or more simple types of deformation. In such cases, structural elements experience complex deformation (combined stresses). The following types of combined stresses take place: unsymmetrical (oblique) bending; eccentric compression (stretching); torsion with bending, etc.

2.2. Unsymmetrical bending

Oblique bending occurs when the force plane passing through the longitudinal axis of the bar does not coincide with any of the principal axes of inertia of the cross section, or a simultaneous combination of two straight bends acting in mutually perpendicular planes.

With oblique bending, four internal force factors act in the cross sections of the beams: Q_X, Q_Y – transverse forces and M_X, M_Y – bending moments. However, as a rule, the influence of transverse forces is insignificant and neglected in calculations. When determining stresses and deflections during oblique bending, the principle of independence of forces is used. Thus, the total normal stresses are determined by the following formula:

$$\sigma=\sigma_{M_X}+\sigma_{M_Y}=\pm\left(\frac{M_X}{I_X}\right)y\pm\left(\frac{M_Y}{I_Y}\right)x,$$

where M_X, M_Y – bending moments; x, y – coordinates of the points at which the σ (stress) is defined; I_X, I_Y – the principal central moments of inertia of the cross section. It is more expedient to set the signs of supplemented stresses based on the deformation nature of the longitudinal layers of the beam.

The calculation for oblique bending strength is reduced to compiling the strength condition for a critical beam section:

$$\sigma_{\substack{max\\min}}=\pm\frac{M_X}{W_X}\pm\frac{M_Y}{W_Y}\le R_c,$$

where W_X, W_Y – axial moments of resistance (section modulus) of cross section during bending; R_c is the design compression (R_t – tensile) strength of the beam material.

According to the condition of strength, as usual, three types of problems can be solved. The most interesting is the design task, since two unknown values are included in the strength condition: W_X, W_Y. Therefore, when solving the design task, the ratio $k=W_X/W_Y$ is preset. Then, the strength condition will take the form:

$$\sigma_{\max} = \frac{1}{W_X}(M_X + kM_Y) \le R.$$

At the same time, the ratio of axial resistance moments of the cross section depends on the shape of the section. So, for example, for rectangular section it is $k=h/b$, for I sections – $k=8\div10$; $k=6\div9$ for channels, etc.

2.3. Eccentric compression

Eccentric compression occurs when the bar is loaded longitudinally by an eccentric force, i.e. at a distance from the geometrical center of the cross section. Deformation of eccentric compression is more characteristic of elements of building structures.

In any cross section of the bar, three internal force factors arise during off-center compression:

$$N=-F; \quad M_X = F \cdot y_F\,; \quad M_Y = F \cdot x_F.$$

Therefore, the general case of eccentric compression is reduced to central compression (N) and net of oblique bending (M_X, M_Y), which, as described in paragraph 2.2, can be represented as two pure uniaxial bends acting in mutually perpendicular planes. Taking into account the principle of independence of forces, normal stresses in the cross section of the bar are equal to the algebraic sum of stresses from each internal force factor:

$$\sigma = \sigma_N + \sigma_{M_x} + \sigma_{M_Y} = \frac{N}{A} + \frac{M_X y}{I_X} + \frac{M_Y x}{I_Y};$$

This formula allows us to determine stresses at any point in the cross section characterized by the coordinates: x, y, the resulting stress formula can be represented after transformation as:

$$\sigma = -\frac{F}{A}\left(1 + \frac{x_F x}{i_y^2} + \frac{y_F y}{i_x^2}\right);$$

where F – external load (with eccentric compression, the sign "–" should be attributed, and with tension "+"); i_x^2 , i_y^2 – squares of the principal radii of inertia of the cross section; x, y are the coordinates of the point at which the stress is determined; x_F , y_F – coordinates of the application point of the external load. At the same time, the coordinates of the points: x, y, x_F, y_F, should be taken into account with their signs.

The eccentric compression strength calculation is limited to compiling the strength condition for hazardous (critical) cross-sectional points. Positions of the critical points are determined by means of a zero line, i.e. the line at the points of which the normal stresses (σ) are zero. To determine zero line position, segments cut off by zero line on coordinate axes are calculated:

$$a_x = -\frac{i_y^2}{x_F};$$

$$a_y = -\frac{i_x^2}{y_F}.$$

According to the obtained segments, a zero line in the section is depicted, and by use of two tangents to the section parallel to the zero line the two most stressed (farthest) points of the cross section in the stretched and compressed zones are found.

The strength conditions take the form:

$$\sigma_{\max} = -\frac{F}{A}\left(1+\frac{x_F x}{i_y^2}+\frac{y_F y}{i_y^2}\right) \le R_t,$$

$$\sigma_{\min} = -\frac{F}{A}\left(1+\frac{x_F x}{i_y^2}+\frac{y_F y}{i_y^2}\right) \le R_C.$$

where x, y – coordinates of critical points of section; R_t, R_c are the design tensile and compression strength of the bar material, respectively. When solving the design problem (selection of sections), in the general case, it is necessary to neglect either the deformation of the central compression or the deformation of the pure cross-bending due to the difficulties of solving the cubic equation. At the same time, the obtained results are rounded up and finally checked according to the general strength condition. But in cases where it is possible to express cross-sectional dimensions through one unknown parameter (for example, "b"), the solution of the problem is somewhat simplified.

2.4. Self-Test Questions

1. What is meant by eccentric compression (tension)?
2. What simple deformations occur with (during) eccentric tension?
3. What is meant by the zero line? What are the zero line properties?
4. What methods are used to determine the position of the zero line during eccentric compression?
5. How do you establish hazardous (critical) points in sections during eccentric compression?
6. How is strength calculation and section selection performed?
7. Which is understood as the core of the section? How to build it?
8. In which cases do you plot a section core?

2.5. Design assignment: "Calculation of Eccentric Compressed Column"

Given: Column (accepted by diagram number) of complex cross section shape is subjected to eccentric compression by external load F (load value and point of application are accepted by variant number, Figure 2.1). Design compression strength of column material R_c = 10 MPa, tensile resistance R_t = 1 MPa.

Required:

1) Determine the geometric characteristics of the cross section of the column;
2) determine the position of the zero line and critical cross-sectional points;
3) calculate the greatest compressive and tensile stresses;
4) select cross section dimensions based on strength;
5) plot diagram of normal stresses;
6) plot and investigate the core of the section;
7) check the correctness of the section core construction using the computer.

Cross-sectional diagrams of the column with application points *F*

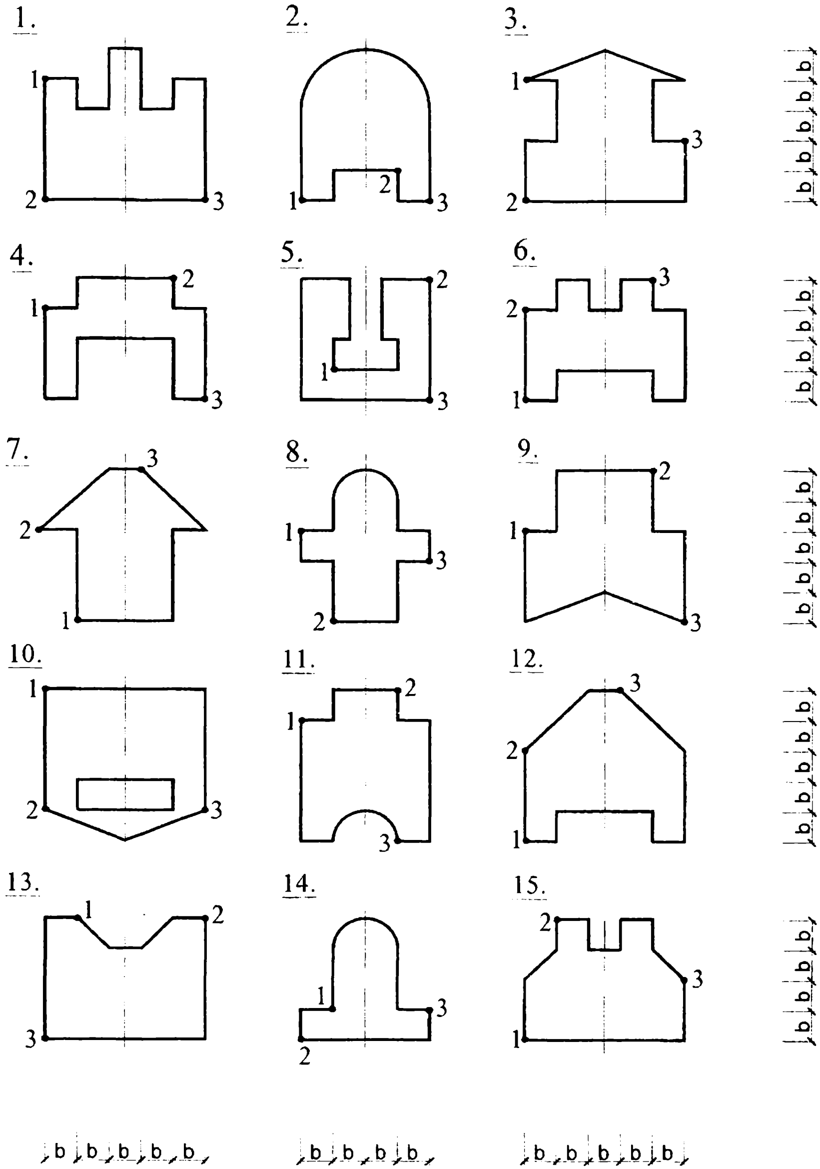


Figure 2.1 – Schemes of transverse sections of the column with points of application of force

2.7. Numeric values of loads and their application points

Table 2.1.

Variant Number	F, kN	Point of application of force
1	450	1
2	480	2
3	490	3
4	460	1
5	440	2
6	430	3
7	420	1
8	470	2
9	410	3
10	550	1
11	530	2
12	540	3
13	505	1
14	515	2
15	525	3

2.8. Example of calculating an eccentrically compressed column

Given: Column of complex cross-section shape (diagram No.), subjected to eccentric compression by external load F=250 kN (variant No.). Design strength of the column material for compression R_c= 10 MPa, for tension R_t = 1 MPa.

Required: (see items 1–7).

<u>Solution</u>

1. Definition the geometry of the cross section of the column.

Let's show the cross section of the column in scale and select the auxiliary axes of coordinates X, Y. We divide the section into two simple ones and determine the coordinates of the centers of gravity of simple sections, their area.

1. for the first section (semicircle):

$$x_1 = 3b + \frac{4b}{3\pi} = 3,43b; \qquad y_1 = 0; \qquad A_1 = 1,57b^2.$$

2. For second section (rectangle):

$$x_2 = 1,5b; \qquad y_2 = 0; \qquad A_2 = 6b^2.$$

The axis X coincides with the axis of symmetry of the section, therefore, it is one of the principal central axes of inertia.

Define the static moment of the cross-section with respect to (w.r.t.) the Y axis:

$$S_y = S_y^I + S_y^{II} = A_1x_1 + A_2x_2 = 1,57b^2 \cdot 3,43b + 6b^2 \cdot 1,5b = 14,38b^3.$$

The total area:

$$A = A_1 + A_2 = 1,57b^2 + 6b^2 = 7,57b^2.$$

Determine the coordinate of the center of gravity of section x_c:

$$x_C = \frac{S_y}{A} = \frac{14,38b^3}{7,57b^2} = 1,9b.$$

The principal central axes X_c, Y_c are shown in the section diagram and the coordinates of the centers of gravity of simple sections relative to the principal central axes are determined:

$x_{C_1} = x_1 - x_C = 1,53b;$ $y_{C_1} = 0;$ $x_{C_2} = x_2 - x_C = -0,4b;$ $y_{C_2} = 0.$

Let's calculate the principal central moments of inertia of the entire section:

$$I_{X_C} = I^I_{X_C} + I^{II}_{X_C} = \frac{\pi(2b)^4}{128} + \frac{3b(2b)^3}{12} = 2,39b^4;$$

$$I_{Y_C} = I^I_{Y_C} + I^{II}_{Y_C} = I_{y_1} + x^2_{C_1}\cdot A_1 + I^{II}_{y_2} + x^2_{C_2}\cdot A_2 = 0,11\,b^4 + (1,53b)^2\cdot 1,57b^2 + \frac{2b(3b)^3}{12} +$$

$$+(0,4b)^2\cdot 6b^2 = 9,2b^4.$$

Define squares of radii of inertia.

$$i_x^2 = \frac{I_{X_C}}{A} = \frac{2,39b^4}{7,57b^2} = 0,32b^2;$$

$$i_y^2 = \frac{I_{Y_C}}{A} = \frac{9,2b^4}{7,57b^2} = 1,22b^2.$$

2. Determination the neutral line (axis) position and critical section points.

Coordinates of external load application point: $x_F = -1,9b;$ $y_F = b$. Calculate the lines that are trimmed by the neutral line on the coordinate axes:

$$a_x = -\frac{i_y^2}{x_F} = \frac{1,22b^2}{1,90b} = 0,64b; \qquad a_y = -\frac{i_x^2}{y_F} = -\frac{0,32b^2}{b} = -0,32b.$$

The neutral line is shown in Figure 2.2.

Find the position of critical points. Having drawn tangents to the section parallel to the zero line, we establish that the most stressed points of the section are points *F* and *D*, which are the most distant from the neutral line. Point *F* has the highest compressive stress and point *D* has the highest tensile stress.

3. Calculation highest stresses.

Stresses in hazardous points shall be determined by the following formulas:

$$\sigma_F = -\frac{F}{A}\left(1 + \frac{y_F y_F}{i_x^2} + \frac{x_F x_F}{i_y^2}\right);$$

$$\sigma_D = -\frac{F}{A}\left(1 + \frac{y_F y_D}{i_x^2} + \frac{x_F x_D}{i_y^2}\right);$$

where $x_F = -1,9b;$ $y_F = b;$ $x_D = (3b - x_C) + b\cdot\cos\alpha;$ $y_D = -b\cdot\sin\alpha;$

$$tg\alpha = \left|\frac{a_x}{a_y}\right| = \frac{0,64b}{0,32b} = 2; \qquad \alpha = 63,4^{\circ}; \qquad \cos\alpha = 0,447; \qquad \sin\alpha = 0,894;$$

$$x_D = 1,1b + 0,447b = 1,547b; \qquad y_D = -0,894b.$$

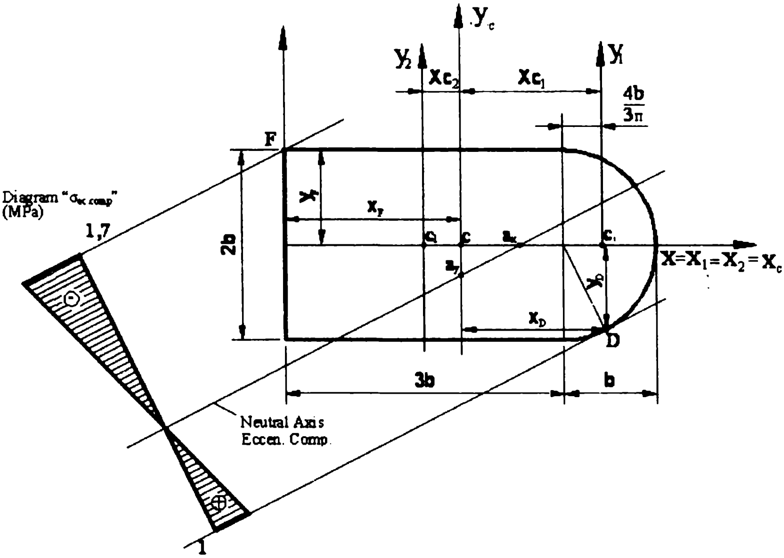


Fig. 2.1. Column cross-section, neutral line, and stress diagram

4. Selection of cross-sectional dimensions based on strength

Compose the strength condition for the critical point in the compressed area of the section:

$$\sigma_F = -\frac{F}{A}\left(1+\frac{y_F^2}{i_x^2}+\frac{x_F^2}{i_y^2}\right) \le R_C;$$

Rewrite the strength condition as follows:

$$\sigma_F = -\frac{F}{A}\left[1+\frac{1}{i_x^2}\left(y_F^2+\frac{i_x^2}{i_y^2}x_F^2\right)\right] \le R_C.$$

Denote: $k=\frac{i_x^2}{i_y^2}=\frac{0,32b^2}{1,22b^2}=0,26,$

Then: $\sigma_F = -\frac{F}{A}\left[1+\frac{1}{i_x^2}(y_F^2+kx_F^2)\right] \le R_C.$

Rewrite stress through unknown parameter "b":

$$\sigma_F = -\frac{F}{7,57b^2}\left[1+\frac{1}{0,32b^2}(b^2+0,26\cdot 1,9^2 b^2)\right] \le R_C,$$

$$\sigma_F = -\frac{F}{7,57b^2}\cdot 7,058 = -0,93\cdot\frac{F}{b^2} \le R_C,$$

$$b=\sqrt{\frac{0,93F}{R_C}}=\sqrt{\frac{0,93\cdot 250\cdot 10^3}{10\cdot 10^6}}=15,3\ cm.$$

Compose the strength condition for the farthest point in the stretched area of the section:

$$\sigma_D=-\frac{F}{A}\left(1+\frac{y_F y_D}{i_x^2}+\frac{x_F x_D}{i_y^2}\right)\le R_t.$$

Rewrite:

$$\sigma_D=-\frac{F}{A}\left[1+\frac{1}{i_x^2}(y_F y_D+k\cdot x_F x_D)\right]\le R_t,$$

$$\sigma_D=-\frac{F}{7,57\cdot b^2}\left[1+\frac{1}{0,32\cdot b^2}\cdot(b\cdot(-0,894b)+0,26\cdot(-1,9b)\cdot 1,547b)\right]\le R_t,$$

$$\sigma_D=\frac{F}{7,57b^2}\cdot 4,18=0,55\frac{F}{b^2}\le R_t,$$ Wherefrom:

$$b=\sqrt{\frac{0,55F}{R_P}}=\sqrt{\frac{0,55\cdot 250\cdot 10^3}{1\cdot 10^6}}=37,2\ \text{cm}.$$

As the design value we take the greater value b = 37.2 cm.

Determine stresses at critical points of section and perform strength check.

$$\sigma_F=-\frac{F}{A}\left(1+\frac{y_F^2}{i_x^2}+\frac{x_F^2}{i_y^2}\right)=-\frac{250\cdot 10^3}{7,57\cdot 37,2^2\cdot 10^{-4}}\cdot\left(1+\frac{b^2}{0,32b^2}+\frac{(1,9b)^2}{1,22b^2}\right)=$$

$$=-1,7\ \text{MPa}<R_C=10\ \text{MPa};$$

$$\sigma_D=-\frac{F}{A}\left(1+\frac{y_F y_D}{i_x^2}+\frac{x_F x_D}{i_y^2}\right)=-\frac{250\cdot 10^3}{7,57\cdot 37,2^2\cdot 10^{-4}}\times$$

$$\left(1+\frac{b(-0,894b)}{0,32b^2}+\frac{(-1,9b)\cdot 1,547b}{1,22b^2}\right)=1\ \text{MPa}\le R_t=1\ \text{MPa}.$$

The stress diagram at eccentric compression is shown in Figure 2.2.

5. Plotting the diagrams of normal stresses.

To plot the diagram σ we will use the graphical method, having previously plotted diagram σ for simple stresses. We construct the normal stress plots from pure uniplanar bends w.r.t. the principal central axes X_c and Y_c. Values of stresses from moments: $M_x = Fy_F$ and $M_y = Fx_F$ at points of section B, F, D, C are equal respectively:

$$\sigma_{M_x}^{B}=-\frac{F\cdot y_F}{I_y}y_B=-\frac{250\cdot 10^3\cdot 37,2\cdot(-37,2)\cdot 10^{-4}}{2.39\cdot 37,2^4\cdot 10^{-8}}=0,75\ \text{MPa};$$

$$\sigma_{M_x}^{F}=-\sigma_{M_y}^{B}=-0,75\ \text{MPa};$$

$$\sigma_{M_x}^{D}=-\frac{F\cdot y_F}{I_y}y_D=-\frac{250\cdot 10^3\cdot 37,2\cdot(-0.894\ )\cdot 10^{-4}}{2.39\cdot 37,2^4\cdot 10^{-8}}=0,57\ \text{MPa};$$

$$\sigma^{o}_{My} = -\frac{F \cdot x_F}{I_y} x_0 = -\frac{250 \cdot 10^3(-1,9) \cdot 37,2 \cdot 2,1 \cdot 37,2 \cdot 10^{-4}}{9,2 \cdot 37,2^4 \cdot 10^{-8}} = 0,78 \text{ MPa};$$

$$\sigma^{F}_{M_y} = -\frac{F \cdot x_F}{I_y} x_F = -\frac{250 \cdot 10^3 \cdot (-1,9)^2 \cdot 37,2^2 \cdot 10^{-4}}{9,2 \cdot 37,2^4 \cdot 10^{-8}} = -0,71 \text{ MPa};$$

$$\sigma^{D}_{M_y} = -\frac{F \cdot x_F}{I_y} x_D = -\frac{250 \cdot 10^3 \cdot (-1,9) \cdot 37,2 \cdot 1,547 \cdot 37,2 \cdot 10^{-4}}{9,2 \cdot 37,2^4 \cdot 10^{-8}} = 0,57 \text{ MPa}.$$

By superimposing the diagram σ_{M_x} on the diagram σ_{My}, we find a zero point i, connecting it with the center of gravity of the cross section C we obtain the position of the neutral axis of the pure oblique bend. Summing up the ordinates of diagram σ_{Mx} and σ_{My}, at points F and D, we obtain an diagram of stresses of pure biaxial bending $\sigma_{b.b}$ (Figure 2.3).

Then the diagram of normal stresses from central compression, at which $N = -F$ can be plotted:

$$\sigma_N = -\frac{F}{A} = -\frac{250 \cdot 10^3}{7,57 \cdot 37,2^2 \cdot 10^{-4}} = -0,24 \text{ MPa}.$$

Adding the diagram $\sigma_{b.b}$ and σ_N we get the diagram of normal stresses from eccentric compression (Figure 2.2). Through the zero point at the diagram $\sigma_{eccen.comp}$, we conduct the zero line at eccentric compression parallel to the neutral axis at pure oblique bending. For control it is necessary to compare the diagram $\sigma_{eccen.comp}$ with the diagram of normal stresses at eccentric compression obtained analytically (Figure 2.2).

6. *Drawing and explore the section core.*

To build a section core, define the lines to be trimmed by zero line:

1. Neutral line 1–1: $a_x = -1,9b$; $a_y = \infty$.

Force application point coordinates (point 1):

$$x_{F_1} = -\frac{i_y^2}{a_x} = \frac{1,22b^2}{1,9b} = \frac{1,22 \cdot 37,2}{1,9} = 23,9 \text{ cm}; \qquad y_{F_1} = -\frac{i_x^2}{a_y} = 0.$$

2. Neutral line 2–2: $a_x = \infty$; $a_y = -b$.

$$x_{F_2} = -\frac{i_y^2}{a_x} = 0; \qquad y_{F_2} = -\frac{i_x^2}{a_y} = \frac{0,32 \cdot b^2}{b} = 12 \text{ cm}.$$

3. Neutral line 3–3: $a_x = 2,1\,b$; $a_y = \infty$;

$$x_{F_3} = -\frac{i_y^2}{a_x} = -\frac{1,22b^2}{2,1b} = -0,58b = -21,6 \text{ cm}.$$

4. Neutral line 4–4: $a_x = \infty$; $a_y = b$;

$$x_{F_4} = -\frac{i_y^2}{a_y} = 0; \qquad y_{F_4} = -\frac{i_x^2}{a_y} = -\frac{0,32b^2}{b} = -12 \text{ cm}.$$

By connecting the resulting points with straight or curved lines, we obtain the core of the section (Figure 2.3).

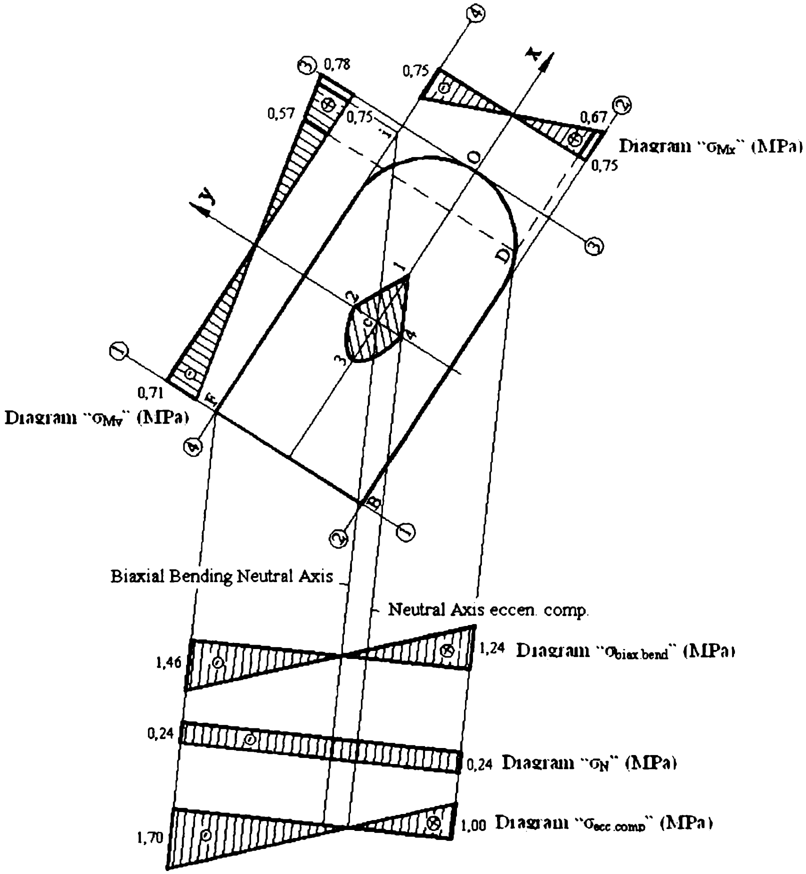


Figure 2.3 – Stress diagrams. Core of cross-section

To examine the core of the section, apply force alternately at points C, 1, 2, 3 (Figure 2.4).

a) Force is applied in the center of gravity of the section (point C).

$$\sigma = -\frac{F}{A} = -\frac{250 \cdot 10^{3}}{7{,}57 \cdot 37{,}2^{2} \cdot 10^{-4}} = -0{,}24 \text{ MPa};$$

b) Force is applied in the area of the section core (point 1), $x_F = 0{,}32b$; $y_F = 0$.

Stresses at the end points of the section will be:

$$\sigma_E = -\frac{F}{A}\left(1 + \frac{x_F x_E}{i_y^2}\right) = -\frac{250 \cdot 10^{3}}{7{,}57 b^{2}}\left(1 + \frac{0{,}32\ b \cdot (-1{,}9\ b)}{1{,}22\ b^{2}}\right) = 0{,}12 \text{ MPa};$$

$$\sigma_0 = -\frac{F}{A}\left(1+\frac{x_F x_0}{i_v^2}\right) = -\frac{250\cdot10^3}{7{,}57b^2}\left(1+\frac{0{,}32\ b\cdot 2{,}1\ b}{1{,}22\ b^2}\right) = -0{,}4\ \text{MPa}.$$

c) Force is applied at the boundary of the core section (point2), $x_F = 0{,}64b$; $y_F = 0$.

$$\sigma_E = -\frac{F}{A}\left(1+\frac{x_F x_E}{i_v^2}\right) = -\frac{250\cdot10^3}{7{,}57b^2}\left(1+\frac{0{,}64\ b\cdot(-1{,}9\ b)}{1{,}22\ b^2}\right) = 0;$$

$$\sigma_0 = -\frac{F}{A}\left(1+\frac{x_F x_0}{i_v^2}\right) = -\frac{250\cdot10^3}{7{,}57b^2}\left(1+\frac{0{,}64\ b\cdot 2{,}1\ b}{1{,}22\ b^2}\right) = -0{,}5\ \text{MPa}.$$

d) Force is applied behind the core of the section (point 3), $x_F = 1{,}1b$; $y_F = 0$.

$$\sigma_E = -\frac{F}{A}\left(1+\frac{x_F\ x_E}{i_v^2}\right) = -\frac{250\ \cdot10^3}{7{,}57\,b^2}\left(1+\frac{1{,}1b\cdot(-1{,}9\ b)}{1{,}22b^2}\right) = 0{,}17\ \text{MPa};$$

$$\sigma_0 = -\frac{F}{A}\left(1+\frac{x_F\ x_0}{i_v^2}\right) = -\frac{250\ \cdot10^3}{7{,}57\cdot37{,}2^2\cdot10^{-4}}\left(1+\frac{1{,}1b\cdot 2{,}1\ b}{1{,}22\ b^2}\right) = -0{,}69\ \text{MPa}.$$

Based on the obtained stress values, we construct their diagrams (Figure 2.4).

Stress diagram analysis, with different arrangement of compressive force F, shows that it is most favorable to load the column with a centrally applied compressive force, within of core. In order to obtain the stress of one sign at all points of the cross section, force should be applied in the zone of the core of the cross section.

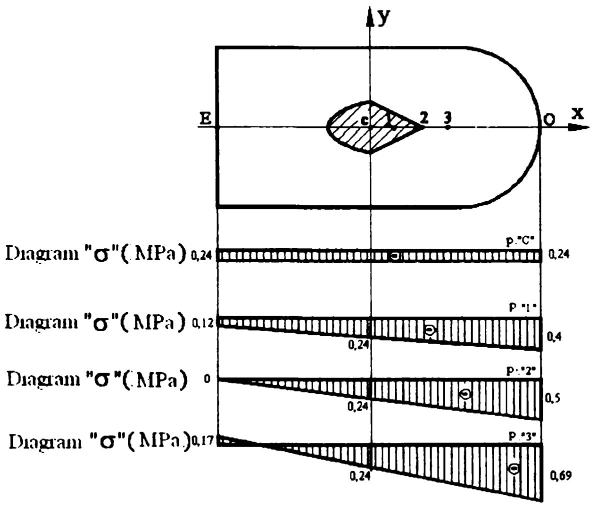


Figure 2.4 – Section Core Study

3. STABILITY OF COMPRESSED MEMBERS

3.1. Concept of stability of the original form of equilibrium

It turns out that the bearing capacity of the compressed column can be exhausted due to the loss of stability, that is, as a result of curvature (bulging), which occurs before the column fails directly from compression deformation. From theoretical mechanics it is known that the equilibrium of an absolutely solid body is stable, indifferent and unstable. A similar phenomenon occurs in the mechanics of a deformable body (strength of materials), with the only difference that the type of equilibrium depends on the amount of external load applied. An example is the equilibrium of a central compression column. At a relatively small value of the compressive load F, the column undergoes compression and is in a stable equilibrium state, since having received a small curvature of the geometric axis due to a transverse "disturbing" force, the column quickly returns to its original position. As the compressive load F increases, the column is slower to return to its original position after the "disturbance" and at some critical value F_{cr} a state of indifferent equilibrium occurs: after some curvature, the column acquires equilibrium in a curved state. There is a bifurcation of equilibrium: the rectilinear shape loses stability, and the curvilinear shape does not yet have time to acquire it, which theoretically becomes stable again at $F > F_{cr}$. However, this condition is practically unacceptable, since the column no longer works on compression, but on compression with bending, and therefore, large deflection and stress occur, which are interconnected with each other by a non-linear dependence. This leads to destruction.

The bending associated with the loss of equilibrium stability of the rectilinear initial form is called longitudinal, since it arises from the longitudinal load. The greatest value of the longitudinal compressive force to which the rectilinear form of the column is maintained is called the buckling (critical) force.

3.2. Euler's buckling force. Euler's formula

The formula for determining the value of the buckling force for a column hinged by both ends was first obtained by L. Euler (1744), so it was called the Euler's formula, and the force is often called the Euler's force. The formula is:

$$F_{cr}=\frac{\pi^2 \cdot E \cdot I}{l^2},$$

I.e. the value of the buckling force is directly proportional to the stiffness (EI) and inversely proportional to the square of the length of the column (l).

For various cases of fixing the ends of compressed columns, the value of the critical force is determined by the formula in the form:

$$F_{cr}=\frac{\pi^2 EI}{(\mu l)^2},$$

where μ – the factor of the reduced length, and the value $\mu \cdot l = l_{red}$ is the reduced length.

3.3 Critical stresses. Slenderness ratio of a column (flexibility)

Taking into account the Euler formula, we get:

$$\sigma_{cr}=\frac{F_{cr}}{A}=\frac{\pi^2 \cdot E \cdot I}{(\mu \cdot l)^2 \cdot A}=\frac{\pi^2 \cdot E}{\lambda^2}.$$

where $\lambda = \frac{\mu \cdot l}{i}$ – slenderness ratio of the column, depends on its geometric dimensions; $i = \sqrt{I / A}$ – inertia radius of cross-section of column. It follows from the obtained expression that the critical stress depends on the elastic constant of the material (E) and the flexibility of the bar (λ).

Given that critical stresses should not exceed the proportionality limit (σ_{pr}), it is easy to determine the ultimate flexibility, i.e. $\sigma_{cr} = \frac{\pi^2 \cdot E}{\lambda^2} \le \sigma_{pr}$; $\lambda_{pr} = \pi \cdot \sqrt{\frac{E}{\sigma_{pr}}}$.

Unlike geometric flexibility, ultimate flexibility (λ_{pr}) depends on physical and mechanical properties of the material from which the column is made, i.e. modulus of longitudinal elasticity (E) and proportionality limit (σ_{pr}).

The Euler's formula applies when the actual flexibility ($\lambda = \frac{\mu \cdot l}{i}$) is greater than the limited (λ_{pr}). If the flexibility of the compressed column is less than the ultimate flexibility ($\lambda < \lambda_{pr}$), then the critical stress and force are determined by the Tetmajer-Yasinskii's formula – empirical formula:

$$\sigma_{cr} = a - b \cdot \lambda; \; F_{cr} = \sigma_{cr} \cdot A,$$

where a and b are material-dependent coefficients (for example, for steel a=310 MPa, b=1,14 MPa).

3.4. Stability analysis

The following stability condition is used for centrally compressed columns:

$$\sigma_{cr} = \frac{N}{A} \le \varphi R,$$

where N is the normal force from the design compressive load; The A – cross-section area of the column: φ – the stress reduction factor or coefficient that reduces the design compression strength R to a value that guarantees the stability of column.

The stability condition allows us to perform three types of calculation, similar to strength calculations. Greatest interest does have the design task, i.e. selection of sections according to the given load and design strength of material R, length of column l and methods of fixing its ends:

$$A \ge \frac{N}{\varphi R}.$$

The use of this inequality is complicated by the fact that it includes two unknown values – A and φ. Therefore, the cross-section is selected by successive approximations. Initially, the value of the coefficient $\varphi \approx 0.5$ is set, the area A is determined, the section is designed so that the principal central moments of inertia are approximately equal: $I_x \approx I_y$ (deviations of up to 10% are allowed). Then, flexibility λ is determined, and the value of the coefficient φ is set according to the tables, taking into account interpolation procedure. After that, the actual stress is determined and compared with the design strength R, taking into account the coefficient φ. If the deviation is more than 5%, the calculation should be repeated.

3.5. Questions for self-test

1. What is meant by the loss of stability of the compressed column?
2. Define the stable and unstable state of the column.
3. Define the critical force.
4. What differential equation is used to derive the Euler's formula?
5. How the critical forces and stresses are determined?
6. What effects do cross-sectional stiffness and column length have on critical force?
7. What is meant by the flexibility of the column? Give the formula for determining geometric flexibility.
8. What is meant by the reduced (free) length of the column? How is the length ratio for the various fixing conditions determined?
9. In which cases do you use the Euler's formula?
10. When is the Tetmajer-Yasinskii's formula used in calculations?
11. What is the calculation of sustainability?
12. What is meant by the stress reduction factor and how is its value determined?
13. How are column sections selected for (from stability conditions)?
14. What are the conditions of equal stability used for, and what is it?
15. Give a diagram of critical stress, refer to ($\sigma_{cr} - \lambda$).

3.6. Assignment for calculation-design work "Calculation of compressed column for stability"

Given: Steel column with length l (need to accept by variant number, Figure 3.1, 3.2) loaded with longitudinal compressive load F. Design strength of column material $R_c = 210$ MPa; modulus of longitudinal elasticity of material $E = 200$ GPa.

It is required:

1) to select the dimensions of the cross section of the column (the cross section of the column should be taken according to the diagram number below);

2) to determine the value of the critical force and compare it with the given load F.

3.7 Column Attachment (Fixation) Diagram.

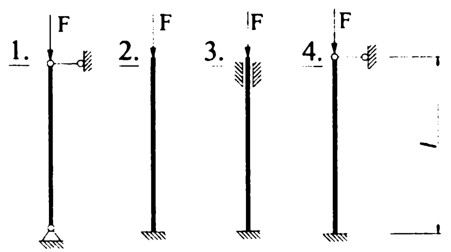


Figure 3.1 – Schemes of fixing column

3.8 Cross-sectional schemes of columns

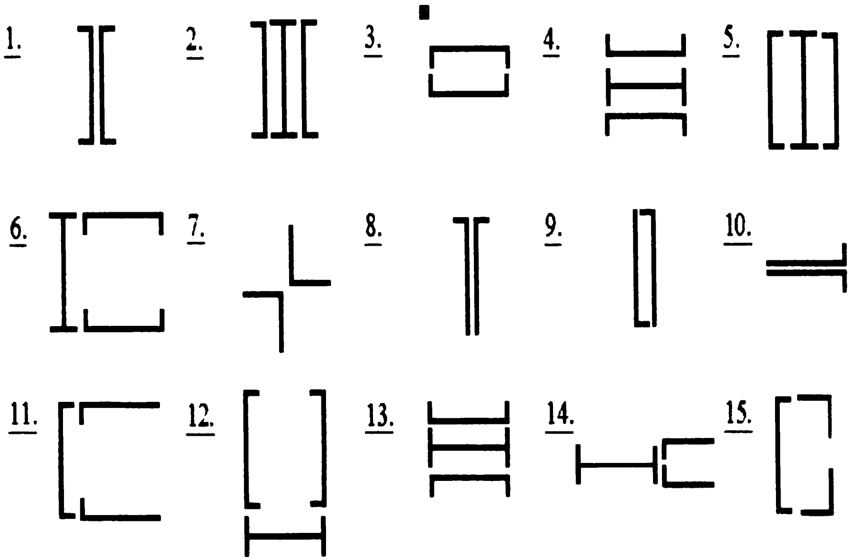


Figure 3.2 – Schemes of transverse sections of composite columns

3.9. Numerical values of column length and loads

Table 3.1.

option	column diagram	l, m	F, kN
1	3	2,4	450
2	1	2,4	480
3	2	3	490
4	3	3,2	460
5	1	2,6	440
6	2	2,5	430
7	3	3,2	420
8	1	3,3	470
9	2	2,9	410
10	3	2,6	550
11	1	3,3	530
12	2	2,8	540
13	3	2,85	505
14	1	2,45	515
15	2	3	525

3.10. Example of stability calculation

Given: Steel column with length – l = 4 m; loaded with longitudinal compressive load – F=540 kN. Design strength of column material – R_c=210 MPa; elastic modulus – E=200 GPa.

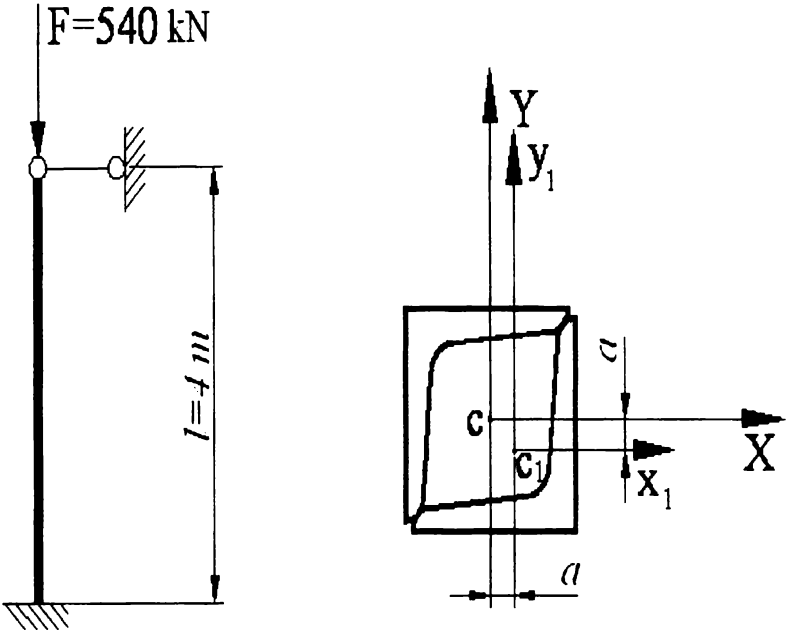


Figure 3.3 – Design diagram of the strut and its cross section

Solution:

1. Cross Section Sizing.

First iteration. We accept $\varphi_1 = 0{,}5$, then from the stability condition:

$$A_1 = \frac{F}{\varphi_1 R} = \frac{540 \cdot 10^3}{0{,}5 \cdot 210 \cdot 10^6} = 51{,}4\ cm^2.$$

According to the tables (grade of rolling profiles – GOST 8509-93) we take two equal angles: 125×125×10, A_1=24,3 cm^2, A=48,6 cm^2, $I_X = 360\ cm^4$, z_0=3,45 cm.

Now we define the geometric characteristics of the section:

$$I_x = I_y = 2(I_{x_1} + a^2 A_1) = 2(360 + 2{,}8^2 \cdot 24{,}3) = 1101\ \text{cm}^4;$$

$$i_{\min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{1101}{48{,}6}} = 4{,}76\ \text{cm}; \qquad \lambda = \frac{\mu \cdot l}{i_{\min}} = \frac{0{,}7 \cdot 400}{4{,}76} = 58{,}8.$$

By the table, we accept – φ:

$$\varphi_1^* = 0{,}89 - \frac{0{,}89 - 0{,}86}{10} \cdot 8{,}8 = 0{,}864;$$

$$\varphi_1^* = 0{,}864 >> \varphi_1 = 0{,}5.$$

Second iteration. Accept:

$$\varphi_2 = \frac{\varphi_1 + \varphi_1^*}{2} = \frac{0{,}5 + 0{,}864}{2} = 0{,}682.$$

Then: $A_2 = \dfrac{F}{\varphi_2 R} = \dfrac{540 \cdot 10^3 \cdot 10^4}{0{,}682 \cdot 210 \cdot 10^6} = 37{,}7\ cm^2.$

We accept two equal angles: $100 \times 100 \times 10$, $A_1 = 19{,}2\ cm^2$,

$I_{x_1} = I_{y_1} = 179\ cm^2, z_0 = 2{,}83\ cm.$

Then define the geometric characteristics of the section:

$$I_x = 2(I_{x_1} + a^2 A_1) = 2(179 + 2{,}17^2 \cdot 19{,}2) = 539\ cm^4;$$

$$i_{\min} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{539}{2 \cdot 19{,}2}} = 3{,}75\ cm, \qquad \lambda = \frac{0{,}7 \cdot 400}{3{,}75} = 74{,}7.$$

By the table, we accept – φ:

$$\varphi_2^* = 0{,}81 - \frac{0{,}81 - 0{,}75}{10} \cdot 4{,}7 = 0{,}782;$$

$$\varphi_2^* = 0{,}782 > \varphi_2 = 0{,}682.$$

Third iteration. Accept:

$$\varphi_3 = \frac{\varphi_2 + \varphi_2^*}{2} = \frac{0{,}682 + 0{,}782}{2} = 0{,}732.$$

Then: $A_3 = \dfrac{540 \cdot 10^3 \cdot 10^4}{0{,}732 \cdot 210 \cdot 10^6} = 35{,}13\ cm^2.$

We accept two equal angles: $110 \times 110 \times 8$, $A_1 = 17{,}2\ cm^2$,

$I_x = 198\ cm^4, z_0 = 3\ cm.$

Define the geometric characteristics of the section:

$$I_x = 2(I_{x_1} + a^2 A_1) = 2(198 + 2{,}5^2 \cdot 17{,}2) = 611\ cm^4;$$

$$i_{\min} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{611}{34{,}4}} = 4{,}21\ cm; \qquad \lambda = \frac{0{,}7 \cdot 400}{4{,}21} = 66{,}5.$$

By the table, we accept – φ:

$$\varphi_3^* = 0{,}86 - \frac{0{,}86 - 0{,}81}{10} \cdot 6{,}5 = 0{,}827.$$

$\varphi_3^* = 0{,}827 > \varphi_3 = 0{,}732$ *the difference*: 13,2 %.

Forth iteration. Accept:

$$\varphi_4 = \frac{\varphi_3 + \varphi_3^*}{2} = \frac{0{,}732 + 0{,}827}{2} = 0{,}78.$$

Then:

$$A_4 = \frac{540\cdot 10^3\cdot 10^4}{0{,}78\cdot 210\cdot 10^6} = 33\ cm^2.$$

We accept two equal angles $110\times 110\times 7$, $A_1 = 15{,}2\ cm^2$, $I_{x_1} = 176\ cm^4, z_0 = 2{,}96\ cm.$

Define the geometric characteristics of the section:

$$I_x = 2(I_{x_1} + a^2 A_1) = 2(176 + 2{,}54^2\cdot 15{,}2) = 548{,}1\ \text{cm}^4;$$

$$i_{min} = \sqrt{\frac{548{,}1}{30{,}4}} = 4{,}25\ cm; \qquad \lambda = \frac{0{,}7\cdot 400}{4{,}25} = 65{,}9.$$

By the table, we accept – φ: $\varphi_4^* = 0{,}86 - \frac{0{,}86 - 0{,}81}{10}\cdot 5{,}9 = 0{,}83.$

Check out the strength:

$$\sigma = \frac{F}{A} = \frac{540\cdot 10^3}{30{,}4\cdot 10^{-4}} = 177{,}6\ \text{MPa};$$

$$\sigma = 177{,}6\ MPa > \varphi R = 0{,}83\cdot 210 = 174{,}3\ \text{MPa}.$$

Overstress is: $\frac{177{,}6 - 174{,}3}{174{,}3}\cdot 100 = 1{,}9\ \%$, the discrepancy is acceptable.

We finally accept the cross section from two angles: $110\times 110\times 7$. $A = 30{,}4\ cm^2, I_x = I_y = 548{,}1\ cm^4.$

2. Critical Force Definition.

For the adopted column $\lambda = 65{,}9 < \lambda_{cr} = 100$, therefore, we use the Tetmajer-Yasinskii's formula to determine the critical force:

$$\sigma_{cr} = a - b\lambda = 310 - 1{,}14\cdot 65{,}9 = 234{,}9\ \text{MPa},$$

Then:

$$F_{cr} = \sigma_{cr}\cdot A = 234{,}9\cdot 10^6\cdot 30{,}4\cdot 10^{-4} = 714\ \text{кN}.$$

The ratio factor F_{cr}/F is: $n = \frac{F_{cr}}{F} = \frac{714}{540} = 1{,}32.$

Table 3.2.

Flexibility of elements $\lambda = \frac{\mu\cdot l}{i}$	Coefficient φ for crdinary steel (Grade C or A 107)	Flexibility $\lambda = \frac{\mu\cdot l}{i}$	Coefficient φ for ordinary steel (Grade C or A 107)
0	1,00	120	0,45
10	0,99	130	0,40
20	0,96	140	0,36
30	0,94	150	0,32
40	0,92	160	0,29
50	0,89	170	0,26
60	0,86	180	0,23

Continuation of table 3.2

70	0,81	190	0,21
80	0,75	200	0,19
90	0,69	210	0,16
100	0,60	220	0,15
110	0,52	230	0,13

BIBLIOGRAPHY:

1. Soprotivlenie materialov /A. F. Smirnov [i dr.]. – M. : Vyssh.shk., 1972. – 480 s.

2. Darkov, A. V. Soprotivlenie materialov / A. V. Darkov, G. S. SHpiro. – M. : Vyssh.shk.,1975. – 654 s.

3. Feodos'ev, V. I. Soprotivlenie materialov / V. I. Feodos'ev. – M. : Nauka, 1986. – 512 s.

4. Sbrnik zadach po soprotivleniyu materialov / Pod red. A. V. Aleksandrova. –M. : Strojizdat, 1977. – 335 c.

5. Teregulov I. S. Soprotivlenie materialov i osnovy teorii uprugosti i plastichnosti / I. S. Teregulov. – M. : Vyssh.shk., 1984. – 472 c.

EDUCATIONAL EDITION

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TASKS AND METHODICAL INSTRUCTIONS FOR PERFORMING CALCULATION-DESIGN WORKS

«STRENGTH OF MATERIALS» FOR STUDENTS OF A SPECIALTY 1 – 70 02 01 «INDUSTRIAL AND CIVIL ENGINEERING»

The text is printed in the author's edition, spelling and punctuation

Responsible for the issue (releaser): Zheltkovich A. E.
Editor: Mitloshuk M. A.
Computer-aided makeup: Rogozhina J. A.

Signed in print 01.06.2022. Format 60x84 $^{1/}{}_{16}$. "Performer" paper.
"Times New Roman" headset. Cond. pr. p. 2,09. Educ. edit. p. 2,25. Order № 518.
Print run 20 copies. Printed on the risograph of the educational institution
"Brest State technical university" 224017, Brest, st. Moscow, 267.
Certificate of state registration of the publisher, manufacturer,
print distributor No. 1/235 of 24.03.2014