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SYSTEMS OF DIFFERENTIAL EQUATIONS IN THE LEBESGUE SPACES

Herein, we investigate systems of nonautonomous differential equations with generalized coefficients using the algebra of new generalized functions. We consider a system of nonautonomous differential equations with generalized coefficients as a system of equations in differentials in the algebra of new generalized functions. The solution of such a system is a new generalized function. It is shown that the different interpretations of the solutions of the given systems can be described by a unique approach of the algebra of new generalized functions. In this paper, for the first time in the literature, we describe associated solutions of the system of nonautonomous differential equations with generalized coefficients in the Lebesgue spaces $L_p(T)$ with functions that satisfy the linear growth condition.

Key words: *algebra of new generalized functions, differential equations with generalized coefficients, functions of finite variation.*

Системы дифференциальных уравнений в пространствах Лебега

Исследуются системы неавтономных дифференциальных уравнений в алгебре новых обобщенных функций. Система неавтономных дифференциальных уравнений с обобщенными коэффициентами рассматривается как система уравнений в дифференциалах в алгебре новых обобщенных функций. Решением таких систем является новая обобщенная функция. Показано, что различные интерпретации решений данных систем могут быть описаны при помощи единственного подхода, использующего новые обобщенные функции. В статье, в отличие от предшествующих работ, описаны ассоциированные решения систем неавтономных дифференциальных уравнений с обобщенными коэффициентами в пространствах Лебега $L_p(T)$, содержащие функции, удовлетворяющие условию линейного роста.

Ключевые слова: *алгебра новых обобщенных функций, дифференциальные уравнения с обобщенными коэффициентами, функции ограниченной вариации.*

Introduction

In this paper, we will consider the following system of equations with generalized coefficients on $T \in [0;a] \subset R$

$$\dot{x}^i(t) = \sum_{j=1}^{q} f^{ij}(t, x(t))\dot{L}^j(t), \; i = \overline{1, p} \tag{1}$$

$$x(0) = x_0, \tag{2}$$

where $f^{ij}, i = \overline{1, p}, \; j = \overline{1, q}$ are some functions, $x(t) = [x^1(t), x^2(t), ..., x^p(t)]$ and $\dot{L}^j(t)$, $j = \overline{1, q}$ are functions of finite variation on T. $\dot{L}^j(t)$ are derivatives in the distributional sense or we can say that $\dot{L}^j(t)$ are derivatives in the Schwartz space. In general, since $\dot{L}^j(t)$ is the distribution and $f^{ij}(t, x(t))$ not smooth functions, the products $f^{ij}(t, x(t))\dot{L}^j(t)$ are not well defined and the solution of system (1) essentially depends on the interpretation. System (1) can describe the model of the rocket flight process or the model of the control problems with impulse actions. Let us recall some approaches to the interpretation of system (1).

The first approach is concerned with considering the system of equations in the framework of the distribution theory. According to this approach, once the product of distributions from some classes is defined, then one tries to find the solution of the system of equations (1) in these classes of distributions. For example, in papers [1; 2] the product of some distributions and discontinuous functions was defined. See also monograph [3] for another definition. Notice that the solutions of system (1) obtained using the products from [1–3] are different.

The second approach is to interpret system (1) as the following system of integral equations:

$$x^i(t) = x_0^i + \sum_{j=1}^{q} \int_0^t f^{ij}(s, x(s))\dot{L}^j(s),\ i = \overline{1, p},$$

where the integrals are understood in the Lebesgue-Stieltjes, Perron-Stieltjes, etc., sense [4; 5].

But in this approach the solution of the system of integral equations depends on the interpretation of the integral and the definition of the functions $x^i(t)$ in the discontinuity points of $L^j(t)$.

The third approach is based on the idea of the approximation of the solution of system (1) by the solutions of the system of ordinary differential equations, which are constructed using the smooth approximation of the functions $L^j(t)$. In monograph [3], it is shown that in this case the limit of the solutions of the smoothed equations exists.

In this paper, we will consider the system of equations (1) using the algebra of new generalized functions from [6]. Thus, we will interpret system of equations (1) as a system of equations in the differentials in the algebra of new generalized functions. Such interpretation says that the solution of system (1) is a new generalized function. In papers [7; 8] an ordinary nonlinear equation with generalized coefficients in the algebra of new generalized functions is considered.

In previous papers [9–12] the general view of system (1) were considered. The coefficients in such systems are generalized derivatives of arbitrary functions of finite variation $L^j(t)$. Using the given sequence of numbers $h_n \to 0$ we construct a sequence of approximating equations, and the generalized solution is defined as the limit of a sequence of the solutions of approximating equations.

It is found that generalized solution exists only under some additional conditions for the behaviour of the sequence h_n in the case of discontinuous functions $L^j(t)$ and different generalized solutions exist for different sequences h_n.

In papers [13; 14] the system of nonlinear differential equations, the coefficients of which are generalized derivatives of the continuous function of finite variation $L^j(t)$ is investigated.

The main purpose of this article is to show that under some conditions this new generalized function associates with some ordinary function, which is natural to call the solution of system (1).

We will describe associated solutions of the approximated systems used in previous similar articles, we will obtain the main results in the Lebesgue spaces $L_p(T)$ as in [15], but we will consider that functions $f^{ij},\ i = \overline{1, p},\ j = \overline{1, q}$ are functions that satisfy the linear growth condition.

The algebra of new generalized functions

In this section, we recall the definition of the algebra of new generalized functions from [6]. At first, we define an extended real line $\tilde{\Re}$ using a construction typical for non-standard analysis.

Let $\overline{\Re} = \{(x_n)_{n=1}^{\infty} : x_n \in R \text{ for all } n \in N\}$ be a set of real sequences. We call two sequences $\{x_n\} \in \overline{\Re}$ and $\{y_n\} \in \overline{\Re}$ equivalent if there is a natural number N such that $x_n = y_n$ for all $n > N$.

The set $\tilde{\Re}$ of equivalence classes is called the extended real line, and any of the classes a generalized real number.

It is easy to see that $R \subset \tilde{\Re}$ because one may associate with any ordinary number $x \in R$ a class containing a stationary sequence with $x_n = x$. It is evident that $\tilde{\Re}$ is an algebra. The product $\tilde{x}\tilde{y}$ of two generalized real numbers is defined as the class of sequences equivalent to the sequence $\{x_n y_n\}$, where $\{x_n\}$ and $\{y_n\}$ are the arbitrary representatives of the classes $\tilde{x}$ and $\tilde{y}$, respectively.

For any segment $T = [0; a] \subset R$ one can construct an extended segment $\tilde{T}$ in a similar way. Let H denote the subset of $\tilde{\Re}$ of nonnegative „infinitely small numbers":

$$H = \{\tilde{h} \in \tilde{\Re} : \tilde{h} = [\{h_n\}], h_n > 0 \text{ for all } n \in N, \lim_{n\to\infty} h_n = 0\}. \quad (2)$$

Consider the set of sequences of infinity differentiable functions $\{f_n(x)\}$ on R. We will call two sequences $\{f_n(x)\}$ and $\{g_n(x)\}$ equivalent if for each compact set $K \subset R$ there is a natural number N such that $f_n(x) = g_n(x)$ for all $n > N$ and $x \in K$. The set of classes of equivalent functions is denoted by $\Im(R)$ and its elements are called new generalized functions. Similarly one can define the space $\Im(T)$ for any interval $T = [0; a]$.

For each distribution f we can construct a sequence $\{f_n\}$ of smooth functions such that f_n converges to f (i.e., one can consider the convolution of f with some δ-sequence). This sequence defines the new generalized function that corresponds to the distribution f. Thus the space of distribution is a subset of the algebra of new generalized functions. However, in this case, infinitely many new generalized functions correspond to one distribution (e. g. by taking a different δ-sequence). We will say that the new generalized function $\tilde{f} = [\{f_n\}]$ associates with the ordinary function or distribution f if f_n converges to f in some sense.

Let $\tilde{f} = [\{f_n\}]$ and $\tilde{g} = [\{g_n\}]$ be generalized functions. Then the composition $\tilde{f} \circ \tilde{g}$ is defined by $\tilde{f} \circ \tilde{g} = [\{f_n \circ g_n\}] \in \Im(R)$. Similarly, one can define the value of the new generalized function $\tilde{f}$ at the generalized real point $\tilde{x} = [\{x_n\}] \in \tilde{\Re}$ as $\tilde{f}(\tilde{x}) = [\{f_n(x_n)\}]$.

For each $\tilde{h} = [\{h_n\}] \in H$ and $\tilde{f} = [\{f_n\}] \in \Im(R)$ we define a differential $d_{\tilde{h}}\tilde{f} \in \Im(R)$ by $d_{\tilde{h}}\tilde{f} = [\{f_n(x + h_n) - f_n(x)\}]$. The construction of the differential was proposed by Lazakovich (see [6]).

Now we can give an interpretation of system of equations (1) using the introduced algebras. Let $L(t), t \in [0; a] = T$ be a right-continuous function of finite variation. We replace ordinary functions in system (1) by corresponding new generalized functions and then write differentials in algebra.

So we have

$$d_{\tilde{h}}\tilde{x}^i(\tilde{t}) = \sum_{j=1}^{q} \tilde{f}^{ij}(\tilde{t}, \tilde{x}(\tilde{t}))d_{\tilde{h}}\tilde{L}^j(\tilde{t}),\ i = \overline{1,p} \qquad (3)$$

with the initial value $\tilde{x}\big|_{[\tilde{0};\tilde{h})} = \tilde{x}_0$, where $\tilde{h} = [\{h_n\}] \in H$, $\tilde{t} = [\{t_n\}] \in T$, $\tilde{x} = [\{x_n\}]$, $\tilde{f} = [\{f_n\}]$, $\tilde{x}_0 = [\{x_{0n}\}]$ and $\tilde{L} = [\{L_n\}]$ are elements of $\Im(R)$. Moreover $\tilde{f}$ and $\tilde{L}$ are associated with f and L, respectively. If $\tilde{x}$ is associated with some function x then we say that x is a solution of system (3).

The following theorem from [16] gives necessary and sufficient conditions for the existence and uniqueness of the solutions of system (3).

Theorem 2.1. If the following equality holds for some representatives

$$\{f_n^{ij}\} \in \tilde{f}^{ij}, \{L_n^j\} \in \tilde{L}^j,\ \{x_n^i\} \in \tilde{x}^i,\ \{x_{0n}^i\} \in \tilde{x}_0^i,$$

for all sufficiently large $n \in N$ and for all $l = 0, 1, ...$

$$\lim_{t\to 0^+}\left(\frac{d^l}{dt^l}[x_{0n}^i(h_n - t) - x_{0n}^i(t)] - \sum_{j=1}^{q}\frac{d^l}{dt^l}[f_n^{ij}(t, x_{0n}(t))[L_n^j(h_n + t) - L_n^j(t)]]\right) = 0,$$

then a solution of system (3) exists and is unique.

The purpose of the present paper is to investigate the case when the solution $\tilde{x}$ of system (3) is associated with some function and to describe all possible associated solutions.

Main results

In this section, we will formulate the main results of this article.

In this paper, we consider specific types of representatives of the new generalized functions (mnemofunctions). We take the following convolutions as representatives of $\tilde{L}$ from system (3)

$$L_n^j(t) = (L^j * \rho_n^j)(t) = \int_0^{\frac{1}{\gamma^j(n)}} L^j(t+s)\rho_n^j(s)ds, \qquad (4)$$

where

$$\rho_n^j(t) = \gamma^j(n)\rho^j(\gamma^j(n)t),\ \rho^j \geq 0, \sup\rho^j \subseteq [0;1], \int_0^1 \rho^j(s)ds = 1$$

and $f_n = f * \tilde{\rho}_n$, $\tilde{\rho} \in C^\infty(R^{z+1})$, $\int_{[0;1]^{z+1}} \tilde{\rho}(x_0, x_1, ..., x_z)dx_0dx_1...dx_z = 1$, $\tilde{\rho} \geq 0, \sup\tilde{\rho} \subseteq [0;1]^{z+1}$.

If the function $\gamma^j(n)$ is some monotonic function such as $\lim\limits_{\substack{n\to\infty \\ h_n\to 0}}\gamma^j(n)=\infty$, we will assume that for $j=\overline{1,w}$ $\lim\limits_{\substack{n\to\infty \\ h_n\to 0}}\gamma^j(n)h_n=\infty$ and for $j=\overline{w+1,q}$ $\lim\limits_{\substack{n\to\infty \\ h_n\to 0}}\gamma^j(n)h_n=0.$

Using representatives, we can rewrite system (3) in the following form:

$$\begin{cases} x_n^i(t+h_n)-x_n^i(t)=\sum\limits_{j=1}^{q} f_n^{ij}(t,x_n(t))[L_n^j(t+h_n)-L_n^j(t)],\ i=\overline{1,z} \\ x_n(t)\big|_{[0;h_n)}=x_{0n}(t) \end{cases}. \tag{5}$$

The solution $\tilde{x}$ of system (3) is associated with some function if and only if the sequence $\{x_n\}$ of the solutions of system (5) converges.

Therefore, we have to investigate the limiting behavior of the sequence $\{x_n\}$.

Let t be an arbitrary point of T. There exist $m_t\in N$ and $\tau_t\in[0;h_n)$ such that $t=\tau_t+m_t h_n$. Set $t_k=\tau_t+kh_n$ for $k=0,1,\ldots,m_t$. Then the solution of system (5) can be written as

$$x_n^i(t)=x_{0n}^i(\tau_t)+\sum_{j=1}^{q}\sum_{k=0}^{m_t-1} f_n^{ij}(t_k,x_n(t_k))[L_n^j(t_{k+1})-L_n^j(t_k)],\ i=\overline{1,z}. \tag{6}$$

Let $L^j(t),\ j=\overline{1,q},\ t\in T=[0;a]$ be a right-continuous function of finite variation. We will assume that $L^j(t)=L(a)$ if $t>a$ and $L^j(t)=L(0)$ if $t<0$. Let us denote by $\operatorname*{var}\limits_{u\in T} L(u)=\sum\limits_{j=1}^{q}\operatorname*{var}\limits_{u\in T} L^j(u)$ the total variation of the function $L=[L^1,L^2,\ldots,L^q]$ on the interval T. Suppose that f is a function that satisfy the linear growth condition with a constant M then for all $x\in R$ and $t\in T$:

$$|f(t,x)|\le M(1+|x|). \tag{7}$$

In order to describe the limits of the sequence x_n, we consider the following system of integral equations

$$x^i(t)=x_0^i+\sum_{j=1}^{q}\int_0^t f^{ij}(s,x(s))dL^{jc}(s)+\sum_{\mu_r\le t} S^i(\mu_r,x(\mu_r-),\Delta L(\mu_r)),\ i=\overline{1,z} \tag{8}$$

where $L^{jc}(t)$ is the continuous part and $L^{jd}(t)$ is discontinuous part of the function $L^j(t)$, $\mu_r,\ r=1,2,\ldots$ – discontinuity points of the function $L^j(t)$, $j=\overline{1,q}$, $\Delta L^j(\mu_r)=L^{jd}(\mu_r+)-L^{jd}(\mu_r-)$, $j=\overline{1,q}$ is the size of the jump

$$S^i(\mu,x,u)=\varphi^i(1,\mu,x,u)-\varphi^i(0,\mu,x,u),$$

where $\varphi^i(t,\mu,x,u)$ is the solution of the integral equation

$$\varphi^i(t,\mu,x,u)=x^i+\sum_{j=1}^{w}u^j\int_0^t f^{ij}(\mu,\varphi(s-,\mu,x,u))dH(s-1)+$$

$$+\sum_{j=w+1}^{q} u^j \int_0^t f^{ij}(\mu,\varphi(s,\mu,x,u))ds,\ i=\overline{1,z}.$$

Here and in what follows all integrals are understood in the Lebesgue-Stieltjes sense.

Theorem 3.1. Let $f^{ij},\ i=\overline{1,z},\ j=\overline{1,q}$ are functions that satisfy the linear growth condition (7) and L^j right-continuous functions of finite variation. Suppose that $\int_T |x_{n0}(\tau_t)-x_0|dt \to 0$ in the space $L_p(T)$ as $n\to\infty$, $h_n \to 0$, $\gamma^j(n)\to\infty$ and $\gamma^j(n)h_n \to\infty$ for $j=\overline{1,w}$ and $\gamma^j(n)h_n \to 0$ for $j=\overline{w+1,q}$, then the solution $x_n(t)$ of (5) converges to the solution $x(t)$ (8) in $L_p(T)$.

Theorem 3.2. Under the condition of theorem 2.1. let $f^{ij},\ i=\overline{1,z},\ j=\overline{1,q}$ are functions that satisfy the linear growth condition (7) and L^j right-continuous functions of finite variation.

Suppose that $\int_T |x_{n0}(\tau_t)-x_0|dt \to 0$ in the space $L_p(T)$ as $n\to\infty,\ h_n \to 0$ $\gamma^j(n)\to\infty$ and $\gamma^j(n)h_n \to\infty$ for $j=\overline{1,w}$ and $\gamma^j(n)h_n \to 0$ for $j=\overline{w+1,q}$, then the associated solution of (3) is the solution of (8) in the space $L_p(T)$.

Similar results for the system of nonautonomous differential equations in the space $L_1(t)$ have been obtained in [10].

Definition 3.3. We say that the function $x(t)$ is an I-associated (S-associated) solution of the system of equations in differentials (3) if it is associated solution (3) under conditions that $\lim\limits_{\substack{n\to\infty \\ h_n\to 0}} \gamma^j(n)h_n=\infty$ ($\lim\limits_{\substack{n\to\infty \\ h_n\to 0}} \gamma^j(n)h_n=0.$) and the representatives of the functions $\tilde{f}$ and $\tilde{L}$ are set by formula (4). In this case, we name $d_{\tilde{h}}\tilde{L}^j$ as an I-associated (S-associated) coefficient.

Let $f: R^z \to R$. We set

$$f_n(t)=(f*\tilde{\rho}_n)(t)=\int_{[0,1/n]^z} f(t+s)\tilde{\rho}_n(s)ds,$$

where $\tilde{\rho}_n(t)\in C^\infty(R^z),\ \tilde{\rho}_n(t)\ge 0,$ supp $\tilde{\rho}_n(t)\subset[0,1/n]^z,\ \int_{[0,1/n]^z}\tilde{\rho}_n(s)ds=1,\ n\in N.$

Consider the case when $\gamma^j(n)=n$ then $\tilde{\rho}_n(t)\in n^z\tilde{\rho}(nt),\ \tilde{\rho}_n(t)\in C^\infty(R^z),$ supp $\tilde{\rho}\subset[0,1]^z,\ \int_{[0,1]^z}\tilde{\rho}_n(s)ds=1,\ n\in N.$

Remark 3.4. Let $\gamma^j(n)=n$, then we can define the set H from (2) using the following subsets:

$$I=\{\tilde{h}\in H: \frac{1}{n}=o(h_n), n\to\infty, h_n\to 0,\ \textit{for all}\ h_n\in\tilde{h}\},$$

$$S=\{\tilde{h}\in H: h_n=o(\frac{1}{n}), h_n\to 0,\ n\to\infty,\ \textit{for all}\ h_n\in\tilde{h}\}.$$

We name the generalized differential $d_{\tilde{h}}$ as I-generalized (S-generalized) differential and denote $d^I_{\tilde{h}}$ ($d^S_{\tilde{h}}$), if $\tilde{h}\in I$ ($\tilde{h}\in S$).

Note, that the I-generalized (S-generalized) differential makes sense only for the new generalized function $\tilde{L}^j$ with representatives (4), where $\gamma^j(n) = n$.

According to equation (3), we will consider the systems of equations with I-generalized and S-generalized differentials:

$$\begin{cases} d^I_{\tilde{h}} \tilde{x}^i(\tilde{t}) = \sum_{j=1}^{q} \tilde{f}^{ij}(\tilde{t}, \tilde{x}(\tilde{t})) d^I_{\tilde{h}} \tilde{L}^j(\tilde{t}), \\ \tilde{x}\big|_{[0,\tilde{h})} = \tilde{x}^0. \end{cases} \tag{9}$$

$$\begin{cases} d^S_{\tilde{h}} \tilde{x}^i(\tilde{t}) = \sum_{j=1}^{q} \tilde{f}^{ij}(\tilde{t}, \tilde{x}(\tilde{t})) d^S_{\tilde{h}} \tilde{L}^j(\tilde{t}), \\ \tilde{x}\big|_{[0,\tilde{h})} = \tilde{x}^0. \end{cases} \tag{10}$$

Remark 3.5. In case $\gamma^j(n) = n$ definition 3.3 will take the following form: we will say that the function $x(t)$ is the I-associated (S-associated) solution of a system of equations in differentials (3) if it is associated solution (9) ((10)).

Let $\gamma^j(n) = n$. In order to describe the limits of the sequence x_n we consider the following system of integral equations

$$x^i(t) = x_0^i + \sum_{j=1}^{q} \int_0^t f^{ij}(s, x(s)) dL^j(s), \; i = \overline{1, z} \tag{11}$$

Theorem 3.6. Let $f^{ij}, \, i = \overline{1, z}, \; j = \overline{1, q}$ are functions that satisfy the linear growth condition (7) and L^j continuous functions of finite variation. Suppose that $\int_T |x_{n0}(\tau_t) - x_0| dt \to 0$ in the space $L_p(T)$ than the solution $x_n(t)$ of (5) converges to the solution $x(t)$ from (11) in the space $L_p(T)$ as $n \to \infty, \; h_n \to 0$.

Theorem 3.7. Under the condition of theorem 2.1. let $f^{ij}, \, i = \overline{1, z}, \; j = \overline{1, q}$ are functions that satisfy the linear growth condition (7) and L^j continuous functions of finite variation. Suppose that $\int_T |x_{n0}(\tau_t) - x_0| dt \to 0$ in the space $L_p(T)$, than the associated solution of (3) is the solution of (11) in the space $L_p(T)$ as $n \to \infty, \; h_n \to 0$.

The proof of a similar theorem in another space and in an autonomous case can be seen in [11].

Let L^j be right-continuous functions of finite variation, $\gamma^j(n) = n$ and $\frac{1}{n} = o(h_n)$ as $n \to \infty, \; h_n \to 0$. In order to describe the limits of the sequence x_n, we consider the following system of integral equations

$$x^i(t) = x_0^i + \sum_{j=1}^{q} \int_0^{t+} f^{ij}(s, x(s-)) dL^j(s), \; i = \overline{1, z} \tag{12}$$

Theorem 3.8. Let $f^{ij}, \, i = \overline{1, z}, \; j = \overline{1, q}$ are functions that satisfy the linear growth condition (7) and L^j right-continuous functions of finite variation. Suppose that

$\int_T |x_{n0}(\tau_t) - x_0| dt \to 0$ in the space $L_p(T)$ than the solution $x_n(t)$ of (5) converges to the solution $x(t)$ in the space $L_p(T)$ (12) as $n \to \infty,\ h_n \to 0$, and $\frac{1}{n} = o(h_n)$.

Theorem 3.9. Under the condition of theorem 2.1. let $f^{ij},\ i = \overline{1,z},\ j = \overline{1,q}$ are functions that satisfy the linear growth condition (7) and L^j right-continuous functions of finite variation. Suppose that $\int_T |x_{n0}(\tau_t) - x_0| dt \to 0$ in the space $L_p(T)$ as $n \to \infty,\ h_n \to 0$ than the I-associated solution of (3) is the solution of (12) in the space $L_p(T)$ as $n \to \infty,\ h_n \to 0$.

Similar results for the system of autonomous differential equations in other spaces have been obtained in [12].

Let L^j be right-continuous functions of finite variation, $\gamma^j(n) = n$ and $h_n = o(\frac{1}{n})$ as $n \to \infty,\ h_n \to 0$. In order to describe the limits of the sequence x_n, we consider the following system of integral equations

$$x^i(t) = x_0^i + \sum_{j=1}^{q} \int_0^t f^{ij}(s, x(s)) dL^{jc}(s) + \sum_{\mu_r \le t} S^i(\mu_r, x(\mu_r -), \Delta L(\mu_r)),\ i = \overline{1,z} \tag{13}$$

where $S^i(\mu, x, u) = \varphi^i(1, \mu, x, u) - \varphi^i(0, \mu, x, u)$, and $\varphi^i(t, \mu, x, u)$ is the solution of the integral equation

$$\varphi^i(t, \mu, x, u) = x^i + \sum_{j=1}^{q} u^j \int_0^t f^{ij}(\mu, \varphi(s, \mu, x, u)) ds,\ i = \overline{1,z}.$$

Theorem 3.10. Let $f^{ij},\ i = \overline{1,z},\ j = \overline{1,q}$ are functions that satisfy the linear growth condition (7) and L^j right-continuous functions of finite variation. Suppose that $\int_T |x_{n0}(\tau_t) - x_0| dt \to 0$ in the space $L_p(T)$, then the solution $x_n(t)$ of (5) converges to the solution $x(t)$ from (13) in the space $L_p(T)$ as $n \to \infty,\ h_n \to 0$ and $h_n = o(\frac{1}{n})$.

Theorem 3.11. Under the condition of theorem 2.1. let $f^{ij},\ i = \overline{1,z},\ j = \overline{1,q}$ are functions that satisfy the linear growth condition (7) and L^j right-continuous functions of finite variation.

Suppose that $\int_T |x_{n0}(\tau_t) - x_0| dt \to 0$ in the space $L_p(T)$ as $n \to \infty,\ h_n \to 0$, then the S-associated solution of (3) is the solution of (13) in the space $L_p(T)$ as $n \to \infty,\ h_n \to 0$.

Similar results for such a system of autonomous differential equations in another spaces have been obtained in [17; 18].

Conclusion

The systems of nonautonomous differential equations with generalized coefficients using the algebra of new generalized functions are investigated. It is shown that different interpretations of the solutions of the given systems can be described by a unique approach of the algebra of new generalized functions.

In this paper, for the first time in the literature, we describe associated solutions of the system of nonautonomous differential equations with generalized coefficients in the Lebesgue spaces $L_p(T)$ with functions that satisfy the linear growth condition.

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