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Reinforced Concrete Structures.
Basis of Design

Recommended by the University Council
as a Textbook for the Foreign students
and for students of Building specialities studied in English

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Implementation of the Eurocodes extends to all of the European countries and there are firm steps toward their adoption internationally. As with any design codes, it is important to have an understanding of the principles and background, as well as design aids to assisst in the design process. It should be pointed the relevance to train in the use of Eurocodes, especially in the engineering schools as a basic course for civil and structural engineering students and as a part of continious professional development courses for engineers and technicians, which be promoted both at national and international levels.

This book is directed primarily on civil and structural engineering students, young designers, who requires and understanding of the basic theory and concise guide for design procedure.

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PREFACE (FOREWORD)

The main purpose of this book is to prove a straight forward and clear introductionto the theoretical background, to present and comment some basical princeples and methods of design for concrete structures in accordance with European Standard EN 1992-1-1 (Eurocode 2).

The Eurocodes are a set of European Standards which provide common princeples and rules for design of a different types of structural elements. The use of these common standards is intended to lower trade barriers. Moreover, these design codes, considered to be the most advanced in the world, will lead to a common understanding of a design principles and rules for concrete structures for design engineers, contractors and the manufacturers of concrete products. The additional advantages of unified codes include the preparation of common design aids and software and the establishment of a common understanding of research and development needs in Europe.

With publication of all of the 58 Eurocode parts, the implementation of the Eurocodes extends to all of the European countries and there are firm steps toward their adoption internationally. As with any design codes, it is important to have an understanding of the principles and background, as well as design aids to assisst in the design process. It should be pointed the relevance to train in the use of Eurocodes, especially in the engineering schools as a basic course for civil and structural engineering students and as a part of continious professional development courses for engineers and technicians, which be promoted both at national and international levels.

Although, the described in this book design methods are generally in accordance with EN 1992-1-1 (EC 2), much of the theory (theoretical background) and practice is of a fundamental nature and should, therefore, be useful for students and engineers in countries outside the Europe.

This book is directed primarily on civil and structural engineering students, young designers, who requires and understanding of the basic theory and concise guide for design procedure.

The authors would like to express sincere gratitude to the colleagues and graduated students: Horbat Yana, Lizahub Aliaksandr, Ratskevich Yulia, Varabei Aliaksandr, Vaskaboinikau Ihar, who pepeared and typed some parts of the figures, diagrams and final draft of the manuscript.

CONTENT

NOTATION

Notation is generally in accordance with EN 1992-1-1 [N3] and principal symbols are listed below. Other symbols are defined in the text where necessary.

Latin upper case letters:

A – Cross sectional area;
A_c – Cross sectyional area of concrete;
A_p – Area of a prestressing tendon or tendons;
A_s – Cross sectional area of reinforcement;
$A_{s,min}$ – minimum cross sectional area of reinforcement;
A_{sw} – Cross sectional area of shear reinforcement;
D – diameter of mandrel;
E – Effect of action;
E_c, $E_{c(28)}$ – Tangent modulus of elasticity of normal weight concrete at a stress of σ_c=0 and at 28 days;
$E_{c,eff}$ – Effective modulus of elasticity of concrete;
E_{cd} – Design value of modulus of elasticity of concrete;
E_{cm} – Secant modulus of elasticity of concrete;
$E_c(t)$ – Tangent modulus of elasticity of normal weight concrete at a stress of σ_c=0 and at time t;
E_s – Design value of modulus of elasticity of reinforcing steel;
EI – Bending stiffness;
F – Action;
F_d – Design value of an action;
F_k – Characterisatic value of an action;
G_k – Characterisatic permanent action;
I – Second moment of area of concrete section;
L – Length;
M_{Ed} – Design value of the applied internal bending moment;
N – Axial force;
N_{Ed} – Design value of the applied axial force (tension or compression);
Q_k – Characterisatic variable action;
R – Resistance;
S – Internal forces and moments;
S – First moment of area;
SLS – Serviceability limit state;
T – Torsional moment;
T_{Ed} – Design value of the applied torsional moment;
ULS – ultimate limit state;
V – Shear force;
V_{Ed} – Design value of the applied shear force.

Latin lower case letters:

a – Distance;

a – Geometrical data;

Δa – Deviation for geometrical data;

b – Overall width of a cross-section, or actual flange width in a T or L beam;

b_w – Width of the web on T, I or L beams;

d – Diameter; Depth;

d – Effective depth of a cross-section;

d_g – Largest nominal maximum aggregate size;

e – Eccentricity;

f_c – Compressive strength of concrete;

f_{cd} – Design value of concrete compressive strength;

f_{ck} – Characteristic compressive cylinder strength of concrete at 28 days;

f_{cm} – Mean value of concrete cylinder compressive strength;

f_{ctk} – Characteristic axial tensile strength of concrete;

f_{ctm} – Mean value of axial tensile strength of concrete;

$f_{0,2k}$ – Characteristic 0,2 % proof-stress of reinforcement;

f_t – Tensile strength of reinforcement;

f_{tk} – Characteristic tensile strength of reinforcement;

f_y – Yield strength of reinforcement;

f_{yd} – Design yield strength of reinforcement;

f_{yk} – Characteristic yield strength of reinforcement;

f_{ywd} – Design yield of shear reinforcement;

h – height;

h – Overall depth of a cross-section;

i – radius of gyration;

k – Coefficient; Factor;

l – (or L) Length; Span;

l_0 – Effective length or lap length;

m – Mass;

r – Radius;

l/r – Curvature at a particular section;

t – Thickness;

t – Time being considered;

t_0 – The age of concrete at the time of loading;

u – Perimeter of concrete cross-section, having area A_c;

u, v, w – Components of the displacement of a point;

x – Neutral axis depth;

x, y, z – Coordinates;

z – Lever arm of internal forces.

Greek lower case letters:

α – Angle; ratio;

β – Angle; ratio; coefficient;

γ – Partial factor;

γ_A – Partial factor for accidential actions, A;

γ_C – Partial factor for concrete;

γ_F – Partial factor for actions, F;

$\gamma_{F,fat}$ – Partial factor for fartigue actions;

$\gamma_{C,fat}$ – Partial factor for fartigue of concrete;

γ_G – Partial factor for permanent actions, G;

γ_M – Partial factor for a material property, taking account of uncertainties in the material property itself, in geometric deviation and in the design model used;

γ_P – Partial factor for actions associated with prestressing, P;

γ_Q – Partial factor for variable actions, Q;

γ_S – Partial factor for reinforcing or prestressing steel;

γ_f – Partial factor for actions without taking account of model uncertainties;

γ_g – Partial factor for permanent actions without taking account of model uncertainties;

γ_m – Partial factor for a material property, taking account only of uncertainties in the material property;

δ – Increment/redistribution ratio;

ζ – Reduction factor/distribution coefficient;

ε_c – Compressive strain in the concrete;

ε_{c1} – Compressive strain in the concrete at the peak stress f_c;

ε_{cu} – Ultimate compressive strain in the concrete;

ε_u – Strain of reinforcement or prestressing steel at maximum load;

ε_{uk} – Characteristic strain of reinforcement or prestressing steel at maximum load;

θ – Angle;

λ – Slenderness ratio;

μ – Coefficient of friction between between the tendons and their ducts;

ν – Poisson's ratio;

ν – Strength reduction factor for concrete cracked in shear;

ζ – Ratio of bond strength of prestressing and reinforcing steel;

ρ – Oven-dry density of concrete in kg/m^3;

ρ_{1000} – Value of relaxation loss (in %), at 1000 hours after tensioning and at a mean temperature of 20°C;

ρ_l – Reinforcement ratio for longitudinal reinforcement;

ρ_w – Reinforcement ratio for shear reinforcement;

σ_c – Compressive strength in the concrete;

σ_{cp} – Compressive strength in the concrete from axial load or prestressing;

σ_{cu} – Compressive strength in the concrete at the ultimate compressive strain ε_{cu};

τ – Torsional shear stress;

– Diameter of a reinforcing bar or of a prestressing duct;

– Equivalent diameter of a bundle of a reinforcing bars;

$\varphi(t,t_0)$ – Creep coefficient, defining creep between times t and t_0, related to elastic deformation at 28 days;

$\varphi(\infty,t_0)$ – Final value of creep coefficient;

ψ – Factors defining representative values of variable actions:

ψ_0 – for combination values;

ψ_1 – for frequent values;

ψ_2 – for quasi-permanent values.

INTRODUCTION

What is the difference between behaviour under the loading of plain concrete and reinforced concrete (RC) members?

In accordance with EN 206 (3.1) [N4] *«concrete is material formed by mixing cement, coarse and fine aggregate and water, with or without the incorporation of admixtures, additions or fibers, which develops its properties by hydration».*
As with most rocklike substances (concrete is an artificial rock), concrete has a high compressive strength and very low tensile strength.

Reinforced Concrete (RC), as a composite material, is a rational combination of concrete and steel, wherein the steel reinforcement provides the tensile strength lacking in the concrete. Steel reinforcing is also capable to resist compression forces and it is used in columns as well as in other structures.

Let's consider free supported beam loaded by two concentrated loads at the span. A beam with two concentrated loads applied at the third points, where the central third between two loads is subjected to constant moment only (pure bending), is shown in Figure I.1.

After initial loading the top half of the beam is subjected to compression and the bottom half is subjected to tension.

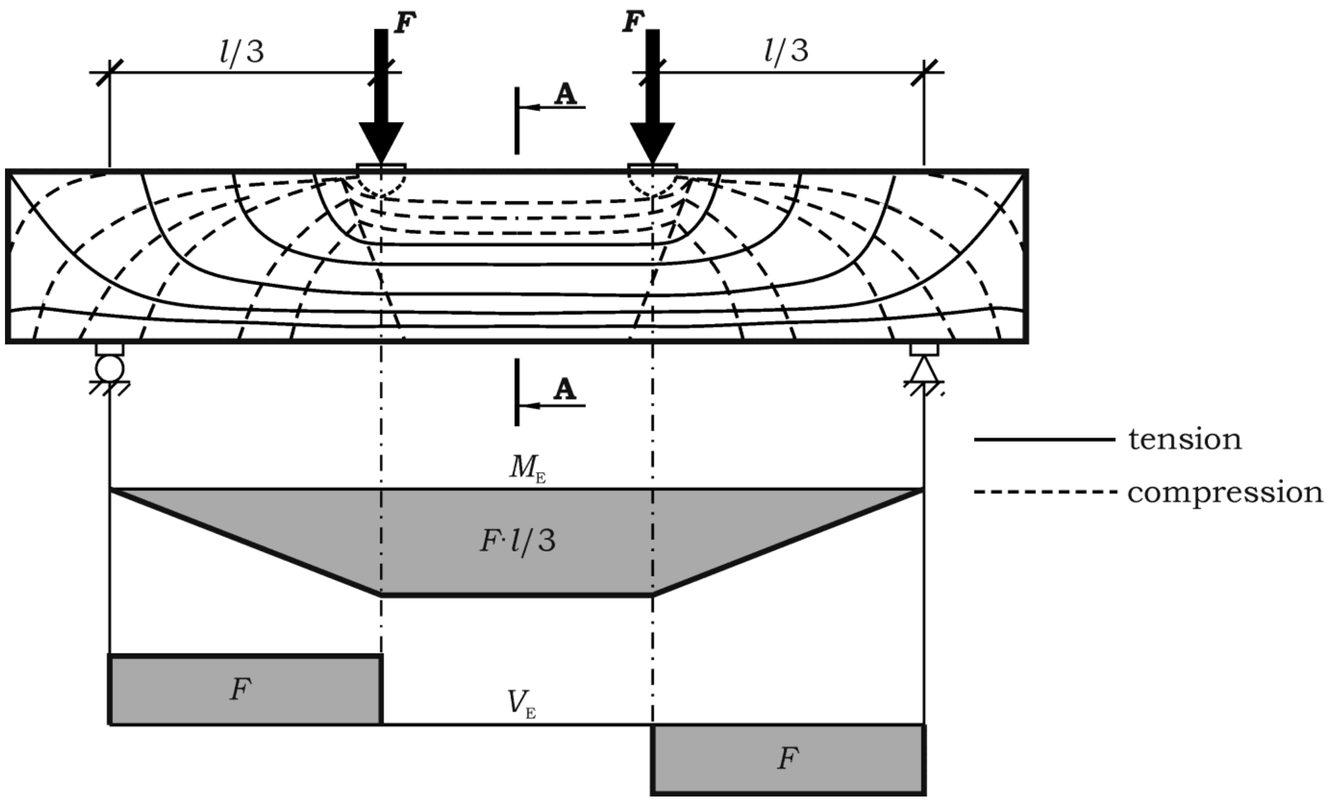


Figure I.1 – Free supported beam loaded by two concentrated loads at the span

There is a plane in the beam, which is not strained and this is known as the ***neutral surface***. The intersection of the neutral surface with a cross-section of the beam defines the ***neutral axis*** (fundamental assumption of bending theory is that plane sections remain plane).

Therefore, the ends of the beam remain plain under the action of the bending moment, resulting in a linear variation of strain with distance from neutral axis, as it shown in Figure I.2.

In addition, there is a linear stress distribution over the depth of the beam if the concrete in the tensile zone (under the neutral axis) works in linear-elastic stage. Before cracking, concrete strains and stresses in compression and tension zones increase with load increasing. In this stage concrete in uncracked sections will resist tension force, but it soon cracks, when strains of the concrete in tensile zone will exceed its ultimate tensile strains ε_{ctu}.

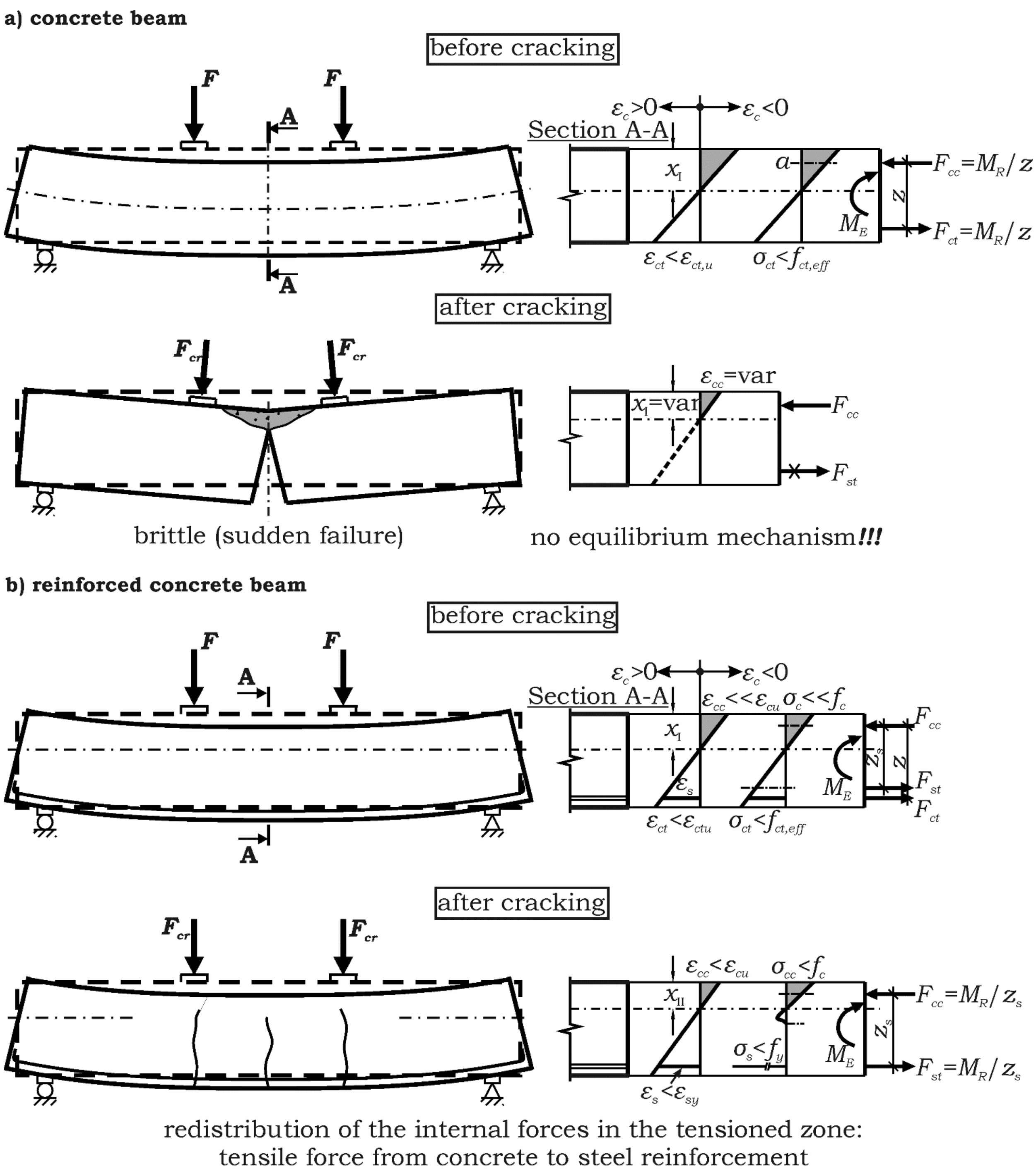
When tensile strain in concrete reaches its ultimate value ε_{ctu} at a particular section, cracking occurs at the applied load P_{crc} (see Figure I.2). First crack occurs at the weakest cross-section (it is equal probability of crack occurring at the length of the pure bending zone!).

When cracking first occurs the stress in the concrete at the section with crack (at the crack) drop to zero and concrete beam *fails suddenly* without any warning (*brittle mode of failure*) (see Figure I.2).

So, reinforcement is required to resist tension force due to the bending moment. When crack occurs in the reinforced beam (RC-beam), the concrete at the top of the section (over the neutral axis) resists compression and steel reinforcement resists tensile force at the cross-section with crack. The effective section resisting moment at cracked position is shown in Figure I.2.

At the conventionally low load (but, after cracking) the concrete in compression and steel in tension are in the elastic range.

The average strains and stresses in the reinforcement just before cracking depend on the amount of the steel reinforcement and bond conditions (between concrete and steel reinforcing bars).



a) – plain concrete beam; b) – reinforced concrete beam

Figure I.2 – Behaviour of the concrete beams under increasing load

The typical load-deflection curve for RC-beam is given in Figure I.3. The behaviour of the cracked section of RC-beam is elastic at low loads and changes to plastic at higher loads (near ultimate), as shown in Figure I.3. Increasing of the load causes displacement of the neutral surface (neutral axis in section) upward, which reduces the compression zone depth and deflection also increases. If the deflection increases without increasing of the load, the reinforced beam fails.

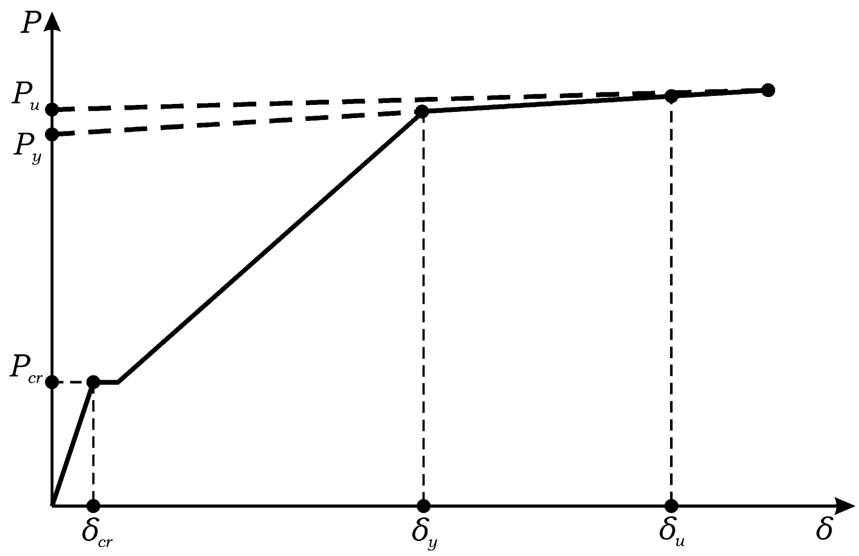


Figure I.3 – Typical load-deflection curve for the reinforced concrete beam

It should be noted, that there are two possible modes of failure of the reinforced concrete beam in the ultimate stage (so-called, "tension failure" and "compression failure"). Failure mode of the RC beam depends on the amount of steel reinforcement present in tension side (see Figure I.4).

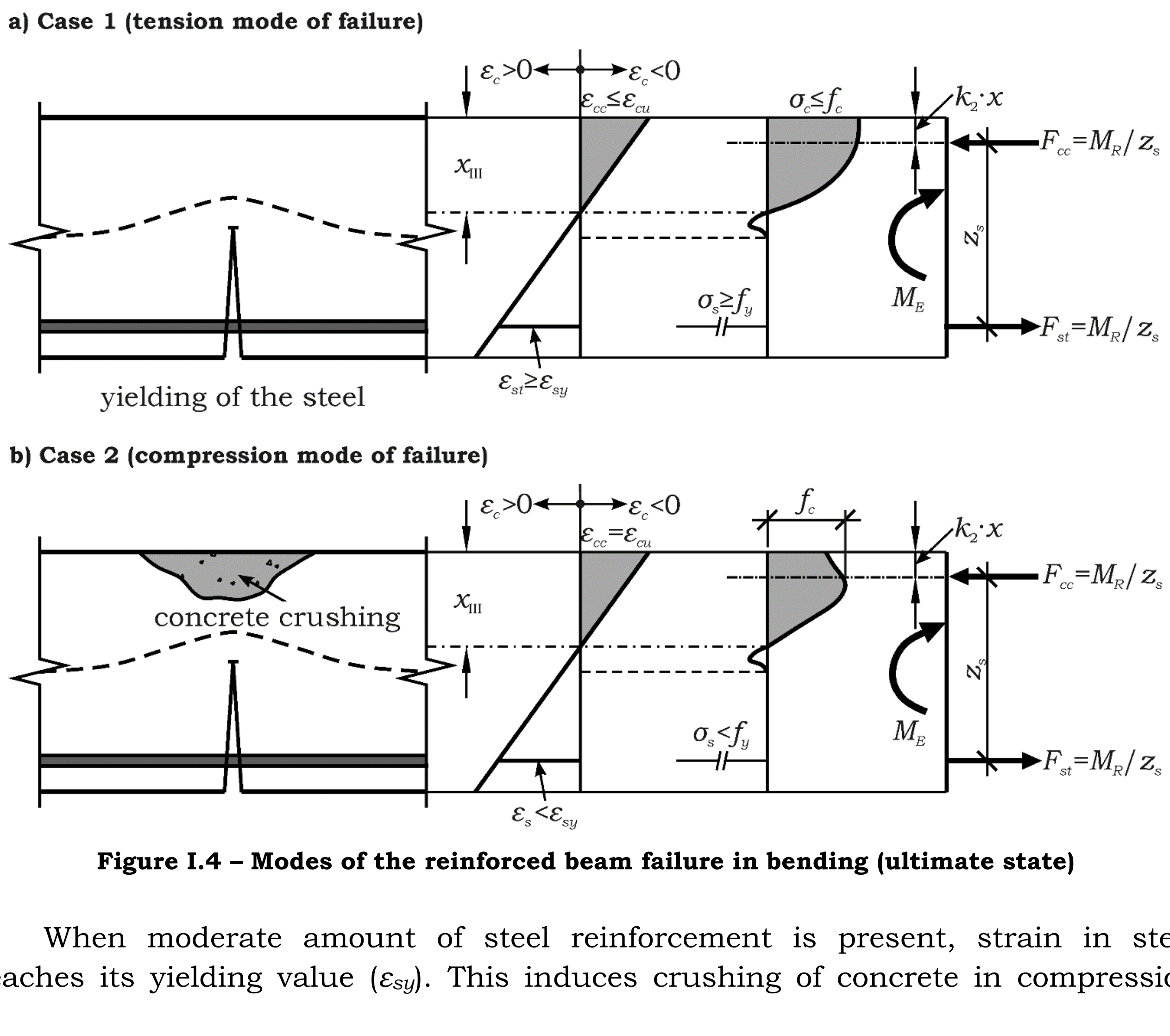


Figure I.4 – Modes of the reinforced beam failure in bending (ultimate state)

When moderate amount of steel reinforcement is present, strain in steel reaches its yielding value (ε_{sy}). This induces crushing of concrete in compression zone and called as "secondary compression failure" (see Figure I.4 a). This failure is gradual as it is preceded by visible signs of collapse. Such a critical sections of the beam are called ***«under reinforced»*** sections.

When amount of steel bars is large or very high strength steel is used, compressive strains in concrete reaches its ultimate value (ε_{cc}=ε_{cu}) before steel yields. Concrete fails by crushing and such a failure is sudden. This mode of failure is almost explosive and occurs without any warnings. Such a critical sections of the beam are called ***«over reinforced»*** sections.

As it was shown, the beams are generally reinforced in the tension zone. Such beams cross-sections are called as a ***«singly reinforced»***. Sometimes rebars are also provided in compression zone in addition to tension rebars to enhance the resistance capacity, then such beams cross-sections are called as a ***«doubly reinforced»***.

Advantages and disadvantages of reinforced concrete as a structural material

As it was shown in [15] reinforced concrete may be the most important material available for construction. It is used in one form or another for almost all of the structures, great or small – buildings, bridges, pavements, dams, retaining walls, tunnels, tanks, and so one.

The tremendous success of this universal material can be understood quite easily if its numerous ***advantages*** are considered. These include the following:

1) It has considerable compression strength per unit cost compared with most other materials;

2) Reinforced Concrete has excellent resistance to the actions of fire and water (in fact, it is the best structural material available for situation when water is presented);

3) Concrete is a low-maintenance material and as compared with other materials, it has a very long service life. Under the proper conditions, reinforced concrete structures cad be used without reduction of their load-carrying capacity;

4) A special feature of concrete is its ability to be cast into an extraordinary variety of shapes: from simple slabs, beams and columns to the great arches and shells;

5) A lower grade of skilled labour is required for erection in comparison with other materials as, for example, steel.

Among reinforced concrete ***disadvantages*** are the following:

1) Concrete has a very low tensile strength;

2) Forms are required to hold the concrete in place until it hardens sufficiently. Formwork is very expansive;

3) The low tensile strength per unit of weight of concrete leads to heavy members, and similarly, the low tensile strength per unit of concrete volume means members will be relatively large. It is an important consideration for tall buildings and long-span structures;

4) The properties of concrete vary widely because of variations in its proportioning and mixing;

5) Concrete demonstrates shrinkage and creep.

Compatibility of concrete and steel

Concrete and steel reinforcement work together in reinforced concrete structures. It should be pointed, that the advantages of each material seem to compensate for disadvantages of the other. The following main causes are determined compatibility of concrete and steel in structure:

1) The excellent bond-slip conditions between steel and surrounding concrete. The two materials bond together as a unit in resisting forces. The excellent bond strength obtained is the result of the chemical adhesion between steel and concrete (cement matrix), the natural roughness of the bars, and the closely spaced rib-shaped deformations rolled onto the bars surface;

2) Excellent protection properties of the fresh and hardened concrete for the steel reinforcement. Reinforcing steel bars are subjected to corrosion, but concrete surrounding them provides them with excellent protection ($pH \geq 13,5$). The strength of exposed steel subjected to the temperatures reached in the fires of ordinary intensity is nil, but enclosing the reinforcing steel in concrete produces very satisfactory fire rating [15];

3) The same value of coefficients of the thermal expansion. Concrete and steel work well together in relation to temperature changes because its coefficients of thermal expansion are quite close (near $10 \cdot 10^{-6}$ K^{-1}).

CHAPTER 1

BASIS OF STRUCTURAL DESIGN

1.1 LIMIT STATE DESIGN BASICAL REQUIREMENTS.

By the fundamental (basical) principle given by EN 1990 [N1] (clause 2.1(P)) a structure shall be designed and executed in such way that it will during its intended life, with appropriate degrees of reliability and in an economical way:

- sustain all actions and influences likely to occur during execution and use, and
- meet the specified serviceability requirements for a structure, or a structural element.

In accordance with this basic principle, the next fundamental requirements can be summarized. A structure shall be designed:

1) to have adequate structural resistance, serviceability and durability;
2) in the event of fire, to have an adequate resistance for the required period of the fire exposure;
3) to have an adequate robustness and not be damaged by accidents (e.g. explosion, impact, consequences of human errors to an extent disproportionate to the original cause.

How it was shown in [3, 8] given the random nature of the basic variables involved in structural design (actions, geometry, strength of materials, etc.), the assessment of structural reliability cannot to be set up by deterministic methods, but requires a probabilistic analysis.

The objective of safety verification is therefore to estimate failure probability, i.e. probability that a certain danger conditions is attained or exceeded, below a fixed value. This value is determined as a function of type of construction, influence on safety of people and damage to goods.

Every situation which is dangerous for a construction is referred as a ***limit state***.

In accordance with EN 1990 [N1] (clause 1.5.2.12) ***limit states*** are defined as states beyond which the structure no longer fulfill the relevant design criteria (***design criteria*** are defined as a quantitative formulations that describe for each limit state conditions to be fulfilled)

Once a construction has exceeded this conditions, it is no longer able to fulfill the functions for which it has been designed.

Limit states are of two types: ***Ultimate Limit States (ULS)*** and ***Serviceability Limit States (SLS)***.

Ultimate Limit States (ULS) are those associated with collapse or failure, and generally govern the strength of the structure or components. They also include loss of equilibrium or stability of the structure as a whole. As the structure will undergo severe deformation prior to reaching collapse conditions (e.g. beams becoming catenaries), for simplicity these states are also prepared, as ultimate limit states, although this condition is between serviceability and ultimate limit states; these states are equivalent to collapse, as they will necessitate replacement of the structure or element EN 1990 [N1].

Serviceability Limit States (SLS) generally correspond to conditions of the structure in use (exploitation). They include deformation, cracking and vibrations, which:

1) damage the structure or non-structure elements (finishes, partitions, etc.) or the contents of buildings (for example, such as machinery);

2) cause discomfort to occupants of buildings;

3) affect adversely appearance, durability or water and weather tightness;

They will generally govern the stiffness of the structure and the detailing of reinforcement within it. Figure 1.1-1 illustrates a typical load-deflection (deformation) relationship of RC structures and the limit states.

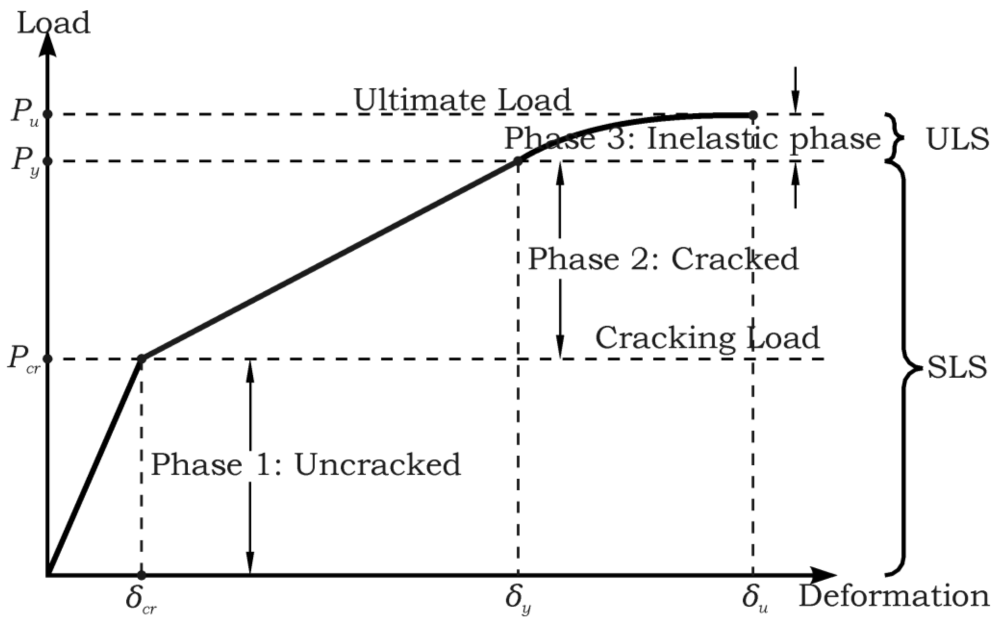


Fig. 1.1-1 – Typical load relationship of RC-element and the limit states

1.2 DEFINITION OF RELIABILITY

As it was shown in [8] a number of definitions of the term «***reliability***» are used in literature and in national and international documents. ISO 2394 [N6] provides a definition of reliability, which is similar to the approach of national standards used in some European countries:

Structural reliability is the ability of structure or a structural member to fulfill the specified requirements for which it has been designed, it includes structural safety, serviceability and robustness.

In quantitative sense reliability may be defined as the complement of the probability of failure.

Note that the above definition of reliability includes four important elements:

1) *given (performance) requirements* – definition of the structural failure;
2) *time period* – assessment of the required service-life *T*;
3) *reliability level* – assessment of the probability of failure p_f;
4) *conditions of use* – limiting input uncertainties.

An accurate determination of performance requirements and an accurate specification of the term failure are of uttermost importance. In many causes, when considering the requirements for stability and collapse of a structure, the specification of the failure is not very complicated. In many other causes, in particular when dealing with various requirements of occupants comfort, appearance and characteristics of the environment, the appropriate definition of failure are dependent of several vaguenesses and inaccuracies. The transformation of these occupants requirements into appropriate technical quantities and precise criteria is very hard and often leads to considerably different conditions [8].

In the following the term failure is being used in a very general sense denoting simply any undesirable state of structure (e.g. collapse or excessive deformation), which is unambiguously given by structural conditions.

The same definition as in ISO 2394 is provided in EN 1990 [N1] including note that the reliability covers the load-bearing capacity, serviceability as well as the durability of structure. Fundamental requirements include the statement (as already mentioned) that «*a structure shall be designed and executed in such way that it will with appropriate degrees of reliability sustain all actions and influences that are likely to occur during execution and use, and remain fit for the intended use*».

Generally a different level of reliability for load-bearing capacity and for serviceability may be accepted for a structures or its parts. In documents [N1, N6] the probability of failure p_f (and reliability index β) are indicated with regard to failure consequences (see Table 1.2-1).

Table 1.2-1 – Definition of consequences classes (Table B1 from EN 1990 [N1])

Consequences Class	**Description**	**Examples of buildings and civil engineering works**
CC3	**High** consequence for loss of human life, or economic, social or environmental consequences **very great**	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequence **considerable**	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	**Low** consequence for loss of human life, economic, social or environmental consequence **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

Table 1.2-1 gives the recommended minimum values for the reliability index associated with reliability classes.

Three reliability classes RC1, RC2 and RC3 may be associated with the three consequences classes CC1, CC2 and CC3 (see Table 1.2-2).

Table 1.2-2 – Recommended minimum values for reliability index *β*, ultimate limit state (Table B2 from EN 1990 [N1])

Reliability Class	**Minimum values for β**	
	1 year reference period	**50 years reference period**
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

The fundamental task of the theory of structure reliability concerns a basic requirements for the relation between the action effect (*E*) and structural resistance (*R*) within in form of inequality:

$$E < R \text{ or } E - R < 0. \tag{1.2-1}$$

Both variables E and R are generally random variables and the validity of Inequality (1.2-1) cannot be guaranteed absolutely, i.e. with the probability equal to 1 (the total certainly). Therefore it is necessary to accept the fact that the limit state described by equation may be exceeded and failure may be occur with certain very small probability.

The most important term used above (and in the theory of structure reliability) is evidently the probability of failure p_f.

In order to define p_f properly it is assumed that structural behavior may be described by a set of basic variables $\boldsymbol{X} = [X_1, X_2, \ldots, X_n]$ characterizing action, mechanical properties, geometrical data and model uncertainties. Furthermore, it is assumed that the limit state (ultimate, serviceability, durability or fatigue) of a structure is defined by the ***Limit State Function*** (or the performance function in accordance with EN 1990 [N1]), usually written in an implicit form:

$$G(\boldsymbol{X}) = 0. \tag{1.2-2}$$

The ***Limit State function*** $\boldsymbol{G(X)}$ should be defined in such way that for a favorable (safe) state of structure the function is positive, $G(\boldsymbol{X}) \geq 0$, and for unfavorable state (failure) of the structure the limit state function is negative, $G(\boldsymbol{X}) < 0$.

For most limit states (ultimate, serviceability, durability and fatigue) the probability of failure can be expressed as:

$$p_f = Prob(G(\boldsymbol{X}) < 0). \tag{1.2-3}$$

The failure probability p_f can be assessed if basic variables $\boldsymbol{X} = [X_1, X_2, ..., X_n]$ are described by appropriate probabilistic (numerical and analytical) models. Assuming that the basic variables $\boldsymbol{X} = [X_1, X_2, ..., X_n]$ are described by time independent joint probability density function $f_x(x)$ then the probability p_f can be determined using the integral:

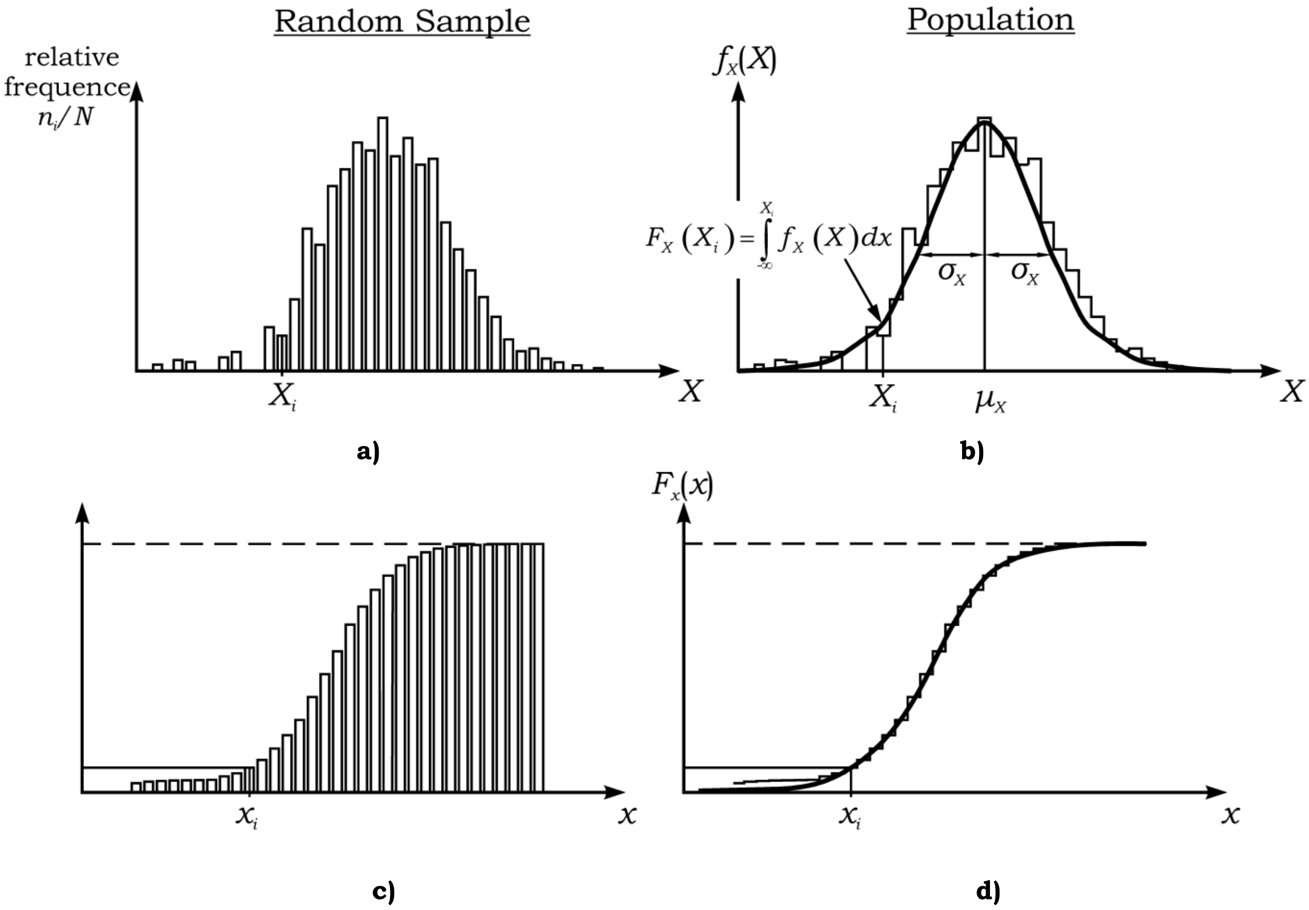
$$\begin{gathered} p_f = \int f_x(x) dx; \\ G(\boldsymbol{X}) < 0. \end{gathered} \tag{1.2-4}$$

More complicated procedures needs to be used when some of the basic variables are time-dependent. However, in many cases the problem may be transformed to a time-independent one for example by considering in Equation (1.2-3) or in Equation (1.2-4) a minimum of the function $G(\boldsymbol{X})$ over the reference period T.

For the simple condition in the form of Inequality (1.2-1), the probability of failure may be formally written as follows:

$$p_f = Prob(E > R). \tag{1.2-5}$$

The random character of the action effect E and the resistance R, both expressed in terms of suitable variable (performance indicator) $\boldsymbol{X}$ (i.e. stress, force, bending moment, deflection) is usually described by appropriate distribution function, i.e. by distribution functions $F_E(X)$, $F_R(X)$ and corresponding probability density functions $f_E(x)$, $f_R(x)$, where x denotes a general point of the considerable variable X used to express both the variables E and R (see Figure 1.2-1). Distribution of variables E and R further depend on appropriate parameters, e.g. moment parameters μ_E, σ_E, μ_R, σ_R, and ω_R.



a) – histogram; b) – probability density function; c) – cumulative frequence;
d) – probability distribution function

Figure 1.2-1 – Discrete empirical distribution of a random sample and probability functions for a random variable population

Let us further assume that E and R are mutually independent (which may be provided by appropriate transformation).

Figure 1.2-2 shows an example of probability density functions of both the variables E and R and their mutual location.

Note, that the probability density functions $f_E(x)$, $f_R(x)$ shown in Figure 1.2-2 overlap each other and therefore it is clear that unfavorable realization of variables E and R, denoted by small letters e and r, may occur in such way that $e > r$, i.e. the load effect is greater than the resistance and failure will occur.

Obviously in order to keep the failure probability $p_f = Prob(E > R)$ within an acceptable limits, the parameters of variables E and R must satisfy certain conditions (concerning the mutual position and variances of both distributions) depending on the types of distribution.

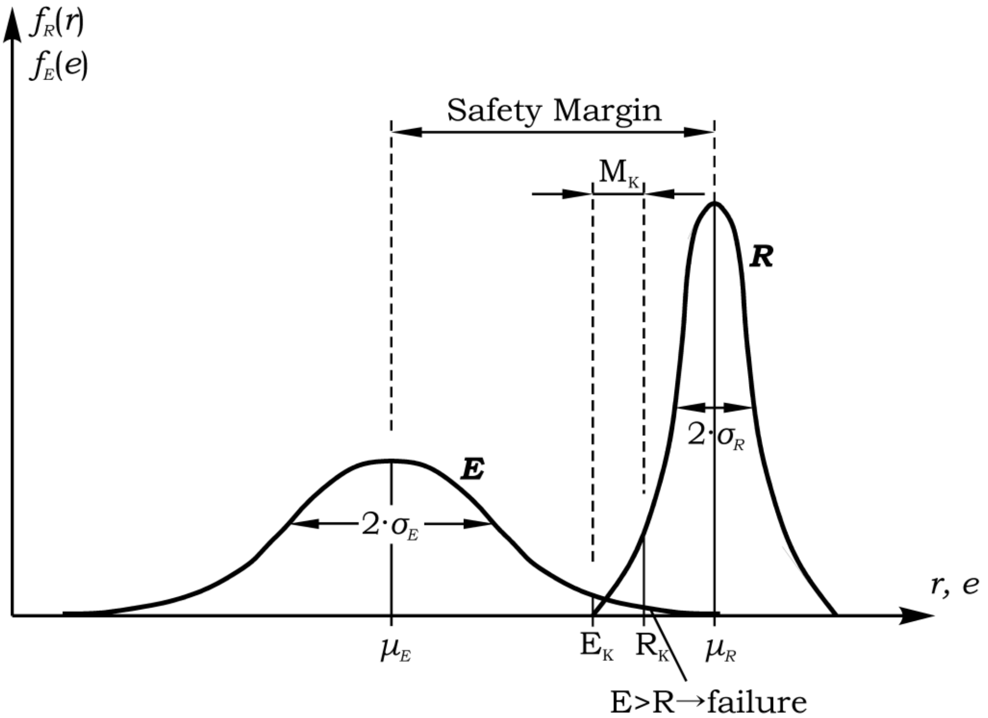
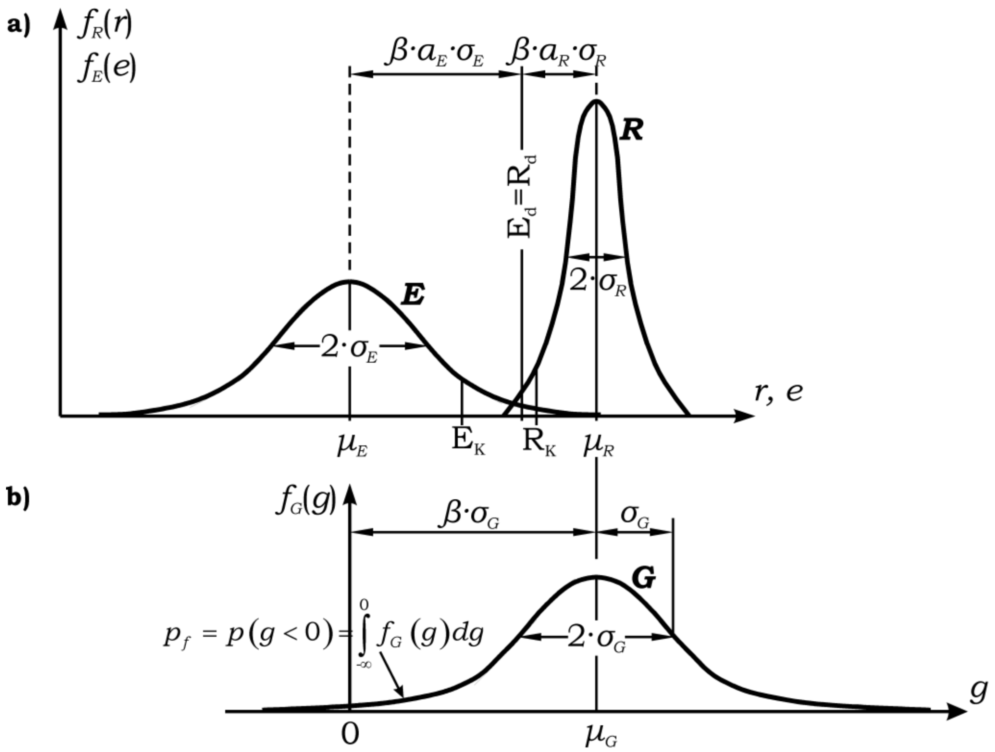


Figure 1.2-2 – Probability density functions for action effects (*E*) and resistance (*R*)

In Figure 1.2-3: p_f represents the probability that failure arises, i.e. that considered limit state is attained or exceeded at least once during *T*.



a) – probability density function; b) – limit state performance function

Figure 1.2-3 – Probability density functions (*E*, *R*) and limit state (perfomance) function

This analysis (known as Level 3 method) is very complex. Because of the difficulty of calculation and of the limitation of available data (data which often fail to give the probabilistic distributions necessary for calculation), this method is of limit applicability to the design practice.

Alternatively, if only the first and second moments (averages and standard deviations) of the random variables *R* and *E*, but not their statistical distributions, are known, the probability of failure can be estimated based on a β index, called «***the reliability index***».

Assume that both basic variables, the action effect *E* and resistance *R* are random variables. A simple solution can be obtained assuming a normal distribution for both *E* and *R*. Then also the difference:

$$G = R - E, \tag{1.2-6}$$

called the safety margin, has a normal distribution with parameters:

$$\mu_G = \mu_R - \mu_E, \tag{1.2-7}$$

$$\sigma_G^2 = \sigma_R^2 + \sigma_E^2 + 2 \cdot \rho_{RE} \cdot \sigma_R^2 \cdot \sigma_E^2, \tag{1.2-8}$$

where: ρ_{RE} is the coefficient of correlation of *R* and *E.*

Assuming that *G* is linear, it was first defined by Cornell as the ratio between the average value μ_G *of* *G* and its standard deviation σ_G:

$$\beta = \frac{\mu_G}{\sigma_G}, \tag{1.2-9}$$

In circumstance where *R* and *E* are not correlated (note, that in case of normal distributions non-correlation is equivalent to statistical independence $\rho_{RE} = 0$), accordance with [N1, N6, 8] reliability index β expressed as follows:

$$\beta = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}}, \tag{1.2-10}$$

where: μ_E, σ_E, μ_G, σ_R are the averages (means) and standard deviations of *R* and *E* respectively.

As was pointed in [N6, 8] this method (known as Level 2 method or «β-method») does not generally allow assessment of the probability of failure, with the exception of the particular case where relation between μ_G and the random variables of the problem is linear and the variables have normal distribution. The probability of failure, i.e. probability that the safety margin *G* assumes non-positive values, is given by the distribution function $f_G(x)$ of *G* calculated in point 0:

$$p_f = Prob(G \le 0) = \Phi_G(0). \tag{1.2-11}$$

Introducing u_0 as the normalized variable of safety margin *G*:

$$u_0 = \frac{G - \mu_G}{\sigma_G}, \tag{1.2-12}$$

the result:

$$G = \mu_G + \sigma_G \cdot u_0. \tag{1.2-13}$$

Substituting Equation (1.2-13) in the Equation (1.2-11) of p_f gives:

$$\begin{aligned} p_f &= Prob(G \le 0) = Prob(\mu_G + \sigma_G \cdot u_0 \le 0) = Prob\left(u_0 \le -\frac{\mu_G}{\sigma_G}\right) = \\ &= Prob(u_0 \le -\beta_0) = \Phi_u(-\beta) = 1 - \Phi_u(\beta), \end{aligned} \tag{1.2-14}$$

where: Φ_u indicates the distribution function *u*. The relation between β and p_f is given in Table 1.2-3.

Table 1.2-3 – Relation between β and p_f

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1,28	2,32	3,09	3,27	4,27	4,75	5,20

The probability density function $f_G(g)$ of the safety margin *G* is shown in Figure 1.2-3, where the grey area under curve $f_G(g)$ corresponds to the failure probability p_f.

As it was shown in [8] the Level 2 method is also difficult to apply in practical design because the necessary data are often not available, so that another method is used: ***the partial factor method or semi-probabilistic method*** (*Level 1 method*).

1.3 PARTIAL FACTORS METHOD

1.3.1 GENERAL

In accordance with EN 1990 [N1] (clause 6.1(1)P) when using the partial factor method, «*it shall be verified that, in the relevant design situations, no relevant limit state is exceeded when design values for actions or effects of actions and resistances are used in the design models*».

Design values should be obtained by using:

- the characteristic, or
- other representative values,

in combination with partial and other factors as defined in EN 1990 [N1], EN 1991 [N2] and EN 1992 [N3].

This method is based on the compliance with a set of rules that ensure the requirement reliability of the structure by using *characteristic values* of the problem variables and a series of *safety elements*. These are represented by partial safety factors «γ_i» which cover the uncertainties in actions and materials, and by additional elements «Δ» for uncertainties in geometry, e.g. to allow for the randomness of cover to reinforcement and therefore of the effective depth of a reinforced concrete section.

The method does not require that the designer has the any probabilistic knowledge, because the probabilistic aspects of the question of the safety are already taken into account in the method calibration process, i.e. in the choice of characteristic value, partial factors, etc., fixed in standards.

The ***partial factors method*** is based on the following ***assumptions***:

1) Resistance (strength) and action effects (stress) are independent random variables;

2) Characteristic values of resistance (strength) and action or action effect (stress) are fixed as fractiles of given order of the respective distributions, on the basis of a given probability;

3) Other uncertainties are taken into account by transforming characteristic values into design values, by applying partial factors and additional elements for reliability;

4) The assessment of safety is possible if the design action (action effects) don't exceed the design resistance.

1.3.2 PRINCIPLES OF LIMIT STATE DESIGN, DESIGN SITUATIONS

Eurocodes adopt the partial factors method, or limit state semi-probabilistic method, as the method for the verification of structure safety.

Design for limit states shall be based on the use of structural and actions models for relevant limit states. It shall be verified that no limit state is exceeded when relevant design value for actions, materials and product properties and geometrical data are used in these models EN 1990 [N1] (clause 6.1(1)P).

The verifications shall be carried out for all relevant design situation and load cases in accordance with EN 1990 [N1].

The situations chosen for design shall cover all situations that can reasonably occurs during the execution and working life of the structure.

In common cases, design situation in EN 1990 [N1] are classified as:

- ***persistent design situation,*** which refer to the conditions of normal use;
- ***transient design situation,*** which refer to temporary conditions applicable to the structure, e.g. during execution or repair;
- ***accidental design situation,*** involving exceptional conditions of the structure or its exposure, including fire, explosion, impact, etc;
- ***seismic design situation,*** which refer to conditions applicable to the structure when subjected to seismic events.

As it is stated in EN 1990 [N1]: «*the selected design situation shall be sufficiently severe and varied so as to encompass all conditions that can reasonably be foreseen to occur during the execution and use of the structure*».

1.3.3 BASIC VARIABLES

1.3.3.1 Actions and environmental influences

As it was pointed in EN 1990 [N1], each design situation is characterized by the presence of several types of actions on the service.

Actions means, as EN 1990 [N1] states, either a set of forces (loads) applied to the structure (*direct actions*), or a set of imposed deformation or accelerations caused, for example, by temperature changes, moisture variation, an even settlement or earthquake (*indirect actions*).

In accordance with EN 1990 [N1], actions are classified as:

- ***permanent actions (G),*** the duration of which is continuous and equal to the design working life of the structure, or for which the variation in magnitude with time is negligible (e.g. self-weight of structures, fixed equipment and road surfacing and indirect actions caused by shrinkage and uneven settlements). Those actions, like prestressing or concrete shrinkage, for which the variation is always in the same direction (monotonic) until the action attains a certain limit value, are also permanent actions;

- ***variable actions (Q),*** divided in variable actions with discrete and regular occurrence in time (e.g. imposed load of people and low-duration imposed load in general on building floor); and variable actions characterized by variable and non-monotonic intensity or direction (e.g. snow, wind, temperature, etc.);
- ***accidental actions (A),*** which are not easily foreseeable and of low duration (e.g. explosions, impacts, fire, etc.). Certain actions, such as seismic actions and snow loads, may be considered as either accidental and/or variable actions, depending on the site location.

Actions caused by water may be considered as permanent and/or variable actions depending on the variation of their magnitude with time.

Additionally, actions shall also be classified:

- by their origin, as ***direct*** or ***indirect***;
- by their spatial variation, as ***fixed*** or ***free***;
- by their nature and/or the structural response, as ***static*** or ***dynamic***.

An action should be described by a model, its magnitude being represented in the most common cases by one scalar which may have several representative values (for some actions and some verifications, a more complex representation of the magnitudes of some actions may be necessary):

$$F_{rep} = \psi \cdot F_k. \tag{1.3-1}$$

The characteristic value F_k of an action in accordance with EN 1990 [N1] (clause p.4.1.2(1)P) representative value and shall be specified:

- as a mean value, an upper or lower value, or a nominal value (which does not refer to a known statistical distribution) (see EN 1991 [N2]);
- in the project documentation, provided that consistency is achieved with methods given in EN 1991 [N2].

Each ***permanent action*** with low variability has a single characteristic value G_k. This is the case of actions due to self-weight: they are generally represented through a nominal value calculated on the basis of the design drawings (structural and non-structural member dimensions) and of the average specific gravity of materials ($G_k = G_m$).

If permanent action has relevant uncertainties (coefficient of variation bigger than 10 %, where the coefficient of variation is the ratio between standard deviation and mean value) and if sufficient statistical information is available, two characteristic values (upper, $G_{k,sup}$ and lower, $G_{k,inf}$) should be used. Value $G_{k,sup}$ is the 95 % fractile and $G_{k,inf}$ is the 5 % fractile of the statistical distribution for *G*, which may be assumed to be Normal (Gaussian). There is a 5 % probability that these two values will be exceeded, the probability that the real value of action is more than $G_{k,sup}$ or less than $G_{k,inf}$ is less than 5 % (see Figure 1.3-1).

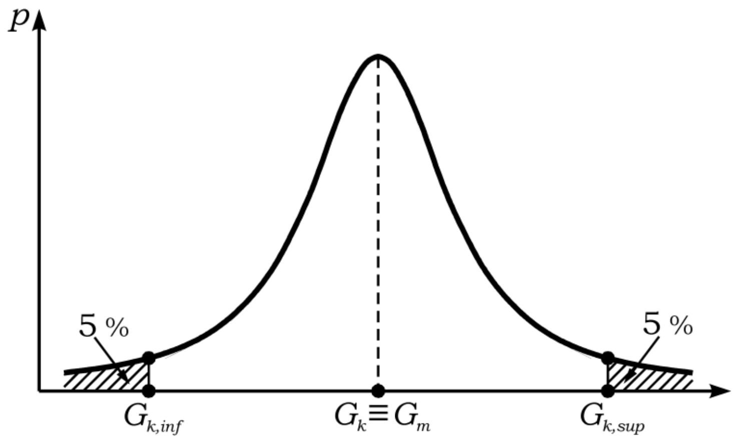


Figure 1.3-1 – Characteristic value of a permanent action.
$G_k \equiv G_m$, if the coefficient of variation is small (negligible); a lower and upper characteristic value of $G_{k,inf}$ and $G_{k,sup}$ are defined if the coefficient of variation is high

For ***variable actions,*** the characteristic value (Q_k) in accordance with EN 1990 [N1] (clause 4.1.2(7)P) shall correspond to either:

• an upper value with an intended probability of not being exceeded or a lower value with an intended probability of being achieved, during some specific reference period *T*;

• a nominal value, which may be specified in cases where a statistical distribution is not known.

The characteristic value of climatic actions is based upon the probability of 0,02 of its time-varying part being exceeded for a reference period of one year. This is equivalent to a mean return period of 50 years for the time-varying part. However in some cases the character of the action and/or the selected design situation makes another fractile and/or return period more appropriate.

Each variable action has four representative values. As it was shown, the main representative value of a variable action is characteristic value Q_k*;* the other representative values are (in decreasing order):

a) ***the combination value,*** represented as a product ψ_0 and Q_k, used for the verification of ultimate limit states and irreversible serviceability limit states;

b) ***the frequent value,*** represented as a product ψ_1 and Q_k, used for the verification of ultimate limit states involving accidental actions and for verifications of reversible serviceability limit states;

c) ***the quasi-permanent value,*** represented as product ψ_2 and Q_k, used for the verification of ultimate limit states involving accidental actions and for the verification of reversible serviceability limit states. Quasi-permanent values are also used for the calculation of long-term effects.

For simplicity, each of these last three values is defined as a fraction of the characteristic value, obtained by applying a reducing factor ψ_i to Q_k. In reality, the frequent value and quasi-permanent value are inherent properties of the variable action, and the ψ_1 and ψ_2 factors are simply the ratios between these values and the characteristic value. On the other hand, the ψ_0 factor, called the combination factor, determines the level of intensity of a variable action when this action is taken into account, in design, simultaneously with another variable action, called ***leading variable action***, which is taken into account by its characteristic value.

The ψ_0 factor takes therefore into account the low probability of simultaneous occurrence of the most unfavorable values of independent variable actions. It is used both for ULS verifications and for irreversible SLS verifications.

The frequent ($\psi_1 \cdot Q_k$) and the quasi-permanent ($\psi_2 \cdot Q_k$) values are used for ULS verification including accidental actions and for reversible serviceability limit states.

For building, for example, the frequent value ($\psi_1 \cdot Q_k$) is chosen so that the time it is exceeded is 0,01 of the reference period. For loads on building floors, the quasi-permanent value ($\psi_2 \cdot Q_k$) is usually chosen so that the proportion of the time it is exceeded is 0,5 of the reference period. The quasi-permanent value can alternatively be determined as the value averaged over a chosen period of time. In the case of wind actions or road traffic loads, the quasi-permanent value is generally taken as zero. Theoretical background for ψ_i coefficients assessment is presented in ISO 2394 [N6].

Figure 1.3-2 resumes the representative values of variable actions.

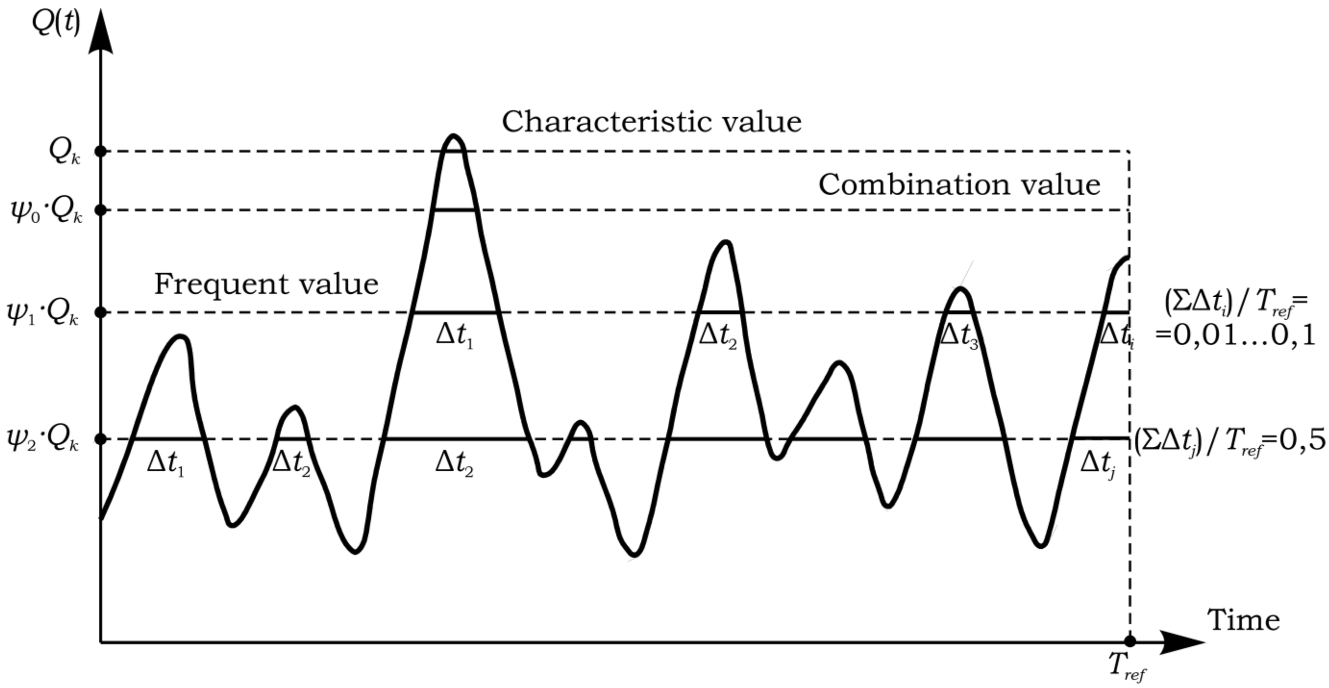


Figure 1.3-2 – Schematic illustration of representative values of the variable actions

Values of ψ_i factors for buildings are defined in National Annexes to EN 1990 [N1]. Table 1.3-1 shows the values of ψ_i factors recommended by EN 1990 [N1].

Table 1.3-1 – Recommended values of ψ_i factors for buildings (Table (A1.1) from EN1990 [N1])

Imposed loads in buildings, category (see EN1991 [N2])	ψ_0	ψ_1	ψ_2
Category A: domestic, residential areas	0,7	0,5	0,3
Category B: office areas	0,7	0,5	0,3
Category C: congregation areas	0,7	0,7	0,6
Category D: shopping areas	0,7	0,7	0,6
Category E: storage areas	1,0	0,9	0,8
Category F: traffic area, vehicle weight ≤30 kN	0,7	0,7	0,6
Category G: traffic area, 30 kN<vehicle weight ≤160kN	0,7	0,5	0,3
Category H: roofs	0	0	0
Snow loads on buildings (see EN1991-1-3)* in Fineland, Iceland, Norway, Sweden and other CEN Member, for sites located at altitude H>1000m a.s.l.	0,7	0,5	0,2
Other CEN Member States, for sites located at altitude H<1000m a.s.l.	0,5	0,2	0
Wind load on buildings (see EN 1991 [N2])	0,6	0,2	0
Temperature (non-fire) in buildings (see EN 1991 [N2])	0,6	0,5	0
Note: *The ψ values may be set by the National annex.* ** For countries not mentioned below, see relevant local conditions.*			

In order to take into account the uncertainties on the choice of characteristic values for actions and some uncertainties concerning the action modeling, design does not use characteristic values, but amplified values, called «***design values***», which are obtained by multiplying characteristic values by a partial factor (see Figure 1.3-3).

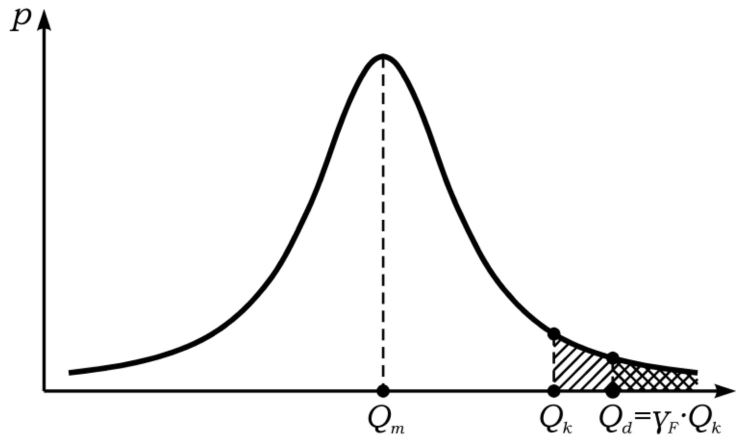


Figure 1.3-3 – Characteristic and design values of a variable actions

Symbols representing the design values are indicated with index «***d***». Table 1.3-2 shows the steps to pass from the representative values of actions to the design values of their effects on construction.

Table 1.3-2 – Procedure to determine the design values of effects on structures starting from the representative values of actions [3, 8]

Expression	Comment
F_i	Actions on the structure are identified
$F_{k,i}$ or $\psi \cdot F_{k,i}$, where ($\psi = \psi_0, \psi_1, \psi_2$).	Representative values are assigned to actions: characteristic values or other (combination, frequent, quasi-permanent) values.
$F_{d,i} = \gamma_{f,i} \cdot F_{k,i}$ (or $\gamma_{f,i} \cdot \psi \cdot F_{k,i}$), where ($\psi = \psi_0, \psi_1, \psi_2$).	Design values of actions are determined by multiplying the representative values $F_{k,i}$ or ψ $\psi \cdot F_{k,i}$ (where $\psi = \psi_0, \psi_1, \psi_2$) by a partial factor $\gamma_{f,i}$. $\gamma_{f,i}$ is a partial factor generally covering the uncertainties related to the choice of characteristic values for actions and, sometimes, part of the uncertainties related action modeling. In case of permanent actions, when it is necessary to split the action into a favorable and an unfavorable part, two different partial factors, indicated as $\gamma_{G,sup}$ and $\gamma_{G,inf}$, are used.
$E_d = E(\gamma_{f,i} \cdot \psi \cdot F_{k,i}; a_d)$	Actions that can occur simultaneously are considered; combinations of actions are calculated and the effects of these combinations on the structure are assessed (e.g. action effect in cross section). a_d represents either the design value of the set of geometrical data (in general, values indicated on the design drawings) or data that take into account the possibility of geometrical imperfection liable to cause second order effects.
$E_d = \gamma_{Ed} \cdot E(\psi \cdot F_{k,i}; a_d)$	The design value of effects is obtained by multiplying the values produced by the design actions, by a partial factor γ_{Ed} mainly covering the uncertainties of the structural model.
$E_d = E(\psi \cdot F_{k,i}; a_d)$	In normal cases, the previous expression is simplified in this one, where: $\gamma_{F,i} = f\left(\gamma_{Ed}, \gamma_{f,i}\right)$ so that the model coefficient γ_{Ed} does not explicitly appear. The product: $F_{d,i} = \gamma_{f,i} \cdot F_{k,i}$ or ($\gamma_{F,i} \cdot \psi \cdot F_{k,i}$; $\psi = \psi_0, \psi_1, \psi_2$) is often directly assumed as the design value of the action $F_{k,i}$.

As was shown in EN 1990 [N1], the values of actions to be used in design are governed by a number of factors. These include:

1) The nature of load. Whether the actions is permanent, variable or accidental, as the confidence in the description of each will vary.

2) The limit state being considered. Clearly the value of an action governing design must be higher for ultimate limit state than for serviceability for persistent and transient design situations. Realistic serviceability loads should be modeled appropriate to the aspect of the behavior being checked (e.g. deflection, cracking or settlement). For example, creep and settlement are function of permanent loads only.

3) The numbers of variable loads acting simultaneously. Statistically, it is improbable that all loads will act at their full characteristic value at the same time. To allow for this, the characteristic values of actions will be needed in modification.

Consider the case of permanent action (G_k) and one variable action (Q_k) only. For the ultimate limit state the characteristic value should be magnified, and the load may be represented as follows:

$$\gamma_G \cdot \sigma_k + \gamma_Q \cdot Q_k, \tag{1.3-2}$$

where: γ_i is a partial factor for actions.

The values of γ_G and γ_Q will be different, and will be a reflection of the variableness of the two loads being different. The γ_i factors are accounted for:

1) the possibility of unfavorable deviation of the loads from the characteristic values;

2) inaccuracies in the analysis;

3) unforeseen redistribution of stresses;

4) variation in the geometry of the structure and its elements, as this affects the determination of the action-effects.

Now let's consider the case of structure subjected to variable action $Q_{k,1}$ and $Q_{k,2}$ simultaneously. If $Q_{k,1}$ and $Q_{k,2}$ are independent, i.e. the occurrence and magnitude of $Q_{k,1}$ does not depend on the occurrence and magnitude of $Q_{k,2}$ and *vice-versa*, then it would be unrealistic to use combination $\gamma_{Q,1} \cdot G_{k,1} + \gamma_{Q,2} \cdot G_{k,2}$ as the two loads are unlikely to act at their maximum at the same time. Joint probabilities will need to be considered to ensure that the probability of occurrence of the two loads is the same as that of a single load.

It will be more realistic and reasonable to consider one load at its maximum in conjunction with a reduced value for the other load. This, we have two possibilities:

$$\gamma_{Q,1} \cdot Q_{k,1} + \psi_{0,2} \cdot \gamma_{Q,2} \cdot Q_{k,2}, \qquad (1.3\text{-}3)$$

or:

$$\psi_{0,1} \cdot \gamma_{Q,1} \cdot Q_{K,1} + \gamma_{Q,2} \cdot Q_{K,2} \qquad .3\text{-}4)$$

Multiplication by ψ_0 is said to produce a ***combination value of the load.*** It should be noted that the values of γ_i and ψ_i vary with each load. The method of deriving ψ_i values is outlined in the addenda to ISO 12394 [N6]. In practice, designer will not have sufficient information to vary the ψ-values in most cases. Table 1.3-1 summarizes the ψ_i values recommended by the EN1990 [N1].

1.3.3.2 Material and product properties

Properties of materials and products should be represented by characteristic values. In accordance with EN1990 [N1] (clause p.1.5.4.1) characteristic value defines as «*the value of material or product property (X_K or R_K) having a prescribed probability of not being attained in a hypothetical unlimited test series*». This value generally corresponds to a specified fractile of the assumed statistical distribution of the particular property of the material or product. When a limit state verification is sensitive to the variability of a material property, upper and lower characteristic values of the material property should be taken into account:

- where a low value of material or product property is unfavorable, the characteristic value should be defined as the 5 % fractile value (see Figure 1.3-4);
- where a high value of material or product property is unfavorable, the characteristic value should be defined as the 95 % fractile value (see Figure 1.3-4).

The structural stiffness parameters (e.g. modulus of elasticity, creep coefficient) and thermal expansion coefficient should be represented by a mean value. Different values should be used to take into account the duration of the load. For the structural stiffness parameters, the characteristic values is taken as a mean value, because depending on the case, these parameters can be favorable or unfavorable.

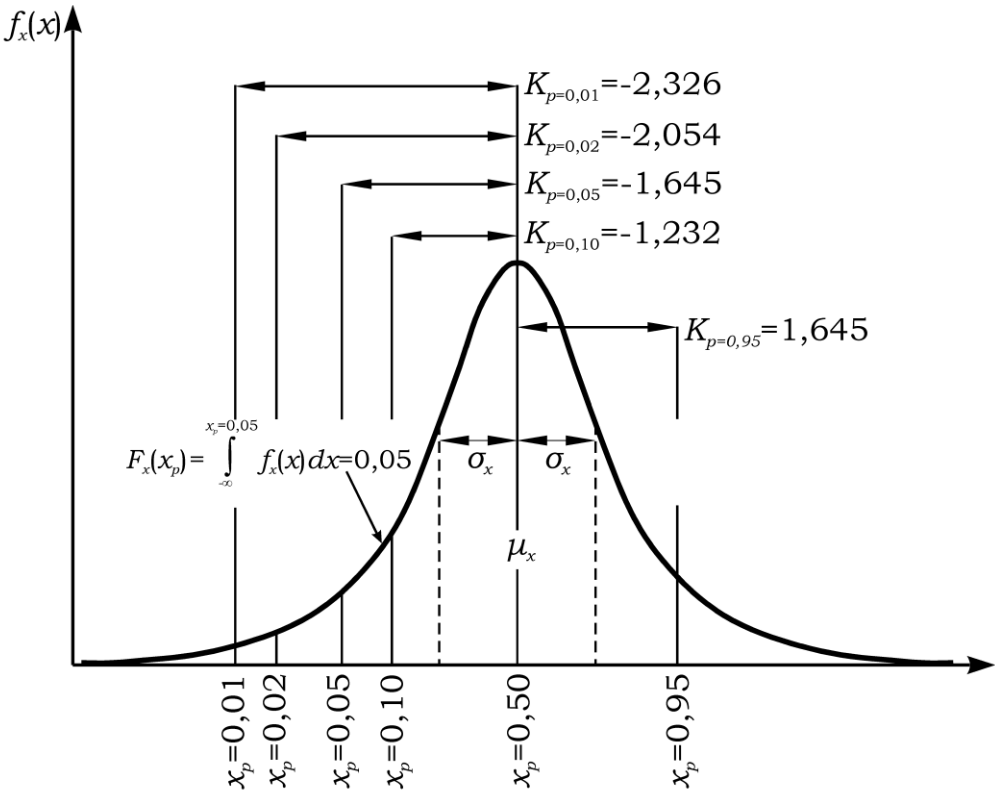


Figure 1.3-4 – Characteristic values (5 % and 95 % fractiles) for material properties

Product properties are also represented by a single characteristic value or a set of characteristic values, according to their constituent materials.

In order to account for the differences between the strength of the test specimens of the structural materials and their strength in-situ, the strength properties will be needed to be reduced. This is achieved by dividing the characteristic values by partial factors for materials (γ_M). Thus the design value $X_d = X_k / \gamma_M$. Uncertainties of the resistance models are also covered by γ_M. Although it is not stated in the code, γ_M also accounts for local weaknesses and inaccuracies in the assessment of resistance of the section.

Table 1.3-3 shows the steps to pass from characteristic values of individual material strengths (X_i) or product resistances (R) (as function of the strength) to the design values of structural resistance.

Table 1.3-4 gives the summarized values of partial factors to be assumed for concrete and reinforcement steel for ULS, in case of persistent, transient and accidental load combinations.

Table 1.3-3 – Procedure to determine the design values of resistances starting from the characteristic values of strength [3]

Expression	Comment
X_i	Material strengths and product resistances involved in the verifications are indentified.
$X_{k,i}$	Characteristic values of material strength and product resistances are introduced.
$X_{d,i} = \eta \cdot \dfrac{X_{k,i}}{\gamma_{m,i}}$	The design value of a material property is determined on the basis of its characteristic value, through the two following operations: a) divide by a partial factor γ_m, to take into account unfavorable uncertainties on characteristic of this property, as well as any local defaults; b) multiply, if applicable, by a conversion factor η mainly aimed at taking into account scale effects.
$R\left(\eta \cdot \dfrac{X_{k,i}}{\gamma_{m,i}}; a_d\right)$	Determine the structural resistance on the basis of design values of individual material properties and geometrical data.
$R_d = \dfrac{1}{\gamma_{Rd}} R\left(\eta \cdot \dfrac{X_{k,i}}{\gamma_{m,i}}; a_d\right)$	Following a procedure similar to the one for calculating the design value of action effects, the design value of structural resistance is determined on the basic of individual material properties and of geometrical data multiplied by a partial factor γ_{Rd} that covers the model uncertainties of resistance and the geometrical data variations, if these are not explicitly taken into account in the model.
$R_d = R\left(\eta \cdot \dfrac{X_{k,i}}{\gamma_{M,i}}; a_d\right)$	As for the action effects, factor γ_{Rd} is often integrated in the global safety factor $\gamma_{m,i}$, by which the characteristic material strength is divided: $\gamma_{m,i} = f\left(\gamma_{Rd}, \gamma_{m,i}\right)$.

Table 1.3-4 – Partial factors for concrete and steel for ultimate limit states (Table (2.1N) from EN 1992 [N3])

Design situation	**γ_c for concrete**	**γ_s for reinforcing steel**	**γ_s for prestressing steel**
Persistent and transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0
Note: *1. The values of partial factors in the table 1.3-4 were determined in accordance with Annex C from EN 1990 [N1]:* $\gamma_M = \exp\left(a_R \cdot \beta \cdot V_R - 1{,}64 \cdot V_f\right)$, *where:* $V_R = \sqrt{V_m^2 + V_G^2 + V_f^2}$, *and* V_R, V_G, V_m, V_f *are coefficients of variation of resistance, geometrical factor, model uncertainty, material strength.*			

For serviceability limit states values of partial factors γ_c and γ_s are equal 1,0 (recommended value for situations not covered by specific parts of EN1992 [N3]). Besides the partial factors for material above, EN1992 [N3] also defines partial factors for shrinkage, prestressing, fatigue loads and materials for foundations. The Table 1.3-5 gives the values recommended in EN1992 [N3].

Table 1.3-5 – Recommended values of partial safety factors [8]

Shrinkage (e.g. for ULS verification of stability when second order effects are relevant)		γ_{SH}	1,0
$\gamma_{P,unfav}$	Favorable in persistent and transient situation	$\gamma_{P,fav}$	1,0
	Stability ULS with external prestressing may be unfavorable	$\gamma_{P,unfav}$	1,3
	Local effects	$\gamma_{P,unfav}$	1,2
Fatigue		$\gamma_{F,fat}$	1,0
Materials for foundations (amplified in order to obtain the design resistance of cast in place piles without permanent scaffolding)		$\gamma_{C,fond}$	$1{,}1 \cdot \gamma_C$

The recommended values of the partial factors given in Table 1.3-5 can be changed in National Annex to EN 1992 [N3].

1.3.4 VERIFICATION OF THE LIMIT STATES BY PARTIAL FACTORS METHOD

1.3.4.1 Ultimate Limit State (ULS)

1.3.4.1.1 General

As it was shown, ***the Ultimate Limit States*** are associated with the loss of the equilibrium of the whole structure, or failure or excessive deformation of a structural member and they generally concern safety of people.

Table 1.3-6 shows the ULS classification according to EN 1990 [N1] (clause (6.4.1)).

Table 1.3-6 – ULS classification in accordance with EN 1990 [N1]

Notation	Definition
EQU	Loss of static equilibrium of the structure or any part of it considered as a rigid body, where: • minor variations in the value or the spatial distribution of the actions from a single source are significant (e.g. self-weight variation), and • the strengths of construction materials or ground are generally not governing.
STR	Internal failure or excessive deformation of the structure or structural members, including footing, piles, basement walls, etc., where the strength of construction materials of the structure governs.
GEO	Failure or excessive deformation of the ground where the strength of soil or rock are significant in providing resistance.
FAT	Fatigue failure of the structure or structural members.

Verification of static equilibrium: when considering a limit state of static equilibrium of the structure EQU (overall stability), it should be verified that the design effects of destabilizing actions are less than the design effects of stabilizing actions:

$$E_{d,dst} \le E_{d,stb}, \tag{1.3-5}$$

where: $E_{d,dst}$ is the design value of the effect of destabilizing;

$E_{d,stb}$ is the design value of the effect of stabilizing actions.

Where appropriate the expression for limit state of static equilibrium may be supplemented by additional terms, including, for example, a coefficient friction between rigid bodies.

When considering ***rupture or excessive deformation of a section, member or connections*** (STR), it should be verified that the design value of internal force or moment (E_d) is less than the design value of resistance (R_d):

$$E_d \le R_d, \tag{1.3-6}$$

where: E_d is the design value of the effect of actions such as internal force, moment or a vector representing several internal forces or moments;

R_d is the design value of the corresponding resistance.

1.3.4.1.2 Combination of actions for persistent or transient design situations

As it was shown below, EN1990 [N1] gives three separate sets of load combinations, namely ***EQU*** (to check against loss of equilibrium), ***STR*** (internal failure of structure governed by the strength of the construction materials) and ***GEO*** (failure of the ground, where the strength of soil provides the significant resistance).

(1) Equilibrium (EQU)

Equilibrium is verified using the load combination Set A in the Annex A from EN1990 [N1], which is follows:

$$\gamma_{G,j,sup} \cdot G_{k,j,sup} \text{"+"} \gamma_{G,j,inf} \cdot G_{k,j,inf} \text{"+"} \gamma_{Q,1} \cdot Q_{k,1} \text{"+"} \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k,i}, \tag{1.3-7}$$

where: $\gamma_{G,j,sup} \cdot G_{k,j,sup}$ is used when the permanent loads are unfavorable;

$\gamma_{G,j,inf} \cdot G_{k,j,inf}$ is used when the permanent actions are favorable.

Numerically, $\gamma_{G,j,sup} = 1{,}1$, $\gamma_{G,j,inf} = 0{,}9$, and γ_Q is equal to 1,5, when unfavorable and 0, when favorable.

The above format applies to the verification of the structure as a right body (e.g. overturning of retaining walls). In cases where the verification of equilibrium also involves the resistance of the structural member (e.g. over hanging cantilevers), the strength verification given below without the above equilibrium check may be adopted. In such verifications, $\gamma_{G,j,inf}$ is equal to 1,15 should be used.

(2) Strength (STR)

When a design does not involve geotechnical actions, the strength of elements should be verified using load combination Set B in the Annex A from EN1990 [N1].

Two options are given. Either combination (6.10) from EN1990 [N1] or less favorable of equation (6.10a) and equation (6.10b) may be used:

$$\gamma_{G,j,\sup} \cdot G_{k,j,\sup} "+" \gamma_{G,j,\inf} \cdot G_{k,j,\inf} "+" \gamma_p{}^p "+" \gamma_{Q,1} \cdot \psi_{0,1} \cdot Q_{k,1} "+" \\ "+" \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k,i}; \tag{1.3-8}$$

$$\xi \cdot \gamma_{G,j,sup} \cdot G_{k,j,sup} "+" \gamma_{G,j,inf} \cdot G_{k,j,inf} "+" \gamma_p{}^p "+" \gamma_{Q,1} \cdot Q_{k,1} "+" \\ "+" \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k,i}. \tag{1.3-9}$$

The above Combination (1.3-8) and Combination (1.3-9) assume that a number of variable actions are present at the same time. Value $Q_{k,1}$ is the dominant load if it is obvious, otherwise each variable load is in turn treated as a dominant load and the others as secondary (accompanying). The dominant load is then combined with combination value of the accompanying (secondary) loads. Both are multiplied by their respective γ_i values.

When design involves geotechnical action, a number of approaches are given in EN1990 [N1].

1.3.4.1.3 Combination of actions for accidental design situation

The load combination recommended is following:

$$G_j + "P" + "A_d" + "\left(\psi_{1,i} \text{ or } \psi_{2,i}\right) \cdot Q_{k,1} + \psi_{2,i} \cdot Q_{k,i}, \tag{1.3-10}$$

where: A_d is the design value of accidental action.

Accidents are unintended events such as explosions, fire or vehicular impact, which are of short duration and which have a very low probability of occurrence (near 10^{-7}/year).

1.3.4.2 Serviceability Limit States (SLS)

1.3.4.2.1 General

Serviceability Limit States (SLS) correspond to conditions beyond which specified service requirements for structure or structural member are no longer met. Exceeding these limits causes limited damage but means that the structures do not meet design requirements: functional requirements (not only of the structure, but also of machines and services), comfort of users, appearance (where the term "appearance", as it was noted in EN 1990 [N1], is concerned with high deformation, extensive cracking, etc.), damage to finishes and to non-structural members. Usually the serviceability requirements are agreed for each individual project.

It should be verified that the design effects of actions do not exceed a nominal value, or a function of certain design properties of materials; for example, deflection under quasi-permanent loads should be less than span/250, and compression stress under a rare combination of loads should not exceed $0,6 \cdot f_{ck}$. In most cases, detailed calculations using various load combinations are unnecessary, as the code stipulates simple compliance rules. It should be verified that:

$$E_d \le C_d, \tag{1.3-11}$$

where: E_d is the design value of the effects of actions specified in the serviceability criterion, determined on the basic of the relevant combination;

C_d is the limiting design value of the relevant serviceability criterion (parameter).

1.3.4.2.2 Combinations of actions for serviceability limit states checking

The combination of actions to be taken into account in the relevant design situation should be appropriate for serviceability requirements and performance criteria being verified. The combinations of actions for serviceability limit states are defined symbolically by the following expressions:

Characteristic combination.

$$\sum G_{K,j}\left(+P\right)"+"Q_{k,1}+\sum_{i>1}\psi_{0,i}\cdot Q_{k,i}. \tag{1.3-12}$$

This represents a combination of service loads, which can be considered rather infrequent. It might be appropriate for checking states such as micro-cracking or possible local non-catastrophic failure of reinforcement leading to large crack in sections.

Frequent combination.

$$\sum G_{k,j}\left(+P\right)"+"\psi_{1,1}\cdot Q_{k,1}+\sum_{i>1}\psi_{2,i}\cdot Q_{k,i},\ i>1. \tag{1.3-13}$$

This represents a combination that is likely to occur relatively frequently in service conditions, and is used for checking cracking.

Quasi-permanent combination.

$$\sum G_{k,j}\left(+P\right)"+"\sum_{i\ge 1}\psi_{2,i}\cdot Q_{k,i},\ i\ge 1. \tag{1.3-14}$$

This will provide an estimate of sustain loads on the structure, a will be appropriate for the verification of creep, settlement, etc.

CHAPTER 2

STRUCTURAL ANALYSIS

2.1 GENERAL REQUIREMENTS

The purpose of structural analysis is the verification of overall stability and establishment of action effects, i.e. the distributions of internal forces and moments. The analysis implies a preliminary idealization of the structure, based on more or less refined assumptions of behavior. In terms of behavior of the structure, there are four types of idealization, as it was shown in EN 1992 [N3]:

- ***linear elastic behavior*** that assumes, for analysis, uncracked cross sections and perfect elasticity. The design procedures for linear analysis are given in Section 2.5.1;
- ***linear elastic behavior with limited redistribution*** (see Section 2.5.2). It is a design (not analysis) procedure based on mixed assumptions, derived from both linear and non-linear analysis;
- ***plastic behavior*** (see Section 2.5.3). Its kinematic approach in accordance with EN1992 [N3] (clause 5.6), assumes at ultimate limit state the transformation of the structure in a mechanism through the formation of plastic hinges; in its statistic approach, the structure is represented by compressed and tensioned elements (strut and tie model);
- ***non-linear behavior,*** that takes into account, for increasing actions, cracking, plastification of reinforcement steel beyond yielding, and plasticization of compressed concrete. The general design procedures for non-linear analysis are given in EN 1992 [N3]. As it was pointed in [8] the first two types of behavior are common for slabs and frames, and plastic analysis is popular in the design of slabs; non-linear analysis is very rarely used in day-to-day design and it is used mainly for structural design in accidental design situations and for the existing structures assessment. The above methods, with the exception of plastic analysis are suitable for both serviceability and ultimate limit states.

2.2 CASES AND LOAD ARRANGEMENTS

In the analysis of the structure, the designer should consider the effects of the realistic combinations of permanent and variable actions.

Within each set of combinations (e.g. dead (permanent) and imposed loads) a number of different arrangements of loads (load cases) throughout the structure (e.g. alternate spans loaded and adjanced spans loaded) will need consideration to identify an envelope of action effects (e.g. bending moments and shear envelopes) to be used in the design of sections.

While the general requirement is that all relevant load cases should be investigated to arrive at the critical conditions for the design on all sections, EN 1992 [N3] permits simplified load arrangements for design of continuous beams and slabs. Accordance with EN1992 [N3] (clause p.5.1.3) the arrangements to be considered are:

1) alternate spans loaded with the design variable and permanent loaded with the design variable and permanent loads $(1{,}35 \cdot G_k + 1{,}5 \cdot Q_k)$ and other spans carrying only the design permanent load $(1{,}35 \cdot G_k)$;

2) any two adjacent spans carrying the design variable and permanent loads $(1{,}35 \cdot G_k + 1{,}5 \cdot Q_k)$, with all other spans carrying only design permanent load $(1{,}35 \cdot G_k)$.

As it was shown in [3, 8], the above arrangements are intended for braced non-sway structures. They also be used in case of sway structures, but the following additional load cases involving the total frame will also need to be considered:

1) all spans loaded with the design permanent loads $(1{,}35 \cdot G_k)$ and the frame subjected to the design wind load $(1{,}5 \cdot W_k)$, when W_k is the characteristic wind load;

2) all spans at all floor levels loaded with $(1{,}35 \cdot G_k + 1{,}5 \cdot Q_k)$ and the frame subjected to the design wind load at $1{,}5 \cdot W_k$;

3) In sensitive structures (sensitivity to lateral deformation), it may be necessary to consider the effect of wind loading in conjunction with patterned imposed loading throughout the frame.

EN 1992 [N3] states that in linear elements and slabs subjected predominantly to bending the effect of shear and axial forces on deformation may be neglected, if these are likely to be less than 10 %. In practice, the designer need not actually calculate these additional deformations to carry out this check.

Deflections are generally of concern only in members with reasonably long spans. In such members, the contribution of shear to the deflections is never significant for members with normal (span/depth) ratios. When the spans are short, EN1992 [N3] provides alternative design models (e.g. truss or strut and tie) in which deflections are rarely, if ever, a consideration. The contribution of the axial loads to deflections may be neglected if the axial stresses do not exceed $0{,}08 \cdot f_{ck}$.

2.3 GEOMETRICAL IMPERFECTIONS

2.3.1 GENERAL

Perfection in buildings exists only in theory; in practice, same degree of imperfection is unavoidable, and designs should recognize this, and ensure that buildings are sufficiently robust to withstand the consequences of such inaccuracies [8]. For example, load-bearing elements may be out of plumb or the dimensional inaccuracies may cause eccentric application of load.

EN 1992 [N3] has a number of provisions in this regard, affecting the design of 1) the structure as a whole, 2) some slender elements and 3) elements which transfer forces to bracing members.

2.3.2 GLOBAL ANALYSIS

For the analysis of the structure as a whole, an arbitrary inclination of the structure $\theta_0 = 1/200$ is prescribed as a basic value. This is then modified for height and for the number of members are involved.

The design value will be:

$$\theta_i = \theta_0 \cdot a_n \cdot a_m, \tag{2.3-1}$$

where:

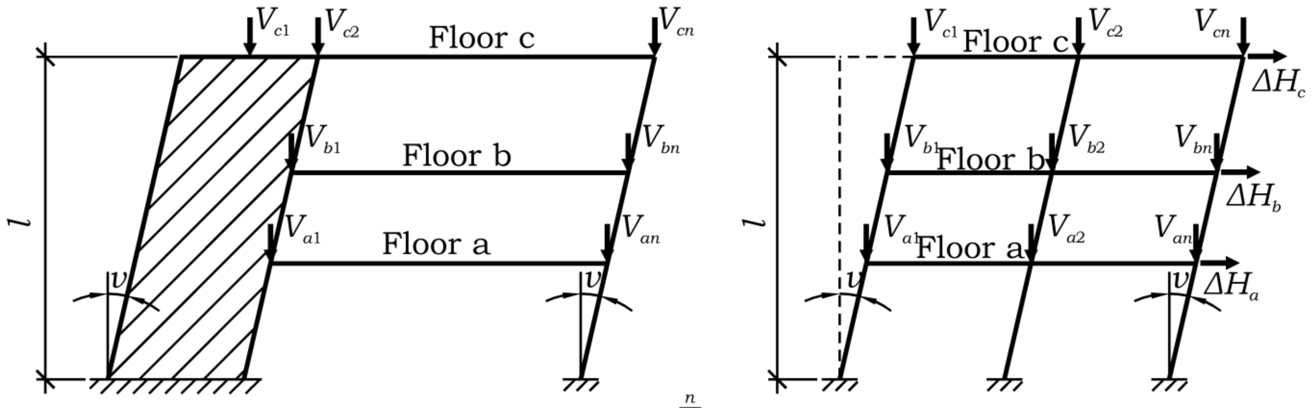
$$a_n = \frac{2}{\sqrt{l}}, \tag{2.3-2}$$

where: l is the total height of structure in meters $(0,67 \leq a_n \leq 1,0,)$.

$$a_m = \sqrt{0,5 \cdot \left(1 + \frac{1}{m}\right)}, \tag{2.3-3}$$

where: m is the number of vertically continuous elements in the storeys contributing to the total horizontal force on the floor. This factor recognizes that the degree of imperfection is statistically unlikely to be the same in all of the members.

As a result of the inclination, a horizontal component of the vertical loads could be through of being applied at each floor level, as shown in Figure 2.3-1 a and in Figure 2.3-1 b.



$$\Delta H_j = \sum_{i=1}^{n} V_i \cdot \upsilon$$

a) – braced structure (number of vertically continuous members m=2);

b) – unbraced structure (number of vertically continuous member m=3)

Figure 2.3-1 – Application of the effective geometrical imperfections

In the design of slender elements, which are prone to fail by buckling (e.g. slender columns), EN 1992 [N3] requires geometrical imperfection to be added to other eccentricities. For example, in the design of the columns, an eccentricity of $(\theta_i \cdot l_0)/2$ is assumed for geometrical imperfection (where l_0 is the effective length of the column).

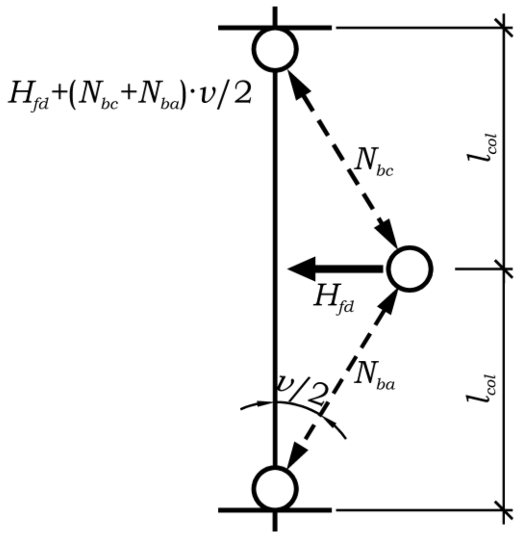


Figure 2.3-2 – Minimum tie force for perimeter columns

In the design of these elements (such as a floor diagram), a force to account for the possible imperfection should be taken into account in addition to other design actions. This additional force is illustrated in Figure 2.3-2. This force need not be taken into account in the design of the bracing element itself [8].

2.4 IDEALIZATION OF THE STRUCTURE

2.4.1 STRUCTURAL MODELS FOR OVERALL DESIGN

In accordance with EN1992 [N3] (clause p.5.3.1), the elements of a structure are classified, by consideration of their nature and function, as: beams; columns; slabs; walls; plates; arches; shells, etc.

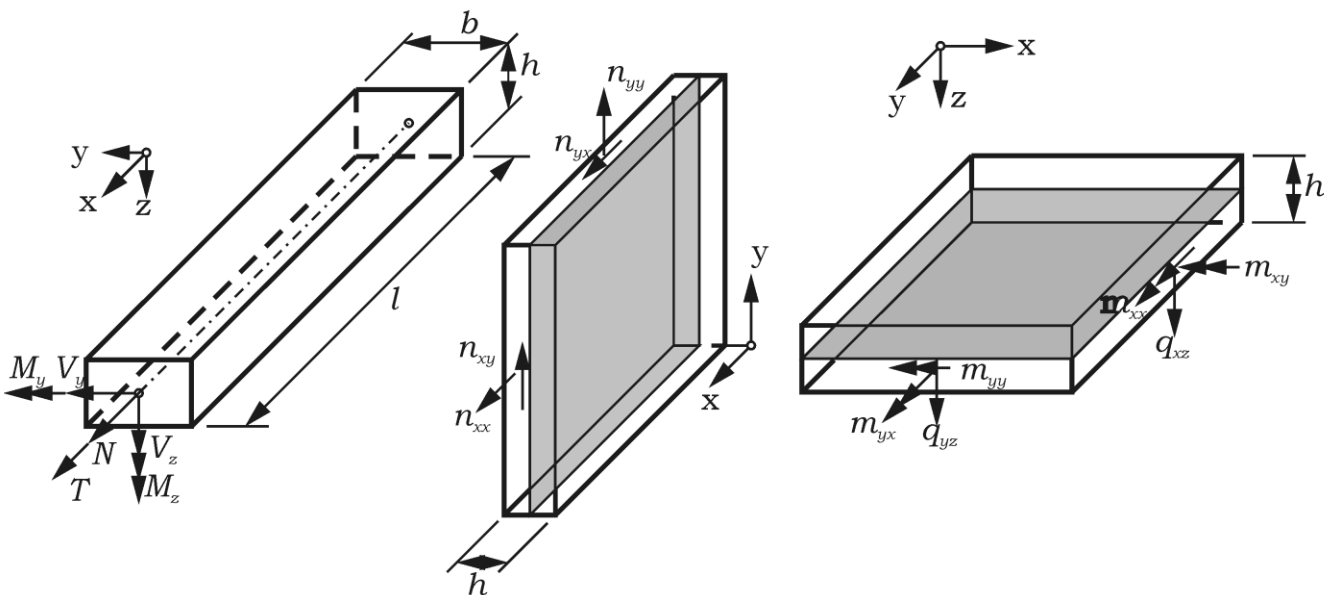
For buildings the following provisions are applicable:

1) ***a beam*** is a member for which span is not less than 3 times of the overall section depth. Otherwise it should be considered as a *deep beam* (Figure 2.4-1 a and Figure 2.4-1 b);

2) ***a slab*** is a member for which a minimum panel dimension is not less than 5 times of the overall slab thickness (Figure 2.4-1 c);

2.1) a slab subjected to dominantly uniform distributed loads may be considered to be ***one-way slab*** (Figure 2.4-2), if either:

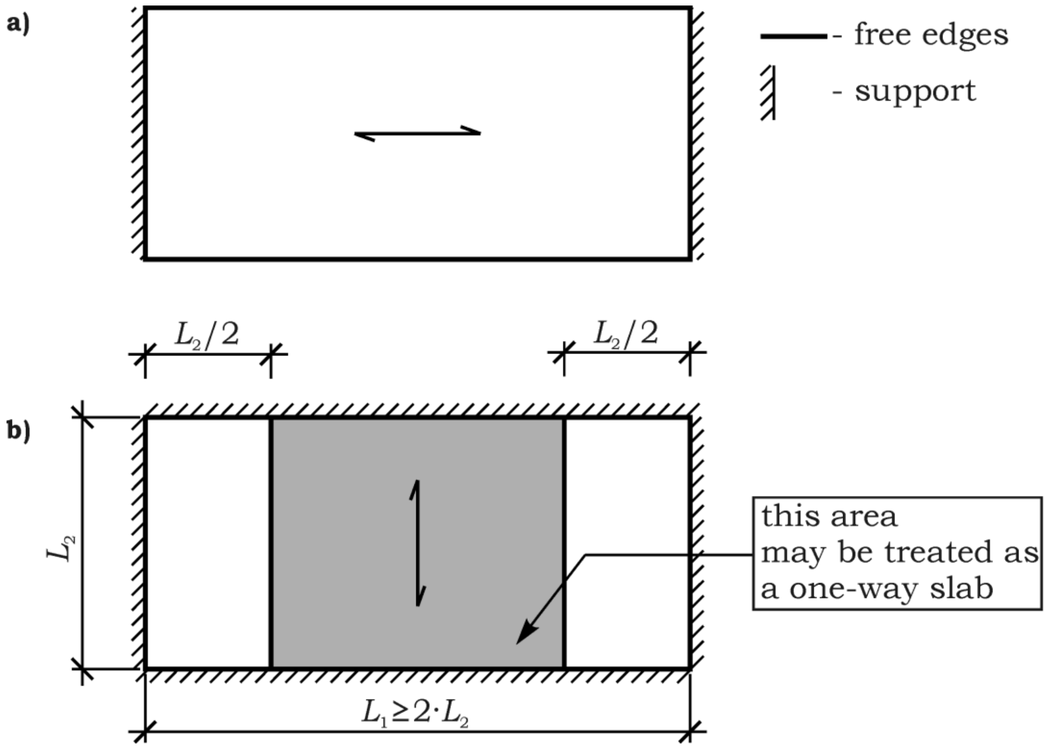
- it possesses two free (unsupported) and sensibly parallel edges, or
- it is the central part of sensibly rectangular slab supported on four edges with ratio of the longer to shorter span greater than 2.



a) b) c)

a) – beam; b) – deep beam; c) – slab

Figure 2.4-1 –Definition of structural members for analysis



a), b) – one-way spanning slab

Figure 2.4-2 – One-way spanning slab (subjected predominantly to ultimate design load)

2.2) ***ribbed or waffle slabs*** need not be treated as discrete elements for the purpose of analysis, provided that the flange or structural topping and transverse ribs have sufficient torsional stiffness. They may be assumed provided that (see Figure 2.4-3):

- The rib spacing does not exceed 1500 mm;
- The depth of the rib below the flange does not exceed 4 times of its width;
- The depth of flange is at least 1/10 of clear distance between ribs or 50mm; whichever is the greater;
- Transverse ribs are provided at a clear spacing not exceeding 10 times of overall depth of the slab.

The minimum flange thickness of 50 mm may be reduced to 40 mm where permanent blocks are incorporated between the ribs.

This exception applies for slabs with clay blocks only. It does not apply for expanded polystyrene blocks.

An exception to this rule is given at EN1992 [N3] (clause 10.9.3(11)) in relation to prefabricated slabs without topping, which may be analyzed as solid slabs provides that the in situ transverse ribs are provided with continuous reinforcement through the precast longitudinal ribs and at a spacing according to Table 10.1 from EN1992 [N3].

3) ***a column*** is a member for which the section depth does not exceed 4 times of its width and the height is at least 3 times of the section depth.

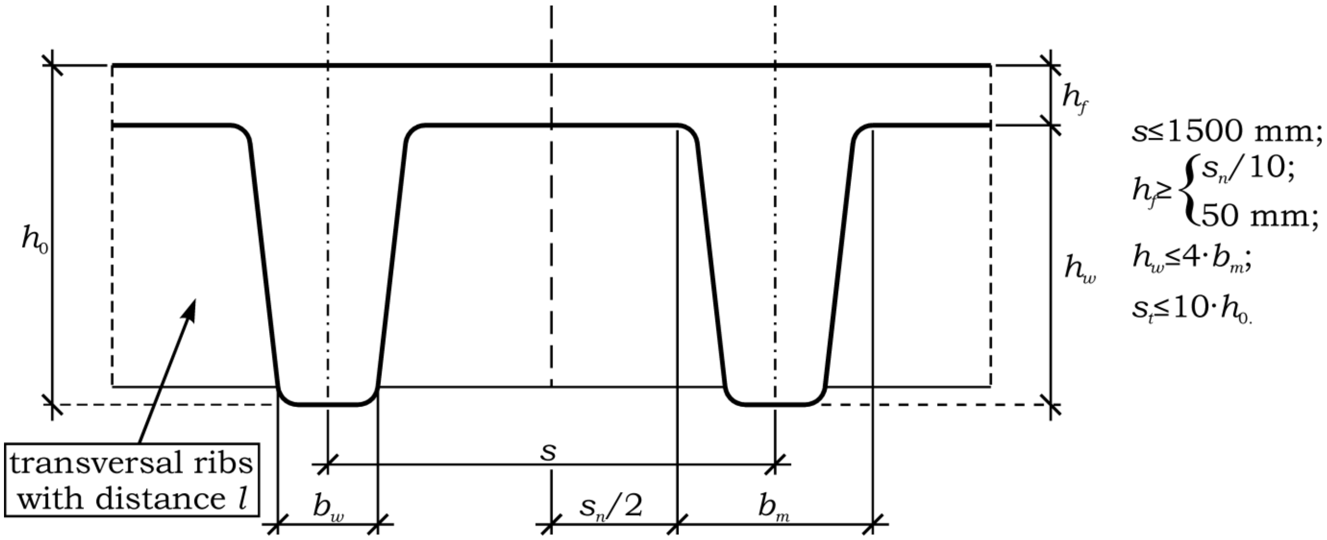


Figure 2.4-3 – Geometrical parameters for slabs (see Figure 5.2 from EN 1990 [N3])

2.4.2 GEOMETRICAL DATA

2.4.2.1 Effective width of flanges (valid for all limit states)

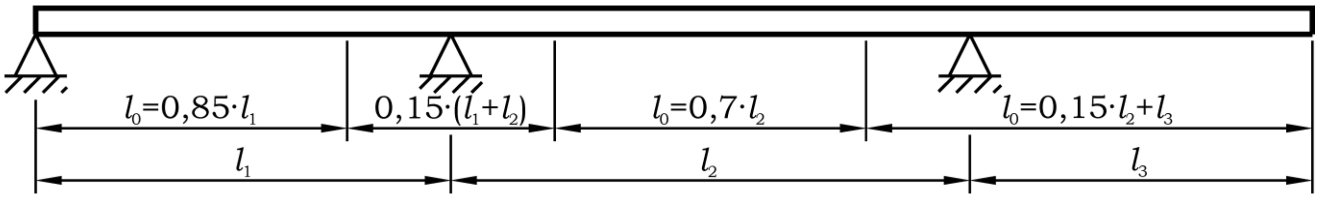
In T-beams the effective flange width, over which uniform conditions of stress can be assumed, depend on the web and flange dimensions, the type of loading, the span, the support conditions and the transverse reinforcement.

As it was noted in [8] if a T-beam with relatively wide flange is subjected to bending moment, the width of flange that effectively works with the rib in absorbing the compressive force (effective width) should be assessed.

An exact calculation shows that the actual distribution of compressive stresses has a higher concentration it the part of flange which is close to the rib and a progressive reduction in the further parts.

This implies that the conservation of plane section is not respected and that the neutral axis is not rectilinear, but is higher on both sides on the rib. In order simplify calculations, the actual distribution of stresses is usually replaced by a conventional block, extended to the effective width. This allows the application of the usual design rules, and in particular the assumption that plane sections remain plane.

In accordance with EN1992 [N3] (clause 5.3.2.1) effective width is defined as a function of the cross-section geometry (b_i – distance between adjacent ribs; b_w – width of ribs) and of the distance l_0 between points of zero moment which may be obtained from the Figure 2.4-4.



Note: *The length of the cantilever, l_3, should be less than the half of the adjacent span and ratio of adjacent spans should lie between 2/3 and 1,5.*

Figure 2.4-4 – Definition of l_0 for calculation of effective flange width (see Figure 5.2 from EN 1992 [N3])

The effective flange width b_{eff} for a T-beam or L-beam may be derived as:

$$b_{eff} = \sum b_{eff,i} + b_w \leq b, \tag{2.4-1}$$

where:

$$b_{eff,i} = 0,2 \cdot b_i + 0,1 \cdot l_0 \leq 0,2 \cdot l_0, \tag{2.4-2 a}$$

and:

$$b_{eff,i} \leq b_i, \tag{2.4-2 b}$$

(for notations see Figure 2.4-4 placed above and Figure 2.4-5 placed below).

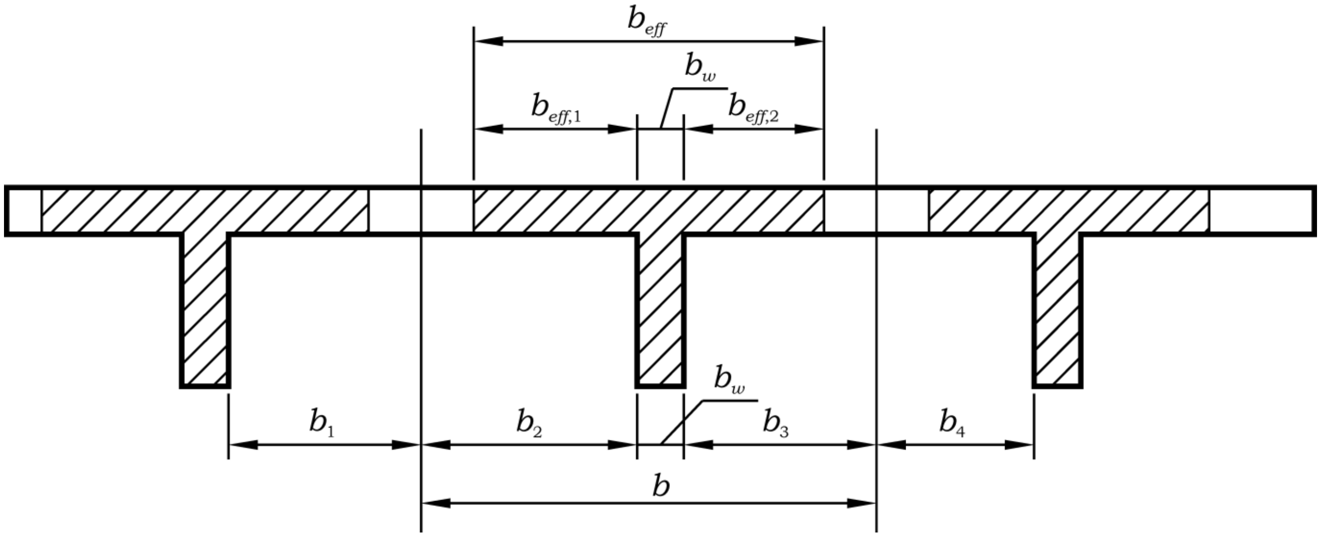
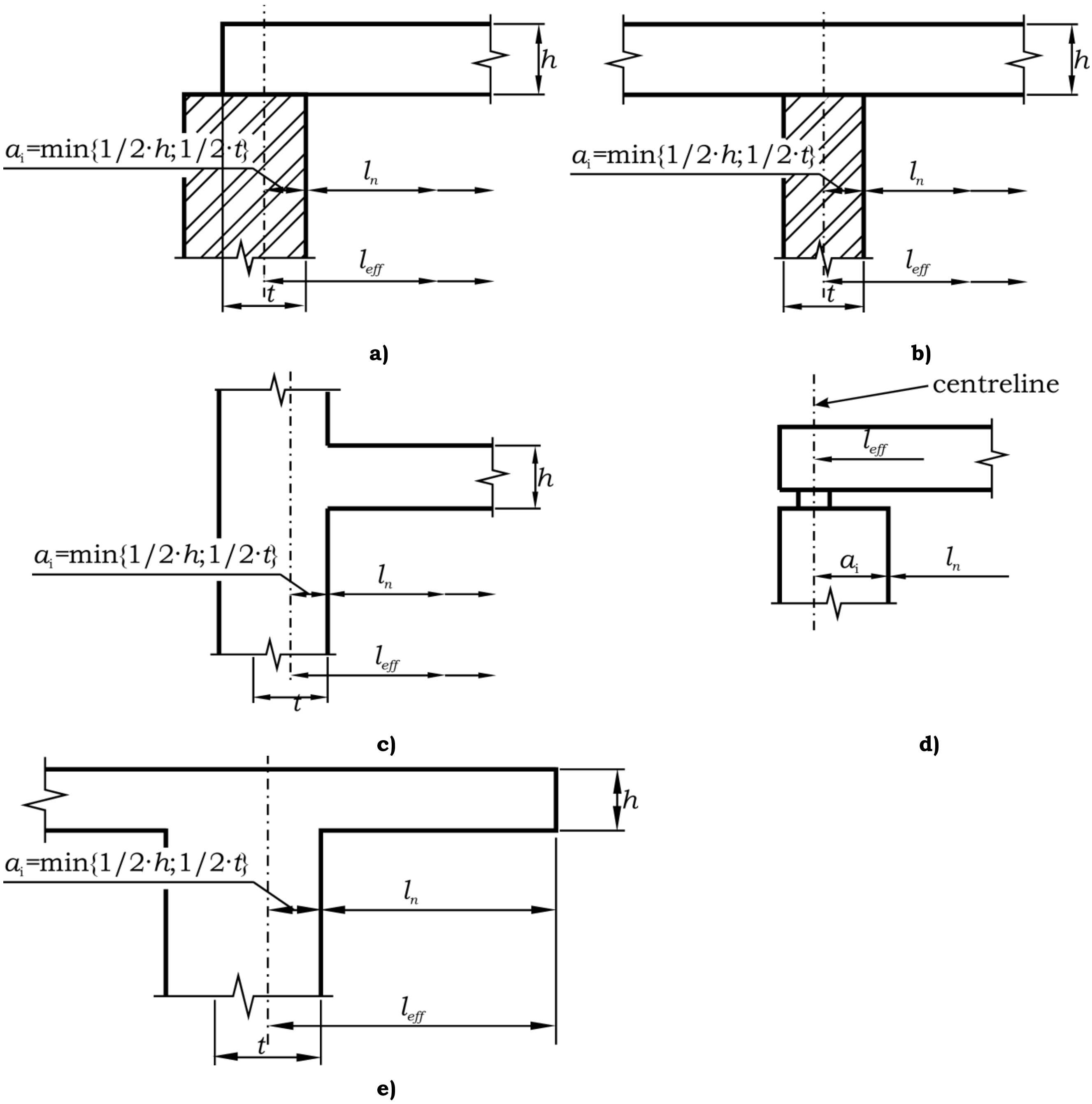


Figure 2.4-5 – Effective flange width parameters (see Figure 5.3 from EN 1992 [N3])

For structural analysis, where a great accuracy is not required, a constant width may be assumed over a whole span. The value applicable to the span section should be adopted.

2.4.2.2 Effective span of beams and slabs in buildings

The following provisions are provided mainly for member analysis, taking into account the different types of support (see Figure 2.4-6). For frame analysis some of these simplifications may be used where it is appropriate.



a) – non-continuous members; b) – continuous members;

c) – supports considered fully restrained; d) – bearing provided; e) – cantilever

Figure 2.4-6 – Effective span (l_{eff}) for different support conditions

(see Figure 5.4 from EN 1992 [N3])

The effective span, l_{eff}, of a member should be calculated as follows:

$$l_{eff} = l_n + a_1 + a_2, \qquad (2.4\text{-}3)$$

where: l_n is the clear distance between the faces of the supports; values for a_1 and a_2, at each end of the span, may be determined from the appropriate a_i values in Figure 2.4-6, where *t* is the width of the supporting element as it is shown.

Continuous slabs and beams may be generally analyzed on the assumption that the supports provide no rotational restraint.

Two important points must be noted. Where a beam or slab is monolithic with its supports, the critical design moment at the support should be taken as that at the face of the support. The design moment and reaction transferred to the supporting element (e.g. column, wall, etc.) should be generally taken as the greater of the elastic or redistributed values. Additionally, in EN1992 [N3] (clause 5.3.2.2) is stated that the moment at the face of support should not be less than 0,65 that of the full-fixed end moment. This ensures a minimum design value for the support moment, particularly, in the case of wide supports.

Regardless of the method of analysis used, where a beam or slab is continuous over a support which may be considered to provide no restraint to rotation (e.g. over walls), the design support moment, calculated on the basis of a span equal to the centre-to-centre distance between supports, may be reduced by an amount ΔM_{Ed} as follows:

$$\Delta M_{Ed} = F_{Ed,sup} \cdot \frac{t}{8}, \qquad (2.4\text{-}4)$$

where: $F_{Ed,sup}$ is the design support reaction;

t is the breadth of the support (see Figure 2.4-6 b).

Note: *Where support bearings are used, t should be taken as the bearing width.*

This Formula (2.4-4) derives by assuming an uniform distribution of the design support reaction $F_{Ed,sup}$ over the breadth of the support:

$$\Delta M_{Ed} = \left(\frac{F_{Ed,sup}}{t}\right) \cdot \frac{t}{2} \cdot \frac{t}{4} = \frac{F_{Ed,sup} \cdot t}{8}. \qquad (2.4\text{-}5)$$

This recognizes the effect of the width of support and arbitrarily rounds of the peak in the bending moment diagram.

2.5 METHODS OF ANALYSIS

2.5.1 LINEAR ELASTIC ANALYSIS

Elastic analysis remains the most popular method for frame (e.g. moment distribution and slope deflection). As it was shown in [3, 8] *braced frames* may be analyzed as a whole frame or may be partitioned into sub-frames (see Figure 2.5-1). The sub-frames may consist of beams at one level with monolithic attachment to the columns. The remote ends may be assumed to be «*fixed*» unless a «*pinned*» end is more reasonable in particular cases. As a further simplification, beams alone can be considered to be continuous over supports providing no restraint to rotation. Clearly this is more conservative. In *unbraced structures,* it is generally necessary to consider the whole structure, particularly when lateral loads are involved. A simplified analysis may be carried out, assuming points of contra flexure at the mid-lengths of beams and columns (see Figure 2.5-2). However, it should be remembered that this method will be inaccurate if: 1) the feet of column are not fixed and/or 2) the beams and columns are not of the similar stiffness.

Linear analysis of the elements based on the theory of elasticity may be used for both the Serviceability and Ultimate Limit States.

For the determination of the action effects, linear analysis may be carried out assuming:

- Untracked cross sections;
- Linear stress-strain relationships, and
- Mean value of the modulus of elasticity.

With these assumptions, stresses are proportional to loads and therefore the superposition principle is applied.

For thermal deformation, settlement and shrinkage effects at the Ultimate Limit State (***ULS***), a reduced stiffness corresponding to the cracked sections, neglecting tension stiffening but including the effects of creep, may be assumed. For the Serviceability Limit State (***SLS***) a gradual evolution of cracking should be considered.

In accordance with EN1992 [N3] (clause 5.4 (2) and clause 5.4 (3)) in calculation of the stiffness of members it is normally satisfactory to assume a «*mean*» value of the modulus of elasticity E_{cm} and the moment of the member. However, when computing the effects of deformation, shrinkage and settlement reduced stiffness corresponding to cracked cross-section should be used.

It is important, that the fact that no limits where set to x_u / d for application of the linear analysis method at the ***ULS***, does not mean than any value of x_u / d may be used in design: it's opportune to observe a limit consistent with the method of linear elastic analysis with limited redistribution, for which $x_u / d \le 0{,}45$. It must be remembered that increasing of the x_u / d values lead to the model uncertainty increasing as well as higher partial factors should be assumed for precaution.

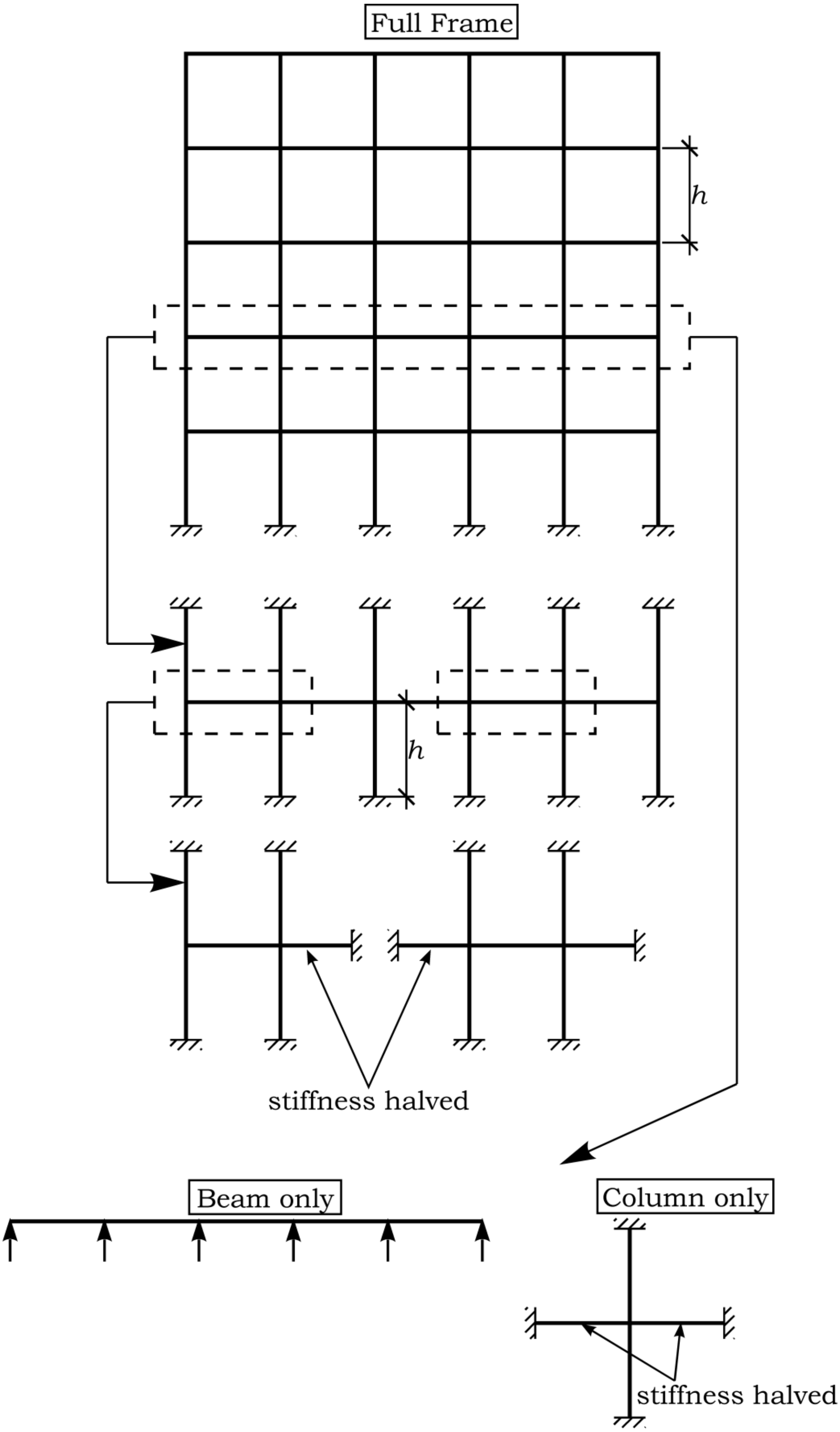


Figure 2.5-1 – Partitioning of multistoried braced structures for analysis (see Figure 3.6 from [8])

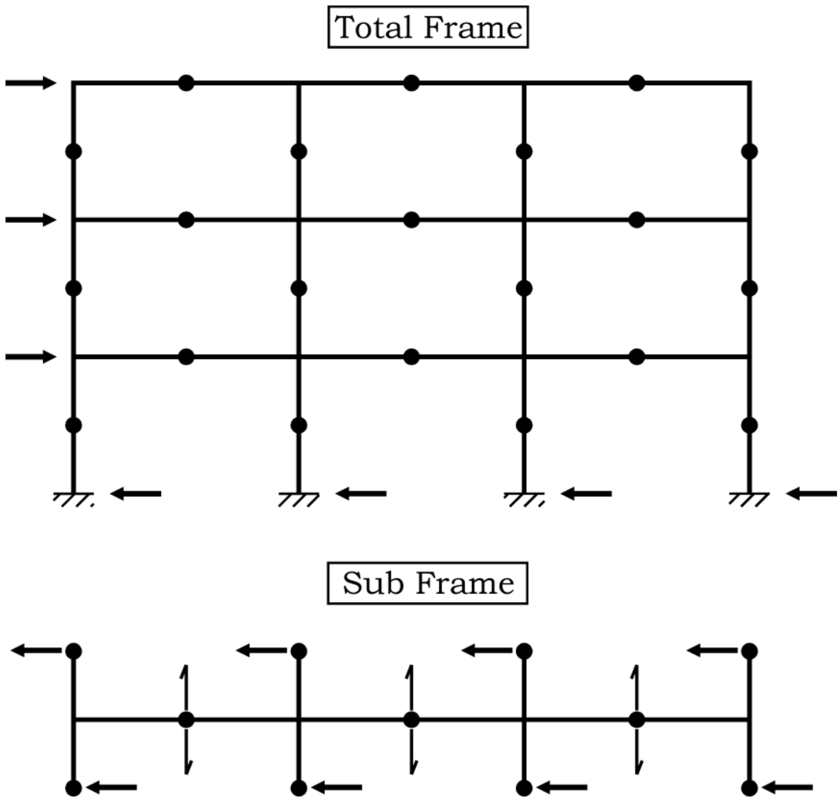


Figure 2.5-2 – Simplified model for the analysis of unbraced structures (see Figure 3.7 from [8])

2.5.2 LINEAR ELASTIC ANALYSIS WITH LIMITED REDISTRIBUTION

At Ultimate Limit State (***ULS***) plastic rotation occur at the most stressed and cracked sections. These rotations transfer to other zones is the effect of the further load increasing, thus it is allowed to take for the design of reinforcement a reduced bending moment δM, smaller than the moment M resulting from elastic linear design, provided that in the other parts of the structure the corresponding variations of load effects (viz. shear) necessary to ensure equilibrium are considered.

As it was shown in [8], «the moment-curvature» response of a true elasto-plastic material will be typically as shown in Figure 2.5-3. The long plateau after moment M_p is reached implies a large rotation capacity.

Considering a continuous beam made of such a material and loaded as shown in Figure 2.5-4, when a bending moment in a critical section (usually at a support) reaches M_p, a plastic hinger is said to be formed. The structure will be able to withstand under the further increasing in loading until the sufficient plastic hinges from to turn the structure into mechanism.

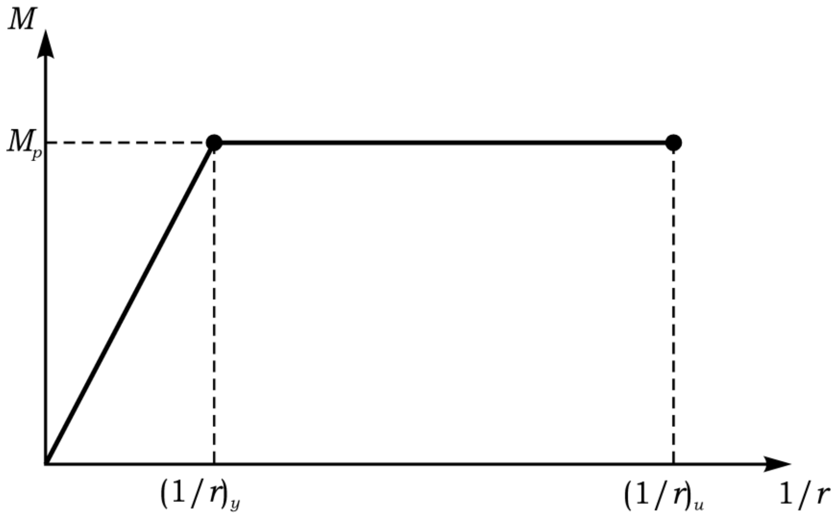


Figure 2.5-3 – «Moment–curvature» bi-linear idealization

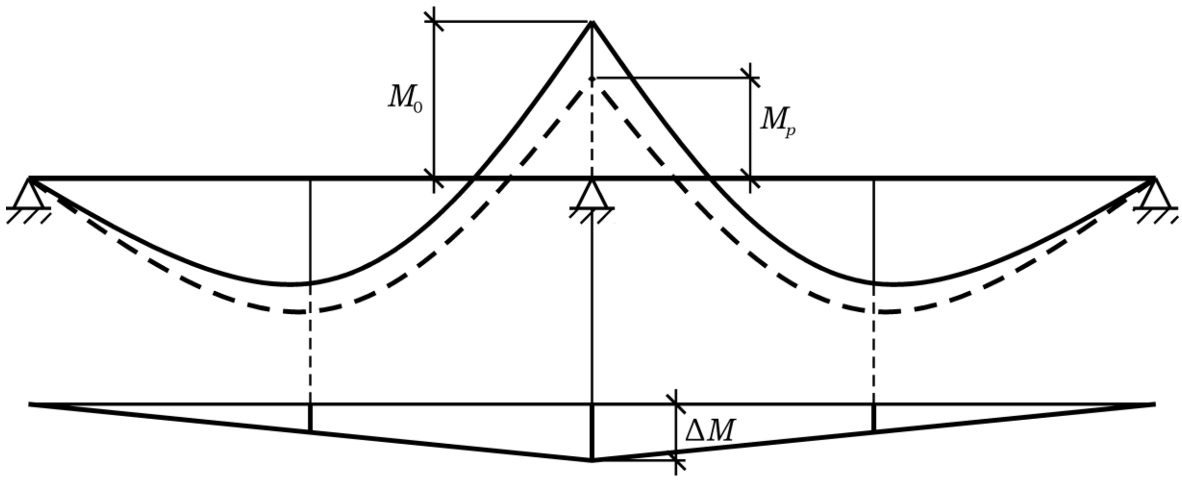


Figure 2.5-4 – Redistribution of the bending moments in continuous beam

Clearly, to exploit plasticity fully, the material must possesses an adequate ductility (rotation capacity), concrete has only limited capacity in this regard. The moment redistribution procedure is an allowance for the plastic hinge analysis. Indirectly, it also ensures that the yield of sections under service loads and large uncontrolled deflections are avoided.

How it was stated in EN1992 [N3] (clause 5.5), linear analysis with limited redistribution may be applied to the analysis of structural members for the verification of ***ULS***.

In accordance with EN1992 [N3], the moments at ***ULS*** calculated using a linear elastic analysis may be redistributed, provided that the resulting distribution of moments remains in equilibrium with the applied loads.

In continuous beams or slabs which:

a) are predominantly subject to flexure and

b) have the ratio of the lengths of adjacent spans in the range of from 0,5 to 2 redistribution of bending moments may be carried out without explicit check on the rotation capacity, provided that:

$$\delta \geq k_1 + k_2 \cdot \frac{x_u}{d} \text{ for } f_{ck} \leq 50\,\text{MPa}, \tag{2.5-1}$$

$$\delta \geq k_3 + k_4 \cdot \frac{x_u}{d} \text{ for } f_{ck} > 50\,\text{MPa}, \tag{2.5-2}$$

$\geq k_5$, where Class B and Class C reinforcement is used (see Annex C from EN1992 [N3]);
$\geq k_6$, where Class A reinforcement is used (see Annex C from EN1992 [N3]),
where: δ is the ratio of the distributed moment to the elastic bending moment;
x_u is the depth of the natural axis at the ultimate limit state after redistribution;
d is the effective section depth;
k_1, k_2, k_3, k_4 are coefficients with the recommended values: $k_1 = 0,44$; $k_2 = 1,25 \cdot \left(0,6 + 0,0014 / \varepsilon_{cu1}^2\right)$; $k_3 = 0,54$; $k_4 = 1,25 \cdot \left(0,6 + 0,0014 / \varepsilon_{cu1}^2\right)$; $k_5 = 0,7$; $k_6 = 0,8$.

Redistribution should not be carried out in circumstances where the rotation capacity cannot be defined with confidence (e.g. in the corners of prestressed frames).

For the design of columns the elastic moments from frame action should be used without any redistribution.

It must be considered that a redistribution carried out in observance of the ductility rules only ensures equilibrium at the ultimate limit state. Specific verifications are needed for the serviceability limit states. Very high redistributions, which may be of advantage at the ultimate limit states, very often must be lowered in order to meet the requirements of serviceability limit states. For the ***design of columns*** the elastic moments from frame action should be used ***without any redistribution***.

2.5.3 PLASTIC ANALYSIS

2.5.3.1 General

As it was shown in EN 1992 [N3] plastic analysis should be based either on the lower bound (static) method or on the upper bound (kinematic) method for the check at ***ULS*** only. The ductility of the critical sections shall be sufficient for the envisaged mechanism to be formed. The effects of previous applications of loading may generally be ignored, and a monotonic increase of the intensity of actions may be assumed.

2.5.3.1.1 Static method

It is based on static theorem of the theory of plasticity, which states: «*whichever load Q, to which a statically admissible tension field corresponds, is lower or equal to the ultimate load* Q_u».

The expression "statically admissible" indicates a field that meets both the conditions of equilibrium and the boundary condition without exceeding the plastic resistance. An important application of this method is the strut-and-tie scheme in accordance EN1992 [N3] (clause p.5.6.4, see section 3.7.4).

Other applications are the management of shear by the method of varying and the analysis of slabs by the equivalent frame analysis method what is presented in Annex E, EN 1992 [N3].

2.5.3.1.2 Kinematic method

In this method, the structure at Ultimate Limit States becomes a mechanism of rigid elements connected by yield hinges. The method is based on the kinematic theorem, which states: «*every load Q, to which corresponds a kinematically admissible mechanism of collapse, is higher or equal to the ultimate load* Q_u» [3, 8]. The method is applied for continuous beams, frames and slabs (in this last case with the theory of yield lines).

2.5.3.2 Plastic analysis for beams, frames and slabs

Plastic analysis without any direct check of rotation capacity may be used for the Ultimate Limit State if the ductility of critical sections will be sufficient for envisaged mechanism to be formed.

The required ductility may be deemed to be satisfied without explicit verification if all the following are fulfilled:

- the area of tensile reinforcement is limited such that, at any section:

$x_u / d \leq 0{,}25$ for concrete strength classes $\leq$C50/60;

$x_u / d \leq 0{,}15$ for concrete strength classes $\geq$C55/67.

- reinforcing steel is either Class B or Class C;
- the ratio of the moments at intermediate supports to the moments in the span should be between 0,5 and 2.

Columns should be checked for the maximum plastic moments which can be transmitted by connecting members. For connections to flat slabs this moment should be included in the punching shear calculation.

When plastic analysis of slabs is carried out account should be taken of any non-uniform reinforcement, corner tie down forces, and torsion at free edges.

Plastic methods may be extended to non-solid slabs (ribbed, hollow, waffle slabs) if their response is similar to that of a solid slab, particularly with regard to the torsional effect.

2.5.3.2.1 Rotation capacity

The simplified procedure for continuous beams and continuous one way spanning slabs is based on the rotation capacity of beam/slab zones over a length of approximately 1,2 times of the section depth. It is assumed that these zones undergo a plastic deformation (formation of yield hinges) under the relevant combination of actions. The verification of the plastic rotation in combination of actions the calculated rotation θ_s is less than or equal to the allowable plastic rotation (see Figure 2.5-5).

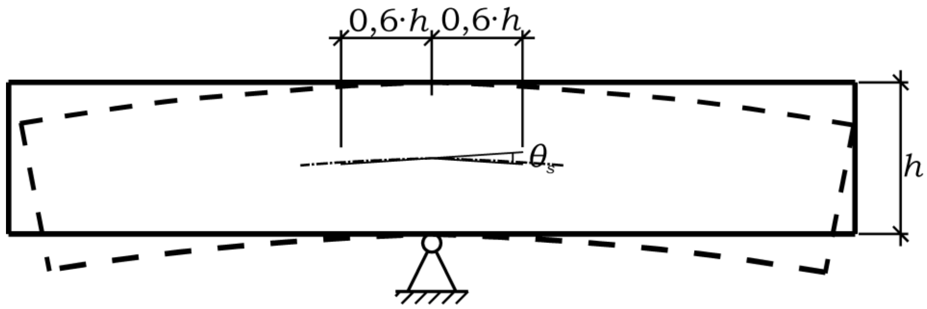


Figure 2.5-5 – Plastic rotation θ_s of reinforced concrete sections for continuous beams and continuous one-way spanning slabs (Figure 5.5 from EN 1992 [N3])

In regions of yield hinges x_u / d shall not exceed the value 0,45 for concrete strength classes less than or equal to C50/60, and 0,35 for concrete strength classes greater than or equal to C55/67.

The rotation θ_s should be determined on the basis of the design values for the actions and materials and on the basis of the mean values for prestressing at the relevant time.

In the simplified procedure, the allowable plastic rotation may be determined by multiplying of the basic value of allowable rotation $\theta_{pl,d}$ by a correction factor k_λ that depends on the shear slenderness.

As it was noted in EN1992 [N3], the recommended values for steel of the Class B and Class C (the use of Class A steel is not recommended for plastic analysis) and concrete strength classes less than or equal to C50/60 and C90/105 are given in Figure 2.5-6.

The values for concrete strength classes C55/67 to C90/105 may be interpolated accordingly. The values apply for a shear slenderness $\lambda = 3{,}0$. For different values of shear slenderness $\theta_{pl,d}$ should be multiplied by k_λ:

$$k_\lambda = \sqrt{\frac{\lambda}{3}}, \tag{2.5-3}$$

where: λ is the ratio of the distance between point of zero and maximum moment after redistribution and effective depth, d.

As a simplification λ may be calculated for the concordant design values of the bending moment and shear:

$$\lambda = \frac{M_{Sd}}{V_{Sd} \cdot d}. \tag{2.5-4}$$

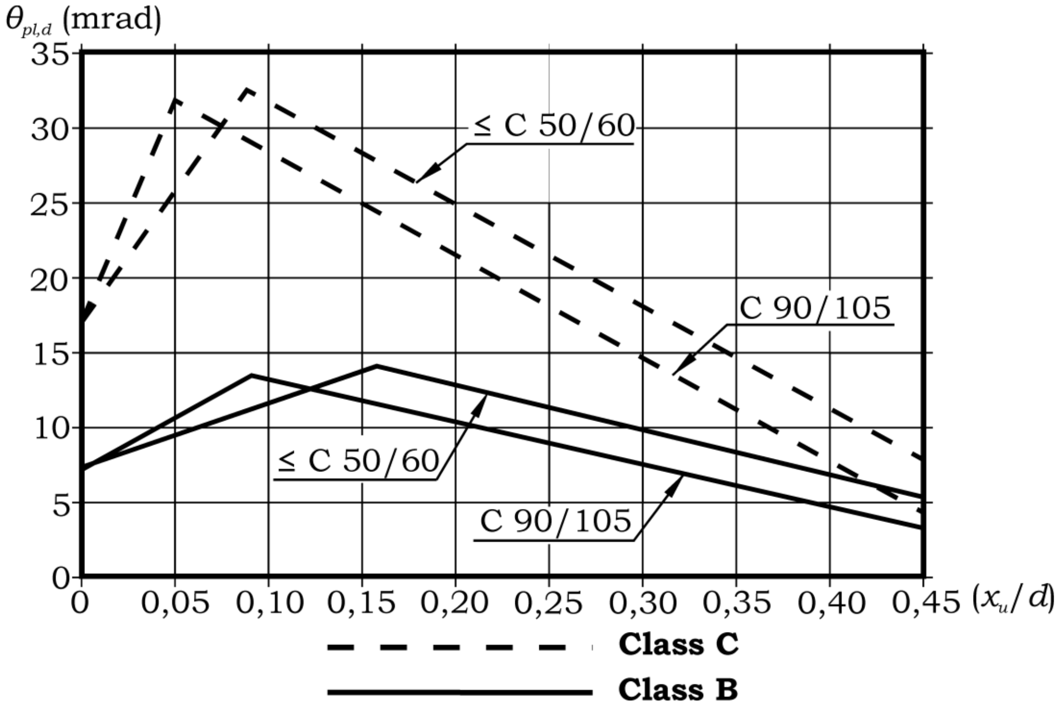


Figure 2.5-6 – Basic value of allowable rotation, $\theta_{pl,d}$, of reinforced concrete sections for Class B and C reinforcement. The values apply for a shear slenderness λ=3,0 (see Figure 5.6 from EN 1992 [N3])

2.5.3.2.2 Analysis with strut and tie models

Strut-and-tie models utilize the lower-bound theorem of plasticity, which can be summarized as follows: for structure under a given system of external loads, if a stress distribution throughout the structure can be found such that 1) all conditions of equilibrium are satisfied and 2) the yield conditions in not violated anywhere, than the structure is safe under the given system of external loads. This approach particularly simplifies the analysis of the parts of the structure where linear distribution of strain is not valid (see Section 2.1).

Strut and tie analysis may also be necessary, particularly when the assumption of linear strain distribution in the plane section is not applied. Examples of this include: anchorage zones; members with significant changes in cross-section, including the vicinity of large holes; beam-column joints; locations adjanced to concentrated loads.

Typical models are shown in Figure 2.5-7. As can be seen, the structure is through of as comprising notional concrete struts and reinforcement ties. Occasionally, concrete ties may also be considered (e.g. slabs without stirrups; anchorages without transverse reinforcement).

Concrete has only a limited plastic deformation capacity; therefore, the model has to be chosen with care to ensure that the deformation capacity is not exceed at any point before the assumed state of stress is reaches in structure. The ties of a strut-and-tie model should coincide in position and direction with the corresponding reinforcement.

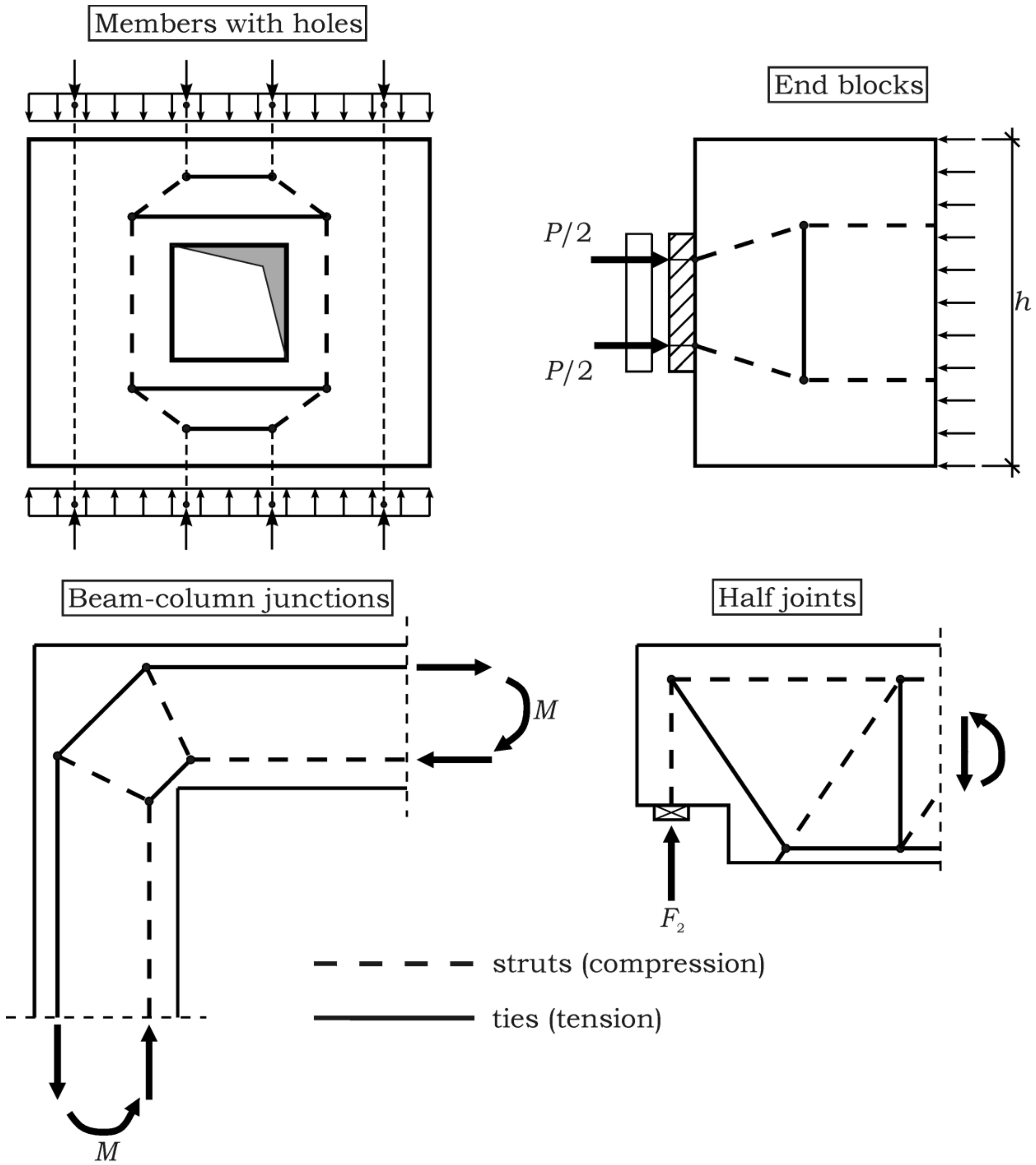


Figure 2.5-7 – Typical strut-and-tie model (see Figure 3.19 from [8])

Possible means for developing suitable strut-and-tie models include the adoption of stress trajectories and distributions from linear-elastic theory or the load path method. All strut-and-tie models may be optimized by energy criteria.

The angels between the struts and ties should generally be greater than 45° in order to avoid incompatibility problems. In this context, the deformation of the struts may be neglected, and the model optimized by minimizing the expression:

$$\sum_{i=1}^{n} F_i \cdot l_i \cdot \varepsilon_{m,i}, \tag{2.5-5}$$

where: F_i is the force in the *i-th* tie;

l_i is the length of the *i-th* tie;

$\varepsilon_{m,i}$ is the mean strain in the *i-th* tie.

Having idealized the structure as struts and ties, it is than a simple matter to arrive at the forces in them based on equilibrium with external loads. Stresses in struts and ties should be verified including those at nodes, where a number of members meet.

In accordance with EN 1992 [N3] limiting stresses is as follows:

1) in ties, f_{yd};

2) in struts with no transverse tension, $\sigma_{Rd,max} = f_{cd}$;

3) in struts with transverse tension, $\sigma_{Rd,max} = 0,6 \cdot \upsilon^{/} \cdot f_{cd}$, where $\upsilon^{/} = 1 - \dfrac{f_{ck}}{200}$;

4) in nodes where no ties are anchored, $\sigma_{Rd,max} = \upsilon^{/} \cdot f_{cd}$;

5) in compression-tension nodes, where ties are anchored in more than one direction, $\sigma_{Rd,max} = 0,85 \cdot \upsilon^{/} \cdot f_{cd}$;

6) in compression-tension nodes, where ties are anchored in more than one direction, $\sigma_{Rd,max} = 0,75 \cdot \upsilon^{/} \cdot f_{cd}$;

7) where a load is distributed uniformly over an area A_0 and it is dispersed to a larger area A_{c1} (which is concentric to A_{c0}), the applied load $F_{Rd,u}$ should be imited as $F_{Rd,u} = A_{c0} \cdot f_{cd} \cdot \left(A_{c1} / A_{c0}\right)^{0,5}$, but limited to $3 \cdot A_{c0} / f_{cd}$.

In accordance with EN1992 [N3], strut and tie models may be used for design in ***ULS*** of continuity regions (cracked state of beams and slabs) and for the design in ***ULS*** and detailing of the discontinuity regions. In general these extend up to a distance *h* (section depth of member) from the discontinuity. Strut and tie models may also be used for members where a linear distribution within the cross section is assumed, e.g. plane strain.

Verification in ***SLS*** may also be carried out using strut-and-tie models, e.g. verification of steel stresses and crack width control, if approximate compatibility for strut-and-tie models is ensured (in particular, the position and direction of important struts should be oriented according to the linear elasticity theory).

Strut-and-tie models consist of struts representing compressive stress fields, of ties representing the reinforcement, and of the connecting nodes. The forces in the elements of a strut-and-tie model should be determined by maintaining the equilibrium with the applied loads in ultimate limit state. The elements of a strut-and-tie models should be dimensioned according to the rules given in Section 6.5 of the EN1992 [N3].

CHAPTER 3

MATERIALS PROPERTIES AND DURABILITY CRITERIA

3.1 CONCRETE

3.1.1 STRENGTH OF CONCRETE

In accordance with EN 206 [N4] normal weight concrete is a concrete in the oven-dry condition having a density greater than 2000 kg/m^3 but not exceeding 2600 kg/m^3. Concrete technology specifications shall satisfy the requirements of EN 206 [N4] and are not discussed at this book.

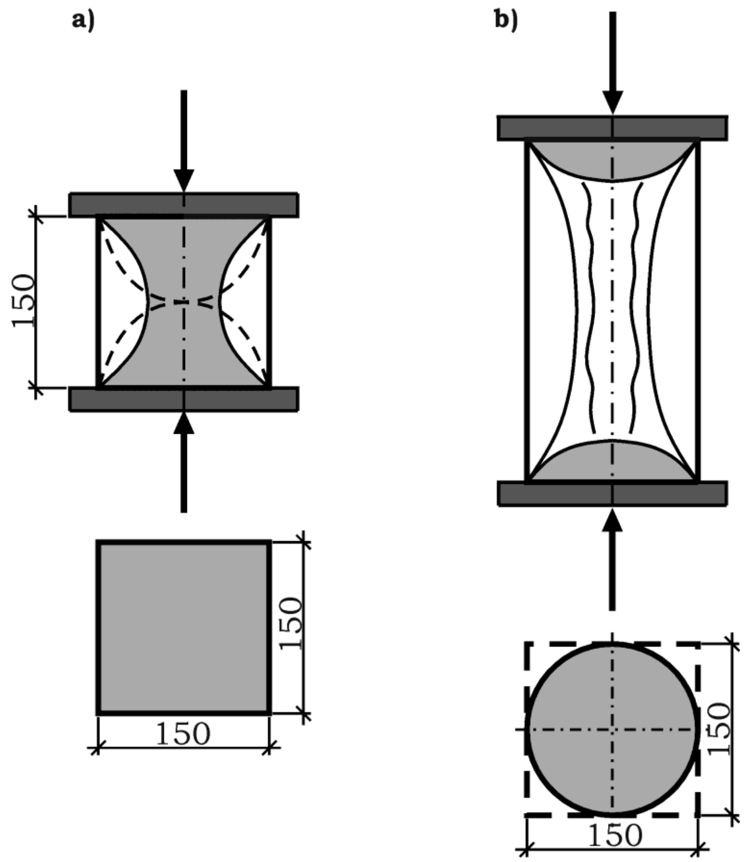
For the hardened concrete strength is the most important property which influences the ultimate resistance forces and moments in analyzed reinforced concrete structures.

Strength of concrete is defined as the maximum load which it can carry per unit area (stress). Under practical conditions concrete is seldom stressed in one direction only (uniaxial stress). Nevertheless, as assumed uniaxial stress-strain conditions can be used in many cases. Concrete strength changes with age and environment; therefore it is not possible to attribute the absolute values to it. Laboratory tests give only an indication of concrete properties in the structure.

3.1.1.1 Compressive strength

Compressive strength of concrete is usually taken as the maximum axial compressive force which it can carries per unit area. It is obtained either from cylinders or cubes with fixed dimentions as it is stated in [N4]. The specimens are loaded longitudinally at a slow rate. In accordance with [N5], testing results is valid for monotonically increasing compressive stresses or strains at a rate of $\dot{\sigma} \approx 0,6 \pm 0,4$ MPa/s or rate of $\dot{\varepsilon} \approx 15 \cdot 10^{-6}$ s^{-1} respectively.

Compressive strength of concrete is obtained through the elaboration of compression tests executed at 28 days on cylindrical specimens of diameter 150 mm and height 300 mm. As in many countries testing is carried out on 150 mm cubic specimens, EN 206 [N4] admits cube compressive strength too (see Figure 3.1-1).



a) – cube; b) – cylinder

Figure 3.1-1 – Specimens for compressive strength testing

For special requirements or in national code test specimens other than cylinders 150/300 mm and stored in other environments may be used to specify the concrete compressive strength. In such cases conversion factor (see Table 3.1-1) should either be determined experimentally or, when given in National Codes, used accordingly for given category of testing equipment.

In the case when concrete cubes 150 mm are used, the characteristic strength value shall be obtained for the various concrete classes of normal weight concrete.

Table 3.1-1 – Conversion factor values

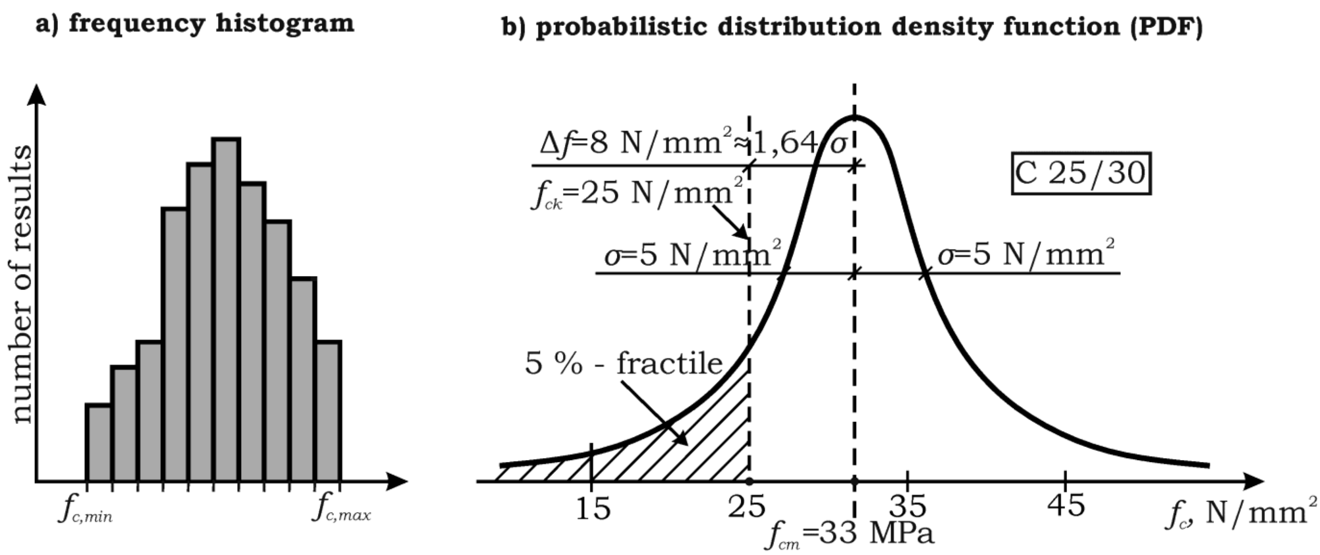
Specimen size (cube)	Conversion factor
150×150×150	1,0
100×100×100	0,95
200×200×200	1,05

So, in accordance with EN 1992 [N3], the compressive strength of concrete is denoted by concrete strength classes which relate to the characteristic (5 %) cylinder strength, f_{ck}, or the cube strength $f_{ck,cube}$.

How it is stated in EN 206 [N4]:

Characteristic strength of concrete, f_{ck} **,** is a value of strength below which 5 % of the population of all possible strength determinations of the volume of concrete under consideration, are expected to fall, and

Compressive strength class is a classification comprising the type of concrete, the minimum characteristic cylinder (150/300 mm) strength ($f_{ck,cyl}$) and the minimum characteristic cube strength (150 mm edge length) and is denoted as **C** $f_{ck,cyl}$ / $f_{ck,cube}$ (for example, C16/20). An exemplary histogram showing the frequency of compressive strength results from testing concrete of the one population is shown in Figure 3.1-2.



a) – frequency histogram; b) – probability density function

Figure 3.1-2 – Frequency histogram and probability density function for concrete compressive strength

The assumption of normal distribution of concrete strength forms the basis of determinations of the characteristic compressive strength, f_{ck}. For Normal (Gaussian) distribution, the probability of the strength lying outside the specified limits either side of the mean strength can be determined.

These limits which are given in Figure 3.1-2 can be expressed in terms of the estimator of the standard deviation *s*, as follows:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(f_{ci} - f_{cm}\right)^2}, \tag{3.1-1}$$

where: f_{ci} is the observed strength result;

f_{cm} is the mean value of compressive strength equal to $\frac{\sum_{i=1}^{n} f_{ci}}{n}$;

n is the number of test results.

The probability of the strength lying outside the range $(f_{cm} \pm t \cdot s)$ for different values t, are given in Table 3.1-2 (also see Figure 3.1-2) depending on the assumed probabilities (as a fractile of probability distribution).

Table 3.1-2 – Fractile values for the concrete strength

Probability of strength lying outside the range f_{cm} - $t \cdot s$	Coefficient t
1 %	2,33
2,5 %	1,96
5 %	1,64
10 %	1,23

Specified characteristic strength, f_{ck}, should be calculated by the expression:

$$f_{ck} = f_{cm} - t \cdot s, \tag{3.1-2}$$

where: s is the standard deviation obtained from Formula (3.1-1).

The characteristic strengths for f_{ck} and the corresponding mechanical characteristics are necessary for design, and given in Table 3.1-3.

In certain situations (e.g. prestressing) it may be appropriate to assess the compressive strength for concrete before or after 28 days, on the basis of test specimens stored under other conditions than standard conditions prescribed in EN 12390 [N7]. It may be required to specify the concrete compressive strength, $f_{ck}(t)$, at time t for a number of stages (e.g. demolding, transfer of prestress), where:

$$f_{ck}(t) = f_{cm}(t) - 8 \text{ for } 3 < t < 28 \text{ days.} \tag{3.1-3}$$

$$f_{ck}(t) = f_{ck} \text{ for } t \geq 28 \text{ days.} \tag{3.1-4}$$

Table 3.1-3 – Strength and deformation characteristics for concrete (Table 3.1 from EN 1992 [N3])

	Strength classes for concrete													
f_{ck} **(MPa)**	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{ck,cube}$ (1) **(MPa)**	15	20	25	30	37	45	50	55	60	67	75	85	95	105
f_{cm} (2) **(MPa)**	20	24	28	33	38	43	48	53	58	63	68	78	88	98
f_{ctm} (3) **(MPa)**	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0
$f_{ctk,0.05}$ (4) **(MPa)**	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5
$f_{ctk,0.95}$ (5) **(MPa)**	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6
E_{cm} (6) **(GPa)**	27	29	30	31	33	34	35	36	37	38	39	41	42	44
ε_{c1} **(‰)** (7)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8
ε_{cu1} **(‰)** (8)	3,5									3,2	3,0	2,8	2,8	2,8
ε_{c2} **(‰)** (9)	2,0									2,2	2,3	2,4	2,5	2,6
ε_{cu2} **(‰)** (10)	3,5									3,1	2,9	2,7	2,6	2,6
n (11)	2,0									1,75	1,6	1,45	1,4	1,4
ε_{c3} **(‰)** (12)	1,75									1,8	1,9	2,0	2,2	2,3
ε_{cu3} **(‰)** (13)	3,5									3,1	2,9	2,7	2,6	2,6

Notes: *Analytical relation/ Explanation:*

1. $f_{ck,cube} = f_{ck,cyl} / 0,8$;
2. $f_{cm} = f_{ck} + 8$, (MPa);
3. $f_{ctm} = 0,3 \cdot f_{ck}^{2/3}$ ≤C50/60; $f_{ctm} = 2,12 \cdot \ln\left(1 + \left(f_{cm} / 10\right)\right)$ >C50/60;
4. $f_{ctk,0.05} = 0,07 \cdot f_{ctm}$, 5 % fractile;
5. $f_{ctk,0.95} = 1,3 \cdot f_{ctm}$, 95 % fractile;
6. $E_{cm} = 22 \cdot \left[\left(f_{cm}\right) / 10\right]^{0,3}$, ($f_{cm}$ in MPa);
7. see Figure 3.1-8, $\varepsilon_{c1}(‰) = 0,7 \cdot f_{cm}^{0,31} \leq 2,8$;
8. see Figure 3.1-8, for $f_{ck} \geq 50$ MPa $\varepsilon_{cu1}(‰) = 2,8 + 27 \cdot \left[\left(98 - f_{cm}\right) / 100\right]^4$;
9. see Figure 3.1-9, for $f_{ck} \geq 50$ MPa $\varepsilon_{c2}(‰) = 2,0 + 0,085 \cdot \left(f_{ck} - 50\right)^{0,53}$;
10. see Figure 3.1-9, for $f_{ck} \geq 50$ MPa $\varepsilon_{cu2}(‰) = 2,6 + 35 \cdot \left[\left(90 - f_{ck}\right) / 100\right]^4$;
11. for $f_{ck} \geq 50$ MPa $n = 1,4 + 23,4 \cdot \left[\left(90 - f_{ck}\right) / 100\right]^4$;
12. see Figure 3.1-10, for $f_{ck} \geq 50$ MPa $\varepsilon_{c3}(‰) = 1,75 + 0,55 \cdot \left[\left(f_{ck} - 50\right) / 40\right]$;
13. see Figure 3.1-10, for $f_{ck} \geq 50$ MPa $\varepsilon_{cu3}(‰) = 2,6 + 35 \cdot \left[\left(90 - f_{ck}\right) / 100\right]^4$.

More precise values should be based on tests especially for $t \leq 3$ days.

The compressive strength of concrete at age t depends on the type of cement, temperature and curing conditions. As it is pointed in EN 1992 [N3], for a mean temperature of 20°C and curing conditions in accordance with EN 12390 [N7], the compressive strength of concrete at various ages $f_{cm}(t)$ is estimated as follows:

$$f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm}, \tag{3.1-5}$$

with:

$$\beta_{cc}(t) = \exp\left\{ s\left[1 - \left(\frac{28}{t} \right)^{1/2} \right] \right\}, \tag{3.1-6}$$

where: $f_{cm}(t)$ is the mean concrete compressive strength at the age of t days;

f_{cm} is the mean compressive strength at 28 days according to Table 3.1-3;

t is the age of concrete in days;

s is the coefficient which depends on the type of cement and it is equal to 0,20 (class R); 0,25 (class N); 0,38 (class S).

3.1.1.2 Tensile strength

The term tensile strength relates to the maximum stress which a concrete can carry when subjected to uniaxial tension. Tensile strength of concrete is generally less than 20 % of the compressive strength. It is difficult to measure there concrete strength in direct tension (see Figure 3.1-3 a) because of the technical problems of holding the specimens in axial load (force). The indirect methods have been developed for assessing this property.

The simplest and most widely used method is the split cylinder test (see Figure 3.1-3 b) in accordance with [N4]. This test entails diametrically loading a cylinder in compression along its entire length. This form of loading shown in Figure 3.1-3 b induces tensile stresses over the loaded diameter plan. Where the tensile strength is determined as splitting tensile strength, $f_{ct,sp}$, an approximate value of the axial tensile strength, f_{ct}, may be taken as:

$$f_{ct} = 0,9 \cdot f_{ct,sp}. \tag{3.1-7}$$

The magnitude of the induced tensile stress $f_{ct,sp}$ at failure is given by the expression:

$$f_{ct,sp} = \frac{2 \cdot F_u}{\pi \cdot d \cdot l}, \tag{3.1-8}$$

where: F_u is the maximum applied load;

l, d are the cylinder length and its diameter, respectively.

Tensile strength of concrete can also be evaluated by means of bending tests conducted on plain concrete prismatic specimens (beams) which have normally 150 mm square cross section. In this test a simply supported beam is loaded at its third points, as it is shown in Figure 3.1-3 c.

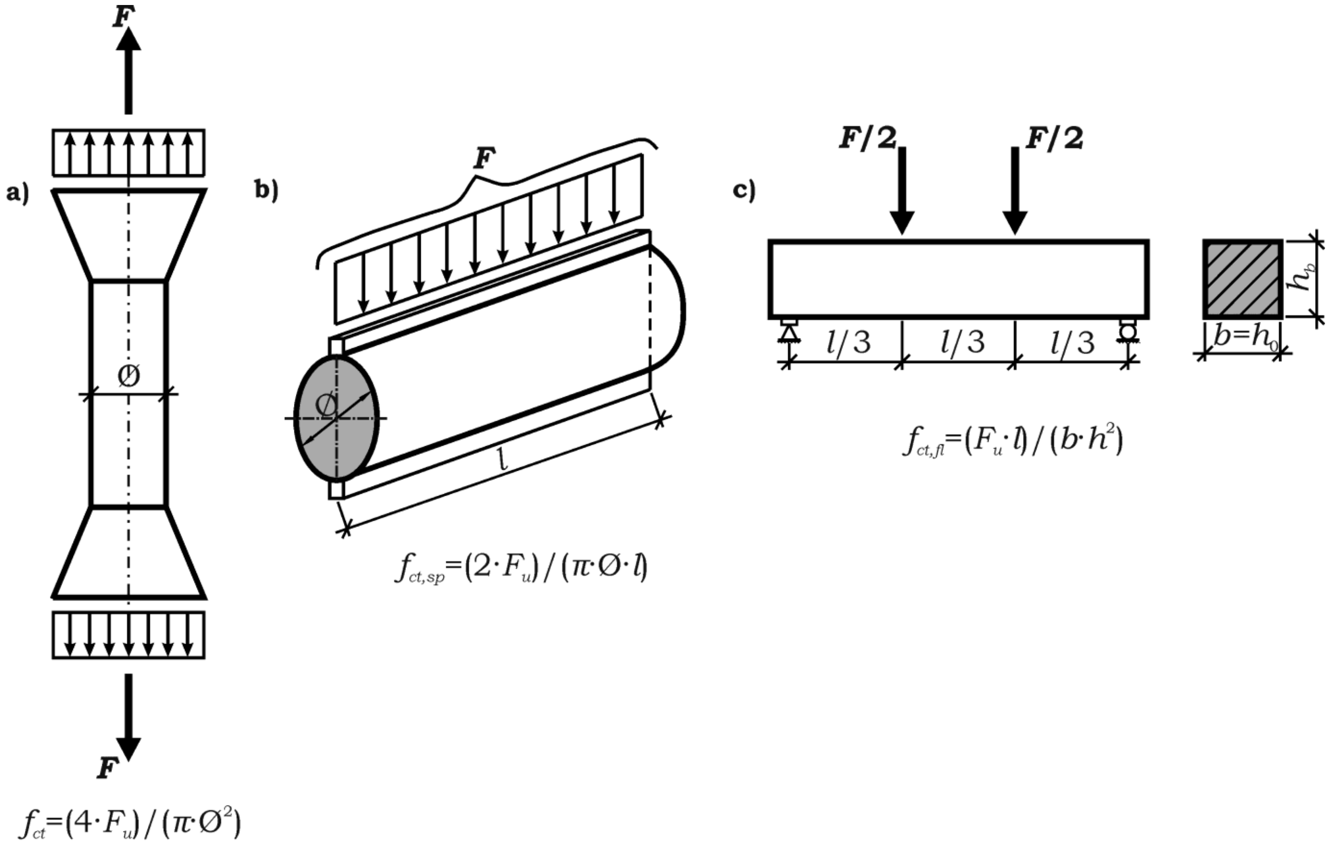
The resulting bending moments induce compressive and tensile stresses at the top and bottom of the beam. The beam fails in tension and the flexural strength $f_{ct,fl}$, is defined by:

$$f_{ct,fl} = \frac{F_u \cdot l}{b \cdot h^2}, \tag{3.1-9}$$

where: F_u is the maximum applied load;

l is a distance between the supports;

b, h are the beam width and depth, respectively.



a) – direct tension; b) – splitting; c) – bending

Figure 3.1-3 – Specimens for concrete tensile strength testing

According to EN 1992 [N3] requirements, these above mentioned methods can be used. However, for prediction of the axial tensile strength f_{ct} it is necessary to use the following conversion factors:

- for results from the flexure test ($f_{ct,fl}$):

$$f_{ct} = 0,5 \cdot f_{ct,fl}; \tag{3.1-10}$$

- for result from splitting test ($f_{ct,sp}$):

$$f_{ct} = 0,9 \cdot f_{ct,sp}. \tag{3.1-11}$$

In the absence of more accurate data, the mean tensile strength of concrete can be obtained as follows:

$$f_{ctm} = 0,30 \cdot \left(f_{ck}\right)^{2/3}, \tag{3.1-12}$$

where: f_{ctm} is the mean value of the tensile strength;

f_{ck} is the characteristic cylinder compressive strength of concrete.

Like for compressive strength the statistical approach for definition of the characteristic tensile strength of concrete f_{ctk} should be used (see Figure 3.1-2) according to Normal distribution. This strength can be determined from the following simplified equations which are constructed on the basis of Formula (3.1-12):

- for lower value of the characteristic strength (5 % fractile):

$$f_{ctk,0.05} = 0,70 \cdot f_{ctm}; \tag{3.1-13}$$

- for upper value of the characteristic strength (95 % fractile):

$$f_{ctk,0.95} = 1,30 \cdot f_{ctm}; \tag{3.1-14}$$

The development of tensile strength with time is strongly influenced by the curing and drying conditions as well as by the dimensions of the structural members. As a first approximation it may be assumed that the tensile strength $f_{ctm}(t)$ is equal to:

$$f_{ctm}(t) = \left[\beta_{cc}(t)\right]^{a} \cdot f_{ctm}, \tag{3.1-15}$$

where: $\beta_{cc}(t)$ follows from the Expression (3.1-6);

a is a coefficient that is equal to 1,0 for t<28 days and equal to 2/3 for t≥28 days.

The values for f_{ctm} are given in Table 3.1-3.

3.1.1.3 Classes of concrete

For design purposes the concrete should be classified into the classes which correspond to a specified value of the characteristic compressive strength. According to EN1992 [N3] the compressive strength of concrete is based on the cylinder strength $f_{ck,cyl}$ and simultaneously it can be associated with the cube strength, $f_{ck,cube}$.

3.1.2 DEFORMATION OF CONCRETE

Concrete deforms under load. Deformations increasing with applied load are commonly known as elastic deformations. If concrete is deformed with time under the constant load this phenomenon is known as time- dependent deformation, or creep.

3.1.2.1 Elastic deformation

The elastic deformations of concrete largely depend on its composition (especially the aggregates). The values given in EN 1992 [N3] should be regarded as indicative for general applications. However, they should be specifically assessed if the structure is likely to be sensitive to deviations from these general values. The modulus of elasticity of a concrete is controlled by the modulus of elasticity of its components. Compared to the use of quartzite aggregates, the modulus of elasticity can be increased by 20 % or decreased by 30 % only by changing the type of aggregate. Table 3.1-4 give the qualitative changes a_E in the modulus of elasticity for different types of aggregate.

Table 3.1-4 – Effect of type of aggregates on modulus of elasticity

Type of aggregate	$\boldsymbol{a_E}$	$\boldsymbol{E_{c0} \cdot a_E}$ **, [GPa]**
Basalt, dense limestone aggregates	1,2	25,8
Quartzite aggregates	1,0	21,5
Limestone aggregates	0,9	19,40
Sandstone aggregates	0,7	15,10

In general case, modulus of elasticity of concrete E_c, is defined as the ratio of load per unit area (stress, σ_c) to elastic deformation per unit of length (strain, ε_c):

$$E_c = \frac{\Delta\sigma_c}{\Delta\varepsilon_c}, \tag{3.1-16}$$

Since the concrete is not a perfectly elastic material the modulus of elasticity depends on adopted definition (see Figure 3.1-4). We can define the following approaches:

- initial tangent modulus E_c;
- secant modulus E_{cm}.

Approximate values for the modulus of elasticity E_{cm} secant value between $\sigma_c = 0$ and $\sigma_c = 0,4 \cdot f_{cm}$ for concretes with quartzite aggregates, are given in Table 3.1-4. As it was shown in Table 3.4, for limestone and sandstone aggregates the value should be reduced by 10 % and 30 % respectively. For basalt aggregates the value should be increased by 20 %. It should be noted, that the modulus of elasticity E_{ci} does not include the initial plastic strain due to its definition. While the limit for the stress σ_c reached in the ***SLS*** it set to $\sigma_c = 0,4 \cdot f_{cm}$ this stress level gives an upper limit for the reduction factor $a_i = E_c / E_c$ is increasing with increasing concrete strength. For concrete strength classes higher than C80/90 the difference between first loading up to $\sigma_c = 0,4 \cdot f_{cm}$ and the unloading branch is smaller than 3 % may be neglected. Note that E_{ci} is considered as the mean value of the tangent modulus of elasticity, hence $E_{ci} = E_{cm}$.

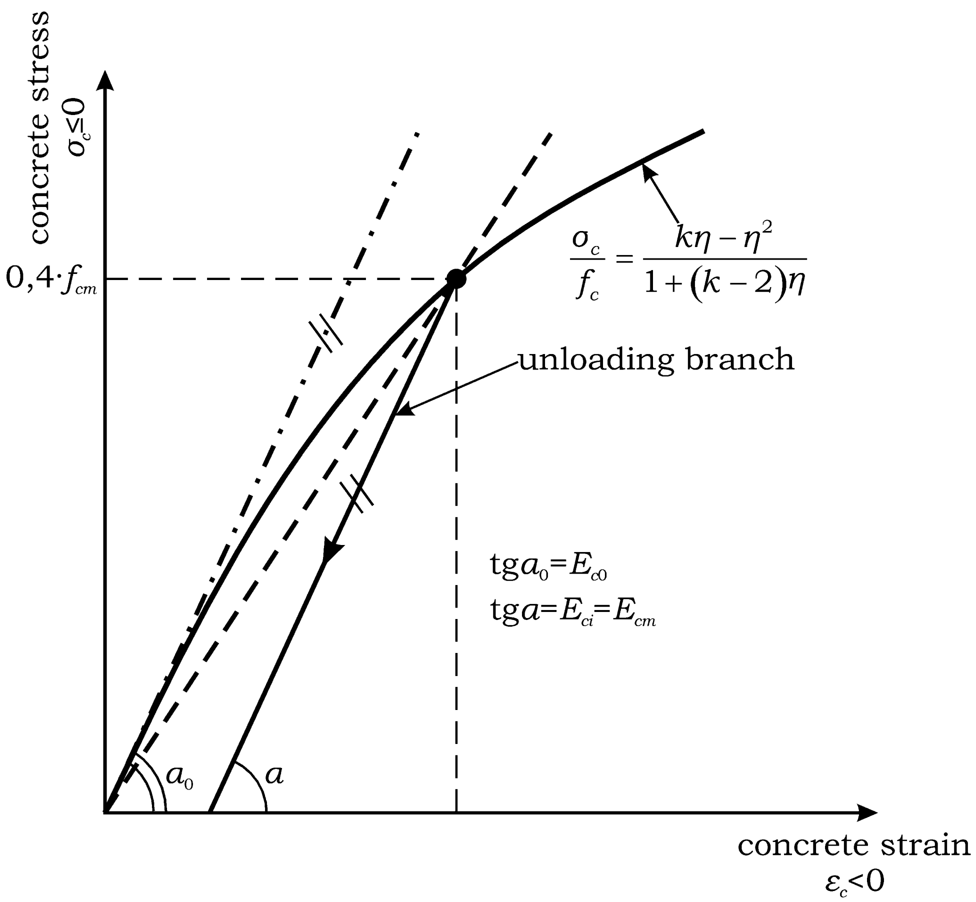


Figure 3.1-4 – Definition of different moduli of elasticity (according to *fib* Bulletin 42)

Where only an elastic analysis of a concrete structure is carried out, a reduced modulus of elasticity E_c according to Equation (3.1-17) should be used in order to account for initial plastic strain, causing some irreversible deformations:

$$E_c = a_i \cdot E_{ci}, \tag{3.1-17}$$

where:

$$a_i = 0,8 + 0,2 \cdot \frac{f_{cm}}{88} \leq 1,0. \tag{3.1-18}$$

How it was shown, the tangent modulus has a significance only for low stress level, whereas the secant modulus takes into account the total deformation (elastic and plastic, or irreversible) at any point. For definition of the secant modulus the relevant standards [N7] require repeated loading and unloading before the specimen is loaded for determination E_{cm} from a tested stress-strain curve according to Formula (3.1-16).

It depends only on strength class of concrete but, as well as, on adequate properties of the aggregate used. For most concrete classes (at 28 days) modulus of elasticity ranges from 27 GPa up to 44 GPa. The modulus E_{cm} in accordance with EN 1992 [N3] may be assumed in term of f_{cm} to be (see Table 3.1-3):

$$E_{cm} = 22 \cdot [f_{cm} / 10]^{0,3}, \tag{3.1-19}$$

where: f_{cm} is in MPa.

Variation of the modulus of elasticity with the time can be estimated by the following expression:

$$E_{cm}(t) = \left[\frac{f_{cm}(t)}{f_{cm}} \right]^{0,3} \cdot E_{cm}, \tag{3.1-20}$$

where: $E_{cm}(t)$ and $f_{cm}(t)$ are the values at an age of t days and both E_{cm} and f_{cm} are the values determined at an age of 28 days. The relation between $f_{cm}(t)$ and f_{cm} follows from Expression (3.1-5).

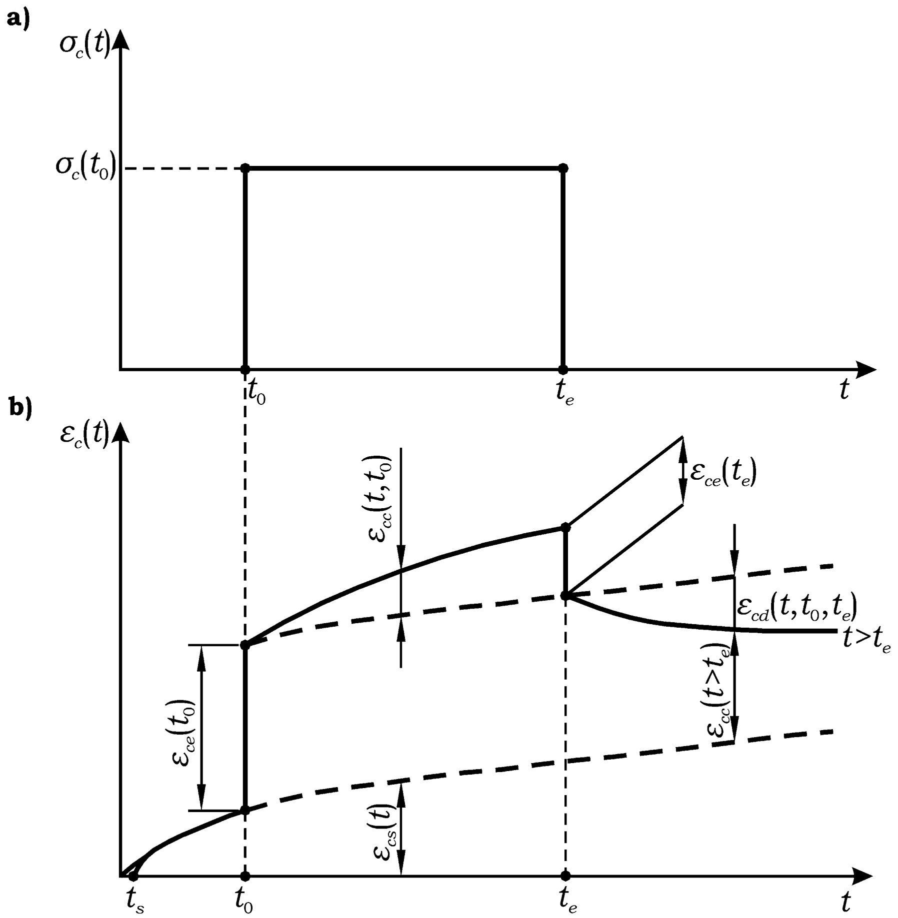
3.1.2.2 Poisson's ratio and coefficient of thermal expansion

The ratio between transverse strain and the strain in the direction of applied uniaxial loading, referred to as *Poisson's ratio* is usually found to be in range from 0,15 to 0,20 for concrete. For design purpose according to EN 1992 [N3] Poisson's ratio may be taken equal to 0,20 for uncracked concrete. If the cracking is permitted for concrete in tension, Poisson's ratio may be taken as zero.

Unless more accurate information is available, for design purpose, the *linear coefficient of thermal expansion* may be taken equal to $10 \cdot 10^{-6}$ K^{-1}.

3.1.2.3 Creep and shrinkage of concrete

Creep and shrinkage of the concrete depend on the ambient humidity, the dimensions of the element and the composition of the concrete. Creep is also influenced by the maturity of the concrete when the load is first applied and depends on the duration and magnitude of the loading. Figure 3.1-5 shows changing in time of the concrete strains under the load. In Figure 3.1-5: $\varepsilon_{ce}(t_0)$, $\varepsilon_{ce}(t_e)$ – elastic strains; $\varepsilon_{sc}(t)$ – shrinkage strain; $\varepsilon_{cc}(t,t_0)$ – creep strain; $\varepsilon_{cd}(t,t_e,t_0)$ – creep recovery.



a) – stresses diagram; b) – strains diagram

Figure 3.1-5 – Stresses and strains diagrams for time dependent deformations

3.1.2.3.1 Creep of concrete

Creep is generally defined as the time-dependent strain caused by a stress which is applied onto the material at certain time t_0, and this sress is maintained constant in time thereafter. According to this definition, if the specimen is simultaneously subjected to drying, temperature changes or other causes of deformation, to measure creep experimentally one must use at least two specimens subjected to exactly the same conditions except that one is loaded and the other remains load-free. Creep strains are than equal to excess strains experienced by the loaded specimen with respect to the unloaded specimen. The resulting strains are partially reversible, which can be measured in a loading/unloading cycle.

The creep deformation of concrete under the constant axial compressive stress is illustrated in Figure 3.1-5. The creep deformation proceeds at a decreasing rate with time. If the load is removed, the elastic strain is immediately recovered, however this recovered elastic strain is less than the initial elastic strain, because of the elastic modulus increasing with the age.

Creep deformations depend mainly on the ambient humidity, composition of the concrete and its maturity and also on the duration and magnitude of the loading.

The following two definition are used:

- ***creep coefficient***, denoted as $\varphi(t,t_0)$, expresses the delayed deformation with respect to the elastic strain (typical values fall in the range from 2,0 to 6,0 for the maximum attained creep strain);

- ***compliance function,*** denoted as $J(t,t_0)$, represents the creep strain per unit of imposed stress and it is used to compare the delayed strain that take place in concretes loaded at different stress levels (although the principle of superposition is valid until approximately 30 % of the peak load in compression test; it includes the elastic instantaneous compliance (also called «***specific creep***»);

- ***specific creep,*** denoted as $C(t,t_0)$, expresses only the delayed strains due to the application of a unit stress (i.e. it excludes the instantaneous elastic strain)

With these definition, the following relations are applied:

$$\varphi(t;t_0) = E_c(t_0) \cdot J(t;t_0), \tag{3.1-21}$$

$$C(t;t_0) = \frac{\varphi(t;t_0)}{E_c(t_0)}, \tag{3.1-22}$$

$$I(t;t_0) = \frac{1}{E_c(t_0)} + C(t;t_0), \tag{3.1-23}$$

where: t_0 is the age at loading;

t is the time at which strains are evaluated.

The creep coefficient $\varphi(t,t_0)$ is related to E_c, the tangent modulus, which one in it's turn may be taken equal to $1{,}05 \cdot E_{cm}$. Where accuracy is required, value of the creep coefficient $\varphi(t,t_0)$ at time t is calculated in accordance with EN 1992 [N3].

The creep coefficient $\varphi(t,t_0)$ may be calculated from the following expression:

$$\varphi(t;t_0) = \varphi_0 \cdot \beta_c(t;t_0), \tag{3.1-24}$$

where: φ_0 is the notional creep coefficient and it can be estimated from the following expression:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0), \tag{3.1-25}$$

where: φ_{RH} is a factor that takes into account the effect of the relative humidity on the value of the notional creep coefficient:

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}}, \text{ for } f_{cm} \leq 35\,\text{MPa}, \tag{3.1-26}$$

$$\varphi_{RH} = \left[1 + \frac{(1 - RH/100)}{0{,}1 \cdot \sqrt[3]{h_0}} \cdot a_1\right] \cdot a_2, \text{ for } f_{cm} > 35\,\text{MPa}, \tag{3.1-27}$$

where: RH is the relative humidity of the ambient environment (in %);
h_0 is the notional size of the member (see Figure 3.1-6) (in mm) and it is equal to $2 \cdot A_c / h$ (where: A_c is the cross-sectional area; h is the perimeter of the member in contact with the atmosphere).

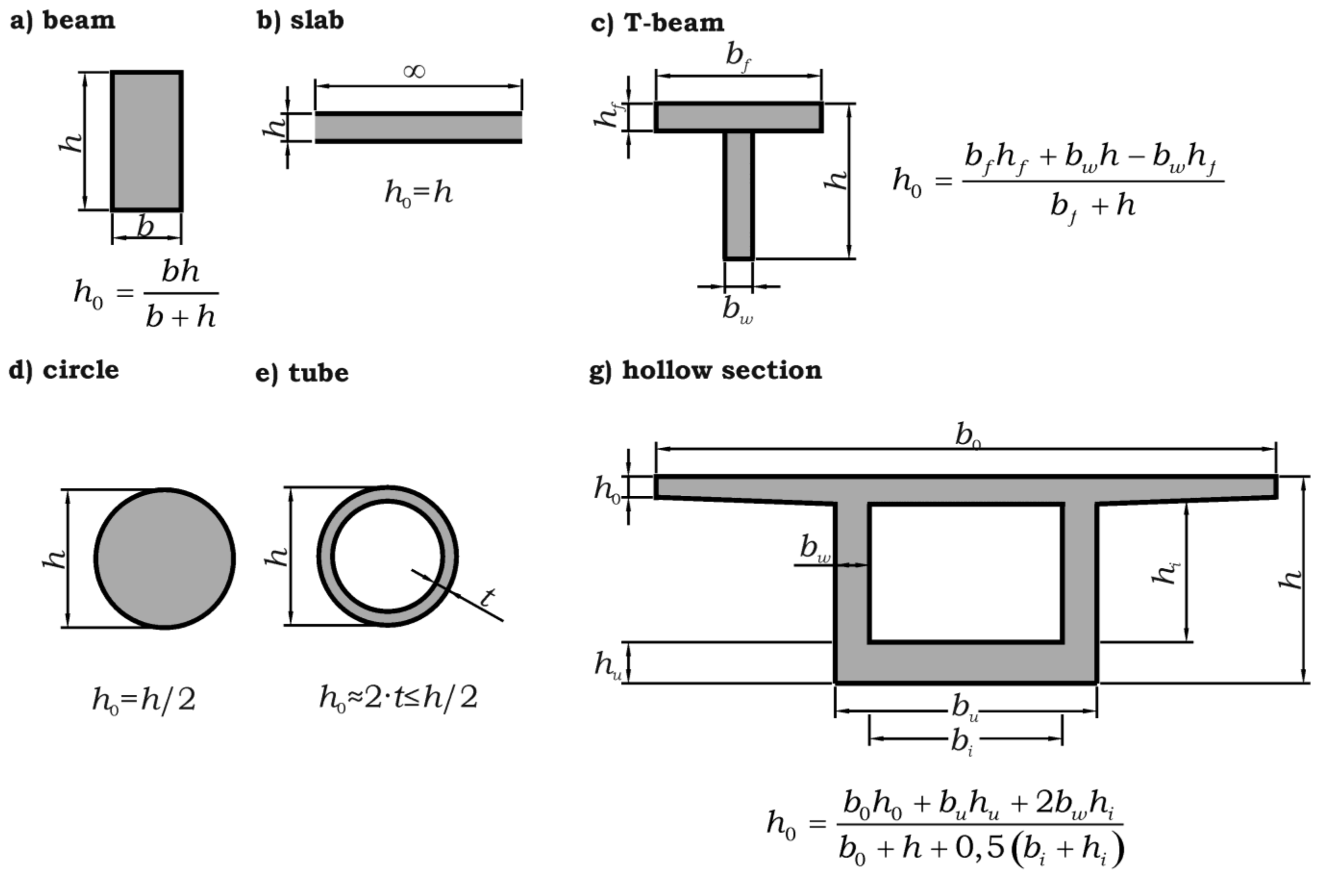


Figure 3.1-6 – For h_0 – value calculation

$\beta(f_{cm})$ is a factor that takes into account the effect of the concrete strength on the value of the notional creep coefficient:

$$\beta(f_{cm}) = \frac{16,8}{\sqrt{f_{cm}}}, \tag{3.1-28}$$

where: f_{cm} is the mean compressive strength of concrete in MPa at the age of 28 days;

$\beta(t_0)$ is a factor that takes into account the effect of the concrete age at loading on the value of the notional creep coefficient:

$$\beta(t_0) = \frac{1}{\left(0,1 + t_0^{0,2}\right)}. \tag{3.1-29}$$

$\beta_c(t;t_0)$ is a coefficient to describe the development of creep with the time after loading, and may be estimated using the following expression:

$$\beta_c(t;t_0) = \left[\frac{t - t_0}{\beta_H + t - t_0}\right], \tag{3.1-30}$$

where: t is the age of concrete in days at the moment considered;
t_0 is the age of concrete at loading in days;
$(t - t_0)$ is the non-adjusted duration of loading in days.

Coefficient β_H is a coefficient depending on the relative humidity (*RH*, %) and the notional member size (h_0 in mm). It may be estimated from the following expressions:

$$\beta_H = 1,5 \cdot \left[1 + (0,012 \cdot RH)^{18}\right] \cdot h_0 + 250 \le 1500, \text{ for } f_{cm} \le 35\,\text{MPa}, \tag{3.1-31}$$

$$\beta_H = 1,5 \cdot \left[1 + (0,012 \cdot RH)^{18}\right] \cdot h_0 + 250 \cdot a_3 \le 1500 \cdot a_3, \text{ for } f_{cm} > 35\,\text{MPa}, \tag{3.1-32}$$

$a_{1/2/3}$ are the coefficients to consider the influence of the concrete strength:

$$a_1 = \left[\frac{35}{f_{cm}}\right]^{0,7}; \quad a_1 = \left[\frac{35}{f_{cm}}\right]^{0,2}; \quad a_1 = \left[\frac{35}{f_{cm}}\right]^{0,5}. \tag{3.1-33}$$

The effect of cement type on the creep coefficient of concrete may be taking into account by modifying the age of loading t_0 in Expression (3.1-30) according to the following expression:

$$t_0 = t_{0,T}\left[\frac{9}{2+t_{0,T}^{1,2}}+1\right] \geq 0{,}5, \qquad (3.1\text{-}34)$$

where: $t_{0,T}$ is the temperature adjusted age of concrete at loading in days adjusted according to following Expression (3.1-35);

α is a power which depends on cement type and it is equal to -1 (for cement class S); 0 (for cement class N); 1 (for cement class R).

The effect of elevated or reduced temperatures within the range 0–80°C on the maturity of concrete may be taken into account by adjusting the concrete age according to the following expression:

$$t_{0,T} = \sum_{i=1}^{n} \exp\left[-\left(\frac{4000}{273+T(\Delta t_i)}+13{,}65\right)\right]\cdot \Delta t_i, \qquad (3.1\text{-}35)$$

where: t_T is the temperature adjusted concrete age which replaces t in the corresponding equation;

$T(\Delta t_i)$ is the temperature in °C during the time period Δt_i;

Δt_i is the number of days where a temperature T prevails.

The mean coefficient of variation of the above predicted creep data, deducted from a computerized data bank or laboratory test results, is of the order of 20 %.

The values of $\varphi(t;t_0)$ are given above should be associated with the tangent modulus E_c.

When a less accurate estimate is considered satisfactory, the value given in Figure 3.1-7 may be adopted for creep of concrete at 70 years.

The values given in Figure 3.1-7 are valid for ambient temperatures between -40°C and +40°C and a mean relative humidity between *RH*=40 % and *RH*=100 %.

$\varphi(\infty;t_0)$ is the final creep coefficient (at 70 years);

h_0 is the notional size (see Figure 3.1-6).

Where great accuracy is not required, the value found from the Figure 3.1-6 may be considered as the creep coefficient, provided that the concrete is not subjected to a compressive stress greater than $0{,}45\cdot f_{ck}(t_0)$ at an age of t_0, the age of concrete at the time of loading. The creep deformation of concrete $\varepsilon_{cc}(\infty;t_0)$ at time $t=\infty$ for a constant compressive stress σ_c applied at the concrete age t_0, is given by the following expression:

$$\varepsilon_{cc}(\infty;t_0) = \varphi(\infty;t_0)\cdot\left(\sigma_c/E_c\right). \qquad (3.1\text{-}36)$$

3.1.2.3.2 Shrinkage of concrete

Shrinkage of concrete is caused by the settlement of solid and the lost of free water from a plastic concrete (plastic shrinkage) by the hydration process (chemical combination of cement with water) (autogenous shrinkage) and by drying of concrete (drying shrinkage). A curve showing the increase in shrinkage strains with time appears in Figure 3.1-5.

Like creep, shrinkage deformations occur at a decreasing rate with time. In accordance with EN1992 [N3], the total shrinkage strains are composed of two components, the ***drying shrinkage*** strains and ***autogenous shrinkage*** strains. The drying shrinkage strains develop slowly, since it is a function of the migration of the water through the hardened concrete. The autogenous shrinkage strains develop during hardening of the concrete: the major part therefore develops in the early days after casting. Autogenous shrinkage is a linear function of the concrete strength. It should be considered specifically when new concrete is casted against hardened concrete.

Hence, the value of the total shrinkage strain ε_{cs} follows from:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}, \tag{3.1-37}$$

where: ε_{cs} is the total shrinkage strain;

ε_{cd} is the drying shrinkage strain;

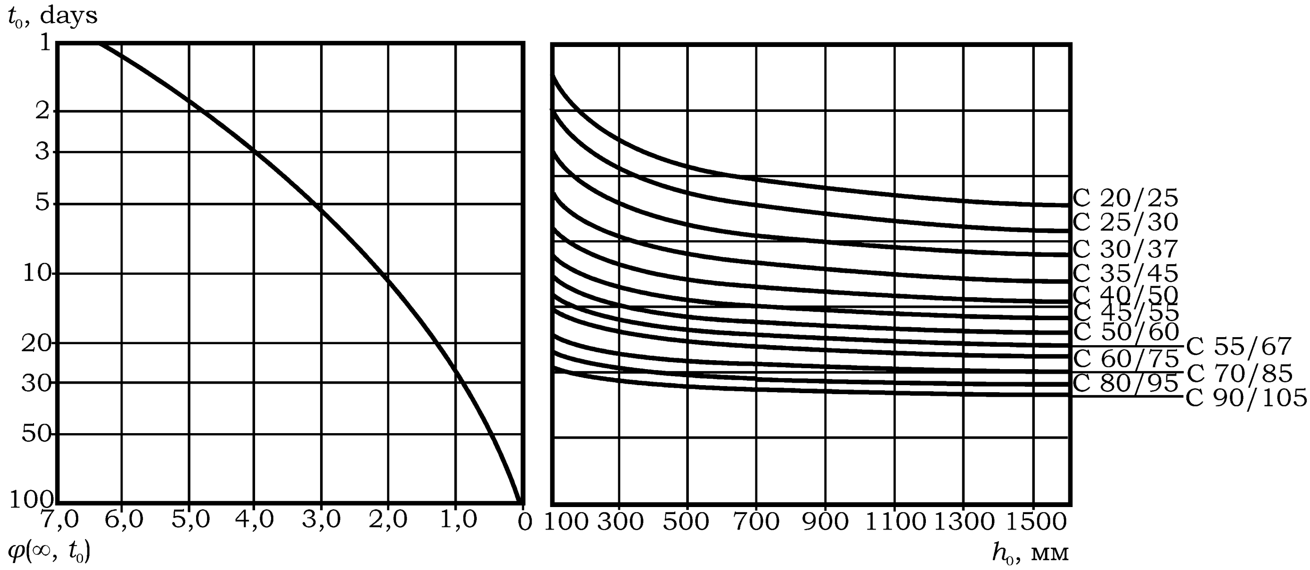
ε_{ca} is the autogenous shrinkage strain.

The final value of the drying shrinkage strain, $\varepsilon_{cd,\infty}$, , is equal to $K_h \cdot \varepsilon_{cd,0}$ .Value of the strain $\varepsilon_{cd,0}$, may be taken from Table 3.1-5 (expected mean values, with a coefficient of variation of about 30 %).

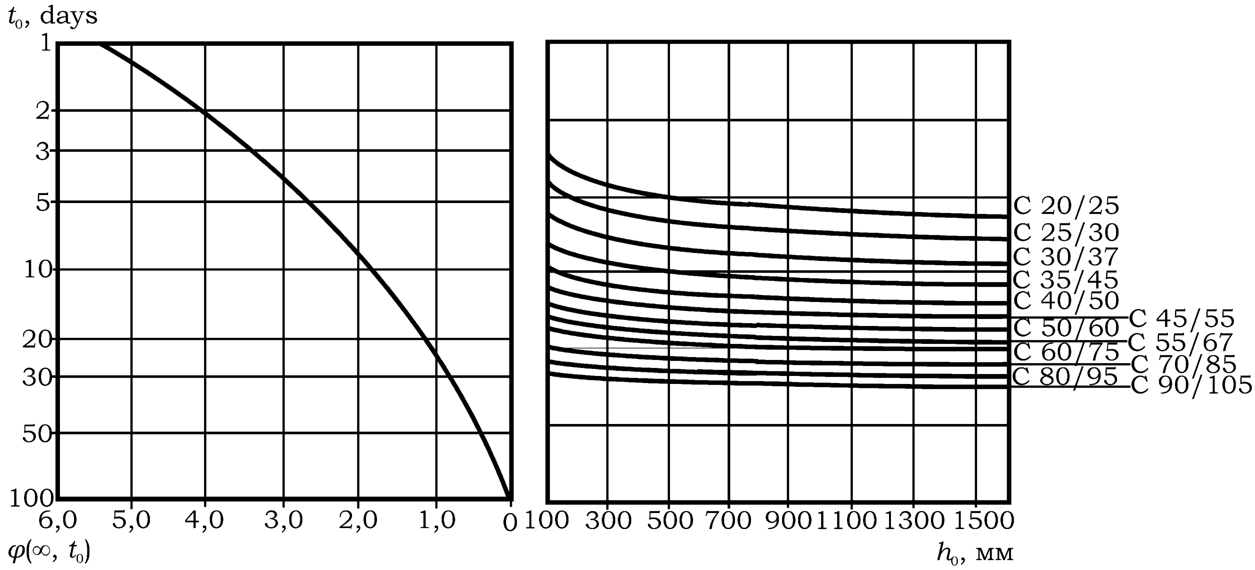
Table 3.1-5 – Nominal unrestrained drying shrinkage values $\varepsilon_{cd,0}$ (in ‰) for concrete with cement CEM Class N (Table 3.2 from EN1992 [N3])

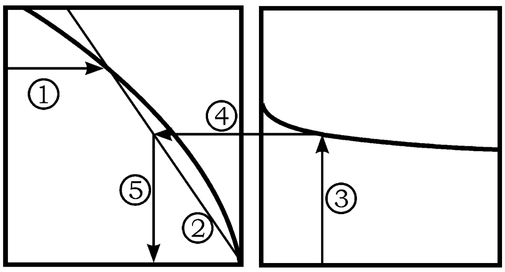
f_{ck} / $f_{ck,cube}$ (MPa)	Relative Humidity (in %)					
	20	**40**	**60**	**80**	**90**	**100**
20/25	0,62	0,58	0,49	0,30	0,17	0,00
40/50	0,48	0,46	0,38	0,24	0,13	0,00
60/75	0,38	0,36	0,30	0,19	0,10	0,00
80/95	0,30	0,28	0,24	0,15	0,08	0,00
90/105	0,27	0,25	0,21	0,13	0,07	0,00

a)



b)





Note:
- intersection point between lines 4 and 5 can also be above point 1;
- for t_0>100 it is sufficiently accurate to assume t_0=100 (and use the tangent line).

a) – inside conditions – RH=50 %; b) – outside conditions – RH=80 %
Figure 3.1-7 – Method for determining the creep coefficient $\varphi(\infty; t_0)$

for concrete under normal environmental conditions (Figure 3.1 from EN 1992 [N3])

The basic drying shrinkage strain $\varepsilon_{cd,0}$ in Table 3.1-5 was calculated in accordance with the following equation:

$$\varepsilon_{cd,0} = 0,85 \cdot \left[\left(220 + 110 \cdot a_{ds1} \right) \cdot \exp\left(-a_{ds2} \cdot \frac{f_{cm}}{f_{cm,0}} \right) \right] \cdot 10^{-6} \cdot \beta_{RH}, \tag{3.1-38}$$

$$\beta_{RH} = 1,55 \cdot \left[1 - \left(\frac{RH}{RH_0} \right)^3 \right], \tag{3.1-39}$$

where: f_{cm} is the mean compressive strength ($f_{cm,0}$ is equal to 10 MPa);

a_{ds1} and a_{ds2} are coefficients which depend on the cement type and equal to 3 and 0,13 respectively (for cement class S); to 4 and 0,12 respectively (for cement class N); to 6 and 0,11 respectively (for cement class R);

RH is the ambient relative humidity (%) (RH_0 is equal to 100 %);

The development of the drying shrinkage strain in time follows from:

$$\varepsilon_{cd,(t)} = \beta_{ds}\left(t;t_s\right) \cdot k_h \cdot \varepsilon_{cd,0}, \tag{3.1-40}$$

where: k_h is a coefficient depending on the notional size h_0 according to Table 3.1-6.

$$\beta_{ds}\left(t;t_s\right) = \frac{\left(t - t_s\right)}{\left(t - t_s\right) + 0,04 \cdot \sqrt{h_0^3}}. \tag{3.41}$$

Table 3.1-6 – Values for k_h in Expression (Table 3.3 from EN1992 [N3])

h_0	k_h
100	1,0
200	0,85
300	0,75
≥500	0,70

The autogenous shrinkage strain follows from:

$$\varepsilon_{ca}\left(t\right) = \beta_{as}\left(t\right) \cdot \varepsilon_{ca}\left(\infty\right), \tag{3.1-42}$$

where:

$$\varepsilon_{ca}\left(\infty\right) = 2,5 \cdot \left(f_{ck} - 10\right) \cdot 10^{-6}, \tag{3.1-43}$$

and:

$$\beta_{as}\left(t\right) = 1 - \exp\left(-0,2 \cdot t^{0,5}\right), \tag{3.1-44}$$

where: *t* is given in days.

3.1.3 STRESS-STRAIN RELATIONS

The relation between stress (σ_c) and strain (ε_c) is shown in Figure 3.1-8, obtained under short term uniaxial loading is considered as the generalized mechanical characteristic of the concrete.

For ***non-linear structural analysis*** the relation «σ_c-ε_c» is shown in Figure 3.1-8 (compressive stress and shortening strain are shown as absolute values) and in accordance with EN1992 [N3] it is described by the following expression:

$$\frac{\sigma_c}{f_{cm}} = \frac{k \cdot \eta - \eta}{1 + (k-2) \cdot \eta}, \tag{3.1-45}$$

where: η is equal to $\frac{\varepsilon_c}{\varepsilon_{c1}}$, where ε_{c1} is the strain at peak stress according to Table 3.1-3;

k is equal to $1{,}05 \cdot E_{cm} \cdot |\varepsilon_{c1}| / f_{cm}$.

Expression (3.1-45) is valid for $0<|\varepsilon_c|<|\varepsilon_{cu1}|$, where ε_{cu1} is the nominal ultimate compressive strain.

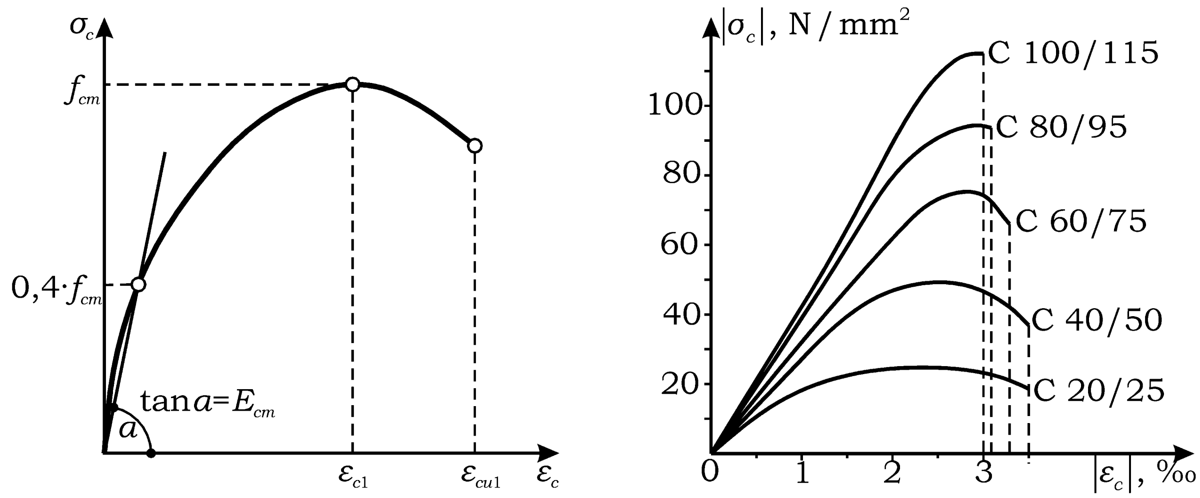


Figure 3.1-8 – Schematic representation of the stress-strain relation for structural analysis (the use $0{,}4 \cdot f_{cm}$ for the definition of E_{cm} is approximate) (Figure 3.2 from EN 1992 [N3])

As it is shown in EN 1992 [N3] other idealized stress-strain relations may be applied, if they adequately represent the behavior of the concrete considered.

For the ***design of cross-sections***, the following stress-strain relationship may be used, see Figure 3.1-9 (compressive strains are shown positive):

$$\sigma_c = f_{cd} \cdot \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^n \right], \text{ for } 0 \le \varepsilon_c \le \varepsilon_{c2}, \tag{3.1-46}$$

$$\sigma_c = f_{cd}, \text{ for } 0 \le \varepsilon_c \le \varepsilon_{c2}, \tag{3.1-47}$$

where: n is the exponent according Table 3.1-3;
ε_{c2} is the strain at reaching the maximum strength according to Table 3.1-3,
ε_{cu2} is the ultimate strain according to Table 3.1-3.

a) general case **b) modification with strength class increasing**

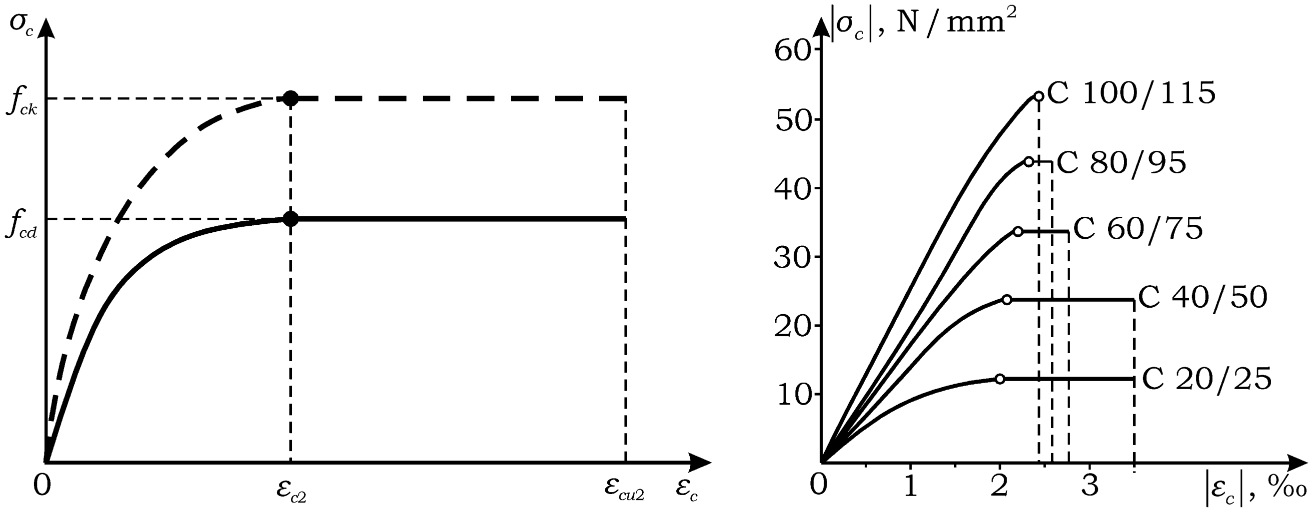


Figure 3.1-9 – Parabola-rectangle diagram for concrete under compression

Other simplified stress-strain relationship may be used if equivalent to or more conservative than the one defined above, for instance bi-linear according to Figure 3.1-10 (compressive stress and shortening strains are shown as absolute values) with values of ε_{c3} and ε_{cu3} according to Table 3.1-3.

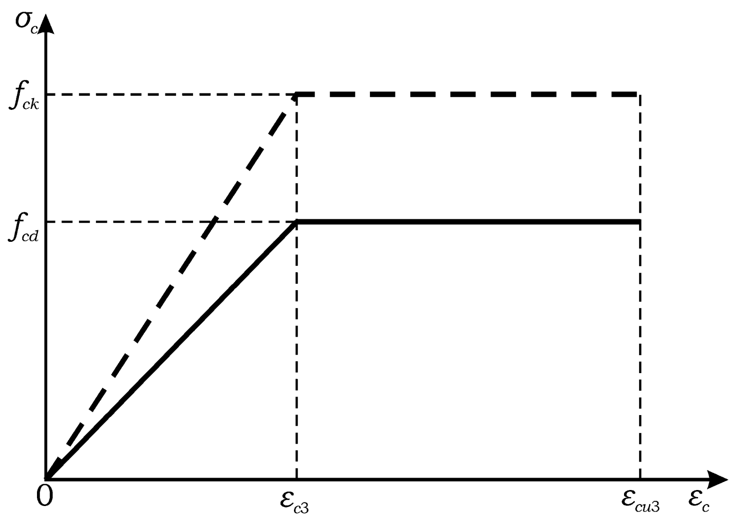


Figure 3.1-10 – Bi-linear stress-strain relation

A rectangular stress distribution (as given in Figure 3.1-11) may be assumed according to EN1992 [N3]. The factor a defining the effective height of the compression zone and the factor η, defining the effective strength, follows from:

$$a = 0{,}8, \text{ for } f_{ck} \leq 50 \text{ MPa}, \tag{3.1-48}$$

$$a = 0{,}8 - (f_{ck} - 50)/400, \text{ for } 50 \text{ MPa} < f_{ck} \leq 90 \text{ MPa}, \tag{3.1-49}$$

and:

$$\eta = 1{,}0, \text{ for } f_{ck} \leq 50 \text{ MPa}, \tag{3.1-50}$$

$$\eta = 1{,}0 - (f_{ck} - 50)/200, \text{ for } 50 \text{ MPa} < f_{ck} \leq 90 \text{ MPa}. \tag{3.1-51}$$

If the width of the compression zone decreases in the direction at the extreme compression fibre, the value $\eta \cdot f_{cd}$ should be reduced by 10 %.

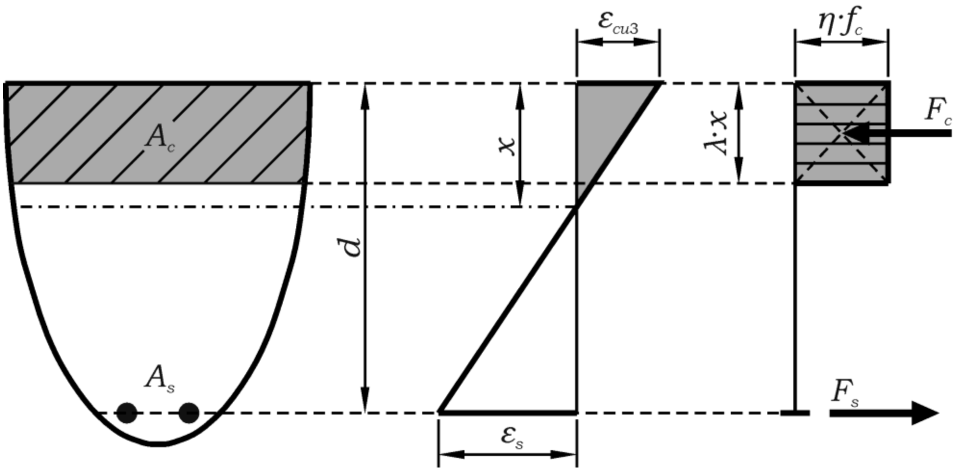


Figure 3.1-11 – Rectangular stress distribution (Figure 3.5 from EN 1992 [N3])

3.1.4 DESIGN COMPRESSIVE AND TENSILE STRENGTHS

The value of the design compressive strength is defined as follows:

$$f_{cd} = \frac{a_{cc} \cdot f_{ck}}{\gamma_c}, \tag{3.1-52}$$

where: a_{cc} is the coefficient, that takes into account long term effects on the compressive strength as well as unfavorable effects resulting from the way of the load applying. Recommended value is 1, but in general case the value of a_{cc} should fall in the diapason between 0,8 and 1,0;

γ_c is the partial safety factor for concrete, and it is equal to 1,5.

The value of the design tensile strength, f_{ctd}, is defined as:

$$f_{ctd} = \frac{a_{ct} \cdot f_{ctk,0.05}}{\gamma_c}, \tag{3.1-53}$$

where: a_{ct} is a coefficient, that takes into account influence of the long term effects on the tensile strength as well as the influence of unfavorable effects, resulting from the way the load applying. The recommended value is 1,0.

3.2 REINFORCING STEEL

3.2.1 GENERAL REQUIREMENTS

Steel reinforcing bars are generally round in cross section. To restrict the longitudinal movement of the bars relative to the surrounding concrete ribs are rolled on the bar surface. The nominal dimensions of a deformed bar are equivalent to those of plain bar having the same weight per unit length as the ribbed bar. The method of production, the specific characteristics, method of testing and attestation are included in EN 10080 [N8]. The requirements for properties of the reinforcement are for the material as placed in the hardened concrete. If site operations can affect the properties of the reinforcement, then those properties shall be verified after such operations.

Where other steels are used, which are not in accordance with EN10080 [N8], the properties shall be verified to be in accordance with EN1992 [N3] (see Sections from 3.2.2 to 3.2.6 and Table 3.2-1).

Table 3.2-1 – Properties of reinforcement (Table C.1 from EN1992 [N3])

Reinforcing steel	B 500A	B 500B	B 450C
Product	Rings, welded meshes	Bars, rings, welded meshes	Bars, rings, welded meshes
Ductility class	A	B	C
Yield strength, f_{yk} [N/mm²]	500	500	≥450 ≤550
Ratio $\left(f_t / f_y\right)_k$	≥1,05[2)]	≥1,08	≥1,15 ≤1,35
Strain at maximum load ε_{uk} [‰]	≥2,5[2)]	≥5,0	≥7,5
Deviation from nominal value of mass [‰]	±4,5 for Ø>8 mm ±4,5 for Ø≤8 mm		
Surface	ribbed		
Relative rib area f_r 5 mm<Ø≤6 mm 6,5 mm<Ø≤12 mm Ø≥12 mm	0,035 0,040 0,056		
Note: *1. The maximum value determined by means of tests may not exceed* $1{,}3 \cdot f_{yk}$. *2. For bars with Ø<6 mm:* $\left(f_t / f_y\right)_k \geq 1{,}03$ *and* $\varepsilon_{uk} \geq 2{,}0\ \%$.			

The values of f_{yk}, k and ε_{uk} in Table 3.2-1 are characteristic values. The maximum % of test results falling below the characteristic value is given for each of characteristic values in the right hand column of Table 3.2-2.

Table 3.2-2 – Properties of reinforcement (Table C.2N from EN1992 [N3])

Product form		Bars and de-coiled rods			Wire fabrics			Requirement or quantile value (%)
Class		A	B	C	A	B	C	–
Fatigue stress range (MPa) (for $N \geq 2 \cdot 10^5$ cycles) with an upper limit of $\beta \cdot f_{yk}$		≥150			≥100			10,0
Bond: Minimum relative rib area, $f_{R,min}$	Nominal bar size (mm)							5,0
	5-6	0,035						
	6,5 to 12	0,040						
	> 12	0,056						
Bond: Where it can be shown that sufficient bond strength is achievable with f_r values less than specified above, the values may be relaxed. In order to ensure that sufficient bond strength is achieved, the bond stresses should satisfy the recommended Expressions (C 1N) and (C 2N) when tested using the CEB/RILEM beam test: $\tau_m \geq 0,098 \cdot (80 - 1,2 \cdot \varnothing)$; $\tau_r \geq 0,098 \cdot (130 - 1,9 \cdot \varnothing)$, where ø is the normal bar size (mm); τ_m is the mean value of bond stress (MPa) at 0,01, 0,1 and 1mm slip; τ_r is the bond stress at failure by slipping.								

3.2.2 Properties of reinforcing steel

The required properties of reinforcing steel shall be verified using the testing procedures in accordance with EN 10080 [N8].

The behavior of reinforcing steel is specified by following properties:

– yield strength (f_{yk});

– maximum actual yield strength ($f_{y,max}$);

– tensile strength (f_t);

– ductility (ε_{uk} and f_t / f_{yk});

– bendability;

– bond characteristics (f_R);

– section sizes and tolerances;

– fatigue strength;

– weldability;

– shear and weld strength for welded fabric and lattice girders.

According by EN1992 [N3], this requirements applies to ribbed and weldable reinforcement, including fabric. The permitted welding methods are given in Table 3.2-1 and in Table 3.2-3.

The surface characteristics of ribbed bars shall be such to ensure adequate bond with the concrete. Adequate bond may be assumed by compliance with the specification of projected rib area, f_R EN 10080 [N8].

The yield strength f_{yk} and the tensile strength f_{tk} are defined respectively as the characteristic value of the yield load, and the characteristic maximum load in direct axial tension, each divided by the nominal cross section area.

Table 3.2-3 – Permitted welding processes and examples of application (Table 3.4 from EN 1992 [N3])

Loading case	Welding method	Bars in tension[1]	Bars in compression[1]
Predominantly static (see 6.8.1 (2))	flash-welding	butt joint	
	manual metal arc welding and metal arc welding with filling electrode	butt joint Ø≥20 mm, splice, lap, cruciform joints, joint with other steel members	
	metal arc active welding	splice, lap, cruciform[3] joints & joint with other steel members	
		–	butt joint with Ø≥20 mm
	friction welding	butt joint, joint with other steels	
	resistance spot welding	lap joint[4] confirm joint[2, 4]	
Not predominantly static (see 6.8.1 (2))	flash-welding	butt joint	
	manual metal arc welding	–	butt joint with Ø≥14 mm
	metal arc active welding	–	butt joint with Ø≥14 mm
	resistance spot welding	lap joint[4] confirm joint[2, 4]	
Notes: *1. Only bars with approximately the same nominal diameter may be welded together;* *2. Permitted ratio of mixed diameter bars ≥0,57;* *3. For bearing joints Ø≤16 mm;* *4. For bearing joints Ø≤28 mm.*			

The reinforcement shall have adequate ductility as defined by the ratio of tensile strength to the yield stress, $\left(f_t / f_{yk}\right)_k$ and the elongation at maximum force, ε_{uk}. values of $\left(f_t / f_{yk}\right)_k$ and ε_{uk} are given in Table 3.2-1 (for class A, B and C).

Figure 3.2-1 shows stress-strain curves for hot rolled and cold worked steel.

a) hot rolled steel **b) cold worked steel**

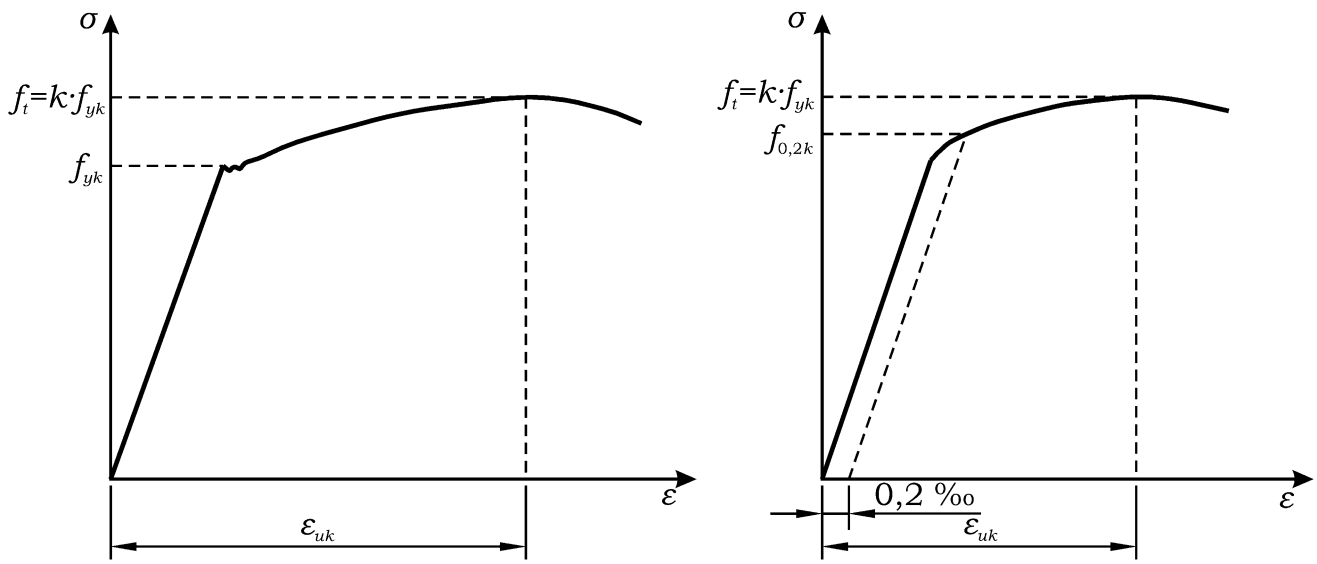


Figure 3.2-1 – Stress-strain diagrams of typical reinforcing steel (absolute values are shown for tensile stress and strain)

3.2.3 DESIGN ASSUMPTIONS

For normal design, either of following assumptions may be made (see Figure 3.2-2):

a) an inclined top branch with strain limit of ε_{ud} and a maximum stress of $k\ f_{yk} / \gamma_s$ at ε_{uk}, where $k = \left(f_t / f_{yk}\right)_k$;

b) a horizontal top branch without the need to check the strain limit.

The design values, f_{yd} of strength steel can be derived from the idealized characteristic diagram by dividing by the partial factor $\gamma_s = 1{,}15$.

The specified characteristic strength, f_{yk} of steel without a pronounced yield stress f_{yk} may be substituted by the stress corresponding to strain equal to 0,02.

For purpose of the designing the mean value of density may be assumed to be equal to 7850 kg/m^3, and design value of the modulus of elasticity, E_s, may be assumed to be 200 GPa.

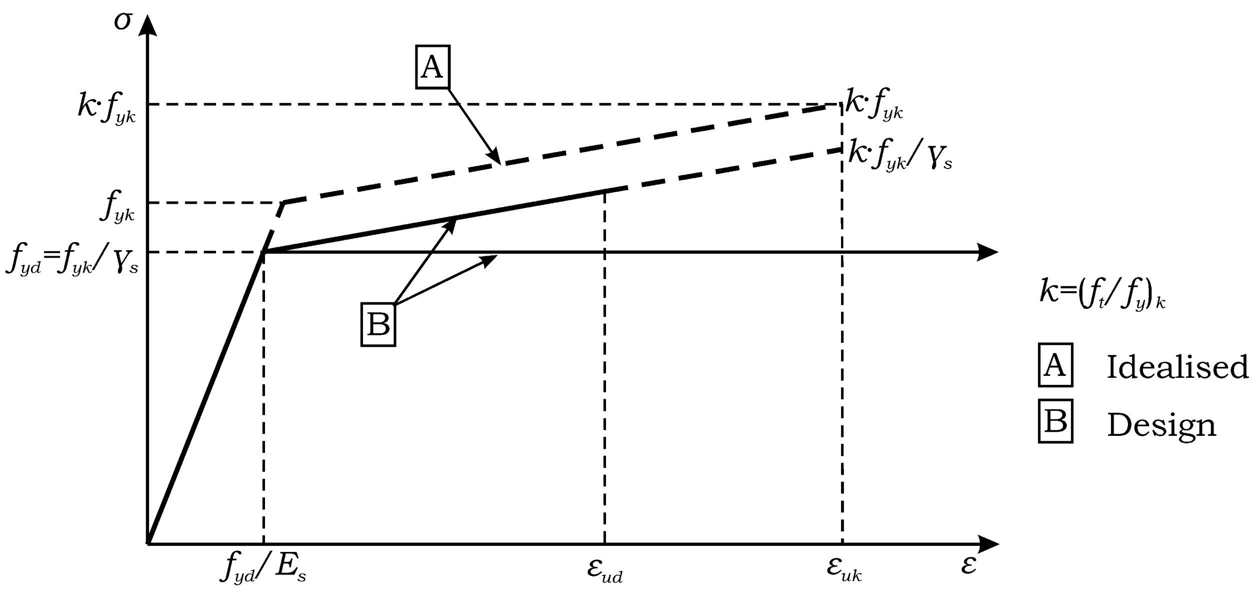


Figure 3.2-2 – Idealised and design stress-strain diagrams for reinforcing steel (for tension and compression) (Figure 3.8 from EN 1992 [N3])

3.3 DURABILITY AND CONCRETE COVER TO REINFORCEMENT

3.3.1 GENERAL REQUIREMENTS

Durability of any reinforced concrete structure may be affected both by the direct actions and by the indirect effects in connection with the structure being considered such as deformations, cracking or water absorption.

In accordance with EN1990 [N1] the structure shall be designed such that deterioration over its design working life does not impair the performance of the structure below that intended having due regard to its environment and the anticipated level of maintenance.

In order to archive an adequately durable structure, the following should be taken into account:

– the intended or foreseeable use of the structure and the expected environmental conditions (the aggressivity of environment to which the member is exposed. This is analogous to establishing the design loading where the ultimate or serviceability limit states are being considered);

– the required design criteria;

– the composition, properties and performance of the materials and products (select materials and design the structure to be able to resist the environment for a reasonable life time);

– the choice of the structural system;

– the shape of members and the structural detailing

– the quality of workmanship, and the level of control;

– the particular protective measures;

– the intended maintenance during the design working life.

3.3.2 CONCRETE DETERIORATION MECHANISM

3.3.2.1 Chemical attack

Chemical attack may be caused by:

– an aggressive environment;

– contact with liquids, gases or chemical solutions during service use (usually from exposure to acidic solution or sulphate salts);

– chlorides contained in concrete

– reactions between the materials in concrete.

In general, concrete has a low resistance to chemical attack. Chemical agents essentially react with certain compounds of the hardened cement paste. The most widely observed chemical corrosive effect are the effect of leaching and carbonation. Where the calcium hydroxide $Ca(OH)_2$ consisted in hardened concrete dissolves in water we have to do with the process of leaching. The effect is enhanced when the water acted on concrete structures is very soft. It has been established that the leaching process reduces the compressive strength of concrete:

– *Corrosion of reinforcement.* As it was shown in [3, 8] normal circumstances, the highly alkaline nature of concrete protects steel embedded within it. Except under the circumstances discussed below, the *pH* value of the pore solution in concrete is in region of 12–14. Steel will not generally corrode in uncontaminated concrete, untie of the pH drops below 10. The protection is afforded by the formulation of a very thin, coherent layer of iron oxide over the surface of the bar under alkaline conditions. Steel protected in this way is described as being in passive state. Two mechanisms can lead to the destruction of the passive state. There are:

– *Carbonation of the concrete.* This is reaction between carbon dioxide in the atmosphere and the alkalis in the cement matrix. The process of carbonation takes place where the water contacting with concrete contains the ion of carbone dioxide – CO_2 or an organic acids. Then free carbon dioxide reacts with water, so: $CO_2 + H_2O \rightarrow H_2CO_3$, and then we have to do with process of carbonation, according to chemical formula: $Ca(OH)_2 + H_2CO_3 \rightarrow CaCO_3 + 2H_2O$. This process starts at the surface, and, with time, penetrates slowly into the concrete. The rate of penetration of carbonation into the concrete is related with the environment and the quality of the concrete. The rate is fastest where the relative humidity is in the range of 50–60 %. It is slower at higher humidities, being effectively zero at 100 %. Good-quality concrete carbonates more slowly than poor quality material. The speed of the phenomenon depends on the rate at which carbon dioxide can diffuse into the concrete. It will decrease with the water-to-cement ratio decreasing and, hence, with the strength increasing. The effect of carbonation is to reduce the alkalinity of the concrete to a level where the natural protection is lost, and corrosion may then occur if the concrete, immediately surrounding the reinforcement, is carbonated. For the most building structures the above given effects may be avoided by adopting an appropriate material specifications for achieving a dense and impermeable concrete.

–*The presence of chlorides* in the concrete. Chlorides have the capacity to destroy the passivity of steel even where alkalinity remains high. It usually occurs locally, giving rise to «pitting corrosion». Chlorides may be involved into the concrete from the various sources, but the commonest are seawater in marine environments, de-icing chemicals on the roads and additives such as a calcium chloride, which was used extensively in the past as an accelerator. The rate at which chlorides penetrate into the concrete depends upon the rate of application of chlorides to the concrete surface and, as with carbonation, on the quality of the concrete.

Once the passivity of the steel has been destroyed, corrosion can occur if there is 1) sufficient moisture and 2) sufficient oxygen. It is found that these two requirements can act against the each other since, if the concrete is wet, oxygen cannot penetrate and, if it is dry, so that there is plentiful oxygen, there is insufficient moisture for the progress of the reaction. As a result, the greatest risk of corrosion is in members subjected to the alternate wetting and drying.

The normal way to design against the corrosion is to ensure that there is an adequate concrete cover to the reinforcement and that the concrete in the cover region is of a high quality and it is well cured. In particularly aggressive environments, however, there are other, more expensive measures which may be taken.

Possibilities are:

–usage of the reinforcement coated with epoxy or similar. Over the recent years, this approach seems to have been somewhat discredited due to some high-profile problem cases;

–usage of the stainless steel reinforcement;

–applying of the surface coating to the concrete to inhibit the ingress of chlorides or carbon dioxide. Such coatings would have to be meticulously maintained to be successful for a long period of time;

–applying of the cathodic protection to the structure.

3.3.2.2 Physical attack

Physical attack may be arise from the following actions:

– abrasion effects: abrasion of concrete surfaces may occur due to trafficking of the concrete or due to sand or gravel suspended in turbular water;

– freeze-thaw actions (frost attack): if saturated concrete is subjected to frequent freezing and thawing, the expansive effects of ice formation will disrupt the concrete.

For typical building structures, physical attack can be neutralized through an appropriate material specification combined with an adequate limitation of cracking.

Physical attack, arising from e.g.:

– temperature change;

– abrasion;

– water penetration (EN206 [N4]).

The composition of the concrete affects both the protection of the reinforcement and the resistance of the concrete to attack. Table 3.3-1 gives an indicative strength classes for the particular environmental exposure classes. This may lead to the choice of higher strength classes than required for the structural design.

Table 3.3-1 – Indicative minimum strength class (Table E.1N from EN1992 [N3])

Exposure Classes according to Table 4.1 from EN 1992 [N3]										
Corrosion										
	Carbonation-included corrosion				Chloride-included corrosion			Chloride-included corrosion from sea-water		
	XC1	XC2	XC3	XC4	XD1	XD2	XD3	XS1	XS2	XS3
Indicative minimum strength class	C20/25	C25/30	C30/37		C30/37		C35/45	C30/37	C35/45	
Damage to Concrete										
	No risk	Freeze/Thaw Attack			Chemical Attack					
	X0	XF1	XF2	XF3	XA1	XA2	XA3			
Indicative minimum strength class	C12/15	C30/37	C25/30	C30/37	C30/37		C35/45			

3.3.3 ENVIRONMENTAL CONDITIONS

Environmental conditions are classified according to Table 3.3-2, based on EN206 2014 [N4]. In addition to the conditions in Table 3.3-2, particular forms of aggressive or indirect action should be considered including:

Chemical attack, arising from e.g.:

– the usage of the building or the structure (storage of liquids, etc.);
– solution of acids or sulfate salts (EN206 [N4]);
– chlorides contained in the concrete (EN206 [N4]);
– alkali-aggregate reaction (EN206 [N4]).

Table 3.3-2 – Exposure classes related to environmental conditions in accordance with EN 206-1 (Table 4.1 from EN 1992 [N3])

Class designation	Description of the environment	Informative examples, where exposure classes may occur
1. No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack. For concrete with reinforcement or embedded metal: very dry.	Concrete inside the buildings with very low air humidity.
2. Corrosion induced by carbonation		
XC1	Dry or permanently wet.	Concrete inside the buildings with low air humidity. Concrete permanently submerged in water.
XC2	Wet, rarely dry.	Concrete surfaces subjected to the long-term contact. Many foundations.

Table 3.3-2 (end)

XC3	Moderate humidity.	Concrete inside the buildings with moderate or high air humidity. External concrete sheltered from the rain.
XC4	Cyclic wet and dry.	Concrete surfaces subjected to the water contact, not within the exposure class XC2.
3. Corrosion induced by chlorides		
XD1	Moderate humidity.	Concrete surfaces exposed to the airborne chlorides.
XD2	Wet, rarely dry.	Swimming pools. Concrete components exposed to the industrial waters containing chlorides.
XD3	Cyclic wet and dry.	Parts of bridges exposed to the spray containing chlorides. Pavements. Car park slabs.
4. Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water.	Structures nears to or on the coast.
XS2	Permanently submerged.	Parts of marine structures.
XS3	Tidal, splash and spray zones.	Parts of marine structures.
5. Freeze/Thaw Attack		
XF	Moderate water saturation, without de-icing agent.	Vertical concrete surfaces exposed to rain and freezing.
XF2	Moderate water saturation, with de-icing agent.	Vertical concrete surfaces of the road structures exposed to freezing and airborne de-icing agents.
XF3	High water saturation, without de-icing agents.	Horizontal concrete surfaces exposed to the rain and freezing.
XF4	High water saturation with de-icing agents or sea water.	Road and bridge decks exposed to the deicing agents. Concrete surfaces exposed to the direct spray containing de-icing agents and freezing. Splash zone of marine structures exposed to the freezing.
6. Chemical attack		
XA1	Slightly aggressive chemical environment according to Table 2 from EN 206 [N4].	Natural soils and ground water.
XA2	Moderately aggressive chemical environment according to Table 2 from EN 206 [N4].	Natural soils and ground water.
XA3	Highly aggressive chemical environment according to Table 2 from EN 206 [N4].	Natural soils and ground water.

3.3.4 CONCRETE COVER

The concrete cover is the distance between the surface of the reinforcement closest to the nearest concrete surface (including links and stirrups and surface reinforcement where relevant) and the nearest concrete surface.

The nominal cover shall be specified on the drawings. It is defined as a minimum cover, c_{min}, plus an allowance in design for deviation, Δc_{dev}:

$$c_{nom} = c_{min} + \Delta c_{dev}. \qquad (3.3\text{-}1)$$

Minimum concrete cover, c_{min}, shall be provided in order to ensure:

– the safe transmission of bond forces;

– the protection of the steel against the corrosion;

– an adequate fire resistance.

The great value for c_{min}, satisfying the requirements for the both bond and environmental conditions shall be used:

$$c_{min} = max\left\{c_{min,b};\ c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add};\ 10\ \text{mm}\right\}, \qquad (3.3\text{-}2)$$

where: $c_{min,b}$ is a minimum cover due to bond requirement;

$c_{min,dur}$ is a minimum cover due to environmental conditions;

$\Delta c_{dur,\gamma}$ is an additive safety element;

$\Delta c_{dur,st}$ is a reduction of minimum cover for use of stainless steel;

$\Delta c_{dur,add}$ is a reduction of minimum cover for use of additional protection.

In order to transmit bond forces safely and to ensure adequate compaction of the concrete, the minimum cover should be no less than $c_{min,b}$ given in Table 3.3-3.

Table 3.3-3 – Minimum cover, $c_{min,b}$, requirements with regard to bond (Table 4.2 from EN 1992 [N3])

Bond Requirement	
Argument of bars	Minimum cover $c_{min,b}$*
Separated	Diameter of bar
Bundled	Equivalent diameter ($Ø_n$)
Note: *If the nominal maximum aggregate size is greater than 32 mm, $c_{min,b}$, should be increased by 5 mm.*	

The minimum cover values for reinforcement in normal weight concrete taking account the exposure classes and the structural classes is given by $c_{min,dur}$ in EN1992 [N3]. The recommended structural class (design working life of 50 years) is *S4* for the indicative concrete strengths given in Table 3.3-1, and the recommended modifications to the structural class is given in Table 3.3-4. The recommended minimum structural class is *S1*. The recommended values of $c_{min,dur}$ are given in Table 3.3-5.

Table 3.3-4 – Recommended structural classification (Table 4.3 N from EN1992 [N3])

Structural Class							
Criterion	**Exposure Class according to Table 4.1 from EN 1992 [N3]**						
	X0	XC1	XC2/ XC3	XC 4	XD1	XD2/XS1	XD3/XS2/ XS3
Design working life of 100 years	Increase class by 2	Increase class by 2	Increase class by 2	Increase class by 2	Increase class by 2	Increase class by 2	Increase class by 2
Strength Class[1)2)]	≥C30/37 Reduce class by 1	≥C30/37 Reduce class by 1	≥C35/45 Reduce class by 1	≥C40/50 Reduce class by 1	≥C40/50 Reduce class by 1	≥C40/50 Reduce class by 1	≥C45/55 Reduce class by 1
Member with slab geometry (position of reinforce-ment not affected by construction process)	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1
Special quality control of the concrete production ensured	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1	Reduce class by 1

Notes to Table 3.3-3: *1. The strength class and w/c ratio are considered to be related values. A special composition (type of cement, w/c value, fine fillers with the intent to produce low permeability may be considered.*

2. The limit may be reduced by one strength class if air entrainment of more than 4 % is applied.

Table 3.3-5 – Values of the minimum cover $c_{min,dur}$ requirements with regard to durability for reinforcement steel in accordance with EN 10080 (Table 4.4N from EN1992 [N3])

Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	**Exposure Class according to Table 4.1 [N3]**						
	X0	XC1	XC2/XC3	XC4	XD1/XS1	XD2/XS2	XD3/XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

Where in-situ concrete is placed against the other concrete elements (precast or in-situ), the minimum concrete cover of the reinforcement to the interface may be reduced to a value corresponding to the requirement for the bond provided that:

– the strength class of concrete is at least C25/30;

– the exposure time of the concrete surface to an outdoor environment is short (<28 days);

– the interface has been roughened.

To calculate the nominal cover, c_{nom}, an addition to the minimum cover shall be made in design to allow for the deviation (Δc_{dev}). The required minimum cover shall be increased by the absolute value is $\Delta c_{dev} = 10$ mm in certain situations, the accepted deviation and, hence, allowance, Δc_{dev}, may be reduced. The recommended values are:

– where fabrication is subjected to a quality assurance system, in which the monitoring includes measurements of the concrete cover, the allowance in design for deviation Δc_{dev} may be reduced: $5 \text{ mm} \leq \Delta c_{dev} \leq 10 \text{ mm}$.

– where it can be assured that a very accurate measurement device is used for monitoring and non-conforming members are rejected (e.g. precast elements), the allowance in design for deviation Δc_{dev} may be reduced as follows: $0 \text{ mm} \leq \Delta c_{dev} \leq 10 \text{ mm}$.

CHAPTER 4

ULTIMATE LIMIT STATES

4.1 BENDING AND BENDING WITH AXIAL LOAD

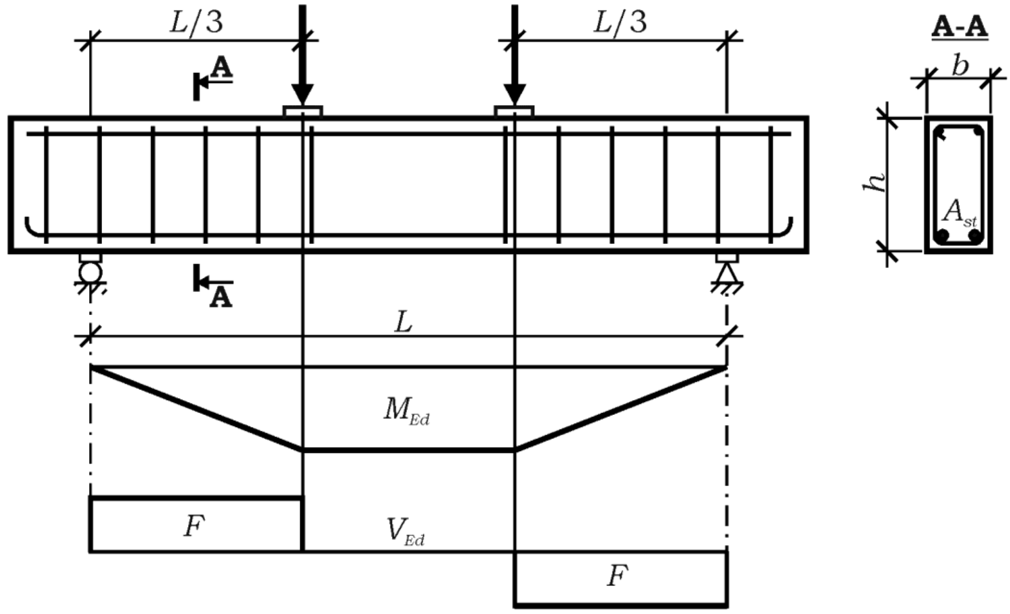
4.1.1 BENDING

4.1.1.1 Flexural behavior of beams

Stage I (Uncracked concrete sections).

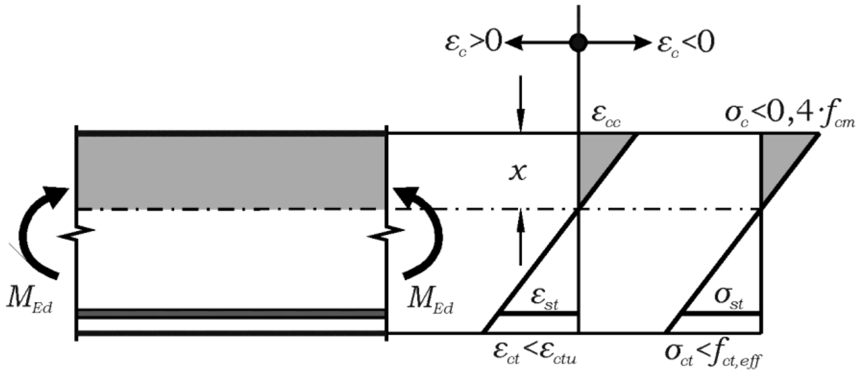
Before cracking, the concrete strains and stresses in compression and tension zones increase with load increasing. At small loads when the tensile strains ε_{ct} are less than value of the concrete ultimate strain in tension ε_{ctu}, the entire cross-section of the beam resists bending moment, with compression on the top face and tension on the bottom side (see Figure 4.1-1). The distribution of strains and stresses in cross-section of the beam for these stage Ia is shown in Figure 4.1-1. As it is shown in Figure 4.1-1 strains and stresses are distributed linearly.

As the load is increased tensile strain ε_{ctu} in the bottom of the cross-section of the beam reaches value close to the ultimate tensile strain value ε_{ctu} and distribution of the tensile stresses becomes to be non-linear in accordance with relationship «$\sigma_{ct} - \varepsilon_{ct}$» for concrete in tension (see Figure 4.1-1, stage Ia). The compressive stresses in concrete are distributed linearly by the height (depth) of the compression zone of the cross-section in accordance with curve «$\sigma_c - \varepsilon_c$» for concrete (see Figure 1, stage Ia). First cracking occurs at the weakest cross-section and this is usually assumed to occur when the concrete tensile strain ε_{ct} reaches the ultimate value ε_{ctu}.

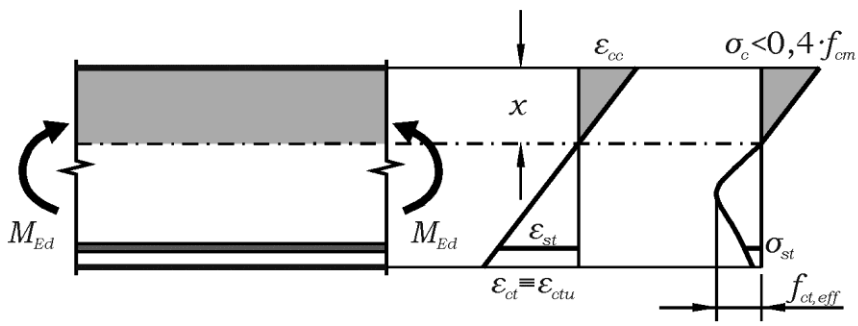


Stage I (Uncracked section)

I a)



I b)



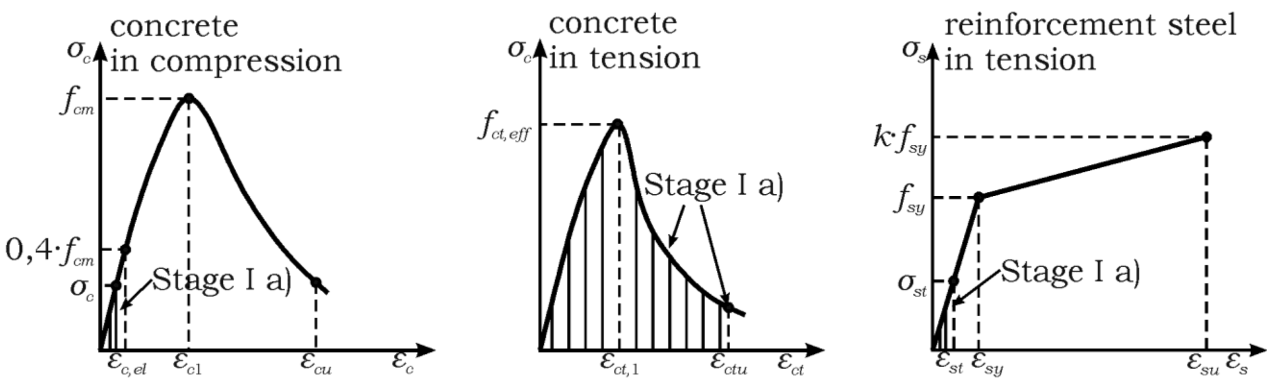


Figure 4.1-1 – Flexural Behavior of Beam. Stage I (Uncracked section), strains and stresses distribution in cross-section

Stage II (Concrete Cracked).

As the load is increased after the ultimate tensile strain in concrete ε_{ctu} is exceeded, cracks begin to develop in the bottom face of the beam. The moment at which these cracks begin to form – that is, when tensile strain in the bottom of the beam ε_{ct} becomes equal to the ultimate tensile strain ε_{ctu} – is reffered to as the cracking moment, M_{cr}.

Now, that the bottom has cracked and another stage is presented because the concrete in cracked zone obviously cannot resist tensile force – steel reinforcement must do it. As it is shown in [6], when first cracks occur, the stress in tensile concrete at the crack drops to zero (see Figure 4.1-2 a). The concrete strains and stresses increase with distance from the crack face due to the steel-concrete bond, until at the same distance *s* from crack, concrete is not longer affected by the crack, as shown in Figure 4.1-2 a. Slip at the concrete-steel interface in the region of significant bond stress (*s* on either side of the crack) causes the crack opening.

A relatively small load increasing will cause a second crack developing at a cross-section at the same distance $x \geq s$ from first crack, there by the reducing of the concrete stress in the vicinity of the crack. Eventually, under the increasing load, primary cracks form at somewhat regular intervals along the member and the primary crack patterns are established.

The concrete tensile stress at the each crack is zero, rising to a maximum value σ_{ct} (less than the concrete tensile strength f_{ctm}) mid-way between adjacent cracks, as it shown in Figure 4.1-2.

After cracking, tensile force is redistributed on the reinforcement at the crack. At the cracked cross-section the concrete at the top resists compression and steel reinforcement resists tension. But in the cross-section, what is situated between two adjacent cracks, concrete and steel reinforcement resist tensile force together.

Cracking is therefore accompanied by a drop in the average tensile stress carried by the concrete and, hence, a reduction in tension stiffening.

After primary crack patterns are established, further load increasing may result in further slip at the concrete-steel interface causing cover-controlled cracks to develop between the primary cracks and gradual breaking down of the bond between the steel and concrete, thereby reducing tension stiffening still further, as it is shown in Figure 4.1-2 b. At this stage, relation between moment and deflection is linear, but bending stiffness of the cracked beam is reduced. As the load increasing hereinafter, these cracks quickly spread up to the vicinity of the neutral axis and then the neutral axis begins to move upward.

This stage will continue as long as the compressive stress in the top face is less then then about one-half of the concrete compressive strength, f_{cm}, and as long as the steel strains are less than its yield strain ε_{sy}. The strains and stresses distribution for this stage are shown in Figure 4.1-2 b. In this stage the compressive strains and stresses in concrete vary linearly with the distance from the neutral axis or as a straight line. The straight-line strain-stress variation normally occurs in reinforced concrete beams under normal service-load conditions.

Stage II (Cracked section)

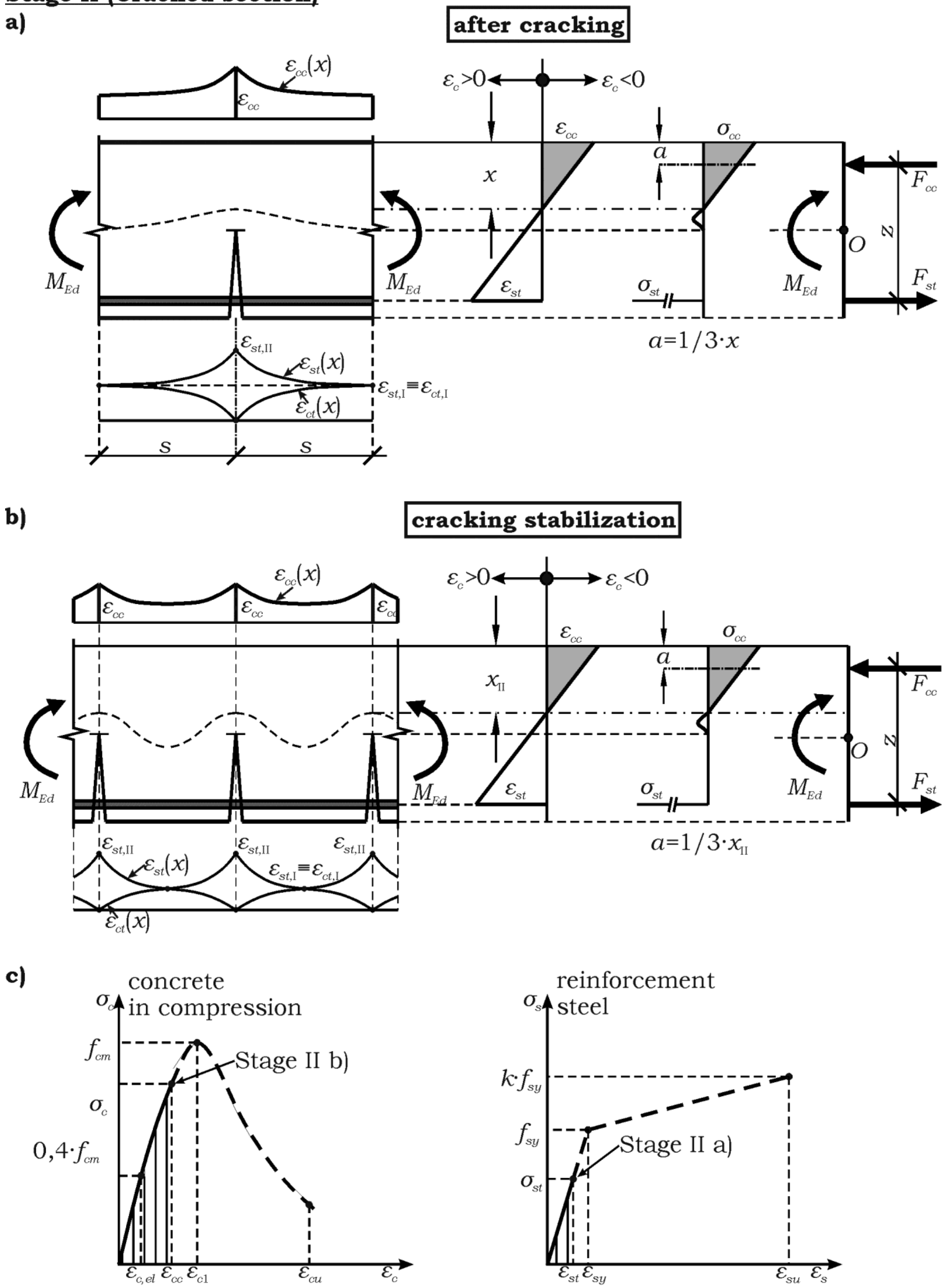


Figure 4.1-2 – Flexural behavior of beam. Stage II (cracked section). Strains and stresses distribution on cross-section

Stage III (ultimate/strength).

As the load increasing herein after so that the compressive stress becomes greater than $0,4 \cdot f_{cm}$ (limit value of the concrete elastic compressive stress), the tensile cracks move further upward, as does to the neutral axis, and the concrete compressive stress begin to change appreciably from straight line. The stress variation in compression zone is much like relationship «$\sigma_c - \varepsilon_c$» that is shown in Figure 4.1-3. Further load increasing on this stage leads to failure of the beam.

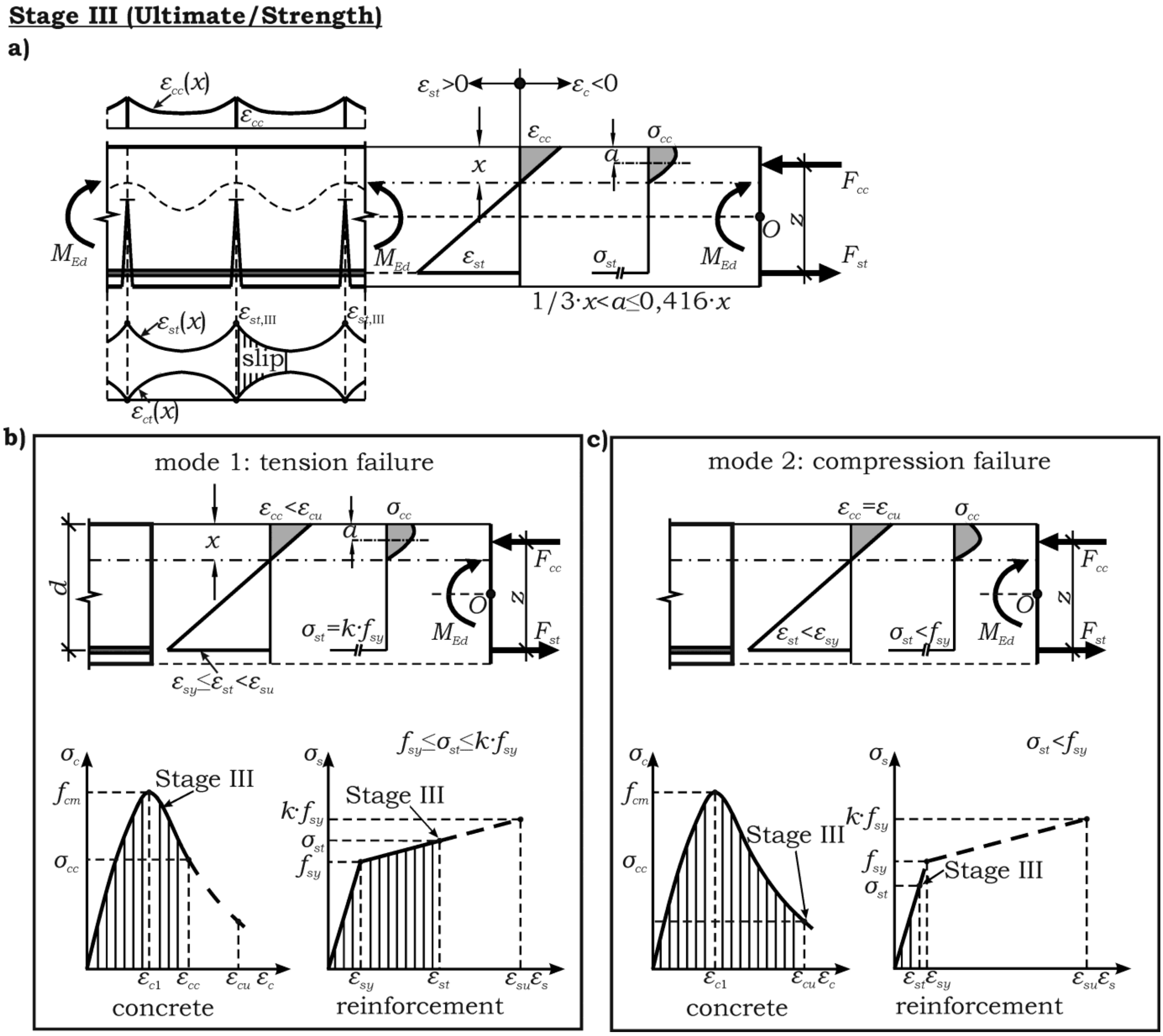


Figure 4.1-3 – Flexural behavior of Beam. Stage III (Ultimate/Strength). Strains and stresses distribution in cross-section

There are two possible ways in which reinforced concrete beam can fail in bending in the ultimate stage and the mode of failure depends on the amount of tensile reinforcement used:

1) Under-reinforced cross-section (element) ($\rho_l \leq \rho_{lim}$; $x \leq x_{lim}$).

In an under-reinforced beam what happens as the load on the beam increases?

If the tensile force that the steel bar can provide is less than compressive force which concrete can provide then the steel will reach its yield point (strains ε_{sy}, see Figure 4.1-3 b) before the concrete crushed ($\varepsilon_s \ge \varepsilon_{sy}; \varepsilon_{cc} < \varepsilon_{cu}$). In this mode of tensile cracks become visible before the steel finally fails ($\varepsilon_s = \varepsilon_{su}; \varepsilon_{cc} < \varepsilon_{cu}$). This mode of failure is preceded by large emptiness deflection (ductile mode);

2) Over-reinforced cross-section (element) ($\rho_l > \rho_{lim}; \xi \le \xi_{lim}$).

In an over-reinfored section steel is stronger than concrete. The tensile force which the steel reinforcement can provide is larger than compressive force which concrete can provide, then the concrete will reach its ultimate strain ε_{cu} before the steel yields (see Figure 4.1-3 c). If it happens the concrete starts to loose strength and the beam fails suddenly without warning with concrete failing in compression. This mode of failure gives little if any prior warning and it is less safe failure mode (brittle failure mode).

3) Balanced cross-section

In balanced section the concrete is almost at its ultimate strain (ε_{cu}) when the steel yields ($\varepsilon_s = \varepsilon_{sy}$). Figure 4.1-4 represents strains and stresses distribution for this case.

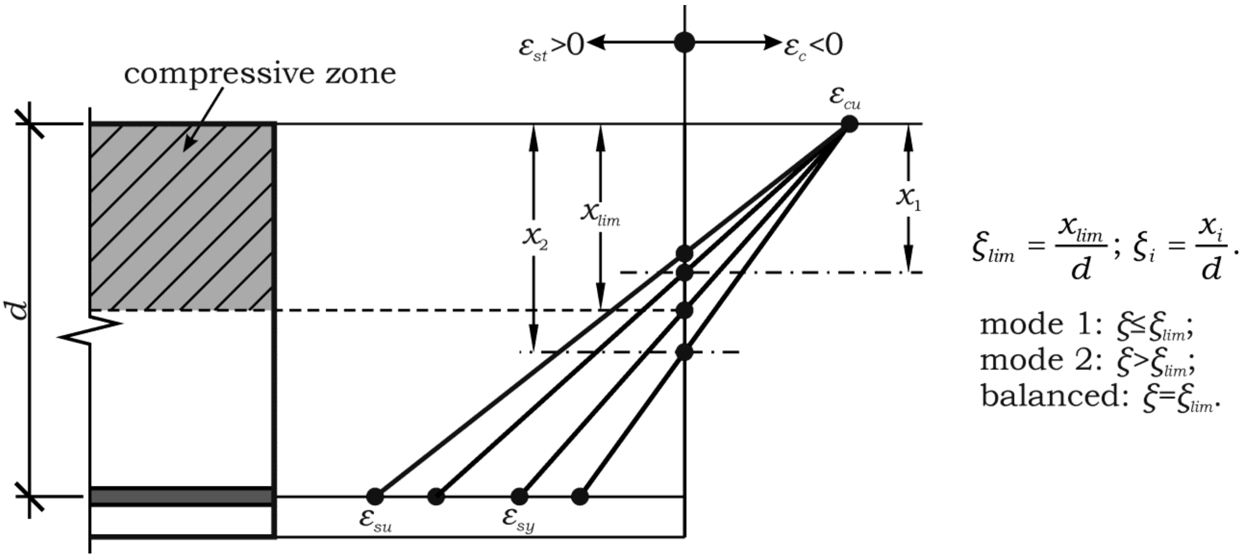


Figure 4.1-4 – Modes of the failure. Balanced state ξ_{lim}

Figure 4.1-5 represents different types of the «moment–curvature» diagrams that represent effect of reinforcement percentage. If Figure 4.1-5 from [5] be examined, there appears two distinct patterns of the beam behavior under the load. The first pattern is where there is a considerable increasing in curvature after the attainment of the maximum moment and where the behavior exhibits characteristics close to elastic-perfectly plastic behavior (see Figure 4.1-5, ductile failure). The second type of the behavior is where failure occurs as the maximum moment is achieved with no subsequent ductility. The first type of behavior clearly allows a large degree of rotation after yield, albeit accompanied by severe and increasing cracking and deflection (ductile mode of failure for under-reinforced sections). The behavior with no post-yield plateau is given by an over-reinforced section where steel remains elastic and the concrete reaches its maximum strain before the steel yields (see Figure 4.1-5, brittle failure).

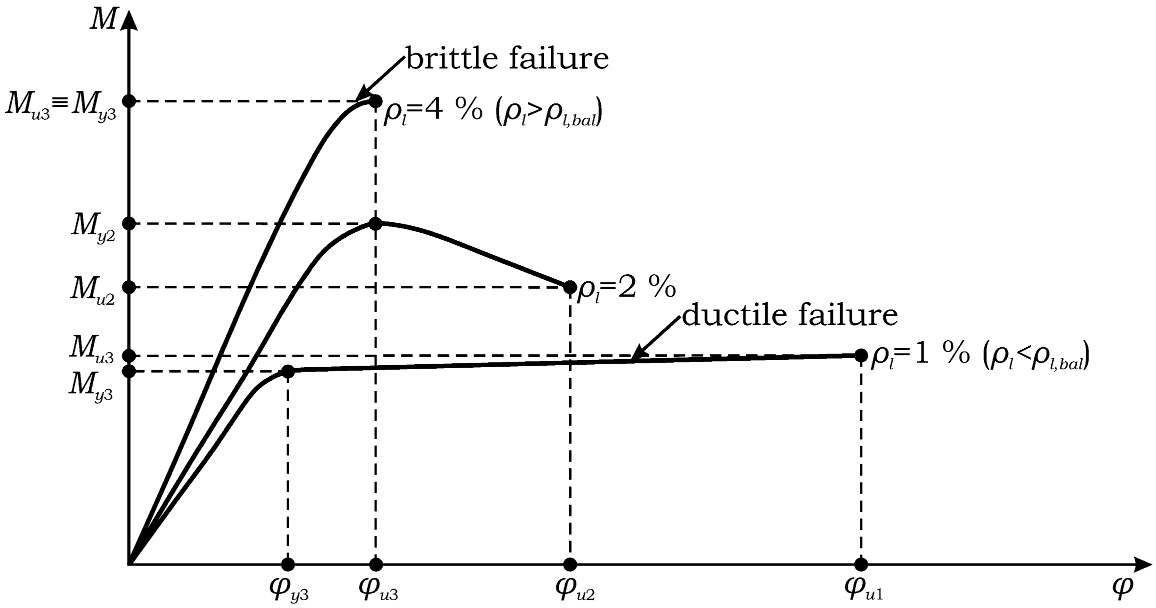


Figure 4.1-5 – «Moment-curvature» diagram showing effect of reinforcement percentage

So, in accordance with EN 1992 [N3], it is recommended to design bending reinforced concrete members based on the under-reinforced sections with a higher ductility.

4.1.1.2 Assumptions

In accordance with EN1992 [N3] (clause 6.1 (2) P), when determining the ultimate moment resistance of reinforced concrete cross-sections, the following assumpions are made:

- the strain in the concrete and reinforcement are derived assumed that plain section remain plane;
- strain in bounded reinforcement, whether in tension or compression, is the same as that in the surrounding concrete;
- the tensile strength of concrete is ignored;
- the stress in concrete in compression are derived from the design «stress-strain» relationship given in Chapter 3 (see Figure 3.1-10 and Figure 3.1-11) with $\gamma_c = 1{,}5$;
- the stress in the reinforcement (reinforcing steel) are derived from the design curve in Chapter 3 (see Figures 3.1-8–3.1-11).

Note that in both cases (rectangular-parabolic and rectangular block) the compressive strain in the concrete shall be limited to $\varepsilon_{cu,2}$ or $\varepsilon_{cu,3}$, depending on the «stress-strain» diagram used (see Figure 3.1-10, Figure 3.1-11 and Table 3.1-3).

The strains in the reinforcing steel shall be limited to ε_{ud} (where applicable).

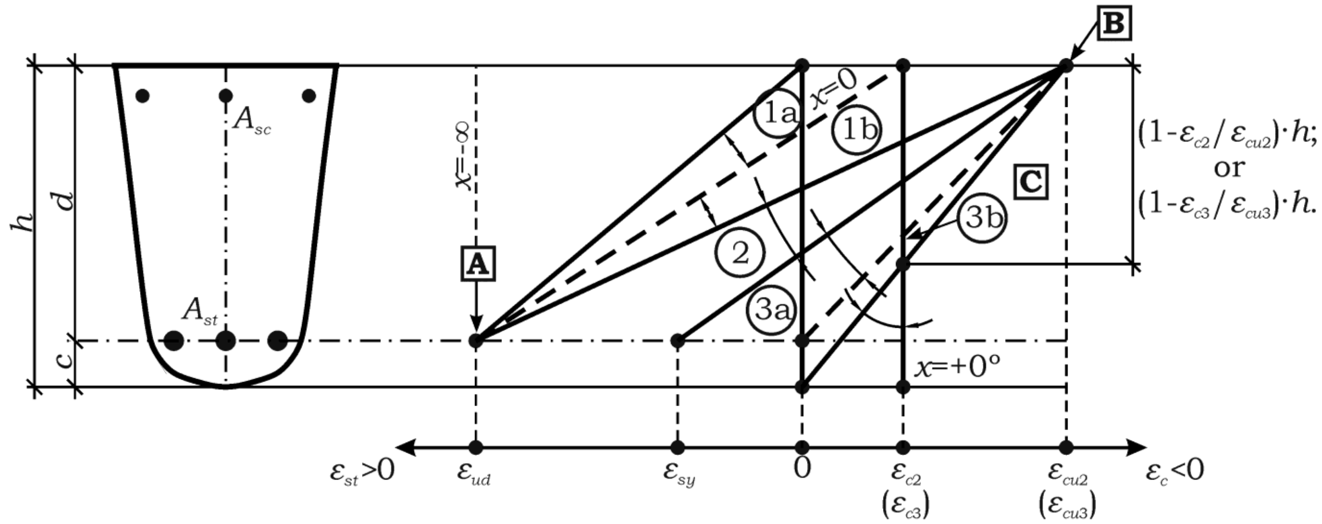
For cross-sections not fully in compression and concrete classes less than C50/60, to fail when the strain reaches $\varepsilon_{cu,2} = 3,5$ ‰ the strain in the tension reinforcement need not to be limited where a horizontal top branch is assumed for the reinforcement stress-strain curve, and limited to ε_{ud}, when relationship «$\sigma_s - \varepsilon_s$» is bilinear, as it was shown in Figure 3.2-2.

For cross-sections that are completely in compression, the strain is limited to $\varepsilon_{c2} = 2$ ‰ at the height of $\left(1 - \dfrac{\varepsilon_{c2}}{\varepsilon_{cu2}}\right) \cdot h$ or $\left(1 - \dfrac{\varepsilon_{c3}}{\varepsilon_{cu3}}\right) \cdot h$ from the most compressive face of section.

For cross-sections loaded by the compressive force it is necessary to assume the minimum eccentricity $e_0 = h/30$ but not less than 20 mm, where h is the depth of section.

In part of cross-sections which are subjected to approximately concentric load ($e_d / h \le 0,1$), such as compression flanges of box girders, the mean compressive strain in the part of the section should be limited to ε_{c2} (or ε_{c3} it the bilinear relations for concrete is used).

The possible range of strain distribution is shown in Figure 4.1-6 and is described in Table 4.1-1.



A - reinforcing steel tension strain limit;
B - concrete compression strain limit;
C - concrete pure compression strain limit.

Figure 4.1-6 – Possible strain distributions in ultimate limit state in accordance with EN1992 [N3]

It should be noted, that some basic assumptions, formulated above (for example, plane cross-section hypothesis) are not strictly true. The deformations within a section is very complex, and, locally in cracks, plane sections don't remain plain. Nor, due to the local bond slip between cracks are the strains in the concrete exactly the same as those in the steel.

It is a universal approach to define failure of concrete in compression by means of a limiting compressive strain ε_{cc}. For concrete strength not exceeding 50 N/mm², the EN 1992 [N3] adopts value of $\varepsilon_{cu} = 3,5$ ‰ for flexure and for combined bending and axial load, where the neutral axis remains within a cross-section, and a limit of between 3,5 ‰ and 2,0 ‰ for cross-section loaded so that whole section is in compression (see Figure 4.1-6). The values for the ultimate compressive strains for situation, where the neutral axis lies within the section and for axial compression vary for higher-strength concretes, with the ultimate strain reducing with increasing strength for cases where the neutral axis is within the section, while the values of the axial compression increases with increasing strength. The logic behind the reduction in the strain limit for axial compression is that, in axial compression, failure will occur at the strain corresponding to the attainment of the maximum compressive stress.

This is 0,002 (2 ‰) for concrete strength is not exceeding 50 N/mm². In flexure, considerably higher strains can be reached before the maximum capacity of the section is reached, and the value of $\varepsilon_{cu} = 3,5$ ‰ has been obtained empirically. A means is needed to be interpolated between the value of $\varepsilon_{cu} = 3,5$ ‰ for flexure and $\varepsilon_{c2} = 2$ ‰ for axial load (see Figure 4.1-6).

Figure 4.1-7 represents cross-sections with strain diagram, stress blocks internal forces for possible ranges of strain distribution in accordance with Table 4.1-1.

Table 4.1-1 – Description of the possible ranges of strain distributions

Range of strain distribution	**Strain**		**Comments**
	Reinforcing steel, A_{st}, tension strain, E_{st}	Concrete compression strain ε_{cc} (most compressive face)	
1a (Figure 4.1-7 a)	$\varepsilon_{st} = \varepsilon_{ud}$	$\varepsilon_{cc} < \varepsilon_{c2}$ or $\varepsilon_{cc} < \varepsilon_{c3}$	Mechanical properties of reinforcing steel are utilized completely; bending; bending with axial force (large eccentricity);
1b (Figure 4.1-7 b)	$\varepsilon_{st} = \varepsilon_{ud}$	$\varepsilon_{c2} \le \varepsilon_{cc} < \varepsilon_{cu2}$ $\varepsilon_{c3} \le \varepsilon_{cc} < \varepsilon_{cu3}$	
2 (Figure 4.1-7 c)	$\varepsilon_{sy} \le \varepsilon_{st} < \varepsilon_{ud}$	$\varepsilon_{cc} = \varepsilon_{cu2}$ or $\varepsilon_{cc} = \varepsilon_{cu3}$	
3a (Figure 4.1-7 d)	$0 \le \varepsilon_{st} < \varepsilon_{sy}$	$\varepsilon_{cc} = \varepsilon_{cu2}$ or $\varepsilon_{cc} = \varepsilon_{cu3}$	Mechanical properties of reinforcing steel are not utilized completely, but concrete properties are used completely bending with axial force with small eccentricity.
3b (Figure 4.1-7 e)	$\varepsilon_{st} < 0$	$\varepsilon_{cc} = \varepsilon_{cu2}$ or $\varepsilon_{cc} = \varepsilon_{cu3}$	
4 (Figure 4.1-7 f)	$\varepsilon_{c2} \le \varepsilon_{st} < 0$	$\varepsilon_{c2} \le \varepsilon_{cc} < \varepsilon_{cu2}$ $\varepsilon_{c3} \le \varepsilon_{cc} < \varepsilon_{cu3}$	

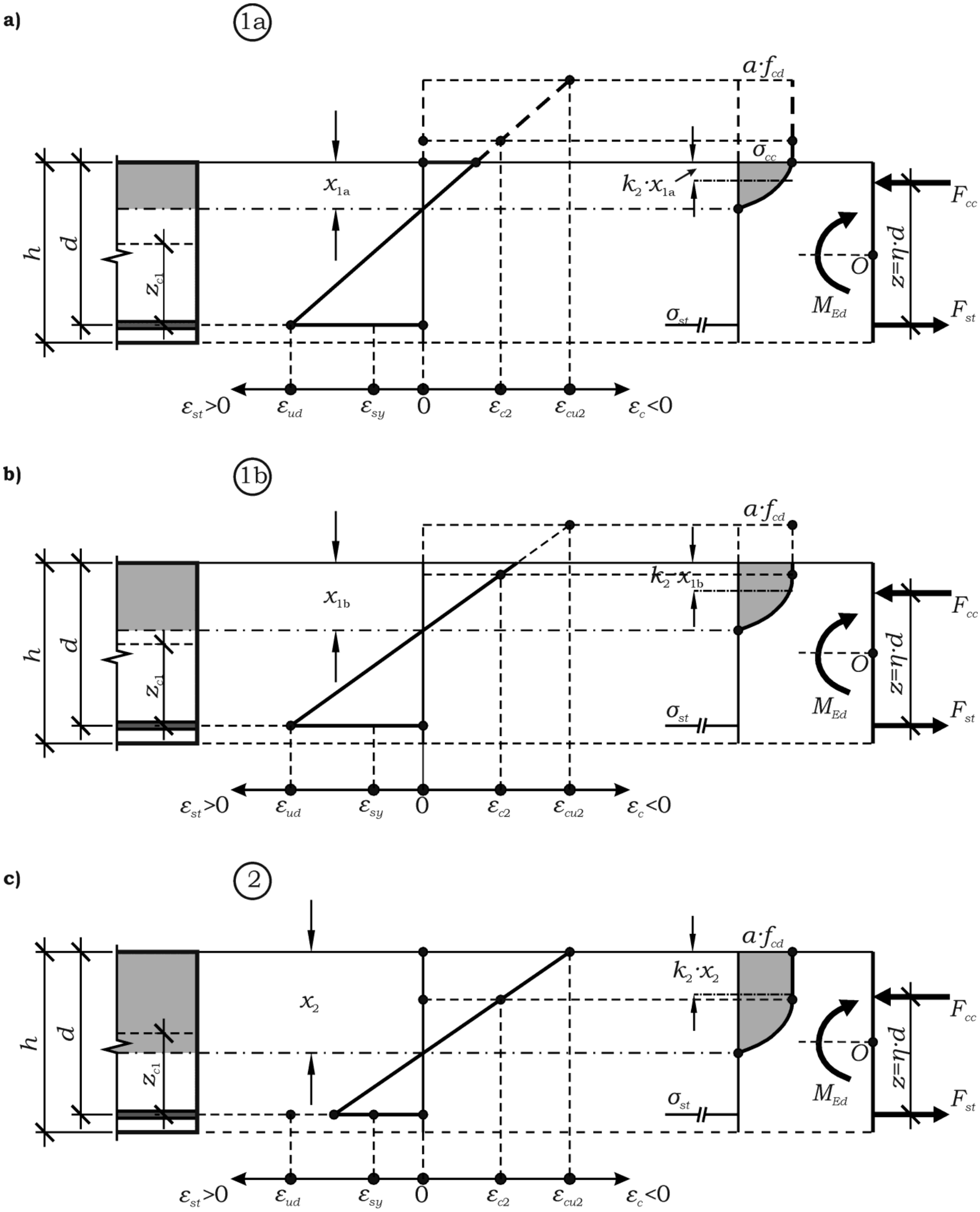


Figure 4.1-7 – Cross-section, strain diagram and stress block
for the possible ranges of strain distribution 1a, 2b and 2 in accordance with Table 4.1-1

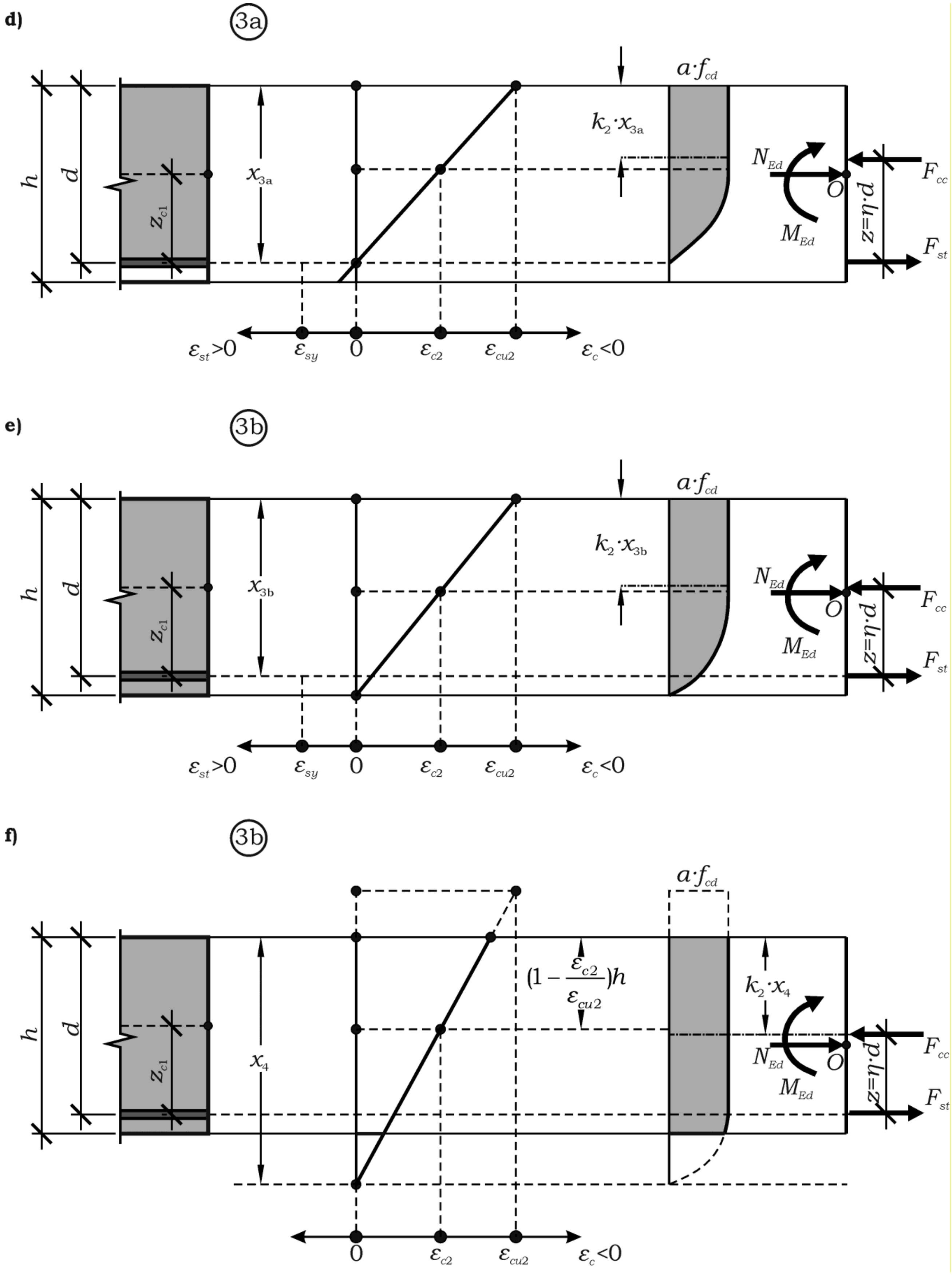


Figure 4.1-7 (end) – Cross-section, strain diagram and stress block for the possible ranges of strain distribution 3a, 3b and 4 in accordance with Table 4.1-1

4.1.1.3 Rectangular-parabolic compressive stress block in concrete

4.1.1.3.1 Rectangular cross-section

(1) Singly reinforced rectangular cross-section. Basic equations

A rectangular-parabolic stress block may be used to provide a more rigorous analysis of the reinforced concrete section. The stress block is similar in shape to the stress-strain curve for concrete in Figure 3.1-9 a, having a maximum stress of f_{cd} at the ultimate strain $\varepsilon_{c2,u} = 3{,}5$ ‰.

A section subjected to an applied design moment M_{Ed} (as the effect of the loading) is shown in Figure 4.1-8 a, and the strain diagram, stress block and internal forces are shown in Figure 4.1-8 b, c, d. Figure 4.1-8 shows the simplified rectangular-parabolic stress block which are used in EN1992 [N3] to develop the design equations for bending.

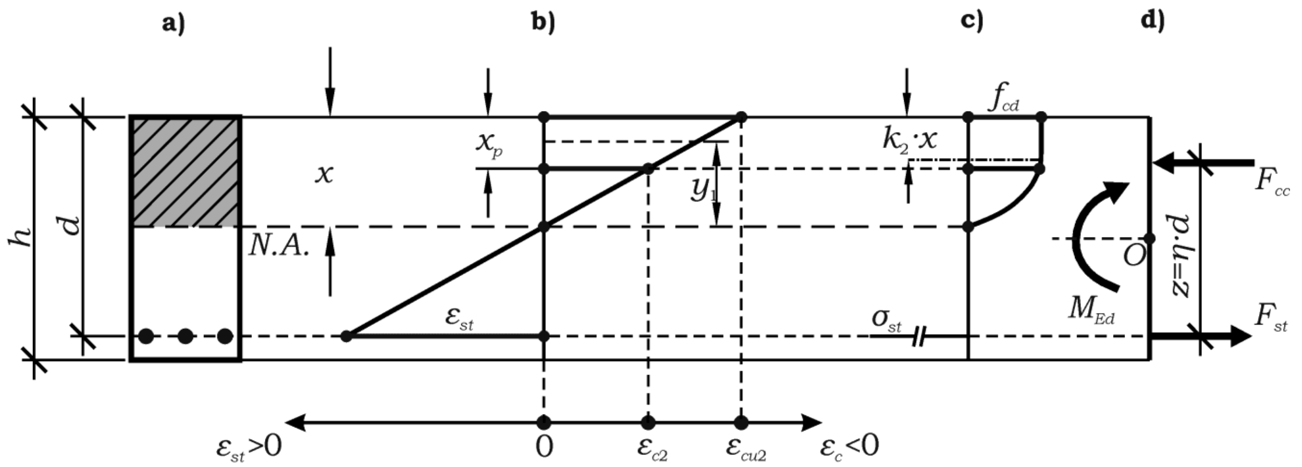
Bending of the section (see Figure 4.1-8) will induce a resultant tensile force F_{st} in the reinforcing steel and a resultant compressive force in the concrete F_{cc} which acts through the centroid of the effective compression zone.

How it was shown previously (see Chapter 1), based on ULS/STR checking by partial factors method for equilibrium, applied design moment M_{Ed} has to be balanced by resisting moment M_{Rd}, so that:

$$M_{Ed} \leq M_{Rd}, \tag{4.1-1}$$

where: M_{Ed} is the design value of the applied moment (as an action effect) at the critical section;

M_{Rd} is the design value of the corresponding resisting moment.



a) – cross-section; b) – strain diagram; c) – stress block; d) – internal forces

Figure 4.1-8 – Rectangular-parabolic concrete compressive stress block, singly reinforced cross-section

The beam section is in equilibrium and, hence, the law of static must hold. The moment of resistance of the section, M_{Rd}, using the rectangular-parabolic stress block (see Figure 4.1-8), is calculated from the equilibrium conditions for cross-section. The sum of the compressive and tensile forces must be zero, and:

$$F_{cc} = F_{st}. \tag{4.1-2}$$

Because the internal forces are equal the moment of resistance with respect to the centroid to the steel reinforcement and centroid of concrete in compression are equal, i.e.:

$$M_{Rd} = F_{cc} \cdot z = F_{st} \cdot z. \tag{4.1-3}$$

where: F_{cc} is the resultant compressive force resisted by concrete (as a resultant of the compressive stress block);

F_{st} is the resultant tensile force in steel reinforcement;

z is the lever arm of internal force.

In general case, resultant compressive force resisted by concrete can be calculated as follows:

$$\begin{aligned} F_{cc} &= \int_0^{x_p} f_{cd} \cdot b \left[1 - \left(1 - \frac{\varepsilon_c(y)}{\varepsilon_{c2}} \right)^n \right] dy + f_{cd} \cdot b \cdot \left(x - x_p \right) = \\ &= f_{cd} \cdot b \left\{ \int_0^{x_p} \left[1 - \left(1 - \frac{\varepsilon_c(y)}{\varepsilon_{c2}} \right)^n \right] dy + \left(x - x_p \right) \right\} \end{aligned} \tag{4.1-4}$$

From strain compatibility diagram (plane section hypothesis, see Figure 4.1-9):

$$\varepsilon_c(y) = y \cdot \varphi = y \cdot \frac{\varepsilon_{c2}}{x_p}, \tag{4.1-5}$$

where: φ is the curvature at the particular section, what is given by (see Figure 4.1-9):

$$\varphi = \frac{\varepsilon_{c2}}{x_p}, \tag{4.1-6}$$

where: x_p is the distance from the neutral axis to strain ε_{c2}.

ε_{c2} is the concrete strain at the end of the parabolic section.

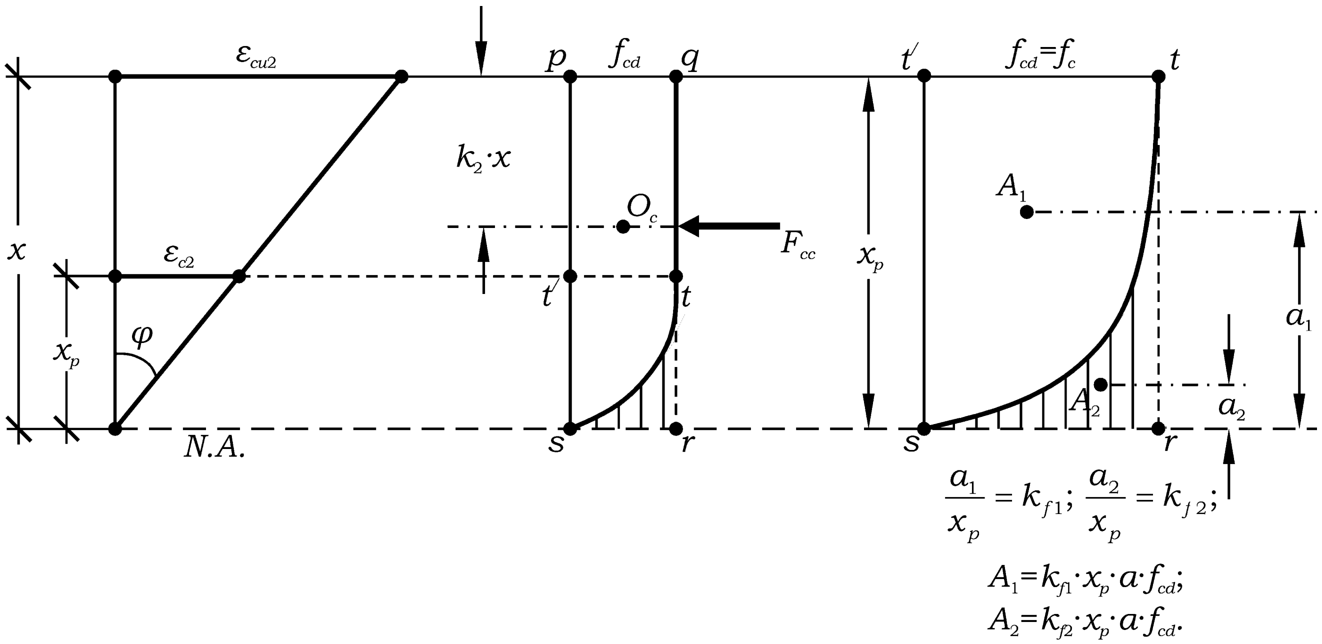


Figure 4.1-9 – Parameters of the rectangular-parabolic stress block

Substituting Equation (4.1-5) in Equation (4.1-4):

$$F_{cc} = f_{cd} \cdot b \left\{ \int_0^{x_p} \left[1 - \left(1 - \frac{y}{x_p} \right)^n \right] dy + \left(x - x_p \right) \right\} \tag{4.1-7}$$

To compute a resultant compressive force resisted by concrete, the rectangular-parabolic stress block is replaced by the equivalent rectangular stress block with mean (average) concrete stress, $f_{c,av}$, that is given by (see Figure 4.1-9).

$$f_{c,av} = \frac{A_{pqrs} - A_{rst}}{x}, \tag{4.1-8}$$

where: $A_{pqrs} = f_{cd} \cdot x$ is the area of the conventional rectangular stress block $(a \cdot f_{cd}; x)$.

$$A_{rst} = A_{st' t \cdot r} - A_{st' t}. \tag{4.1-9}$$

The area of compressive stress block $A_{st' t}$ is defined as follows:

$$A_{st' t} = \int_0^{x_p} f_{cd} \left[1 - \left(1 - \frac{y}{x_p} \right)^n \right] dy, \tag{4.1-10}$$

and the area $A_{st' t \cdot r}$ is given by:

$$A_{st/t\cdot r} = f_{cd} \cdot x_p. \tag{4.1-11}$$

Therefore:

$$A_2 = f_{cd} \cdot x_p - \int_0^{x_p} f_{cd} \cdot \left[1-\left(1-\frac{y}{x_p}\right)^n\right] dy = f_{cd}\left\{x_p - \int_0^{x_p} \left[1-\left(1-\frac{y}{x_p}\right)^n\right] dy\right\}. \tag{4.1-12}$$

From strain compatibility diagram the distance x_p from the neutral axis to strain ε_{c2} is given by:

$$A_2 = f_{cd} \cdot x_p - \frac{x}{\varepsilon_{c2,u}} = \frac{x_p}{\varepsilon_{c2}} \longrightarrow x_p = x \cdot \frac{\varepsilon_{c2}}{\varepsilon_{c2,u}}. \tag{4.1-13}$$

Hence, in general case, the mean compressive stress, $f_{c,av}$, can be calculated:

$$f_{c,av} = \frac{f_{cd} \cdot x - A_2}{x}. \tag{4.1-14}$$

Table 4.1-2 gives values of the coefficients k_{f1}, k_{f2}, which are used to calculate the areas A_1 and A_2 according following expressions:

$$A_1 = k_{f1} \cdot x_p \cdot f_{cd}; \tag{4.1-15}$$

$$A_2 = k_{f2} \cdot x_p \cdot f_{cd}. \tag{4.1-16}$$

The values of the coefficients k_{f1}, k_{f2}, k_w, k_{w2}, in Table 4.1-2 were obtained from Equation (4.1-10) by integration for range $0 \le y \le x_p$.

Combining of the Equation (4.1-14) and Equation (4.1-16) gives:

$$\begin{aligned} f_{c,av} &= \frac{f_{cd} \cdot x - k_{f2} \cdot x_p \cdot f_{cd}}{x} = \frac{f_{cd} \cdot x - f_{cd} \cdot k_{f2} \cdot \left(\frac{\varepsilon_{c2}}{\varepsilon_{c2,u}}\right) \cdot x}{x} = \\ &= f_{cd} \cdot \left[1 - k_{f2} \cdot \left(\frac{\varepsilon_{c2}}{\varepsilon_{c2,u}}\right)\right] = \omega_c \cdot f_{cd}, \end{aligned} \tag{4.1-17}$$

where:

$$\omega_c = \left[1 - k_{f2} \cdot \left(\frac{\varepsilon_{c2}}{\varepsilon_{c2,u}}\right)\right]. \tag{4.1-18}$$

For strength classes for concrete from C12/15 to C50/60 in accordance with Table 4.1-2: $\varepsilon_{c2,u} = 3{,}5$ ‰; $\varepsilon_{c2} = 2{,}0$ ‰, $k_{f2} = 0{,}333$, value of the coefficient ω_c is given by:

$$\omega_c = \left[1 - 0{,}333 \cdot \left(\frac{2{,}0}{3{,}5}\right)\right] = 0{,}809 \simeq 0{,}81. \qquad (4.1\text{-}19)$$

(***Note:*** this value is used for simplified rectangular block: λ=0,8).

Table 4.1-2 gives values of the coefficient ω_c for different strength classes for concrete. Values of ω_c, as it can be seen from Table 4.1-2 decrease with increasing of strength class for concrete (for high strength classes (more C50/60) value $\omega_c \to 0{,}5$).

Table 4.1-2 – Value of the coefficients that are used for calculation of the rectangular parabolic stress block parameters

Parameters	≤C50/60	C55/67	C60/75	C70/85	C80/95	C90/105
n	2,0	1,75	1,60	1,45	1,4	1,4
$k_{f1} = \dfrac{A_1}{\omega \cdot f_{cd}} = \dfrac{n}{n+1}$	0,667	0,635	0,615	0,597	0,590	0,590
$k_{f2} = \dfrac{A_2}{\omega \cdot f_{cd}} = \dfrac{1}{n+1}$	0,333	0,364	0,385	0,403	0,410	0,410
$k_{\omega 1} = \dfrac{a_1}{\omega} = \dfrac{n+3}{2 \cdot (n+2)}$	0,625	0,595	0,576	0,559	0,552	0,552
$k_{\omega 2} = \dfrac{a_2}{\omega} = \dfrac{1}{n+2}$	0,250	0,274	0,289	0,303	0,308	0,308
ε_{c2},[‰]	-2,0	-2,2	-2,3	-2,4	-2,5	-2,6
ε_{cu2},[‰]	-3,5	-3,1	-2,9	-2,7	-2,6	-2,6
$\dfrac{x_p}{x} = \dfrac{\varepsilon_{c2}}{\varepsilon_{cu2}}$	0,571	0,710	0,793	0,888	0,961	1,0
$\omega_c = \left[1 - k_{f2} \cdot \left(\dfrac{\varepsilon_{c2}}{\varepsilon_{c2,u}}\right)\right]$	0,810	0,754	0,694	0,642	0,605	0,590
$k_2 = 1 - \dfrac{0{,}5 \cdot k_{f2} \cdot k_{\omega 2} \cdot \left(\dfrac{\varepsilon_{c2}}{\varepsilon_{c2,u}}\right)^2}{\omega_c}$	0,416	0,403	0,380	0,371	0,360	0,366
$C_0 = \dfrac{\omega_c}{k_2}$	1,947	1,870	1,826	1,730	1,650	1,612

In general case location of the resultant compressive force (depth of the $k_2 \cdot x$ centroid for compression stress block) is calculated by taking area moments of the stress block about neutral axis (see Figure 4.1-9):

$$y_c = \left(x - k_2 \cdot x\right) = \frac{\int_0^x \left[1 - \left(1 - y / x_p\right)^n\right] \cdot y dy}{\int_0^x \left[1 - \left(1 - y / x_p\right)^n\right] dy}. \tag{4.1-20}$$

For rectangular-parabolic stress block distance ($x - k_2 \cdot x$) is determined:

$$\left(x - k_2 \cdot x\right) = \frac{A_{pqrs} \cdot 0,5x - A_{rst} \cdot a_2}{\omega_c \cdot a \cdot f_{cd} \cdot x}. \tag{4.1-21}$$

In accordance with Table 4.1-2 $a_2 = k_{\omega 2} \cdot x_p$, and distance ($x - k_2 \cdot x$) is equal:

$$\left(x - k_2 \cdot x\right) = \frac{f_{cd} \cdot 0,5x^2 - f_{cd} \cdot k_{f2} \cdot k_{\omega 2} \cdot x_p^2}{\omega_c \cdot f_{cd} \cdot x}. \tag{4.1-22}$$

Substituting $x_p = \left(\dfrac{\varepsilon_{c2}}{\varepsilon_{c2,u}}\right) \cdot x$ in Equation (4.1-22) and solving for k_2 gives:

$$k_2 = 1 - \frac{0,5 \cdot k_{f2} \cdot k_{\omega 2} \cdot \left(\varepsilon_{c2} / \varepsilon_{c2,u}\right)^2}{\omega_c}. \tag{4.1-23}$$

For concrete classes from C12/15 to C50/60: $\varepsilon_{c2,u} = 3,5‰$; $k_{f2} = 0,333$; $k_{\omega 2} = 0,25$; $\omega_c = 0,81$ (see Table 4.1-2), value k_2 is obtained from Equation (4.1-23):

$$k_2 = 1 - \frac{0,5 \cdot 0,333 \cdot 0,25 \cdot \left(2,0/3,5\right)^2}{0,81} = 0,416. \tag{4.1-24}$$

Table 4.1-2 contains values of the k_2-coefficient for different concrete strength classes.

The moment of resistance, M_{Rd}, with respect to steel reinforcement is calculated by:

$$M_{Rd} = \omega_c \cdot f_{cd} \cdot b \cdot x \cdot z, \tag{4.1-25}$$

and taking in account, that $z = d - k_2 \cdot x$, Equation (4.1-25) can be rewritten as follows:

$$M_{Rd} = \omega_c \cdot f_{cd} \cdot b \cdot \left(d - z\right) \cdot \frac{1}{k_2} \cdot z, \tag{4.1-26}$$

or:

$$M_{Rd} = C_0 \cdot f_{cd} \cdot b \cdot (d - z) \cdot z, \qquad (4.1\text{-}27)$$

where: $C_0 = \dfrac{\omega_c}{k_2}$.

Defining $\eta = \dfrac{z}{d}$, and replacing M_{Rd} by M_{Ed} Equation (4.1-27) can be rewritten as:

$$\eta^2 - \eta + \frac{M_{Ed}}{C_0 \cdot f_{cd} \cdot b \cdot d^2} = 0. \qquad (4.1\text{-}28)$$

Solving of the quadratic equation for η gives:

$$\eta = \frac{z}{d} = 0,5 + \sqrt{0,25 - \frac{a_m}{C_0}} = 0, \qquad (4.1\text{-}29)$$

where: $a_m = \dfrac{M_{Ed}}{f_{cd} \cdot b \cdot d^2}$.

If the lever arm of internal forces, $z = \eta \cdot d$, is known, then the depth of neutral axis x can be calculated from following expression:

$$\frac{x}{d} = (1 - \eta) \cdot \frac{1}{k_2}, \qquad (4.1\text{-}30)$$

From strain compatibility diagram (for plane cross-section) follows:

$$\frac{\varepsilon_{c2,u}}{x} = \frac{\varepsilon_{st}}{(d - x)}, \qquad (4.1\text{-}31)$$

or:

$$\varepsilon_{st} = \varepsilon_{c2,u} \cdot \left(\frac{d - x}{x} \right). \qquad (4.1\text{-}32)$$

Substituting Equation (4.1-30) into Equation (4.1-32), the following expression is obtained:

$$\varepsilon_{st} = \varepsilon_{c2,u} \cdot \left(\frac{k_2}{1 - \eta} - 1 \right). \qquad (4.1\text{-}33)$$

The failure mode can be estimated based on the following inequality:

$$\varepsilon_{sy} \le \varepsilon_{st} \le \varepsilon_{s,R}, \qquad (4.1\text{-}34)$$

where: ε_{sy} is a yield strain for the steel reinforcement ($\varepsilon_{sy} = \frac{f_{yk}}{E_s}$);

$\varepsilon_{s,R}$ is an ultimate value of the strain for the steel reinforcement in accordance with EN1992 [N3] ($\approx 0,9 \cdot \varepsilon_{su}$).

If Inequality (4.1-34) is satisfied, then the reinforcement steel yields before concrete in compression zone reaches its maximum strain capacity (ultimate compression strain $\varepsilon_{c2,u}$) and crushes (under-reinforced section; «*tension failure mode*»). If Inequality (4.1-34) is not satisfied, then the concrete reaches its maximum (ultimate) strain $\varepsilon_{c2,u}$ before the steel reinforcement yields (over-reinforced section; «*compression failure mode*»).

Limit value of the neutral axis depth x_{lim} is defined by the following equation (see Figure 4.1-4):

$$x_{lim} = \left(\frac{\varepsilon_{c2,u}}{\varepsilon_{sy} + \varepsilon_{c2,u}} \right) \cdot d, \tag{4.1-35}$$

or:

$$x_{lim} = \left(\frac{\varepsilon_{c2,u}}{f_{yk} / E_s + \varepsilon_{c2,u}} \right) \cdot d, \tag{4.1-36}$$

If, $x \le x_{lim}$, Inequality (4.1-34) is satisfied, and $x > x_{lim}$, Inequality (4.1-34) is not satisfied.

If, $x = x_{lim}$, moment of resistance $M_{Rd,lim}$ is given by:

$$M_{Rd,lim} = \omega_c \cdot f_{cd} \cdot b \cdot d^2 \cdot \left(\frac{\varepsilon_{c2,u}}{\varepsilon_{sy} + \varepsilon_{c2,u}} \right) \cdot \left[1 - k_2 \cdot \left(\frac{\varepsilon_{c2,u}}{\varepsilon_{sy} + \varepsilon_{c2,u}} \right) \right], \tag{4.1-37}$$

Defining: $\xi_{lim} = \frac{x_{lim}}{d} = \frac{\varepsilon_{c2,u}}{\varepsilon_{sy} + \varepsilon_{c2,u}}$, Equation (4.1-37) can be rewritten as follows:

$$M_{Rd,lim} = \omega_c \cdot f_{cd} \cdot b \cdot d^2 \cdot \xi_{lim} \cdot (1 - k_2 \cdot \xi_{lim}), \tag{4.1-38}$$

Hence,

$$\frac{M_{Rd,lim}}{f_{cd} \cdot b \cdot d^2} = \omega_c \cdot \xi_{lim} \cdot (1 - k_2 \cdot \xi_{lim}), \tag{4.1-39}$$

or:

$$a_{m,lim} = \omega_c \cdot \xi_{lim} \cdot (1 - k_2 \cdot \xi_{lim}). \tag{4.1-40}$$

For the concrete strength classes from C12/15 to C50/60: $\varepsilon_{c2,u} = 3,5‰$ and for the steel reinforcement, for example S500 grade: $f_{yk} = 500$ N/mm^2, $\varepsilon_{sy} = \dfrac{500}{200 \cdot 10^3} = 2,5‰$, and: $\omega_c = 0,81$ and $k_2 = 0,416$ (see Table 4.1-2):

$$a_{m,lim} = 0,81 \cdot \left(\frac{3,5}{2,5+3,5}\right) \cdot \left(1 - 0,416 \cdot \left(\frac{3,5}{2,5+3,5}\right)\right) = 0,368.$$

(a) Determination of the required bending reinforcement

If inequality $a_m \le a_{m,lim}$ is satisfied, then required steel reinforcement area is calculated from following equation (moment resistance with respect to compression concrete):

$$M_{Ed} = M_{Rd} = F_{st} \cdot z = f_{yd} \cdot A_{st} \cdot z. \tag{4.1-41}$$

Solving Equation (4.1-41) for A_{st} gives:

$$A_{st} = \frac{M_{Ed}}{f_{yd} \cdot z} = \frac{M_{Ed}}{f_{yd} \cdot \eta \cdot d}. \tag{4.1-42}$$

(b) Checking of ULS for bending

According with the basic requirement of the partial factors method the following inequality shall be satisfied:

$$A_{st} = \frac{M_{Ed}}{f_{yd} \cdot z} = \frac{M_{Ed}}{f_{yd} \cdot \eta \cdot d}. \tag{4.1-43}$$

Equilibrium of axial forces gives:

$$F_{st} = F_{cc}, \tag{4.1-44}$$

or:

$$\omega_c \cdot f_{cd} \cdot b \cdot x = f_{yd} \cdot A_{st}, \tag{4.1-45}$$

Neutral axis depth is calculated from the following equation:

$$x = \frac{f_{yd} \cdot A_{st}}{\omega_c \cdot f_{cd} \cdot b}. \tag{4.1-46}$$

If $x \le x_{lim}$, then moment of resistance, M_{Rd}, is given by:

$$M_{Rd} = \omega_c \cdot f_{cd} \cdot b \cdot x \cdot z = \omega_c \cdot f_{cd} \cdot b \cdot x \cdot \left(b - k_2 \cdot x\right). \qquad (4.1\text{-}47)$$

If, $x > x_{lim}$, then the reinforced steel does not yield and moment of resistance, M_{Rd}, is calculated, assuming $x = x_{lim}$:

$$M_{Rd} = M_{Rd,lim} = a_{m,lim} \cdot f_{cd} \cdot b \cdot d^2. \qquad (4.1\text{-}48)$$

(c) Short calculation algorithm

Required area of the steel reinforcement.

The required area of the steel reinforcement can be calculated based on stream line procedure using coefficient from Table 4.1-3.

Table 4.1.3 – Coefficients for bending members section design

Range of strain distribution	**Design parameters**					
	Coefficients				**Strains, ‰**	
	$\xi = x/d$	**ω**	**ζ**	**μ**	**ε_{cc} in concrete (compression)**	**ε_{s1} in reinforcement (tension)**
Ia	0,02	0,002	0,993	0,002	0,20	10,0
	0,03	0,004	0,990	0,004	0,31	10,0
	0,04	0,008	0,986	0,008	0,42	10,0
	0,05	0,012	0,983	0,012	0,53	10,0
	0,06	0,017	0,979	0,017	0,64	10,0
	0,07	0,023	0,976	0,022	0,75	10,0
	0,08	0,030	0,972	0,029	0,87	10,0
	0,09	0,037	0,969	0,036	0,99	10,0
	0,10	0,045	0,965	0,044	1,11	10,0
	0,11	0,054	0,961	0,052	1,24	10,0
	0,12	0,063	0,957	0,061	1,36	10,0
	0,13	0,073	0,953	0,070	1,49	10,0
	0,14	0,083	0,949	0,079	1,63	10,0
	0,15	0,093	0,945	0,088	1,76	10,0
	0,16	0,104	0,940	0,098	1,90	10,0
Limit values for range Ia	0,167	0,111	0,938	0,104	2,00	10,0
Ib	0,17	0,115	0,936	0,107	2,05	10,0
	0,18	0,125	0,931	0,117	2,20	10,0
	0,19	0,136	0,927	0,126	2,35	10,0
	0,20	0,147	0,922	0,135	2,50	10,0
	0,21	0,157	0,917	0,144	2,66	10,0
	0,22	0,168	0,912	0,153	2,82	10,0
	0,23	0,179	0,907	0,162	2,99	10,0
	0,24	0,189	0,902	0,171	3,16	10,0
	0,25	0,200	0,897	0,179	3,33	10,0
Limit values for range Ib	0,259	0,211	0,892	0,187	3,50	10,0
	0,26	0,210	0,892	0,188	3,50	9,96
	0,27	0,219	0,888	0,194	3,50	9,46
	0,28	0,227	0,884	0,200	3,50	9,00
	0,29	0,235	0,879	0,206	3,50	8,57

Table 4.1-3 (cont.)

Ib	0,30	0,243	0,875	0,213	3,50	8,17
	0,31	0,251	0,871	0,219	3,50	7,79
	0,32	0,259	0,867	0,225	3,50	7,44
	0,33	0,267	0,863	0,230	3,50	7,11
	0,34	0,275	0,859	0,236	3,50	6,79
	0,35	0,283	0,854	0,242	3,50	6,50
	0,36	0,291	0,850	0,248	3,50	6,22
	0,37	0,300	0,846	0,252	3,50	5,96
	0,38	0,308	0,842	0,259	3,50	5,71
	0,39	0,316	0,838	0,264	3,50	5,47
	0,40	0,324	0,834	0,270	3,50	5,25
II	0,41	0,332	0,829	0,275	3,50	5,04
	0,42	0,340	0,825	0,281	3,50	4,83
	0,43	0,348	0,821	0,286	3,50	4,64
	0,44	0,356	0,817	0,291	3,50	4,45
	0,45	0,364	0,813	0,296	3,50	4,28
	0,46	0,372	0,809	0,301	3,50	4,11
	0,47	0,380	0,805	0,306	3,50	3,95
	0,48	0,388	0,800	0,311	3,50	3,79
	0,49	0,397	0,796	0,316	3,50	3,64
	0,50	0,405	0,792	0,321	3,50	3,50
	0,51	0,413	0,788	0,325	3,50	3,36
	0,52	0,421	0,784	0,330	3,50	3,23
	0,53	0,429	0,779	0,334	3,50	3,10
	0,54	0,437	0,775	0,339	3,50	2,98
	0,55	0,445	0,771	0,343	3,50	2,86
	0,56	0,453	0,767	0,348	3,50	2,75
	0,57	0,461	0,763	0,352	3,50	2,64
	0,58	0,469	0,759	0,356	3,50	2,53
	0,59	0,478	0,755	0,360	3,50	2,43
	0,60	0,486	0,750	0,364	3,50	2,33
	0,61	0,494	0,746	0,368	3,50	2,24
	0,62	0,502	0,742	0,372	3,50	2,15
III	0,79	0,640	0,671	0,429	3,50	0,93
	0,80	0,648	0,667	0,432	3,50	0,87
	0,81	0,656	0,663	0,435	3,50	0,82
	0,82	0,664	0,659	0,437	3,50	0,77
	0,83	0,672	0,655	0,440	3,50	0,72
	0,84	0,680	0,651	0,442	3,50	0,67
	0,85	0,688	0,646	0,445	3,50	0,62
	0,86	0,696	0,642	0,447	3,50	0,57
	0,87	0,704	0,638	0,449	3,50	0,52
	0,88	0,712	0,634	0,452	3,50	0,48
	0,89	0,720	0,630	0,454	3,50	0,43
	0,90	0,729	0,626	0,456	3,50	0,39
	0,91	0,737	0,621	0,458	3,50	0,35
	0,92	0,745	0,617	0,460	3,50	0,30
	0,93	0,753	0,613	0,462	3,50	0,26
	0,94	0,761	0,609	0,463	3,50	0,22
	0,95	0,769	0,605	0,465	3,50	0,18
	0,96	0,777	0,601	0,467	3,50	0,15
	0,97	0,785	0,597	0,468	3,50	0,11
	0,98	0,793	0,592	0,470	3,50	0,07
	0,99	0,801	0,588	0,471	3,50	0,04
	1,00	0,810	0,584	0,473	3,50	0,00
	1,01	0,818	0,580	0,474	3,50	-0,04
	1,02	0,826	0,576	0,476	3,50	-0,07

Table 4.1-3 (end)

	1,03	0,834	0,572	0,477	3,50	-0,10
	1,04	0,842	0,568	0,478	3,50	-0,13
	1,05	0,850	0,564	0,479	3,50	-0,17
	1,06	0,858	0,560	0,480	3,50	-0,20
	1,07	0,866	0,555	0,481	3,50	-0,23
	1,08	0,874	0,550	0,481	3,50	-0,26
	1,09	0,882	0,546	0,482	3,50	-0,29
	1,10	0,890	0,543	0,483	3,50	-0,32
	1,11	0,899	0,538	0,484	3,50	-0,35
	1,12	0,907	0,534	0,484	3,50	-0,38
	1,13	0,915	0,530	0,485	3,50	-0,40
	1,14	0,923	0,525	0,485	3,50	-0,43
	1,15	0,931	0,522	0,486	3,50	-0,46

Table 4.1-3 gives values of $\xi = {}^{x}\!/_{d}$, η and ω for singly reinforced beams sections as a function a_m. In this case following procedure can be utilized:

1. Calculate $a_m = \dfrac{M_{Ed}}{f_{cd} \cdot b \cdot d^2}$.
2. Choose coefficients ξ, η and ω from Table 4.1-3 as a function of a_m.
3. Check on steel yielding:

3.1. If $a_m \le a_{m,lim}$ or $\xi \le \xi_{lim}$ – steel yields before concrete crushes and singly reinforced section will suffice.

Hence, $A_{st} = \dfrac{M_{Ed}}{f_{yd} \cdot \eta \cdot d}$.

3.2. If $a_m > a_{m,lim}$ or $\xi > \xi_{lim}$ – design doubly reinforced section in accordance with 4.1.1.3.1 (2) (compression reinforcement must be supplied), or increase effective depth of the section, d, or increase strength class of the concrete.

Checking of the bending capacity (for a given input data).

1. Calculate $x = \dfrac{A_{st} \cdot f_{yd}}{\omega_c \cdot f_{cd} \cdot b \cdot d}$ and $\xi = \dfrac{x}{d}$.
2. Table 4.1.3 gives values of a_m, η as a function of ξ.
3. Check the steel yielding:

3.1. If $\xi \le \xi_{lim}$ or $a_m \le a_{m,lim}$ – steel yields before concrete crushes, and: $M_{Rd} = a_m \cdot f_{cd} \cdot b \cdot d^2$, or $M_{Rd} = A_{st} \cdot f_{yd} \cdot \eta \cdot d$.

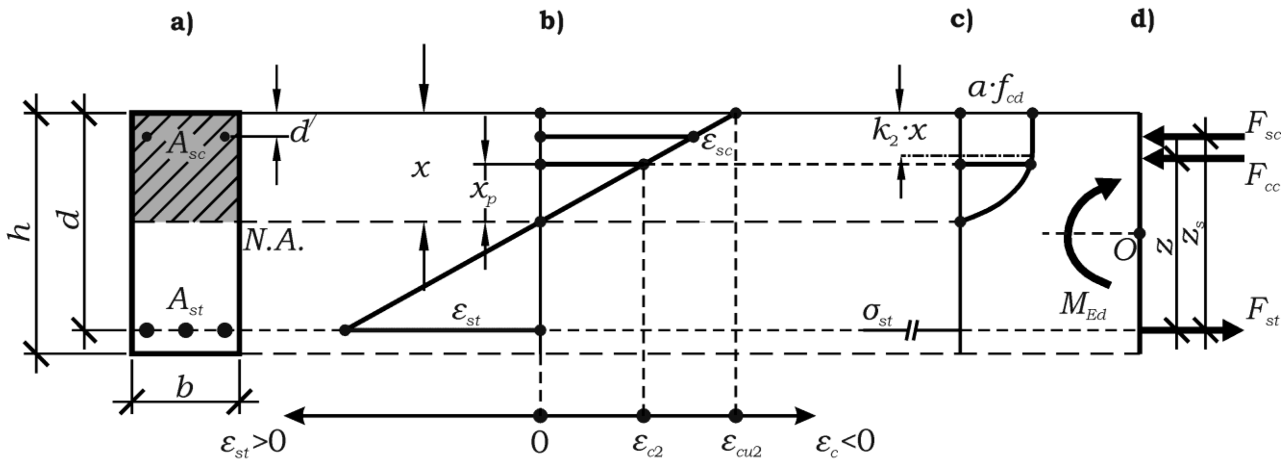
3.2. If $\xi > \xi_{lim}$ or $a_m > a_{m,lim}$ – concrete crushes before steel yields, $a_m = a_{m,lim}$: $M_{Rd} = M_{Rd,lim} = a_{m,lim} \cdot f_{cd} \cdot b \cdot d^2$.

4. Check ULS/STR conditions: $M_{Ed} \le M_{Rd}$.

(2) Doubly reinforced rectangular cross-section. Basic equations

If the moment applied to the cross-section is such that the limiting value $\xi_{lim} = \dfrac{x_{lim}}{d \cdot a_{m,lim}}$ is exceeded when it is attempted to design the section as singly reinforced, then compression reinforcement must be supplied in order to resist the excess moment.

Doubly reinforced cross-section is given in Figure 4.1-10. The applied moment on the section M_{Ed} is resisted in part by the limiting moment $M_{Rd,lim}$ determined from the maximum allowed value of ξ and the remainder by the couple introduced by the forces in the compression reinforcement (and the additional equal force in the tension reinforcement).



a) – cross-section; b) – strain diagram; c) – stress block; d) – internal forces

Figure 4.1-10 – Doubly reinforced cross-section

Equilibrium of axial forces and moments gives (see Figure 4.1-10):

$$M_{Rd} = F_{cc} \cdot z + F_{sc} \cdot z_s; \qquad (4.1\text{-}49)$$

$$F_{st} = F_{cc} + F_{sc}, \qquad (4.1\text{-}50)$$

where: F_{cc} is the resultant compressive force resisted by concrete;

F_{sc} is the resultant force in the compression reinforcement;

F_{st} is the resultant tensile force in steel reinforcement;

z is the lever arm of internal forces;

z_s is the lever arm as a distance between the centroid of the tension and compression reinforcement.

Ultimate moment of resistance respect to tension steel reinforcement is given by:

$$M_{Rd} = \omega_c \cdot f_{cd} \cdot b \cdot x \cdot z + F_{sc} \cdot z_s. \qquad (4.1\text{-}51)$$

Taking in account, that $x = (d - z) \cdot \frac{1}{k_2}$; $F_{sc} = \sigma_{sc} \cdot A_{sc}$ and $z_s = d - c_1$, Equation (4.1-51) can be rewritten as follows:

$$M_{Rd} = \omega_c \cdot f_{cd} \cdot b \cdot \frac{1}{k_2} \cdot (d - z) \cdot z + \sigma_{sc} \cdot A_{sc} \cdot (d - c_1). \tag{4.1-52}$$

Defining $\eta = z/d$ and $C_0 = \omega_c / k_2$, Equation (4.1-52) can be rewritten as follows:

$$\eta^2 - \eta + \frac{M_{Rd} - \sigma_{sc} \cdot A_{sc} \cdot (d - c_1)}{C_0 \cdot f_{cd} \cdot b \cdot d^2}, \tag{4.1-53}$$

where: σ_{sc} is the stress in compression reinforcement in general case $\sigma_{sc} = k_{s2} \cdot f_{yd}$ ($k_{s2} = 1$, if $\varepsilon_{sc} \geq \varepsilon_{sy}$ and $k_{s2} = \frac{\varepsilon_{sc}}{\varepsilon_{sy}}$, if $\varepsilon_{sc} < \varepsilon_{sy}$).

Equation (4.1-53) can be rewritten as follows (with M_{Rd} replaced by M_{Ed}):

$$\eta^2 - \eta + \frac{a_m}{C_0} = 0, \tag{4.1-54}$$

where: $a_m = \frac{M_{Ed} - k_{s2} \cdot f_{yd} \cdot A_{sc} \cdot (d - d_1)}{f_{cd} \cdot b \cdot d^2}$.

Solving Equation (4.1-54) for η to gives:

$$\eta = \frac{z}{d} = 0{,}5 + \sqrt{0{,}25 - \frac{a_m}{C_0}}. \tag{4.1-55}$$

Limit value of the moment following of resistance (if $x = x_{lim}$)can be calculated from the following equation:

$$M_{Rd,lim} = \omega_c \cdot f_{cd} \cdot b \cdot x_{lim} \cdot (d - k_2 \cdot x_{lim}) + k_{s2} \cdot f_{yd} \cdot A_{sc} \cdot (d - c_1). \tag{4.1-56}$$

Defining $x_{lim} = \left(\frac{\varepsilon_{c2,u}}{\varepsilon_{sy} + \varepsilon_{c2,u}} \right) \cdot d$, Equation (4.1-56) can be rewritten as follows:

$$M_{Rd,lim} = f_{cd} \cdot b \cdot d^2 \cdot a_{m,lim} + k_{s2} \cdot f_{yd} \cdot A_{sc} \cdot (d - c_1), \tag{4.1-57}$$

or:

$$a_{m,lim} = \frac{M_{Rd} - k_{s2} \cdot f_{yd} \cdot A_{sc} \cdot (d - c_1)}{f_{cd} \cdot b \cdot d^2}. \tag{4.1-58}$$

(a) Determination of the required bending reinforcement

Equation (4.1-57) can be rewritten (with M_{Rd} replaced by M_{Ed}):

$$M_{Ed} - k_{s2} \cdot f_{yd} \cdot A_{sc} \cdot (d - c_1) = f_{cd} \cdot b \cdot d^2 \cdot a_{m,lim}. \quad (4.1\text{-}59)$$

From Equation (4.1-59) compression steel area can be calculated as:

$$A_{sc} = \frac{M_{Ed} - a_{m,lim} \cdot f_{cd} \cdot b \cdot d^2}{k_{s2} \cdot f_{yd} \cdot (d - c_1)}. \quad (4.1\text{-}60)$$

The resultant compression force F_{cc} is given by:

$$F_{cc} = \omega_c \cdot f_{cd} \cdot b \cdot \frac{1}{k_2} \cdot (d - z) = \frac{\omega_c}{k_2} \cdot f_{cd} \cdot b \cdot d \cdot \left(1 - \frac{z}{d}\right) = C_0 \cdot f_{cd} \cdot b \cdot d \cdot (1 - \eta). \quad (4.1\text{-}61)$$

Tensile steel area can be calculated from equilibrium of axial forces:

$$A_{st} = \frac{C_0 \cdot f_{cd} \cdot b \cdot d \cdot (1 - \eta) + k_{s2} \cdot f_{yd} \cdot A_{sc}}{f_{yd}}, \quad (4.1\text{-}62)$$

or defining $C_0 \cdot (1 - \eta) = \omega_c \cdot \dfrac{x}{d} = \omega_c \cdot \xi$, Equation (4.1-62) can be rewritten as follows:

$$A_{st} = \frac{\omega_c \cdot f_{cd} \cdot b \cdot d \cdot \xi + k_{s2} \cdot f_{yd} \cdot A_{sc}}{f_{yd}}. \quad (4.1\text{-}63)$$

(b) Checking of *ULS* for bending. Ultimate moment of resistance

Checking of ***ULS*** for bending is performed based on Inequality (4.1-1). In this case ultimate moment of resistance M_{Rd} can be calculated using Table 4.1-3 and Table 4.1-4.

Table 4.1-4 – Coefficients for calculation reinforcement area A_{sc}

Range of strain distribution	Neutral axis depth, ξ	Coefficient k_{s2}							
		Steel grade 400				Steel grade 500			
		a_2/d= =0,04	a_2/d= =0,08	a_2/d= =0,12	a_2/d= =0,16	a_2/d= =0,04	a_2/d= =0,08	a_2/d= =0,12	a_2/d= =0,16
Ia	0,04	0	-40,23	-0,47	-0,72	0	-0,19	-0,39	-0,60
	0,05	0,06	-0,18	-0,42	-0,66	0,05	-0,15	-0,35	-0,55
	0,06	0,12	-0,12	-0,36	-0,61	0,10	-0,10	-0,30	-0,51
	0,07	0,19	-0,06	-0,31	-0,55	0,16	-0,05	-0,26	-0,46
	0,08	0,25	0	-0,25	-0,50	0,21	0	-0,21	-0,42
	0,09	0,31	0,06	-0,19	-0,44	0,26	0,05	-0,16	-0,37
	0,10	0,38	0,12	-0,12	-0,38	0,32	0,10	-0,14	-0,32
	0,11	0,45	0,19	-0,06	-0,32	0,37	0,16	-0,05	-0,27
	0,12	0,51	0,26	0	-0,26	0,42	0,22	0	-0,22
	0,13	0,59	0,32	0,06	-0,20	0,49	0,27	0,05	-0,17
	0,14	0,66	0,40	0,13	-0,13	0,55	0,33	0,11	-0,11
	0,15	0,74	0,47	0,20	-0,07	0,62	0,39	0,17	-0,06
	0,16	0,81	0,54	0,27	0	0,67	0,45	0,22	0

Table 4.1-4 (end)

	0,167	0,86	0,59	0,32	0,04	0,72	0,49	0,27	0,03
	0,90	0,62	0,34	0,34	0,07	0,75	0,52	0,28	0,06
	0,98	0,70	0,42	0,42	0,14	0,82	0,58	0,35	0,12
	1	0,77	0,50	0,50	0,21	0,88	0,64	0,42	0,17
	1	0,86	0,58	0,58	0,28	0,95	0,72	0,48	0,23
Ib	1	0,94	0,64	0,64	0,36	1	0,78	0,53	0,30
	1	1	0,74	0,74	0,44	1	0,85	0,62	0,37
	1	1	0,82	0,82	0,52	1	0,93	0,68	0,43
	1	1	0,90	0,90	0,60	1	1	0,75	0,50
	1	1	0,99	0,99	0,68	1	1	0,82	0,57
	1	1	1	1	0,76	1	1	0,89	0,63
	1	1	1	1	0,77	1	1	0,90	0,64
	1	1	1	1	0,81	1	1	0,93	0,67
	1	1	1	1	0,86	1	1	0,95	0,71
	1	1	1	1	0,90	1	1	0,98	0,75
	1	1	1	1	0,93	1	1	1	0,78
	1	1	1	1	0,97	1	1	1	0,81
	1	1	1	1	1	1	1	1	0,83
II	1	1	1	1	1	1	1	1	0,86
	1	1	1	1	1	1	1	1	0,88
	1	1	1	1	1	1	1	1	0,90
	1	1	1	1	1	1	1	1	0,93
	1	1	1	1	1	1	1	1	0,95
	1	1	1	1	1	1	1	1	0,97
	1	1	1	1	1	1	1	1	0,98
	1	1	1	1	1	1	1	1	1
Note. *In table signs with «+» – compression.*									

For given section and areas of steel reinforcement A_{st}, A_{sc} relative depth of neutral axis can be calculated from equilibrium of axial forces:

$$\xi = \frac{x}{d} = \frac{A_{st} \cdot f_{yd} - A_{sc} \cdot k_{s2} \cdot f_{yd}}{\omega_c \cdot f_{cd} \cdot b \cdot d}. \qquad (4.1\text{-}64)$$

Table 4.1-3 gives values of η and a_m as a function ξ and Table 4.1-4 gives value of k_{s2} as a function of ξ too.

Ultimate moment of resistance M_{Ed} is given by:

$$M_{Rd} = a_m \cdot a \cdot f_{cd} \cdot b \cdot d^2 + k_{s2} \cdot f_{yd} \cdot A_{sc} \cdot \left(d - c_1\right). \qquad (4.1\text{-}65)$$

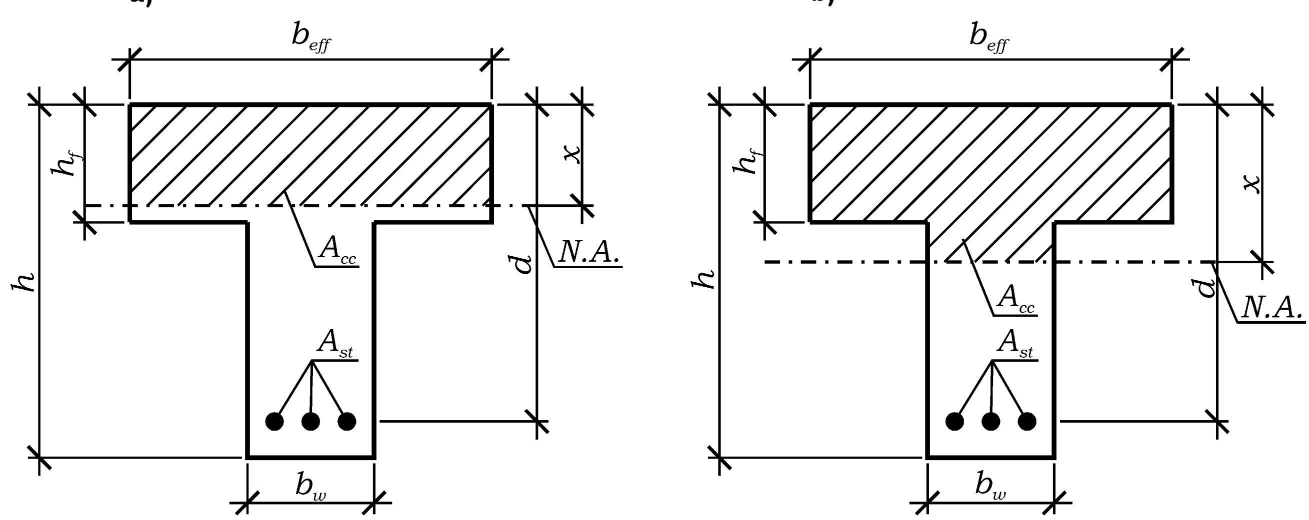
4.1.1.3.2 Flanged cross-section

Flanged cross-section beams occur where it is necessary to deepen a cross-section in order to carry the reaction from one or two spanning slabs and any additional loads from cladding or permanent partions. Such beams are cast as a part of the slab system, and the flexural resistance is calculated with an allowance of part of the slab acting integrally with the beam. For one way spanning slabs half of total loading from the slab is taken by each beam and slab system. For two spanning slab the load is determined on the basis of a 45° dispersion or where the Hillerborg strip.

For singly reinforced flanged cross-section it is necessary to consider two conditions (two cases must be considered, see Figure 4.1-11):

a) If, $x \le h_f$ or $\xi = \dfrac{x}{d} \le \beta = \dfrac{h_f}{d}$, then the neutral axis is in the flange, and

b) If, $x > h_f$ or $\xi = \dfrac{x}{d} > \beta = \dfrac{h_f}{d}$, then the neutral axis is in the web.



a) – neutral axis is in the flange $x \le h_f$; b) – neutral axis is in the web $x > h_f$

Figure 4.1-11 – Flanged cross-section

For case (a) the design follows the calculations for rectangular cross-sections with the width of the section taken as the effective width of the flange $b = b_{eff}$ (determination of effective width b_{eff}, see Chapter 2). This is because the non-rectangular cross-section below the neutral axis is in tension and is, therefore, considered to be cracked and inactive (according in assumption, the tensile strength of concrete is ignored). If x is less than the flange thickness (h_f), the stress block does lie within the flange as assumed and the area of reinforcement is given by the following expression:

$$A_{st} = \frac{M_{Ed}}{f_{yd} \cdot \eta \cdot d}. \tag{4.1-66}$$

For case (b) general solution can be obtained by assuming two superimposed sections ① and ②, as it shown in Figure 4.1-12. In accordance with the stress block given for the section ① a compressive zone is situated in the flange ($b_{eff} - b_w$) only, and for the section ② it is in the web (see Figure 4.1-13).

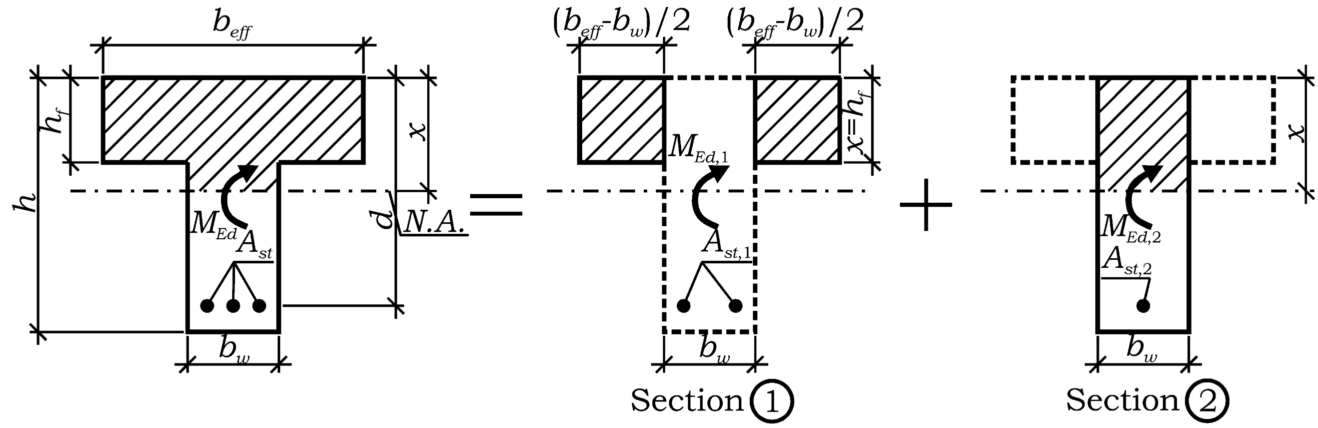


Figure 4.1-12 – Flanged cross-section. Dividing on the two superimposed sections ① and ②

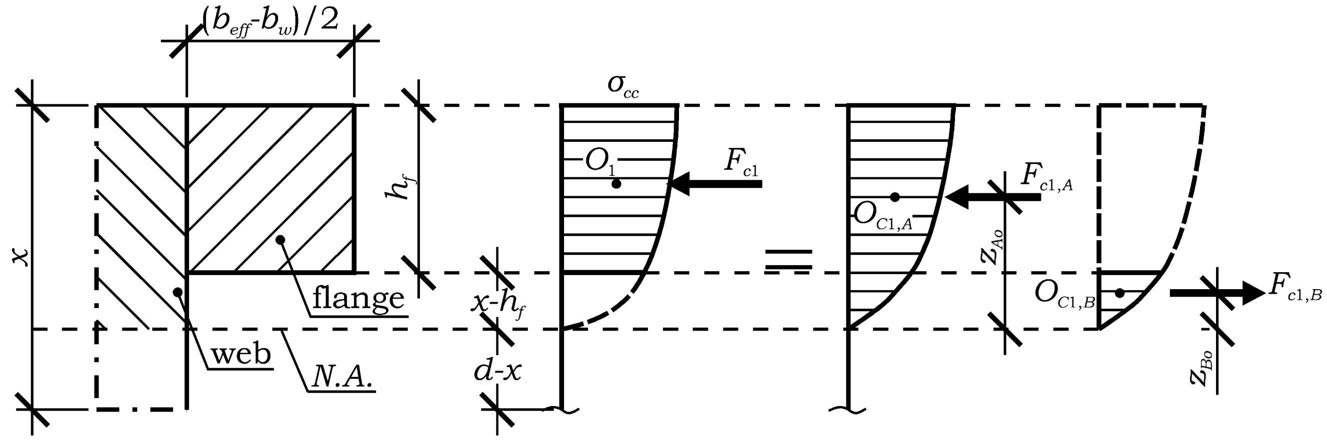


Figure 4.1-13 – Flanged cross-section. Stress blocks in concrete

For section 1 from the proportions of the strain distribution diagram follows:
If, $0 \le \xi \le 0{,}259$, (strain ranges 1a, 1b), then

$$\frac{x-h_f}{\varepsilon_f} = \frac{d-x}{\varepsilon_{s,R}} \rightarrow \varepsilon_f = \varepsilon_{s,R} \cdot \frac{x-h_f}{d-x}, \text{ [‰]}. \tag{4.1-67}$$

If, $\xi > 0{,}259$, (strain range 2 or 3), then

$$\frac{x-h_f}{\varepsilon_f} = \frac{x}{\varepsilon_{c2,u}} \rightarrow \varepsilon_f = \varepsilon_{c2,u} \cdot \frac{x-h_f}{x}, \text{ [‰]}. \tag{4.1-68}$$

Defining $\xi = \dfrac{x}{d}$ and $\beta = \dfrac{h_f}{d}$, following design conditions can be written:
If, $0 \le \xi \le 0{,}259$, (strain ranges 1a, 1b), then:

$$\xi < \frac{1}{6} + \frac{5}{6} \cdot \beta. \tag{4.1-69}$$

If, $\xi > 0{,}259$, (strain range 2 or 3), then:

$$\xi < \frac{7}{3} \cdot \beta. \tag{4.1-70}$$

In general case, for section 1 equilibrium conditions of axial forces and moments about the neutral axis (stress block A and B, see Figure 4.1-13) can be written as:

$$F_{c1} = F_{c1,A} - F_{c1,B}, \tag{4.1-71}$$

$$M_{Rd,1} = F_{c1,A} \cdot z_{c1,A} - F_{c1,B} \cdot z_{c1,B}. \tag{4.1-72}$$

For rectangular-parabolic stress block the resultant compressive force in concrete (see Figure 4.1-13) is expressed as follows:

$$F_{c1} = \left(b_{eff} - b_w\right) \cdot f_{cd} \cdot \left\{ \int_0^x \left[1 - \left(1 - \frac{\varepsilon_c(y)}{\varepsilon_{c,2}} \right)^n \right] dy - \int_0^{x-h_f} \left[1 - \left(1 - \frac{\varepsilon_c(y)}{\varepsilon_{c,2}} \right)^n \right] dy \right\}. \tag{4.1-73}$$

For strength classes of the concrete from C16/20 to C50/60, ($n = 2, \varepsilon_{c2} = 2{,}0$ ‰), Equation (4.1-73) can be rewritten as follows:

$$F_{c1} = \left(b_{eff} - b_w\right) \cdot f_{cd} \cdot \left[\int_0^x \left(\varepsilon_c(y) - \frac{\left(\varepsilon_c(y)\right)^2}{4} \right) dy - \int_0^{x-h_f} \left(\varepsilon_c(y) - \frac{\left(\varepsilon_c(y)\right)^2}{4} \right) dy \right], \tag{4.1-74}$$

and, after integration can be written:

$$F_{c1} = \omega_T \cdot \beta \cdot d \cdot \left(b_{eff} - b_w\right) \cdot f_{cd}, \tag{4.1-75}$$

where: ω_T is the dimensionless coefficient in accordance with Table 4.1-5. Table 4.1-5 gives value of ω_T for singly reinforced section as a function β and ξ.

Table 4.1-5 – Values of the ω_T coefficient

Range of strain distribution	Neutral axis depth, ξ	Coefficient ω_T						
		β=0,08	β=0,10	β=0,12	β=0,14	β=0,16	β=0,18	β=0,20
Ia	0,08	–	–	–	–	–	–	–
	0,09	0,037	–	–	–	–	–	–
	0,10	0,043	–	–	–	–	–	–
	0,11	0,049	0,054	–	–	–	–	–
	0,12	0,055	0,061	–	–	–	–	–
	0,13	0,060	0,068	0,073	–	–	–	–
	0,14	0,065	0,074	0,081	–	–	–	–
	0,15	0,068	0,080	0,088	0,093	–	–	–
	0,16	0,072	0,085	0,095	0,102	–	–	–
	0,167	0,073	0,088	0,099	0,107	0,111	–	–

Table 4.1-5 (end)

	0,17	0,075	0,090	0,101	0,110	0,115	–	–
	0,18	0,076	0,093	0,106	0,116	0,123	–	–
	0,19	0,078	0,095	0,110	0,122	0,131	0,136	–
	0,20	0,079	0,098	0,114	0,127	0,138	0,145	–
Ib	0,21	0,079	0,098	0,116	0,131	0,143	0,152	0,157
	0,22	0,080	0,099	0,118	0,134	0,148	0,159	0,166
	0,23	0,080	0,100	0,119	0,137	0,152	0,165	0,174
	0,24	0,080	0,100	0,119	0,138	0,154	0,168	0,179
	0,25		0,100	0,120	0,139	0,157	0,172	0,185
	0,26			0,120	0,139	0,158	0,175	0,189
	0,27			0,120	0,140	0,159	0,177	0,192
	0,28			0,120	0,140	0,160	0,178	0,194
	0,29				0,140	0,160	0,178	0,195
	0,30				0,140	0,160	0,179	0,196
	0,31				0,140	0,160	0,179	0,197
	0,32				0,140	0,160	0,179	0,197
	0,33				0,140	0,160	0,179	0,198
	0,34					0,160	0,179	0,198
	0,35					0,160	0,180	0,199
II	0,36					0,160	0,180	0,199
	0,37					0,160	0,180	0,200
	0,38					0,160	0,180	0,200
	0,39						0,180	0,200
	0,40						0,180	0,200
	0,41						0,180	0,200
	0,42						0,180	0,200
	0,43							0,200
	0,44							0,200
	0,45							0,200
	0,46							0,200
	0,47							0,200

The lever arm, $z_{c1,A}\left(z_{c1,B}\right)$ is given by:

$$z_{c1,A} = z_{A,0} + \left(d - x\right). \tag{4.1-76}$$

$$z_{c1,B} = z_{B,0} + \left(d - x\right). \tag{4.1-77}$$

Moment of resistance $M_{Rd,1}$ (for section 1) is given by:

$$M_{Rd,1} = a_T \cdot d^2 \cdot \left(b_{eff} - b_w\right) \cdot a \cdot f_{cd}, \tag{4.1-78}$$

where: a_T is the dimension less coefficient according to Table 4.1-6. Table 4.1-6 gives value of a_T as a function β and ξ.

Table 4.1-6 – Values of the a_T coefficient

Range of strain distribution	Neutral axis depth, ξ	Coefficient a_T						
		β=0,08	β=0,10	β=0,12	β=0,14	β=0,16	β=0,18	β=0,20
Ia	0,08	–	–	–	–	–	–	–
	0,09	0,036	–	–	–	–	–	–
	0,10	0,042	–	–	–	–	–	–
	0,11	0,048	0,052	–	–	–	–	–
	0,12	0,053	0,059	–	–	–	–	–
	0,13	0,058	0,066	0,070	–	–	–	–
	0,14	0,062	0,071	0,077	–	–	–	–
	0,15	0,066	0,076	0,084	0,088	–	–	–
	0,16	0,069	0,081	0,090	0,096	–	–	–
	0,167	0,071	0,084	0,094	0,101	–	–	–
Ib	0,17	0,072	0,085	0,095	0,103	0,107	–	–
	0,18	0,074	0,089	0,100	0,109	0,115	–	–
	0,19	0,075	0,091	0,104	0,114	0,122	0,126	–
	0,20	0,076	0,092	0,107	0,118	0,127	0,133	–
	0,21	0,076	0,093	0,109	0,122	0,132	0,140	0,144
	0,22	0,077	0,094	0,111	0,125	0,137	0,145	0,151
	0,23	0,077	0,095	0,112	0,127	0,140	0,150	0,158
	0,24	0,077	0,095	0,113	0,129	0,143	0,155	0,163
	0,25		0,095	0,113	0,129	0,144	0,157	0,167
II	0,26			0,113	0,130	0,146	0,159	0,171
	0,27			0,113	0,130	0,146	0,161	0,173
	0,28			0,113	0,130	0,147	0,162	0,174
	0,29				0,130	0,147	0,162	0,175
	0,30				0,130	0,147	0,163	0,176
	0,31				0,130	0,147	0,163	0,177
	0,32				0,130	0,147	0,163	0,178
	0,33				0,130	0,147	0,163	0,178
	0,34					0,147	0,163	0,178
	0,35					0,147	0,163	0,179
	0,36					0,147	0,163	0,179
	0,37					0,147	0,163	0,179
	0,38					0,147	0,163	0,180
	0,39						0,163	0,180
	0,40						0,163	0,180
	0,41						0,163	0,180
	0,42						0,163	0,180
	0,43							0,180
	0,44							0,180
	0,45							0,180
	0,46							0,180
	0,47							0,180

Under consideration section ② (see Figure 4.1-13) was assumed that moment $M_{Rd,2}$, is calculated for rectangular section with $b = b_w$. Finally, moment of resistance, M_{Rd}, for flanged section, can be calculated:

$$M_{Rd,1} = \left[a_T \cdot \left(b_{eff} - b_w \right) + a_m \cdot b_w \right] \cdot d^2 \cdot f_{cd}, \qquad 4.1\text{-}79)$$

or:

$$M_{Rd,1} = f_{cd} \cdot \left[a_T \cdot \left(b_{eff} - b_w \right) + a_m \cdot b_w \right] \cdot d^2. \tag{4.1-80}$$

4.1.1.4 Rectangular compressive stress block in concrete

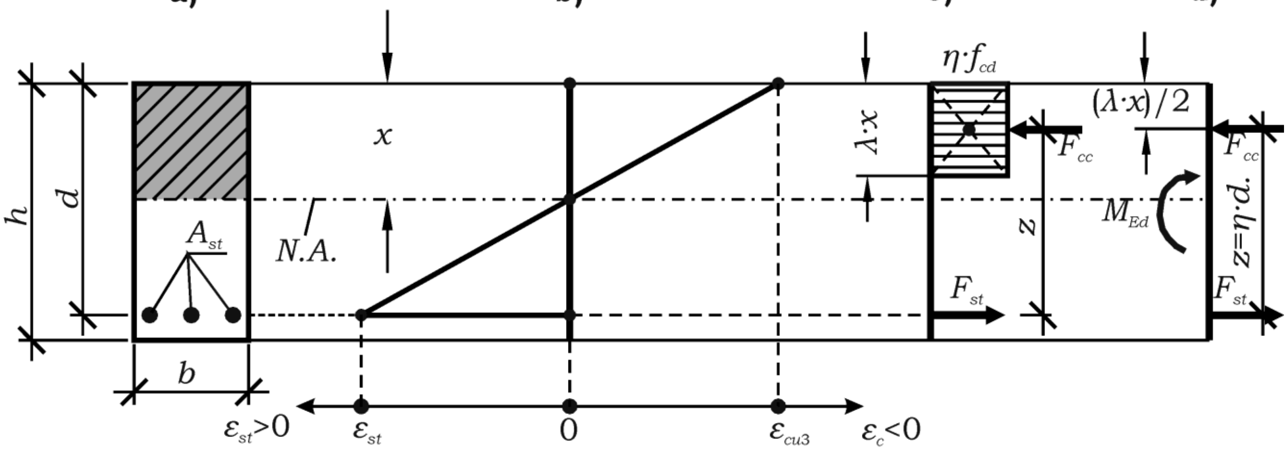
In accordance with EN 1992 [N3], the rectangular stress block, as shown in Figure 3.1-11, may be used in preference to the more rigorous rectangular-parabolic stress block. This simplified stress distribution will facilitate the analysis and provide more manageable design equation in particular when dealing with non-rectangular cross-sections or when undertaking hand calculations [6].

It can be seen from Figure 3.1-11 that in this case the stress block does not extend to the neutral axis of the section but has a depth $x_{eff} = \lambda \cdot x$. This will result in the centroid of the stress block being $x_{eff} / 2 = \lambda \cdot x / 2$ from the top (compressive) edge of the section, which is very nearly the same location as for the more precise rectangular-parabolic stress block (as it was shown in previous Section 4.1.1.3.1, $k_2 = 0,416$ for rectangular-parabolic block). Also the areas of the two types of the stress block are approximately equal. This the moment of resistance of the cross-section, M_{Rd}, will be similar using calculation based on either of the two stress blocks.

4.1.1.4.1 Rectangular cross-section

(1) Singly reinforced rectangular cross-section. Determination of the required bending reinforcement

Bending of the section will induce a resultant force F_{st} in the reinforcement steel, and a resultant compressive force F_{cc} in the concrete, which act through the centroid of the effective area of concrete in compression, as it is shown in Figure 4.1-14.



a) – cross-section; b) – strain diagram; c) – stress block; d) – internal forces

Figure 4.1-14 – Singly reinforced cross-section with rectangular stress block

For equilibrium, the ultimate design moment, M_{Ed}, must be balanced by the moment of the resistance of the section M_{Rd}, so that:

$$M_{Ed} = M_{Rd} = F_{cc} \cdot z = F_{st} \cdot z, \tag{4.1-81}$$

where: z is the lever arm between the resultant forces F_{cc} and F_{st};

F_{cc} is the resultant compressive force, that is given by the following expression:

$$F_{cc} = f_{cd} \cdot b \cdot \lambda \cdot x = 0,8 \cdot f_{cd} \cdot b \cdot x, \tag{4.1-82}$$

and:

$$z = d - 0,5 \cdot \lambda \cdot x = d - 0,4 \cdot x. \tag{4.1-83}$$

Substituting Equation (4.1-82) and Equation (4.1-83) into Equation (4.1-81):

$$M_{Ed} = M_{Rd} = f_{cd} \cdot b \cdot \lambda \cdot x \cdot \left(d - 0,5 \cdot \lambda \cdot x\right), \tag{4.1-84}$$

and, replacing $\lambda{\cdot}x$ from Equation (4.1-83) gives:

$$M_{Ed} = M_{Rd} = f_{cd} \cdot b \cdot d^2 \cdot \left(1 - \frac{z}{d}\right) \cdot \frac{z}{d} \cdot 2. \tag{4.1-85}$$

Defining $\eta = \dfrac{z}{d}$ and $a_m = M_{Ed} / f_{cd} \cdot b \cdot d^2$, Equation (4.1-85) can be rewritten as follows:

$$a_m = 2 \cdot \left(1 - \eta\right) \cdot \eta. \tag{4.1-86}$$

From Equation (4.1-86) it follows that:

$$\eta = \frac{z}{d} = 0,5 + \sqrt{0,25 - 0,5 \cdot a_m}, \tag{4.1-87}$$

or rewritten in another way:

$$\eta = \frac{z}{d} = 0,5\left(1 + \sqrt{1 - 2 \cdot a_m}\right). \tag{4.1-88}$$

This Equation (4.1-88) is valid under the assumption that reinforcing steel yields before the concrete crushes. As it was shown in Section 4.1.1.1, at ultimate state (ULS/STR) it is important that member sections in flexure should be ductile and that failure should occur with the gradual yielding of the tension steel and not by a sudden catastrophic failure of the concrete.

Hence, to ensure yielding of the tension steel at the ultimate limit state (ULS):

$$\xi = \frac{x}{d} \le \xi_{lim} = \frac{x_{lim}}{d}, \tag{4.1-89}$$

or:

$$a_m \le a_{m,lim}, \tag{4.1-90}$$

where:

$$\xi_{lim} = \frac{x_{lim}}{d} = \frac{\varepsilon_{cu,3} \cdot d}{\varepsilon_{cu,3} + \varepsilon_{sy}} = \frac{\varepsilon_{cu,3}}{\varepsilon_{cu,3} + \frac{f_{yd}}{E_s}}, \tag{4.1-91}$$

and:

$$a_{m,lim} = 0,8 \cdot \xi_{lim} \cdot (1 - 0,4 \cdot \xi_{lim}). \tag{4.1-92}$$

For steel with $f_{yk} = 500$ N/mm² the yield strain is $\varepsilon_{sy} = 2,17$ ‰. Inserting these values for $\varepsilon_{cu,3} = 3,5$ ‰ and ε_{sy} into Equation (4.1-91):

$$\xi_{lim} = \frac{x_{lim}}{d} = \frac{3,5}{3,5 + 2,17} = 0,617. \tag{4.1-93}$$

$$a_{m,lim} = 0,8 \cdot 0,617 \cdot (1 - 0,4 \cdot 0,617) = 0,372. \tag{4.1-94}$$

From the other hand, EN1992 [N3] (clause 5.6.3) limits the depth of the neutral axis to 0,45·d for concrete strength classes less than or equal to C50/60 (and 0,35·d for concrete classes C55/67 and greater) in order to provide a ductility (possibility to moment redistribution) i.e. under reinforced section, Thus: $\xi_{lim} = \frac{x_{lim}}{d} = 0,45$, and $a_{m,lim} = 0,295$.

Area of tensile reinforcement steel (A_{st}) can be calculated from Equation (4.1-81):

$$M_{Ed} = M_{Rd} = F_{st} \cdot z = A_{st} \cdot f_{yd} \cdot z. \tag{4.1-95}$$

Defining $z = \eta \cdot d$, Equation (4.1-95) can be rewritten as follows:

$$M_{Ed} = A_{st} \cdot f_{yd} \cdot \eta \cdot d. \tag{4.1-96}$$

And area of tensile reinforcement is expressed from Equation (4.1-96) as follows:

$$A_{st} = \frac{M_{Ed}}{f_{yd} \cdot \eta \cdot d}. \tag{4.1-97}$$

Equation (4.1-97) can be used to calculate the area of tension reinforcement provided that the design ultimate moment, $M_{Ed} \le M_{Rd}$.

(2) Doubly reinforced rectangular cross-section

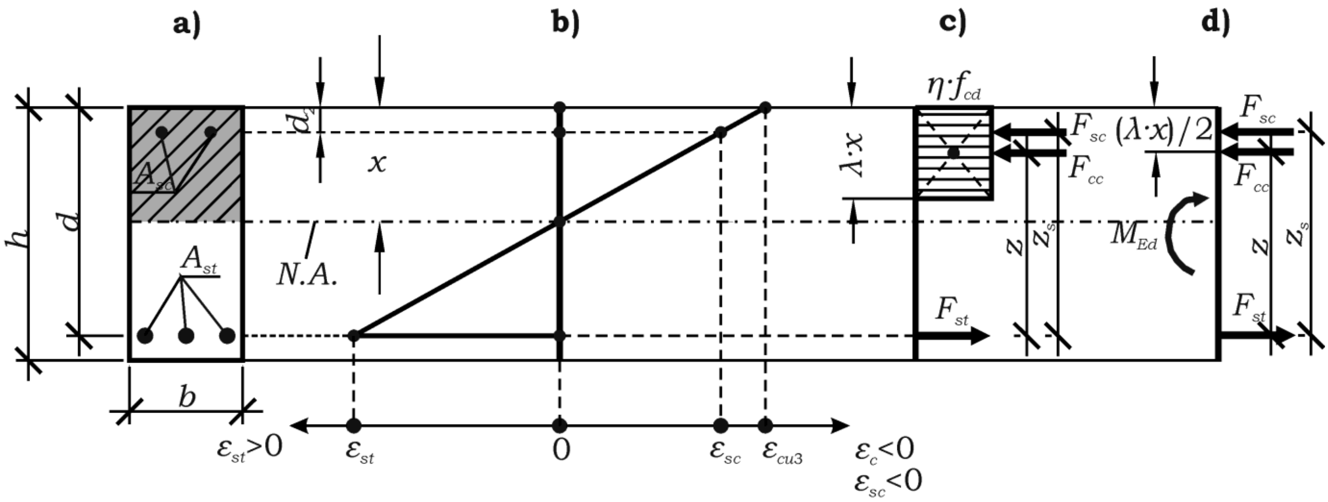
For equilibrium of the section in Fugure 4.1-15:

$$F_{st} = F_{sc} + F_{cc}, \tag{4.1-98}$$

and taking moment about the centroid of the tension steel reinforcement:

$$M_{Ed} = M_{Rd} = f_{cd} \cdot b \cdot \lambda \cdot x \cdot z + A_{sc} \cdot f_{yd} \cdot z_s, \tag{4.1-99}$$

$$M_{Ed} = M_{Rd} = f_{cd} \cdot b \cdot \lambda \cdot x \cdot (d - 0{,}5 \cdot \lambda \cdot x) + A_{sc} \cdot f_{yd} \cdot (d - d_2). \tag{4.1-100}$$



a) – cross-section; b) – strain diagram; c) – stress block; d) – internal forces

Figure 4.1-15 – Doubly reinforced cross-section with rectangular stress block

If the design ultimate moment M_{Ed} is greater than the ultimate limit moment of resistance $M_{Rd,lim}$, i.e $M_{Ed} > M_{Rd,lim}$ (or $a_m > a_{m,lim}$) then compression reinforcement is required.

Provided that: $\frac{d_2}{x} \le 0{,}38$, i.e. compressive steel yield (where: d_2 – is the depth of the compression steel from the must compression face), and:

$$x = \frac{d - z}{0,4}. \tag{4.1-101}$$

The area of compression reinforcement A_{sc} is given by:

$$A_{sc} = \frac{M_{Ed} - M_{Rd,lim}}{f_{yd} \cdot (d - d_2)}. \tag{4.1-102}$$

And the area of tension reinforcement A_{st} is given by:

$$A_{st} = \frac{M_{Rd,lim}}{f_{yd} \cdot \eta \cdot d} + A_{sc}, \tag{4.1-103}$$

where: $\eta = \frac{z}{d} = 0,5\left(1 + \sqrt{1 - 2 \cdot a_{m,lim}}\right)$;

$a_{m,lim}$ – is calculated in accordance with Equation (4.1-92);

$M_{Rd,lim}$ – limit moment of resistance, that is given by:

$$M_{Rd,lim} = a_{m,lim} \cdot f_{cd} \cdot b \cdot d^2 = 0,372 \cdot f_{cd} \cdot b \cdot d^2. \tag{4.1-104}$$

Equation (4.1-102) and Equation (4.1-103) have been derived using the rectangular stress block shown in Figure 4.1-15. This is similar to that used to derive equations for the design of singly reinforced cross-sections, except for the additional force due to the steel in the compression face.

The values of the coefficients η, ξ and $a_{m,lim}$ which are used for calculating of the sections in bending can be obtained from Table 4.1-7 as a function of the a_m.

Table 4.1-7 – Coefficients for sections design, based on rectangular stress block

ξ	η	a_m	ξ	η	a_m
0,02	0,990	0,020	0,33	0,835	0,275
0,03	0,985	0,030	0,34	0,830	0,282
0,04	0,980	0,039	0,35	0,825	0,289
0,05	0,975	0,048	0,36	0,820	0,295
0,06	0,970	0,058	0,37	0,815	0,301
0,07	0,965	0,067	0,38	0,810	0,309
0,08	0,960	0,077	0,39	0,805	0,314
0,09	0,955	0,085	0,40	0,800	0,320
0,10	0,950	0,095	0,41	0,795	0,326
0,11	0,945	0,104	0,42	0,790	0,332
0,12	0,940	0,113	0,43	0,785	0,337
0,13	0,935	0,121	0,44	0,780	0,343
0,14	0,930	0,130	0,45	0,775	0,349
0,15	0,925	0,139	0,46	0,770	0,354
0,16	0,920	0,147	0,47	0,765	0,359
0,17	0,915	0,155	0,48	0,760	0,365
0,18	0,910	0,164	0,49	0,755	0,370

Table 4.1-7 (end)

0,19	0,905	0,172	0,50	0,750	0,375
0,20	0,900	0,180	0,51	0,745	0,380
0,21	0,895	0,188	0,52	0,740	0,385
0,22	0,890	0,196	0,53	0,735	0,390
0,23	0,885	0,203	0,54	0,730	0,394
0,24	0,880	0,211	0,55	0,725	0,400
0,25	0,875	0,219	0,56	0,720	0,403
0,26	0,870	0,226	0,57	0,715	0,408
0,27	0,865	0,233	0,58	0,710	0,412
0,28	0,860	0,241	0,59	0,705	0,416
0,29	0,855	0,248	0,60	0,700	0,420
0,30	0,850	0,255	0,61	0,695	0,424
0,31	0,845	0,262	0,62	0,690	0,428
0,32	0,840	0,269	0,63	0,685	0,432

4.1.1.4.2 Flanged cross-section

For the singly reinforced cross-sections it is necessary to consider two conditions:

1) The stress block lies within the compressive flange;
2) The stress block extends below the flange.

(1) Flanged cross-section – the depth of stress block $\boldsymbol{\lambda \cdot x < h_f}$ (see Figure 4.1-16)

(a) Required area of reinforcement

For this depth of stress block, the beam flanged cross-section can be considered as an equivalent rectangular cross-section of width be equal to the flange width, $b = b_f$. This is because of the non-rectangular part of the section below the neutral axis is in tension and it is, therefore, considered to be cracked and inactive.

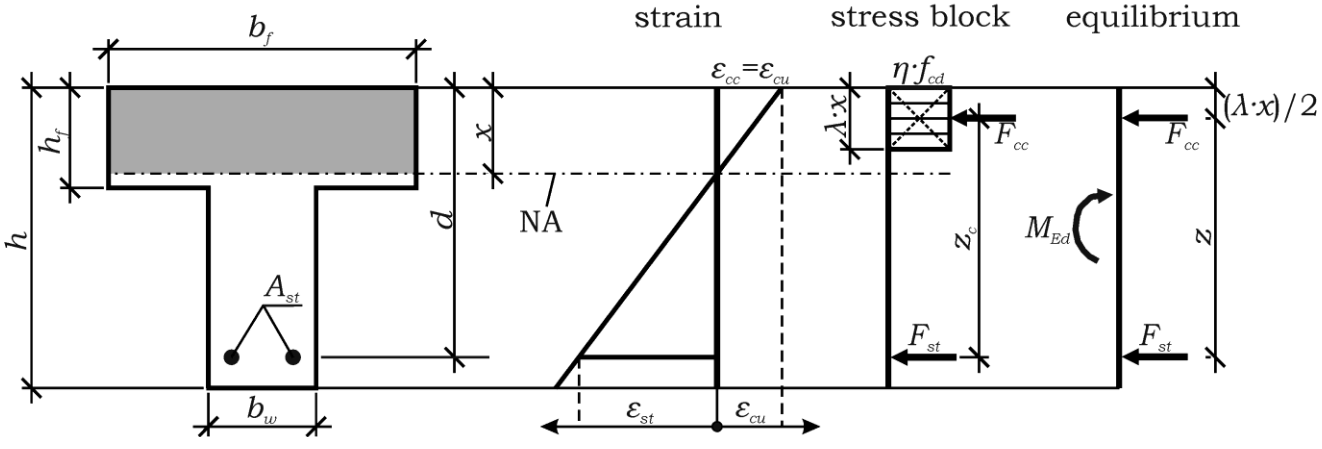


Figure 4.1-16 – Flanged cross-section – rectangular stress block within the flange (case: $\boldsymbol{\lambda \cdot x < h_f}$)

Thus, $a_{m,eff} = \dfrac{M_{Ed}}{f_{cd} \cdot b_f \cdot d}$ can be calculated and the lever arm η, depth of the neutral axis ξ is determined from Table 4.1-7. As it was shown, relation between the lever arm $z = \eta \cdot d$ and the depth x of the neutral axis (see Figure 4.1-16) is given by:

$$z = d - \frac{\lambda \cdot x}{2}, \tag{4.1-105}$$

or:

$$\lambda \cdot x = 2 \cdot (d - z) = 2 \cdot (d - \eta \cdot d) = 2 \cdot d \cdot (1 - \eta), \tag{4.1-106}$$

and:

$$\xi = \frac{x}{d} = \frac{2}{\lambda}(1 - \eta), \tag{4.1-107}$$

If $\xi_{eff} = \dfrac{\lambda \cdot x}{d}$ is less than the flange relative thickness $\beta = \dfrac{h_f}{d}$, the stress block does lie within the flange as it was assumed and it is necessary to check on the following conditions: $\xi_{eff} \le \xi_{lim}$. If this condition is satisfied, than the area of tensile reinforcement A_{st} is given by:

$$A_{st} = \frac{M_{Ed}}{f_{yd} \cdot \eta \cdot d}, \tag{4.1-108}$$

If $\xi_{eff} \ge \xi_{lim,eff}$, than doubly reinforced cross-section should be designed. If $\xi_{eff} = \dfrac{\lambda \cdot x}{d} > \beta$, then the depth of the stress block extends below the flange and design procedure according in section 4.1.1.4.1 should be utilized.

(b) Checking of *ULS* for bending

For the cross section equilibrium (see Figure 4.1-16):

$$F_{cc} = F_{st}. \tag{4.1-109}$$

Therefore:

$$f_{cd} \cdot \lambda \cdot x \cdot b_f = f_{yd} \cdot A_{st}, \tag{4.1-110}$$

and solving for the depth of the stress block:

$$x = \frac{f_{yd} \cdot A_{st}}{f_{cd} \cdot \lambda \cdot b_f}. \tag{4.1-111}$$

If $x \le h_f$, the stress block lies within the flange and with this depth of neutral axis, the steel reinforcement A_{st} will have yielded as it was assumed. Lever arm is equal to:

$$z = d - \frac{\lambda \cdot x}{2}. \tag{4.1-112}$$

Taking moments about the centroid of the reinforcement A_{st}, the moment of resistance is calculated as follows:

$$M_{Rd} = F_{cc} \cdot z = \eta \cdot f_{cd} \cdot \lambda \cdot x \cdot b_f \cdot z. \tag{4.1-113}$$

The ultimate limit state (ULS) criterion should be satisfied:

$$M_{Rd} \ge M_{Ed}, \tag{4.1-114}$$

otherwise → in the flanged cross-section the depth of the stress block extents below the flange, $\lambda \cdot x > h_f$.

(2) Flanged cross-section – the depth of stress block $\boldsymbol{\lambda \cdot x > h_f}$ (see Figure 4.1-17)

(a) Required area of reinforcement

As it was shown, for design of the flanged cross-section, the procedure described in previous section, will check if the depth of the stress block extends below the flange. As an alternative procedure, proposed in [6], is to calculate the moment of resistance M_{Rd} of the cross section with $\lambda \cdot x = h_f$, the depth of the flange. Hence, if the design moment M_{Ed} is such that:

$$M_{Ed} \ge M_{Rd}, \tag{4.1-115}$$

then, the stress block must be extended below the flange, and $\lambda \cdot x > h_f$.

In this case the design can be carried out either:

(1) using an exact method to determine the depth of the neutral axis, as it was shown in Section 4.1.1.4.2.

(2) designing for the conservative condition of $x = 0,45 \cdot d$, which is the maximum value for x for singly reinforced section and concrete classes less than C50/60 in accordance with EN 1992 [N3].

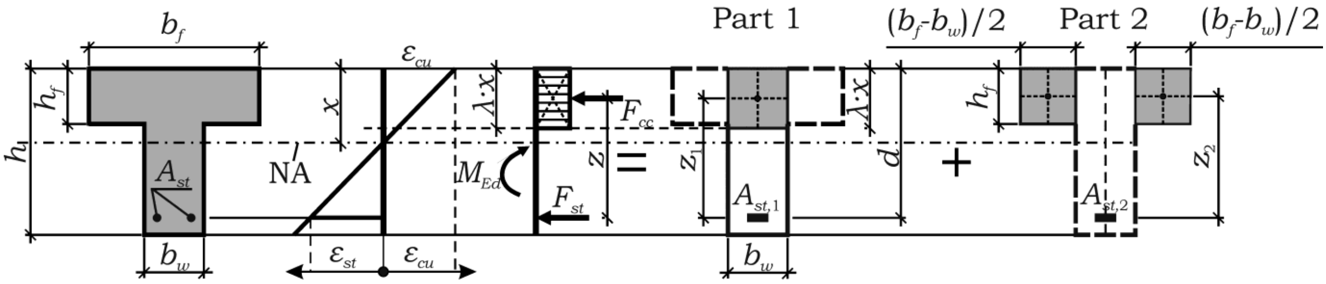


Figure 4.1-17 – Flanged cross-section – the depth of the stress block extends below the flange (case: $\boldsymbol{\lambda \cdot x > h_f}$)

In case of the rectangular stress block, solution can be the most easily achieved by considering the base stress block to be made up of two parts (subblock 1 and subblock 2) as it is shown in Figure 4.1-17.

It will be assumed that the neutral axis is large enough for the whole flange to be at a stress f_{cd}.

Hence, by equilibrium (see Figure 4.1-17):

$$A_{st,2} = \left(b_f - b_w\right) \cdot h_f \cdot f_{cd} / f_{yd}, \tag{4.1-116}$$

and:

$$M_{Rd,2} = A_{st,2} \cdot f_{yd} \cdot \left(d - \frac{h_f}{2}\right). \tag{4.1-117}$$

The steel area required for the rectangular web $A_{st,1}$ can now be obtained by using Table 4.1-3 to assess the reinforcement area needed for the rectangular section of the width b_w to support a moment $M_{Rd,1} = M_{Ed} - M_{Rd,2}$. Although very unlikely to be exceeded, the limiting (balanced) moment for flange section, where $\xi_{lim} = \left(x / d\right)_{lim}$ exceed $\beta = h_f / d$ is given by:

$$M_{Rd,lim} = M_{Rd,lim,1} + b_f \cdot \eta \cdot f_{cd} \cdot h_f \cdot \left(d - \frac{\eta}{2}\right). \tag{4.1-118}$$

The procedure for the design of flanged section is summarized as follows:

1. Calculate $a_m = \dfrac{M_{Ed}}{b_f \cdot d^2 \cdot \eta \cdot f_{cd}}$.
2. Follow Table 4.1-7 to obtain values of the ω, η, ξ.
3. If $\xi \le \beta = h_f / d$, calculate A_{st} as for the rectangular cross section with the width $b = b_f$.

If $\xi > \beta$, then:

- calculate $A_{st,2}$ and $M_{Rd,2}$ in accordance with Equation (4.1-116) and Equation (4.1-117).

- calculate $M_{Rd,1} = \Delta M = M_{Ed} - M_{Rd,2}$.

4. Use the provisions from Section 4.1.1.3.1 to calculate steel reinforcement area $A_{st,1}$ for the moment $\Delta M = M_{Rd,1}$ (rectangular cross-section of width b_w to resist the moment $M_{Rd,1}$).

5. Calculate area of the tensile steel reinforcement $A_{st} = A_{st,1} + A_{st,2}$.

(b) Checking of *ULS* for bending

In case, when $\lambda \cdot x > h_f$ $\left(\xi_{eff} > \beta = \dfrac{h_f}{d}\right)$ the following design procedure should be used to obtain the moment of resistance of the flanged cross-section. As it is shown in Figure 4.1-17, the resultant of compressive force $F_{cc,2}$, developed in concrete and acting through the centroid of the stress subblock 2 is equal:

$$F_{cc,2} = \left(b_f - b_w\right) \cdot \beta \cdot d \cdot \eta \cdot f_{cd} = \left(b_f - b_w\right) \cdot h_f \cdot \eta \cdot f_{cd}, \tag{4.1-119}$$

and resultant of compressive force $F_{cc,1}$ acting throughout the centroid of the stress subblock (subblock 1, Figure 4.1-17):

$$F_{cc,1} = \xi_{eff} \cdot d \cdot b_w \cdot \eta \cdot f_{cd}. \tag{4.1-120}$$

Applying the principle of superposition to stress subblock 1 and subblock 2 (see Figure 4.1-17), the equilibrium of longitudinal forces can be written as:

$$F_{cc,1} + F_{cc,2} - F_{st} = 0, \tag{4.1-121}$$

and taking into account moments about the centroid of reinforcement A_{st}, the moment of resistance M_{Rd} can be written as follows:

$$M_{Rd} = F_{cc,1} \cdot z_{c1} + F_{cc,2} \cdot z_{c2}, \tag{4.1-122}$$

or:

$$M_{Rd} = \left[\beta \cdot (1 - 0{,}5 \cdot \beta) \cdot \left(b_f - b_w\right) + \xi_{eff} \cdot \left(1 - 0{,}5 \cdot \xi_{eff}\right) \cdot b_w\right] \cdot \eta \cdot f_{cd} \cdot d^2. \tag{4.1-123}$$

Further, Criterion (4.1-1) of the Ultimate Limit States (ULS/STR) should be checked.

In case, when $\xi_{eff} \le \xi_{eff,lim}$, section should be designed in accordance with the provisions, that were formulated for the tension failure mode, otherwise – cross-section should be designed as a flanged cross-section with double reinforcement (doubly reinforced flanged section).

4.1.2 BENDING WITH AXIAL LOAD

As it is shown in [13], reinforced concrete elements (for example, short columns) under the axial load with uniaxial bending behave in different manner than when it is subjected to the axial load, though the column subjected to axial load can also carry some moment that may appear during construction or otherwise.

4.1.2.1 Behaviour of short column under the axial load and uniaxial moment

As it is shown in [13], depending on position of the neutral axial, the column (compressed element) may or may not have tensile stress to be taken by longitudinal reinforcement. In the compression region, however, longitudinal steel reinforcement will carry the compression force along with the concrete as in case of axially loaded column (member).

The position of the neutral axis and eccentricities of the axial force N_{Ed} are widely varying as follows (see Figure 4.1-18):

1) for the strain profile *EF*, the depth of neutral axis *x* is infinity and the eccentricity of the load is zero (point ①);

2) for the strain profile *LM* (point ②), the depth of the neutral axis is outside the section ($x > h$), with an appropriate eccentricity having compressive strain in the section;

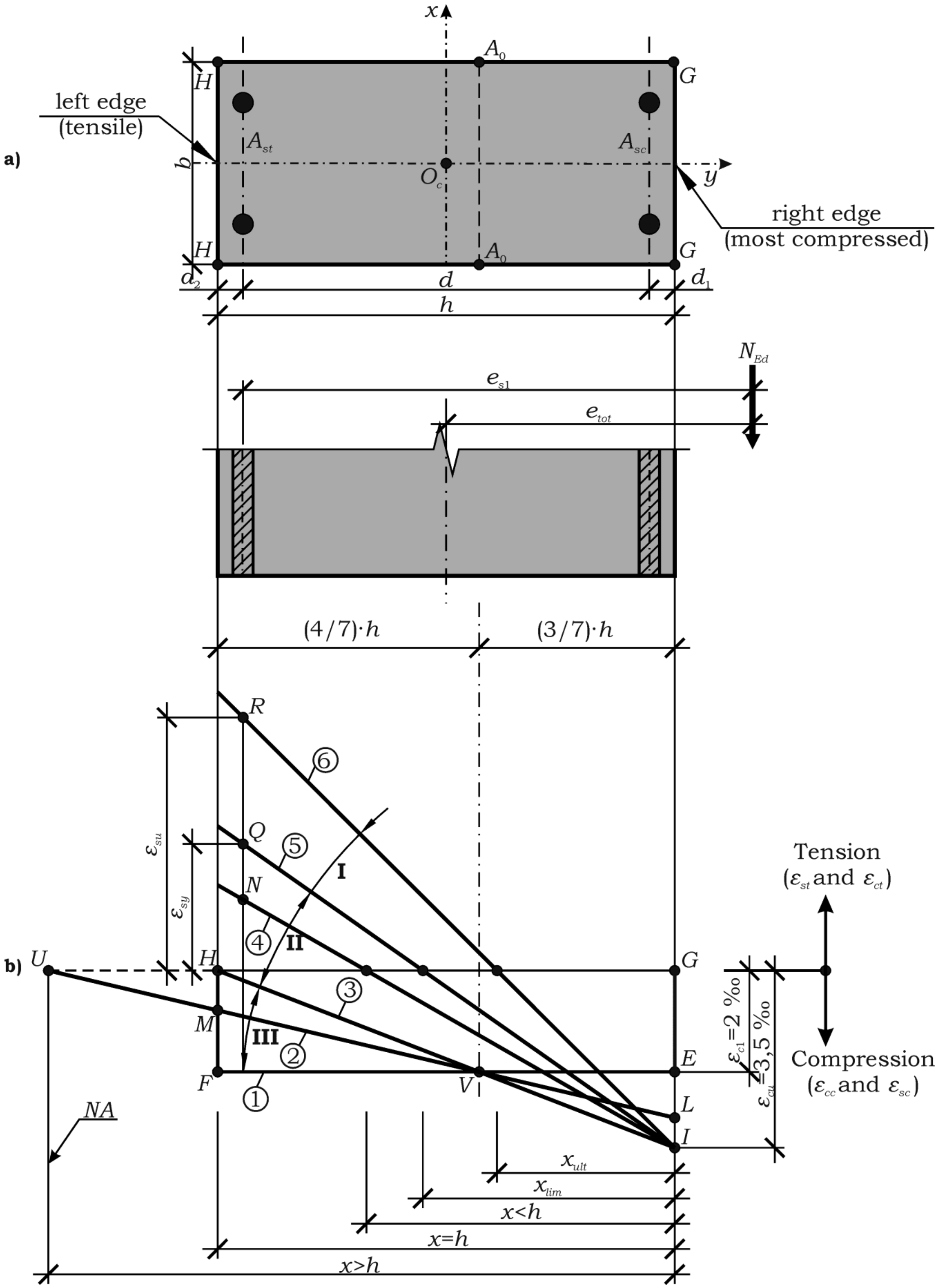
3) for the strain profile *IH* (point ③), the depth of the neutral axis is just at the left edge of the section ($x = h$), with an appropriate eccentricity, having zero and $\varepsilon_{cu,2}$=3,5 ‰ compressive strain at the left and right edges of the section respectively;

4) for the strain profile *IN* (point ④), the depth of the neutral axis is within the section ($x < h$), with an appropriate eccentricity, having tensile strains of the left of the neutral axis and $\varepsilon_{cu,2} = 3,5$ ‰ compressive strain at the right edge.

As it is shown in [13], it is evident that gradual increase of the eccentricity of the load N_{Ed} from zero is changing the strain profiles (strain distributions across the section height) from *EF* to *LM*, *IH* and then to *IN* (point ④) (see Figure 4.1-18). Therefore, we can accept that if we increase the eccentricity of the load to infinity, there will be only M_{Rd} acting on the column (compressed RC-element).

Designing by $N_{Rd,u}$ as the load causes collapse of the column when acting alone and $M_{Rd,u}$ as the moment that also causes collapse when acting alone, mark them in Figure 4.1-19 in vertical and horizontal axis respectively.

These two points are extreme points of the diagram of «$N_{Rd}-M_{Rd}$» (see Figure 4.1-19), any point on which of $M_{Rd,i}$ and $N_{Rd,i}$ (of different magnitudes) that will cause collapse of the same element having the neutral axis either outside or within the member cross-section.



a) – cross-section; b) – strains profile

Figure 4.1-18 – Cross-section and strains profile for short compressed member under the axial load with bending moment

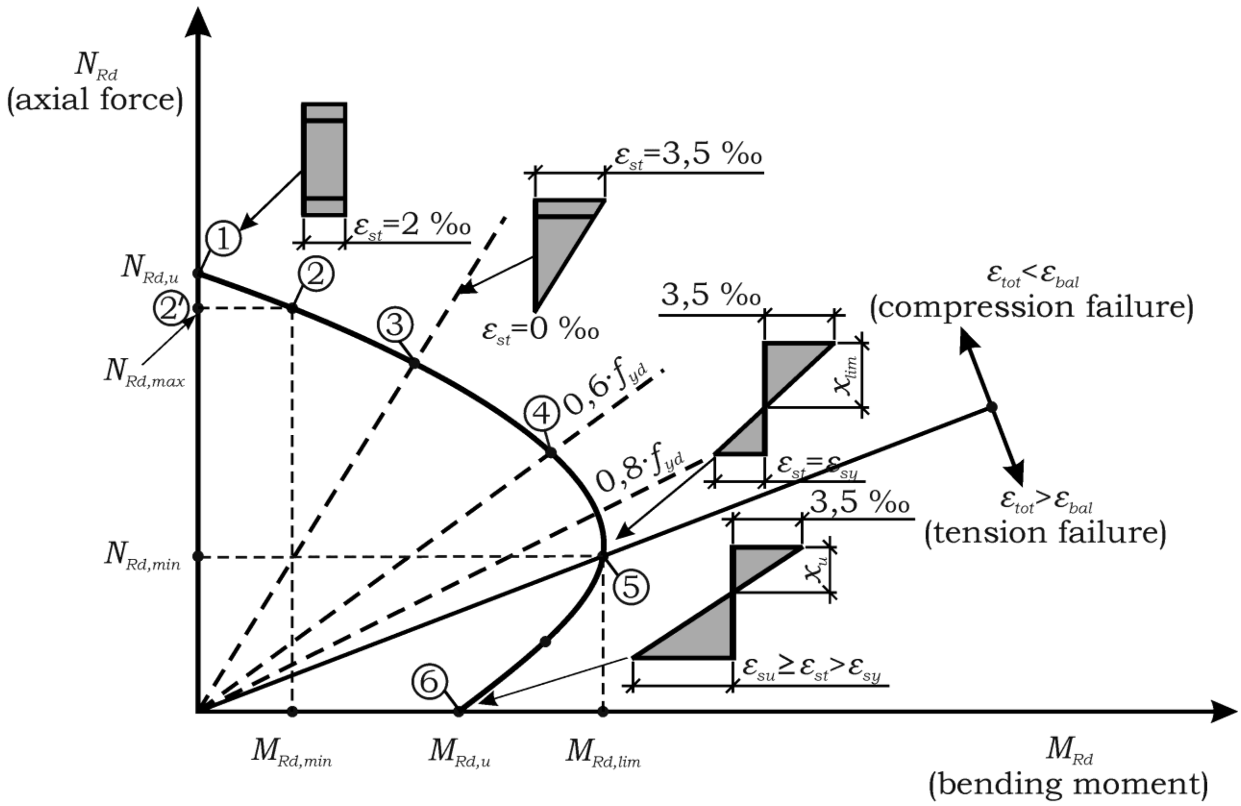


Figure 4.1-19 – Typical interaction diagram

The plot of «N_{Rd}–M_{Rd}» on Figure 4.1-19 is designated as interaction diagram since any points on the diagram give a pair of values of $N_{Rd,i}$ and $M_{Rd,i}$ causing collapse of the same compressed element in an interactive manner.

4.1.2.2 Rectangular-parabolic stress block in concrete

4.1.2.2.1 Modes of failure of columns and the basic equations formulation

In according with [6, 13], two distinct categories of the location of the neutral axis clearly indicate the following types of failure modes:

– ***compression failure***, when the neutral axis is outside the cross-section, causing compression throughout the section, and

– ***tension failure***, when the neutral axis is within the cross-section developing tensile strain on the left of the neutral axis.

Balanced failure mode. Before taking up these two failure modes, let us discuss about the third model of failure, i.e. the ***balanced failure***. Under this model of failure, yielding of outer most row of longitudinal steel reinforcement near the tensile fiber of the cross section occurs simultaneously with the attainment of the maximum compressive strain of $\varepsilon_{cu,2} = 3,5$ ‰ in concrete at the compressive edge of the cross-section. As a result, yielding of longitudinal steel reinforcement at the outermost row near tensile edge and crushing of concrete at the opposite edge of the cross-section occurs simultaneously. Such a strain profile (strain distribution) is known as «*balanced*» strain profile which is shown by the strain profile IQ in Figure 4.1-18 and point ⑤ (see Figure 4.1-19).

The depth of the neutral axis is designated as $x_{lim} = 3,5$ ‰ and it is shown in Figure 4.1-18. The balanced strain profile *IQ* (point ⑤) (see Figure 4.1-18) also shows the strain $\varepsilon_{st} = \varepsilon_{sy}$, whose numerical value would change depending on the grade of steel reinforcement.

It is important to observe that this balanced profile *IQ* does not pass through the fulcrum point *V* in Figure 4.1-18, while other points ①, ②, ③, i.e. profiles *EF*, *LM*, *IH*-lines pass through the fulcrum point *V* as none of them produce tensile strain anywhere in the cross section of the element.

To have the balanced strain profile *IQ* (see Figure 4.1-18) caused balanced failure of the structural element, the required load and moment are designated as $N_{Rd,lim}$ and $M_{Rd,lim}$ respectively are shown in Figure 4.1-19, as the coordinates of point ⑤.

The corresponding eccentricity of the load (force) $N_{Ed,lim}$ is defined by the notation $e_{tot,lim} = \dfrac{M_{Rd,lim}}{N_{Rd,lim}}$. The four parameters of the balanced failure are, therefore, $N_{Rd,lim}$, $M_{Rd,lim}$, $e_{tot,lim}$ and x_{lim} (the coefficient of the neutral axis depth ξ_{lim}).

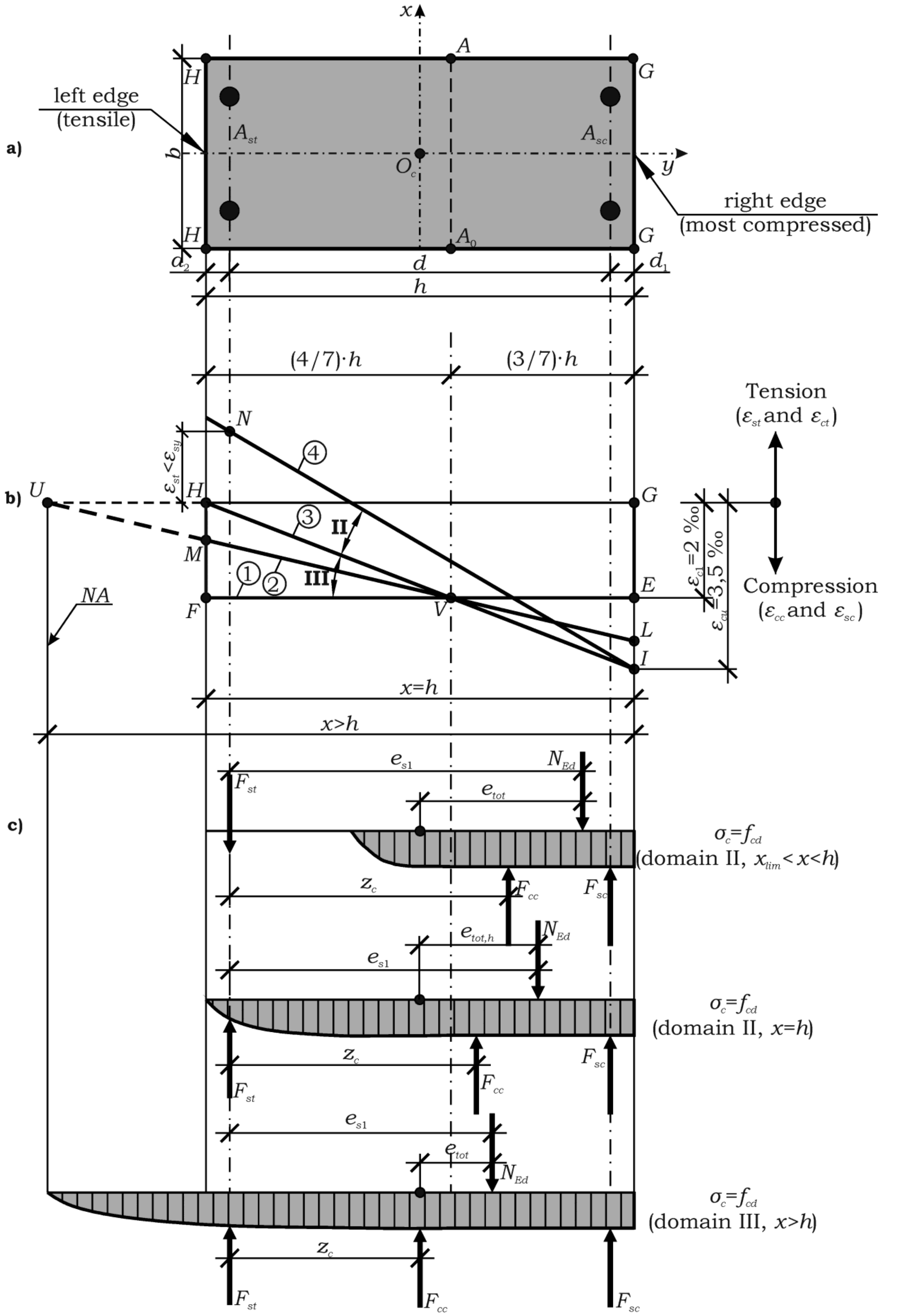
Compression failure mode. Compression failure of the column occurs when the eccentricity of the axial force N_{Ed} is less than that of balanced eccentricity ($e_{tot} < e_{tot,lim}$) and the depth of the neutral axis is more than that of balanced failure (the limit value of the x_{lim}, $x > x_{lim}$).

It is evident from the Figure 4.1-20, that these strain profiles may develop tensile strain of the left on the neutral axis depth $x = h$. All these strain profiles having $x_{lim} < x < h$ will not pass through the fulcrum point *V* (see Figure 4.1-20).

On the other hand, all of the strain profiles having *x* greater than *h* pass through the fulcrum point *V* and cause compression failure (see Figure 4.1-20).

The axial loads (forces) causing compression failure are higher than the balanced load $N_{Rd,lim}$ having respective eccentricities less than that of the load of balanced failure.

The extreme strain profile is *EF* marked by the point with number 1 in Figure 4.1-20. Some of these points causing compression failure are shown in Figure 4.1-20 and marked as ①, ②, ③ and ④, having $x > x_{lim}$ (or $\xi > \xi_{lim}$), either within or outside the section. Three such strain profiles are interesting and need for the further elaboration. One of them is the profile *IH* marked by point ③ (see Figure 4.1-20), for the which one $x = h$. Denoting the depth of the neutral axis by *h* and eccentricity of the load for this profile by $e_{tot,h}$, we observe that the strain profiles *LM* and *EF* (see Figure 4.1-20) marked by ② and ①, have the respective $x > h$ and $e_{tot} < e_{tot,h}$. The second profile is *EF* marked by point 1 in Figure 4.1-20, is for the maximum capacity of the column to carry out the axial load N_{Ed}, when eccentricity is zero and for the which one moment M_{Ed} is zero and the neutral axis theoretically is at infinity.



a) – cross-section; b) – strains profile; c) – compressive concrete stress block

Figure 4.1-20 – Compression failure mode (the neutral axis is within and outside the section)

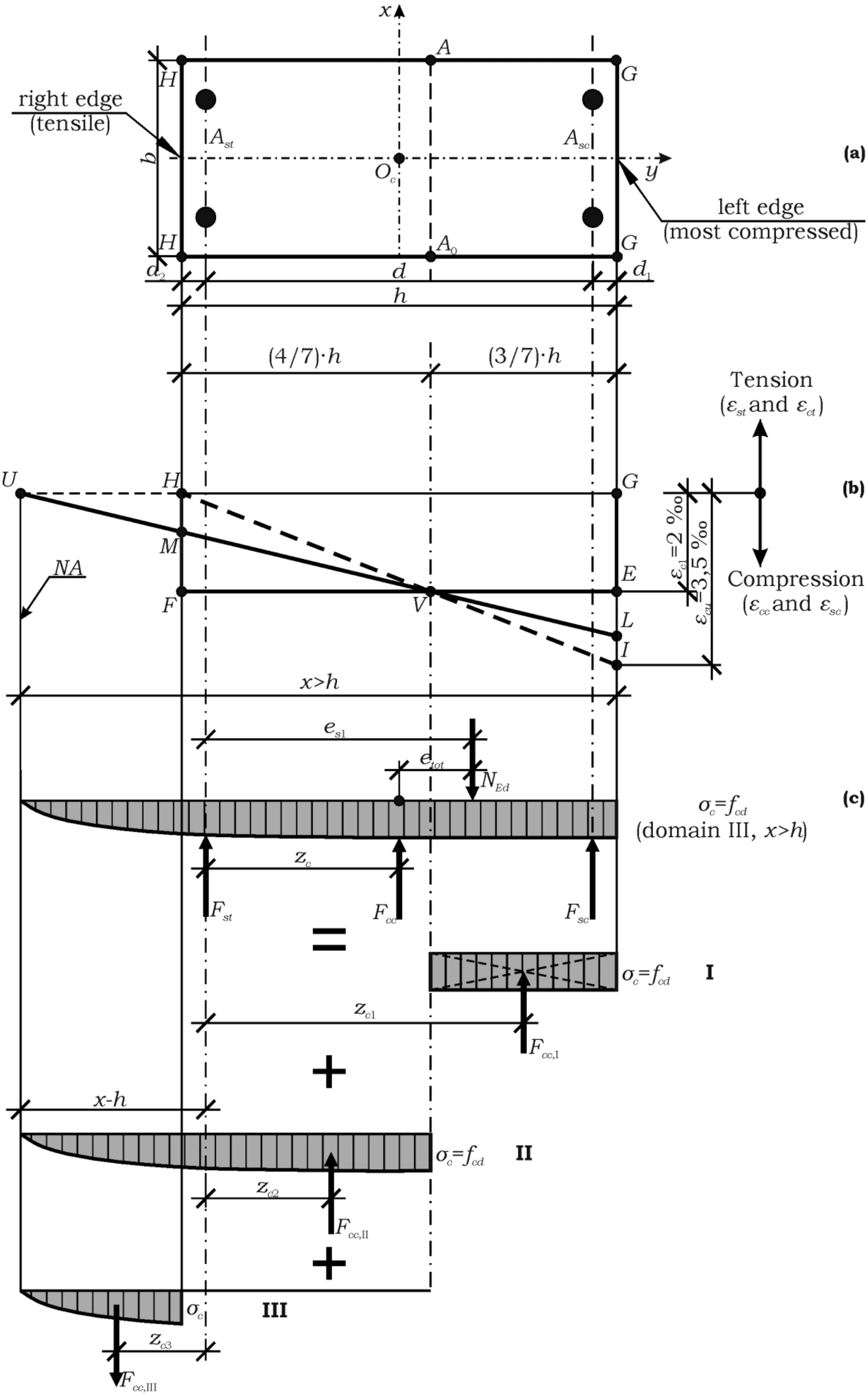
The third important strain profile *LM*, marked by point ② in Figure 4.1-20, is also due to the another pair of collapse internal forces N_{Ed} and M_{Ed}, having the capacity to accommodate the minimum eccentricity of the load, which hardly can be avoided in practical construction or for the other reason.

The load $N_{Ed,max}$, as it is seen from the Figure 4.1-19, is less then $N_{Ed,u}$ and column can carry $N_{Ed,min}$ and M_{Ed} in an interactive mode to cause collapse. Hence, a column having a capacity to carry the truly concentric force $N_{Rd,u}$ (when $M_{Rd,u} = 0$), shall not be allowed in the design.

Instead, its maximum load (force) shall be resisted up to $N_{Rd,max}$ ($< N_{Rd,u}$) along with $M_{Rd,min}$ (due to the minimum eccentricity). As it was shown in [13], accordingly the actual interaction diagram to be used for the purpose of the design shall terminate with horizontal line $2^{/}$ at point 2 on the Figure 4.1-19. Point 2 on the interaction diagram has the capacity $N_{Rd,max}$ with $M_{Rd,min}$ having eccentricity of e_{min} ($= \dfrac{M_{rd,min}}{N_{rd,max}}$) and depth of the neutral axis $x \gg h$ (see Figure 4.1-19).

It is seen that from point ① to point ⑤ (i.e. from compression failure to balanced failure) of interaction diagram (see Figure 4.1-19), the forces are gradually decreasing and the moments are correspondingly increasing. The eccentricities of the successive loads are also increasing and the depth of the neutral axis is decreasing from infinity to finite but outside and then within the section up to x_{lim} at balanced failure (point ⑤, see Figure 4.1-19). Moreover, this region of compression failure can be subdivided into two zones: 1) zone from point ① to point ②, where eccentricity of the load is less than minimum eccentricity that should be considered in design, and ②) zone from point ② to point ⑤, where the eccentricity of the load is equal or more than the minimum that specified in EN 1992 [N3]. It has been mentioned also that the first zone *x* from point ① to point ② should be avoided in the design of the column.

Figure 4.1-21 represents the stress block for a typical strain profile LM having neutral axis depth *x* outside the section ($x > h$, $\xi > \dfrac{h}{d}$). The strain profile LM in Figure 4.1-21 shows that up to a distance $\left(\dfrac{3}{7}\right) \cdot h$ from the right (the most compressed) edge (point A_0) the compressive strain is $\varepsilon_{cc} \geq 0{,}02$ and, therefore, the compressive stress shall remain constant at f_{cd}. The remain part of the column cross-section of the length $\left(\dfrac{4}{7}\right) \cdot h$, i.e. up to the left edge, has been reducing compressive strains (*but not to zero!*). The stress block is, therefore, parabolic from A_0 to H, which becomes zero at U (outside the section).



a) – cross-section; b) – strains profile; c) – compressive concrete stress block

Figure 4.1-21 – Compression failure mode (the neutral axis is outside the section, *x>h*)

In this case, the applied design force N_{Ed} and moment M_{Ed} must be balanced by the resistance force N_{Rd} and moment of the resistance M_{Rd} of the forces developed within the cross-section. In general case, equilibrium conditions can be written as follows (see Figure 4.1-21):

$$F_{cc} + F_{sc} + F_{st} - N_{Ed} = 0; \tag{4.1-124}$$

$$N_{Ed} \cdot e_{s1} = F_{cc} \cdot z_c + F_{sc} \cdot \left(d - a_2\right), \tag{4.1-125}$$

where:

$$e_{s1} = e_{tot} + 0,5 \cdot h - a_1. \qquad 4.1\text{-}126)$$

For linear strain distribution (see Figure 4.1-21) values of the strain can be expressed as follows:

$$\frac{\varepsilon_{cc}}{x} = \frac{\varepsilon_{cc,1}}{x-h} = \frac{\varepsilon_{st}}{x-d} = \frac{\varepsilon_{sc}}{x-a_2} = \frac{0,002}{x - \left(\frac{3}{7}\right) \cdot h}. \tag{4.1-127}$$

For the design purpose, the Ratio (4.1-127) can be rewritten as a function of the relative depth of the neutral axis $\xi = \dfrac{x}{d}$ and relative height (depth) of the cross-section β_h (see Table 4.1-3) can be calculated as follows:

$$\beta_h = \frac{h}{d} = 1 + \frac{a_1}{d}. \tag{4.1-128}$$

With the usage of the dimensionless parameters ξ and β_h values of the strains can be calculated from the Ratios (4.1-127) as:

- concrete compressive strain at the most stressed (compressed) fiber (right edge) of the cross-section:

$$\varepsilon_{cc} = 2,0 \cdot \frac{\xi}{\xi - \left(\frac{3}{7}\right) \cdot \beta_h} = \frac{14 \cdot \xi}{7 \cdot \xi - 3 \cdot \beta_h} \; [‰]; \tag{4.1-129}$$

- compressive strain in steel reinforcement A_{sc}, which is situated near the most stressed (compressed) fiber (right edge) of the cross-section:

$$\varepsilon_{sc} = \frac{14 \cdot \left(\xi - {}^{a_2}\!/_{d}\right)}{7 \cdot \xi - 3 \cdot \beta_h} \; [‰]; \tag{4.1-130}$$

- compressive strain in steel reinforcement A_{st}, which is situated near the less stressed (compressed) fiber (left edge) of the cross-section:

$$\varepsilon_{sc} = \frac{14 \cdot (\xi - 1)}{7 \cdot \xi - 3 \cdot \beta_h} \text{ [‰]}. \tag{4.1-131}$$

At the state, when $x = h$ and $\xi = \beta_h$, compressive strain in reinforcement A_{st} is equal to:

$$\varepsilon_{st} = -\frac{7 \cdot (\beta_h - 1)}{2 \cdot \beta_h} \text{ [‰]}. \tag{4.1-132}$$

Resultant compressive force resisted by concrete F_{cc} (see Figure 4.1-21) can be calculated based on the superposition principle applying to the resultant forces $F_{cc,1}$, $F_{cc,2}$, $F_{cc,3}$, obtained by the division of the initial (original) stress block to the three subblocks: I, II and III, as it is shown in Figure 4.1-21:

$$F_{cc} = F_{cc,1} + F_{cc,2} + F_{cc,3}, \tag{4.1-133}$$

where:

$$F_{cc,1} = \frac{3}{7} \cdot \beta_h \cdot b \cdot d \cdot f_{cd}, \tag{4.1-134}$$

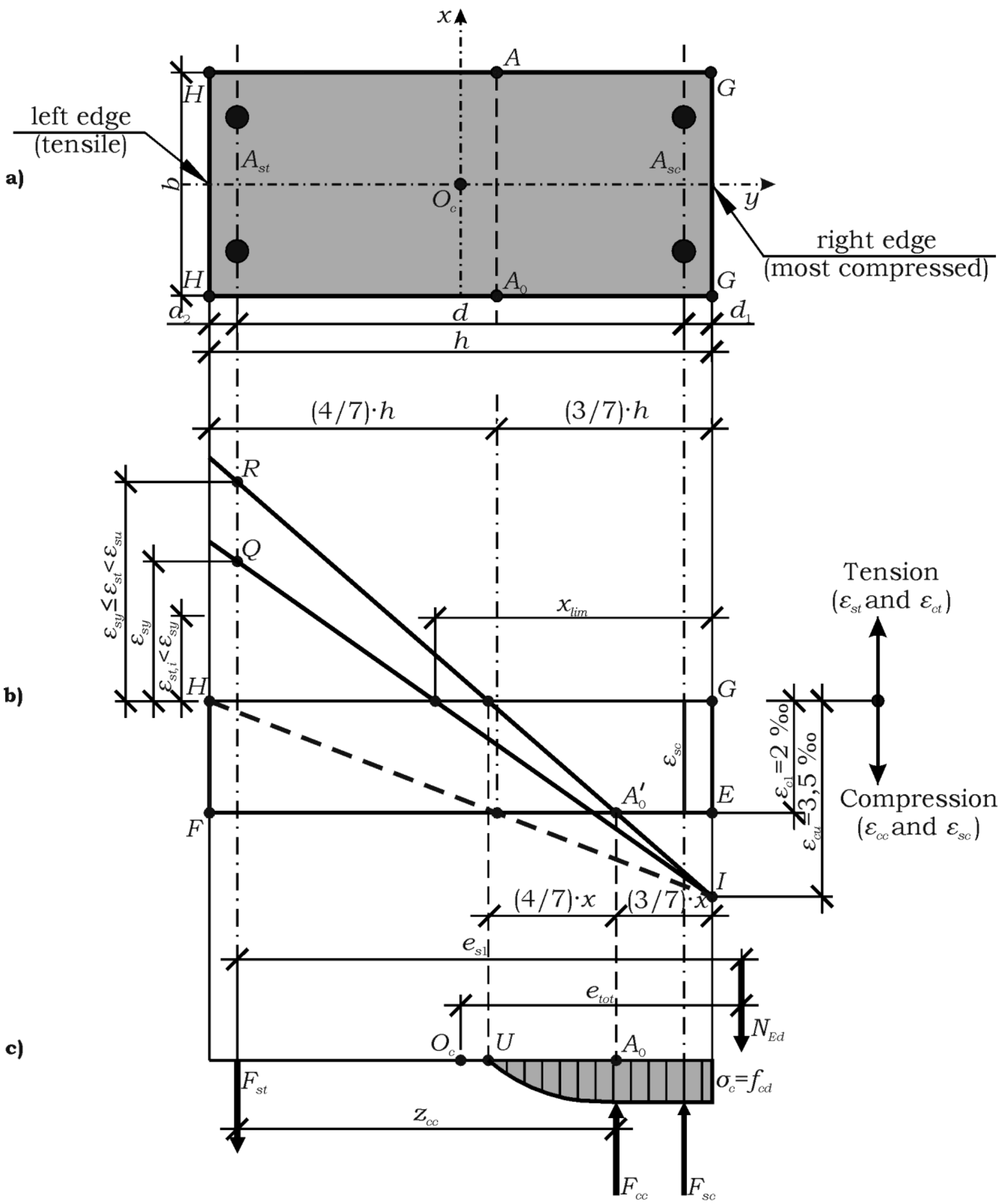
$$F_{cc,2} = \frac{2}{3}\left(x - \frac{3}{7} \cdot h\right) \cdot b \cdot f_{cd} = \frac{2}{3}\left(\xi - \frac{3}{7} \cdot \beta_h\right) \cdot b \cdot d \cdot f_{cd}, \tag{4.1-135}$$

$$F_{cc,3} = f_{cd} \cdot b \cdot \int_0^{x-h} \left(\varepsilon_{cy} - \frac{\varepsilon_{cy}^2}{4}\right) dy. \tag{4.1-136}$$

From the linear strain distribution:

$$\frac{\varepsilon_c(y)}{y} = \frac{2{,}0}{x - \frac{3}{7} \cdot h} \rightarrow \varepsilon_c(y) = \frac{2 \cdot y}{x - \frac{3}{7} \cdot h} \text{ [‰]}. \tag{4.1-137}$$

Figure 4.1-22 represents the stress block for a typical strain profile *IN* having neutral axis depth $x < h$ within the section. The strain profile *IN* in Figure 4.1-22 shows that from point *E* to point $A_0^{/}$, i.e. up to distance $\left(\frac{3}{7}\right) \cdot x$ from the right edge, the compressive strain ε_{cc} is ≥ 2 ‰, and, therefore, the compressive stress remain constant at f_{cd}. From $A_0^{/}$ to U, i.e. for a distance $\left(\frac{4}{7}\right) \cdot x$, the strain is reducing from $\varepsilon_{c1} = 2$ ‰ to zero and stress in this zone is parabolic as it is shown in Figure 4.1-22.



a) – cross-section; b) – strains profile; c) – compressive concrete stress block
Figure 4.1-22 – Strain profile and stress distribution, when $x \leq x_{lim}$

Substituting Equation (4.1-137) in Equation (4.1-136), the resultant compressive force $F_{cc,3}$ can be written:

$$F_{cc,3} = \frac{2}{3} \cdot \frac{(\xi - \beta_h)^2 \cdot \left(\xi - \frac{1}{7} \cdot \beta_h\right)}{\left(\xi - \frac{3}{7} \cdot \beta_h\right)} \cdot b \cdot d \cdot f_{cd}, \qquad (4.1\text{-}138)$$

and after substituting Equations (4.1-134), (4.1-135), and (4.1-138) to the Equation (4.1-133), the resultant compressive force resisted by concrete F_{cc} is expressed as follows:

$$F_{cc} = \frac{2}{3}\left[\xi + \frac{3}{14} \cdot \beta_h - \frac{(\xi - \beta_h)^2 \cdot \left(\xi - \frac{1}{7} \cdot \beta_h\right)}{\left(\xi - \frac{3}{7} \cdot \beta_h\right)}\right] \cdot b \cdot d \cdot f_{cd}. \qquad (4.1\text{-}139)$$

Moment of resistance M_{Rd}, taking about the centroid of the reinforcement A_{st} is calculated as:

$$M_{Rd} = F_{cc,1} \cdot z_1 + F_{cc,2} \cdot z_2 + F_{cc,3} \cdot z_3. \qquad (4.1\text{-}140)$$

The values of the internal force F_{cc}, lever arm z and moment of the resistance M_{Rd} can be obtained with the usage of the coefficients a_m, ω and η from the Tables 4.1-3. In this case:

– resultant compressive force in concrete:

$$F_{cc} = \omega(\xi, \beta_h) \cdot b \cdot d \cdot f_{cd}; \qquad (4.1\text{-}141)$$

– lever arm between F_{cc} and F_{st}:

$$z_c = \eta(\xi, \beta_h) \cdot d; \qquad (4.1\text{-}142)$$

– moment of resistance M_{Rd}, taking about centroid of the reinforcement A_{st}:

$$M_{Rd} = a_m(\xi, \beta_h) \cdot b \cdot d^2 \cdot f_{cd}. \qquad (4.1\text{-}143)$$

If $x_{lim} < x \le h$ or $\xi_{lim} < \xi \le \frac{h}{d}$ (see Figure 4.1-22) strain in tensile reinforcement ε_{st} is less, than ε_{sy}:

$$\varepsilon_{st} < \varepsilon_{sy} = \left|\frac{f_{yd}}{E_s}\right|, \tag{4.1-144}$$

and stress σ_{st} in reinforcement A_{st} is equal:

$$\sigma_{st} = k_{s1} \cdot f_{yd}, \quad 4.1\text{-}145)$$

where: k_{s1} is coefficient, which is obtained as a function of the relative neutral axis depth:

$$k_{s1} = \frac{\varepsilon_{st}}{\varepsilon_{sy}} = \frac{0{,}0035 \cdot (1-\xi)}{\xi} \cdot \frac{E_s}{f_{yd}}. \tag{4.1-146}$$

Tension failure mode. Tension failure occurs when the eccentricity of the load is greater than the balanced eccentricity ($e_{tot} > e_{tot,lim}$). The depth of the neutral axis is less than that of the balanced failure ($x \le x_{lim}$ and $\xi \le \xi_{lim}$). The longitudinal steel in the outermost row in the tension zone of the cross-section yields first.

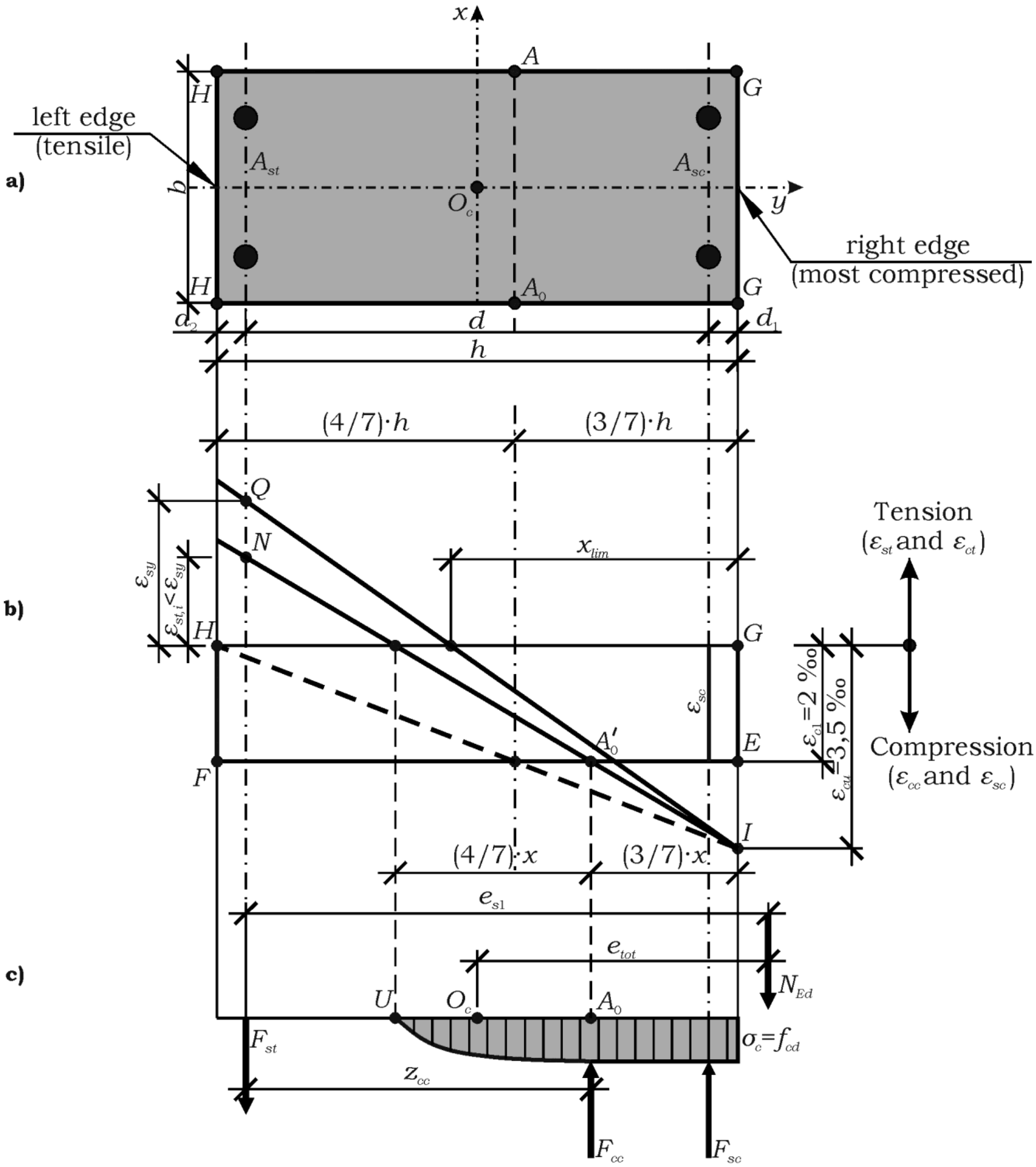
Gradually, with increasing of the tensile strain, longitudinal steel of the inner row, if provided, starts yielding till the compressive strain reaches $\varepsilon_{cu} = 3{,}5$ ‰ at the most compressed fiber (edge) of the cross section.

The line *IR* in Figure 4.1-23 represents such a profile for the which some of the inner row of steel bars have yielded and concrete compressive strain has reached $\varepsilon_{cu} = 3{,}5$ ‰ at the most compressed fiber of the cross-section. The depth of the neutral axis is designated by the x_{ult} or $\varepsilon_{ult} = \dfrac{x_{ult}}{d}$.

It is interesting to note that in this region of interaction diagram (from point ⑤ to point ⑥, see Figure 4.1-19), both of the load (force) and the moment are found to decrease till point ⑥ when the column fails due to $M_{Rd,u}$ is acting alone.

At point 6, let us to consider that the column is loaded in simple beginning to the point (when $M_{Ed} = M_{Rd,u}$) at which yielding of the steel in tension begins. Addition of some axial compressive force N_{Ed} at this stage will reduce the previous tensile stress of steel reinforcement to the value less than its yielding strength. As a result, it can carry additional moment. This increasing of the moment carrying capacity with the increasing of the load shall continue till the combined stress in steel, due to the additional axial load and increased resistance moment, reaches the yield strength.

For the case, when $x < x_{lim}$ ($\xi \le \xi_{lim}$ or $e_{tot} \ge e_{tot,lim}$), design procedure is the same as it is in case of the pure uniaxial bending without axial force.



a) – cross-section; b) – strains profile; c) – compressive concrete stress block
Figure 4.1-23 – Strain profile and stress distribution, when $x > x_{lim}$

4.1.2.2.2 Calculation of the required area of the reinforcement

The required area of the steel reinforcement can be calculated based on the streamlined procedure using coefficients from Table 4.1-3. Table 4.1-3 give values of the depth of the neutral axis $\xi = x/d$, dimensionless parameters η and ω for reinforced concrete element as a function of a_m. In this case the following procedure can be utilized:

1. Calculate:

$$a_m = \frac{M_{Ed,1}}{b \cdot d^2 \cdot f_{cd}}, \quad (4.1\text{-}147)$$

where: $M_{Ed,1}$ is the substituted moment obtained from the following equations:

$$M_{Ed,1} = M_{Ed} + N_{Ed} \cdot (0{,}5 \cdot h - c_1), \quad (4.1\text{-}148)$$

or:

$$M_{Ed,1} = N_{Ed} \cdot (e_{tot} + 0{,}5 \cdot h - c_1), \quad (4.1\text{-}149)$$

with $e_{tot} = \frac{M_{Ed}}{N_{Ed}}$.

2. Choose the parameters ξ, η, ω from Table 4.1-3 or Table 4.1-8 a, b, c and value of the coefficient $a_{m,lim}$ from Table 4.1-3.

Table 4.1-8 a) – Coefficients for calculation of the rectangular section under compression for $x > h$, and $\beta_h = 1{,}05$

ξ=x/d	Coefficients			Strains, ‰	
	ω	η	a_m	ε_{cc} in concrete (compression)	ε_{st} in reinforcement (tension)
1,06	0,857	0,559	0,479	3,475	-0,179
1,08	0,869	0,554	0,481	3,429	-0,254
1,10	0,880	0,550	0,482	3,385	-0,308
1,12	0,890	0,543	0,483	3,343	-0,358
1,14	0,899	0,538	0,484	3,304	-0,406
1,16	0,907	0,534	0,484	3,268	-0,451
1,18	0,915	0,530	0,485	3,233	-0,493
1,20	0,922	0,527	0,486	3,200	-0,533
1,25	0,938	0,520	0,487	3,125	-0,625
1,30	0,950	0,515	0,489	3,059	-0,706
1,35	0,961	0,510	0,490	3,000	-0,778
1,40	0,970	0,506	0,491	2,947	-0,842
1,45	0,978	0,503	0,492	2,900	-0,900
1,50	0,985	0,499	0,492	2,857	-0,952
1,55	0,990	0,498	0,493	2,818	-1,000
1,60	0,996	0,495	0,493	2,783	-1,043
1,65	1,000	0,494	0,494	2,750	-1,083
1,70	1,004	0,492	0,494	2,720	-1,120
1,75	1,007	0,491	0,494	2,692	-1,154
1,80	1,010	0,490	0,495	2,667	-1,185
1,85	1,013	0,489	0,495	2,643	-1,214
1,90	1,016	0,488	0,495	2,621	-1,241
1,95	1,018	0,487	0,496	2,600	-1,267
2,00	1,020	0,486	0,496	2,581	-1,290
2,10	1,024	0,484	0,496	2,545	-1,330
2,20	1,026	0,483	0,496	2,514	-1,371
2,30	1,029	0,482	0,497	2,486	-1,405
2,40	1,031	0,482	0,497	2,462	-1,436

Table 4.1-8 a) (end)

2,50	1,033	0,481	0,497	2,439	-1,463
2,60	1,034	0,481	0,497	2,419	-1,488
2,70	1,035	0,480	0,497	2,400	-1,511
2,80	1,037	0,480	0,497	2,383	-1,532
2,90	1,038	0,480	0,498	2,367	-1,551
3,00	1,039	0,479	0,498	2,353	-1,569
3,10	1,040	0,479	0,498	2,340	-1,585
3,20	1,040	0,479	0,498	2,327	-1,600
3,30	1,041	0,478	0,498	2,316	-1,614
3,40	1,042	0,478	0,498	2,305	-1,627
3,50	1,042	0,478	0,498	2,295	-1,639
3,60	1,043	0,477	0,498	2,286	-1,651
3,70	1,043	0,477	0,498	2,277	-1,662
3,80	1,044	0,477	0,498	2,269	-1,672
3,90	1,044	0,477	0,498	2,261	-1,681
4,00	1,044	0,477	0,498	2,254	-1,690
5,00	1,047	0,476	0,498	2,198	-1,758
∞	1,050	0,475	0,499	2,000	-2,000

Table 4.1-8 b) – Coefficients for calculation of the rectangular section under compression for $x > h$, and $\beta_h = 1,10$

$\xi = x/d$	Coefficients			Strains, ‰	
	ω	η	a_m	ε_{cc} in concrete (compression)	ε_{st} in reinforcement (tension)
1,12	0,903	0,534	0,484	3,454	-0,370
1,14	0,915	0,530	0,484	3,410	-0,419
1,16	0,925	0,524	0,485	3,369	-0,465
1,18	0,935	0,520	0,486	3,331	-0,508
1,20	0,944	0,515	0,486	3,294	-0,549
1,25	0,963	0,506	0,487	3,211	-0,642
1,30	0,979	0,498	0,488	3,138	-0,724
1,35	0,993	0,492	0,489	3,073	-0,797
1,40	1,004	0,488	0,490	3,015	-0,862
1,45	1,014	0,483	0,490	2,964	-0,920
1,50	1,022	0,480	0,491	2,917	-0,972
1,55	1,029	0,477	0,491	2,874	-1,020
1,60	1,035	0,474	0,491	2,835	-1,063
1,65	1,040	0,473	0,492	2,800	-1,103
1,70	1,045	0,471	0,492	2,767	-1,140
1,75	1,049	0,496	0,492	2,737	-1,173
1,80	1,053	0,467	0,492	2,700	-1,204
1,85	1,056	0,467	0,493	2,684	-1,233
1,90	1,059	0,466	0,493	2,660	-1,260
1,95	1,062	0,464	0,493	2,638	-1,285
2,00	1,065	0,463	0,493	2,617	-1,308
2,10	1,069	0,461	0,493	2,579	-1,351
2,20	1,072	0,460	0,493	2,545	-1,388
2,30	1,075	0,460	0,494	2,516	-1,422
2,40	1,078	0,458	0,494	2,489	-1,452
2,50	1,080	0,457	0,494	2,465	-1,479
2,60	1,082	0,456	0,494	2,443	-1,503
2,70	1,083	0,456	0,494	2,423	-1,526
2,80	1,085	0,455	0,494	2,405	-1,546
2,90	1,086	0,455	0,494	2,388	-1,565
3,00	1,087	0,454	0,494	2,373	-1,582
3,10	1,088	0,454	0,494	2,359	-1,598

Table 4.1-8 b) (end)

3,20	1,089	0,454	0,494	2,346	-1,613
3,30	1,090	0,453	0,494	2,333	-1,626
3,40	1,090	0,453	0,494	2,322	-1,639
3,50	1,091	0,453	0,494	2,311	-1,651
3,60	1,092	0,453	0,495	2,301	-1,662
3,70	1,092	0,453	0,495	2,292	-1,673
3,80	1,093	0,453	0,495	2,283	-1,682
3,90	1,093	0,453	0,495	2,275	-1,692
4,00	1,093	0,453	0,495	2,267	-1,700
5,00	1,096	0,452	0,495	2,208	-1,767
6,00	1,097	0,451	0,495	2,171	-1,809
8,00	1,099	0,450	0,495	2,125	-1,860
10,00	1,099	0,450	0,495	2,099	-1,889
∞	1,100	0,450	0,495	2,000	-2,000

Table 4.1-8 c) – Coefficients for calculation of the rectangular section under compression for $x > h$, and $\beta_h = 1{,}15$

$\xi = x/d$	Coefficients			Strains, ‰	
	ω	η	a_m	ε_{cc} in concrete (compression)	ε_{st} in reinforcement (tension)
1,16	0,937	0,519	0,486	3,478	-0,480
1,18	0,950	0,512	0,486	3,435	-0,524
1,20	0,961	0,506	0,486	3,394	-0,566
1,25	0,985	0,493	0,486	3,302	-0,660
1,30	1,005	0,485	0,487	3,221	-0,743
1,35	1,021	0,477	0,487	3,150	-0,817
1,40	1,035	0,470	0,487	3,087	-0,882
1,45	1,047	0,465	0,487	3,030	-0,940
1,50	1,057	0,462	0,488	2,979	-0,993
1,55	1,065	0,458	0,488	2,932	-1,041
1,60	1,073	0,455	0,488	2,890	-1,084
1,65	1,079	0,452	0,488	2,852	-1,123
1,70	1,085	0,450	0,488	2,817	-1,160
1,75	1,090	0,448	0,488	2,784	-1,193
1,80	1,095	0,446	0,488	2,754	-1,224
1,85	1,099	0,444	0,488	2,726	-1,253
1,90	1,101	0,443	0,488	2,701	-1,279
1,95	1,105	0,442	0,488	2,676	-1,304
2,00	1,108	0,440	0,488	2,654	-1,327
2,10	1,113	0,438	0,488	2,613	-1,391
2,20	1,118	0,436	0,488	2,577	-1,406
2,30	1,121	0,435	0,488	2,545	-1,439
2,40	1,124	0,434	0,488	2,517	-1,468
2,50	1,127	0,433	0,488	2,491	-1,495
2,60	1,129	0,432	0,488	2,468	-1,519
2,70	1,131	0,431	0,488	2,447	-1,540
2,80	1,132	0,431	0,488	2,427	-1,560
2,90	1,134	0,431	0,489	2,409	-1,579
3,00	1,135	0,431	0,489	2,393	-1,595
3,10	1,136	0,430	0,489	2,378	-1,611
3,20	1,137	0,430	0,489	2,364	-1,625
3,30	1,138	0,430	0,489	2,351	-1,639
3,40	1,139	0,429	0,489	2,339	-1,651
3,50	1,140	0,429	0,489	2,328	-1,663
3,60	1,140	0,429	0,489	2,317	-1,674
3,70	1,141	0,429	0,489	2,307	-1,684
3,80	1,141	0,429	0,489	2,298	-1,693

Table 4.1-8 c) (end)

3,90	1,142	0,428	0,489	2,289	-1,702
4,00	1,142	0,428	0,489	2,281	-1,711
5,00	1,145	0,427	0,489	2,219	-1,775
6,00	1,147	0,426	0,489	2,179	-1,816
8,00	1,148	0,426	0,489	2,131	-1,865
10,00	1,149	0,426	0,489	2,104	-1,893
∞	1,150	0,425	0,489	2,000	-2,000

3. Check on design case – failure mode:

3.1. If $a_m \le a_{m,lim}$, then failure mode is tension failure mode (large eccentricity case) and steel yields before concrete crushing. In this case reinforcement A_{sc} is not required and $A_{sc,prov}$ can be taken without any calculation as $A_{sc,prov} \ge A_{sc,min}$. Taking into account area of reinforcement $A_{sc,prov}$, new value of the coefficient a_m can be calculated as follows:

$$a_m = \frac{M_{Ed,1} - A_{s2,prov} \cdot (d - a_2) \cdot k_{s2} \cdot f_{yd}}{b \cdot d^2 \cdot f_{cd}}, \quad (4.1\text{-}150)$$

where: k_{s2} is taken from Table 4.1-4. For calculated value of a_m, dimensionless parameters ξ, ω can be obtained from Table 4.1-3.

Required area of the reinforcement A_{st} is calculated from the following equation:

$$A_{st} = \left(\omega \cdot d \cdot b \cdot f_{cd} + A_{sc} \cdot k_{s2} \cdot f_{yd} - N_{Ed}\right) \cdot \frac{1}{k_{s1} \cdot f_{yd}}, \quad (4.1\text{-}151)$$

where: $k_{s1} = 1$ for the case of tension failure mode (case of the «*large*» eccentricity).

3.2. If $a_m > a_{m,lim}$, then failure mode is compression failure mode (case of the «*small*» eccentricity).

In this case value of Δa_m is calculated as follows:

$$\Delta a_m = a_m - a_{m,lim}, \quad (4.1\text{-}152)$$

where: $a_{m,lim}$ limit value is obtained from Table 4.1-3 as a function of ξ_{lim}.

Required area of reinforcement A_{sc} in this case can be calculated from the following equation:

$$A_{s2} = \frac{\Delta a_m}{1 - \frac{d_1}{d}} \cdot d \cdot b \cdot \frac{f_{cd}}{k_{s2} \cdot f_{yd}}. \quad (4.1\text{-}153)$$

If assume, that relative depth of neutral axis ξ is equal to ξ_{lim} (i.e. $\xi = \xi_{lim}$), independently from ratio a_1/d, value of the coefficient $k_{s2} = 1$ and required area of the reinforcement A_{st}:

$$A_{st} = \left[\left(\omega_{lim} + \frac{\Delta a_m}{1 - \frac{d_1}{d}} \right) \cdot d \cdot b \cdot f_{cd} - N_{Ed} \right] \cdot \frac{1}{k_{s1} \cdot f_{yd}}, \qquad (4.1\text{-}154)$$

or, if the area of reinforcement $A_{sc,prov}$ is taken greater, then calculated value A_{sc}:

$$A_{st} = \left[\omega_{lim} \cdot d \cdot b \cdot f_{cd} + A_{sc,prov} \cdot f_{yd} - N_{Ed} \right] \cdot \frac{1}{k_{s1} \cdot f_{yd}}. \qquad (4.1\text{-}155)$$

If calculated from Equation (4.1-155) area of reinforcement $A_{st} < 0$, then strength properties of steel is not utilized completely. In this case it is necessary to find the corresponding value of the coefficient $k_{s2} < 1$ (compression failure mode – case of the «*small*» eccentricity). The depth of the neutral axis can be determined by assuming, that whole axial force N_{Ed} is resisted by concrete in cross section:

$$\omega = \frac{N_{Ed}}{b \cdot d \cdot f_{cd}}. \qquad (4.1\text{-}156)$$

For obtained value of dimensionless parameters ω, the depth of neutral axis ξ and αm should be chosen from the Table 4.1-3 in such way, that equilibrium conditions (Equation (4.1-124) and Equation (4.1-125) will be satisfied.

The area of the reinforcement A_{st} is calculated from Equation (4.1-151) taking into account $k_{s2} = 1$, and the area of reinforcement A_{sc} is calculated from the following equation:

$$A_{sc} = \frac{M_{Ed,1} - a_m \cdot d^2 \cdot b \cdot f_{cd}}{f_{yd} \cdot (d - d_1)}. \qquad (4.1\text{-}157)$$

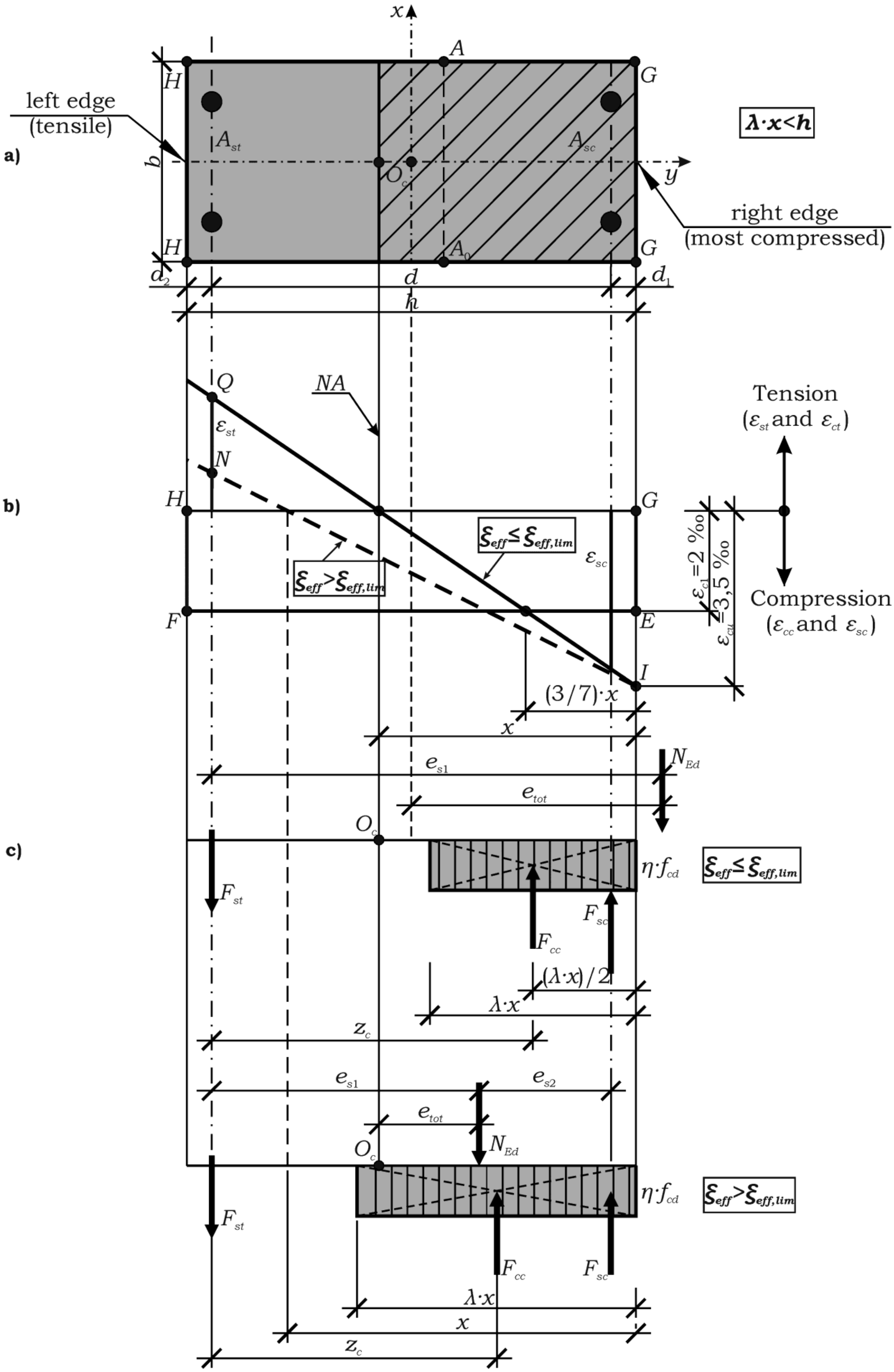
4.1.2.3 Equivalent rectangular compressive stress block in concrete

In general case, the applied axial force may be tensile or compressive. In the analysis that follows, a compressive force is considered. For the tensile load, the same basic principles of equilibrium, compatibility of strains, stress-strain relationships would apply, but it could be necessary to change the sign of the applied load N_{Ed}, when we consider the equilibrium of forces on the cross-section.

Figure 4.1-24 represents the cross section of a member with typical strain and stress distribution for the varying position of the neutral axis. The cross section is subjected to a bending moment M_{Ed} and axial compressive force N_{Ed}, and in the Figure 4.1-24 and Figure 4.1-25 the direction of the moment is such as to cause compression on the right edge of section and tension on the left edge.

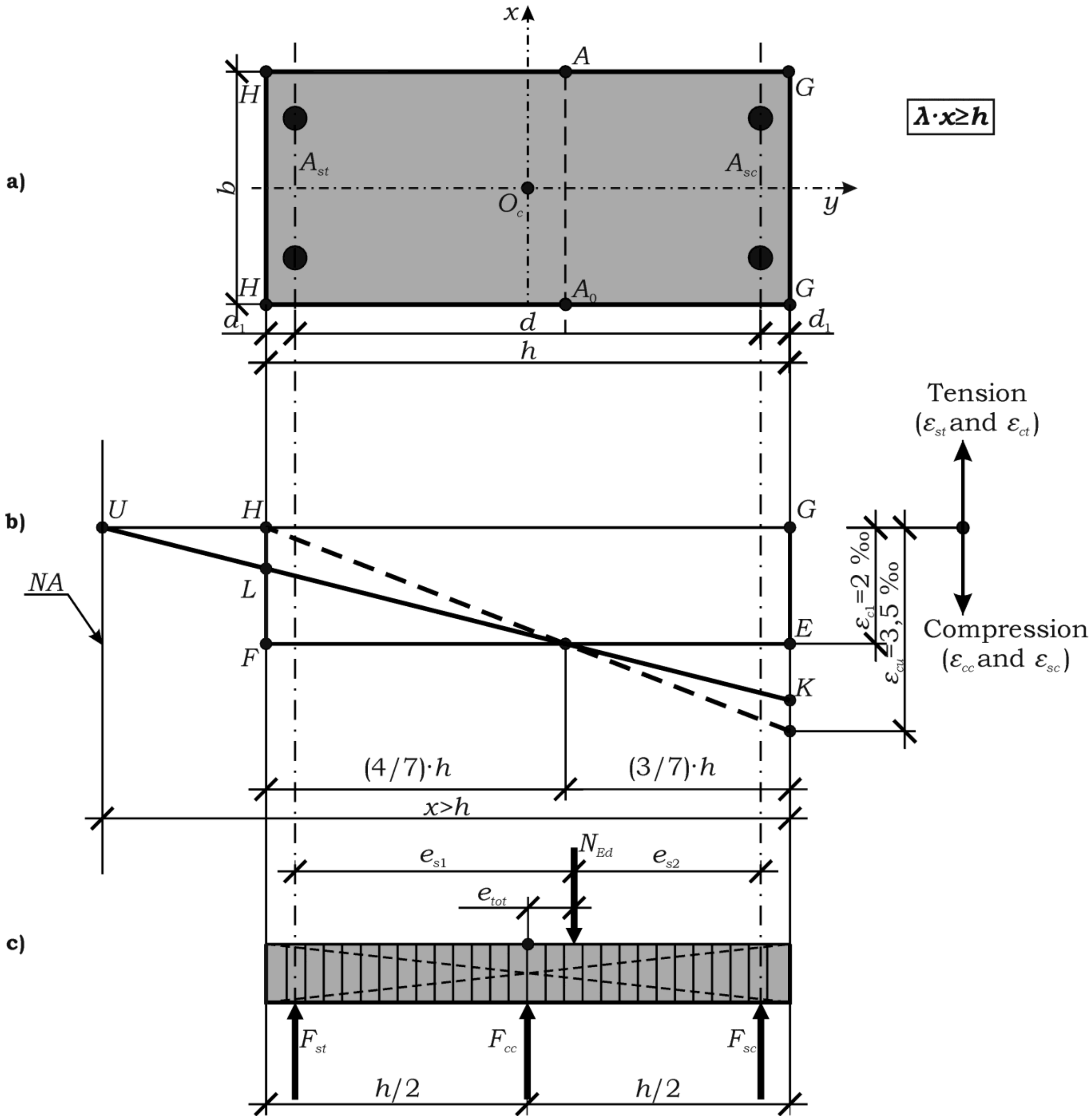
For cases where is tension in the section (see Figure 4.1-23), the limiting concrete strain is taken as $\varepsilon_{cu} = 3,5$ ‰ – the value used in design and analysis of section for bending.

However, for cases where it is no tension in section (see Figure 4.1-25), the limiting strain is taken as a value of $\varepsilon_{c,2} = 2$ ‰ at the level of $\left(\frac{3}{7}\right) \cdot h$ of the depth of the section.



a) – cross-section; b) – strains profile; c) – stress block

Figure 4.1-24 – Strains profile and stress distribution for the case when $\lambda \cdot x < h$



a) – cross-section; b) – strains profile; c) – stress block

Figure 4.1-25 – Strains profile and stress distribution for the case when $\lambda \cdot x \geq h$

4.1.2.3.1 Modes of failure and the basic equations formulation

The applied force N_{Ed} must be balanced by the forces developed within the cross section, therefore:

$$N_{Ed} = F_{cc} + F_{sc} + F_{st}, \tag{4.1-158}$$

where: F_{cc} is the resultant compressive force developed in the concrete and acting through the centroid of the stress block;

F_{sc} is the resultant compressive force developed in the reinforcement A_{sc} and acting through its centroid;

F_{st} is the resultant tensile or compressive force developed in the reinforcement A_{st} and acting through its centroid.

In the Equation (4.1-158), force F_{st} will be negative whenever the position of the neutral axis is such that reinforcement A_{st} is in tension, as it is shown in Figure 4.1-24. Substituting into Equation (4.1-158) the term for the stress and areas:

$$N_{Rd} = N_{Ed} = \eta \cdot f_{cd} \cdot b \cdot \lambda \cdot x + \sigma_{sc} \cdot A_{sc} + \sigma_{st} \cdot A_{st}, \qquad (4.1\text{-}159)$$

where σ_{sc} is the compressive stress in reinforcement A_{sc};

σ_{st} is the tensile or compressive stress in reinforcement A_{st}.

Thus, $\dfrac{N_{Ed}}{\eta \cdot f_{cd} \cdot b \cdot h}$ and $\dfrac{M_{Ed}}{\eta \cdot f_{cd} \cdot b \cdot h^2}$ can be calculated for the specified ratios of $\dfrac{A_{st}}{b \cdot h}$ and x/h, so that the column design charts for a symmetrical reinforcement arrangement such as one shown in Figure 4.1-26 can be plotted.

a)

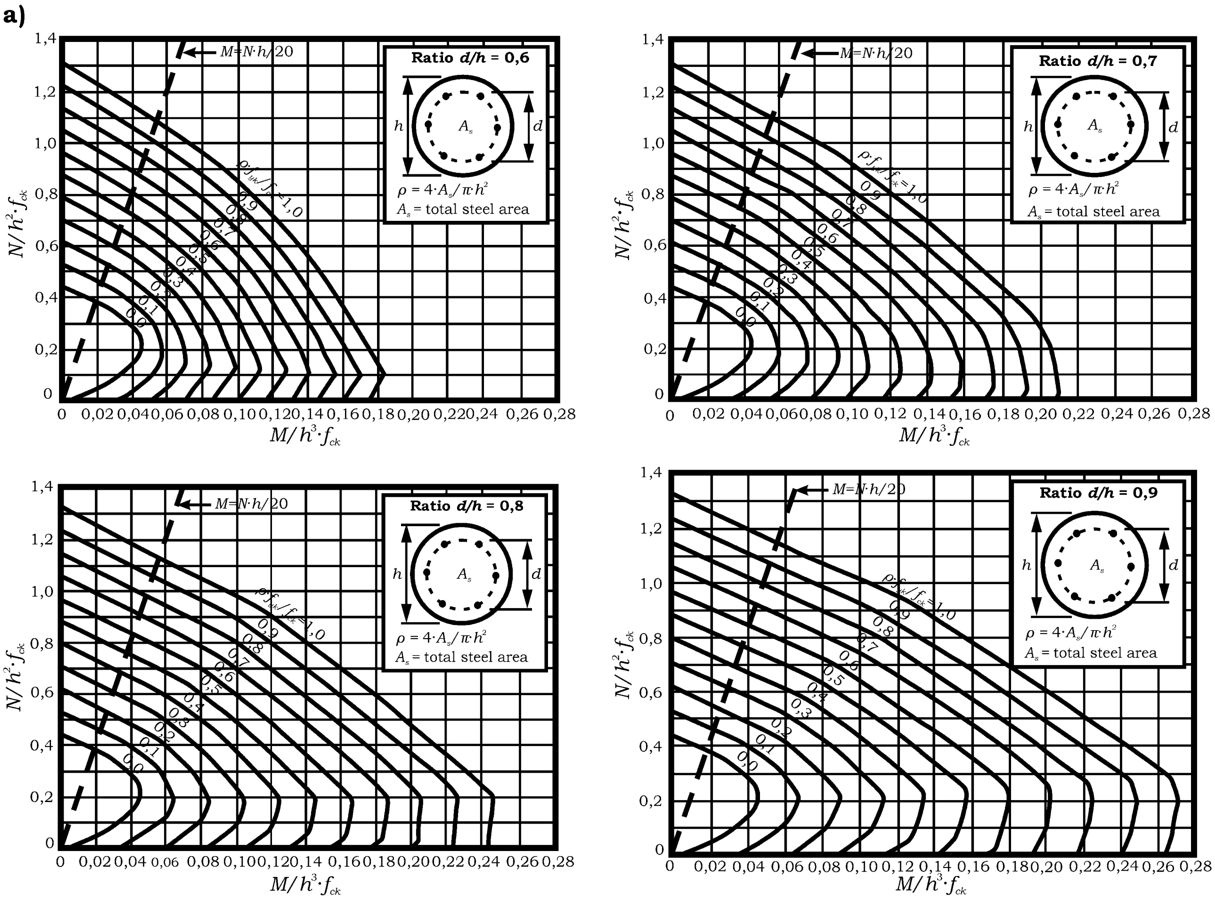
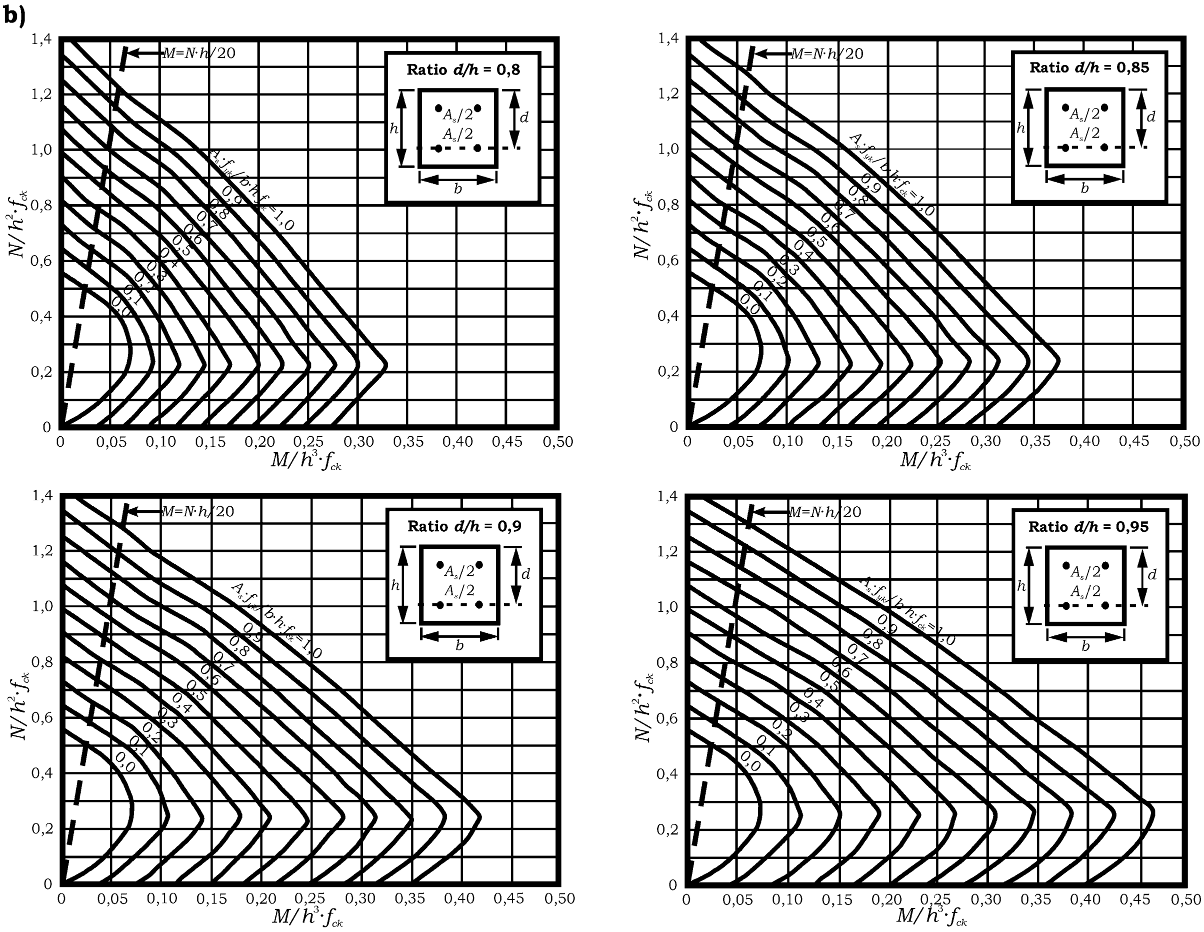


Figure 4.1-26 – Design charts for columns



a) – for circular columns with d/h=0,6; 0,7; 0,8; 0,9;

b) – for rectangular columns with d/h=0,8; 0,85; 0,9; 0,95

Figure 4.1-26 (end) – Design charts for columns

The direct solution of the Equation (4.1-162) and Equation (4.1-163) for the design of column reinforcement would be very tendinous and, therefore, a set of design charts for the usual case of symmetrical sections is available in special publications, such as [8].

As it was shown earlier the magnitude of the eccentricity ($e_{tot} = \frac{M_{Ed}}{N_{Ed}}$) affects the position of the neutral axis and, hence, the strains and stresses in the reinforcement.

For the linear strain distribution (see Figure 4.1-24 and Figure 4.1-25):

$$\varepsilon_{sc} = \varepsilon_{cu} \cdot \left(\frac{x - d_1}{x} \right), \tag{4.1-164}$$

and:

$$\varepsilon_{st} = \varepsilon_{cu} \cdot \left(\frac{d - x}{x} \right). \tag{4.1-165}$$

For values of x greater than h, when neutral axis extends below the cross-section, as it is shown in Figure 4.1-25, the steel strains are given by the alternative expression:

$$\varepsilon_{sc} = 0{,}002 \cdot \frac{7 \cdot (x - d_1)}{(7 \cdot x - 3 \cdot h)}, \tag{4.1-166}$$

and:

$$\varepsilon_{st} = 0{,}002 \cdot \frac{7 \cdot (x - d)}{(7 \cdot x - 3 \cdot h)}. \tag{4.1-167}$$

In the Equation (4.1-165) and Equation (4.1-166):
ε_{sc} is the compressive stress in reinforcement A_{sc};
ε_{st} is the tensile or compressive strain in reinforcement A_{st}.

Let's consider the following modes of failure of the section as it is shown on the interaction diagram of Figure 4.1-19 for the case of rectangular compressive stress block in concrete.

a) tensile failure $\varepsilon_{st} \geq \varepsilon_{sy}$.

This type of failure is associated with large eccentricity (e_{tot}) and small depth of the neutral axis (x).

Failure begins with yielding of the tensile reinforcement, followed by crushing of the concrete as the tensile strains rapidly increase.

b) balanced failure $\varepsilon_{st} = \varepsilon_{sy}$, between points ⑤ and ⑥ on Figure 4.1-19.

When failure occurs with yielding of the tension steel and crushing of the concrete at the same instant it is describes as a «*balanced*» failure. With $\varepsilon_{st} = \varepsilon_{sy}$:

$$x = x_{lim} = \frac{d}{1 + \frac{\varepsilon_{sy}}{\varepsilon_{cu}}}. \tag{4.1-168}$$

Substituting the value of $\varepsilon_{sy} = 2{,}17$ ‰ (for steel grade S500):

$$x_{lim} = 0{,}617 \cdot d. \tag{4.1-169}$$

Equation (4.1-158) and Equation (4.1-160) become:

$$N_{lim} = F_{cc} + F_{sc} - F_{st} = \eta \cdot f_{cd} \cdot b \cdot 0{,}8 \cdot x_{lim} - \sigma_{sc} \cdot A_{sc} - f_{yd} \cdot A_{st}, \tag{4.1-170}$$

and:

$$M_{lim} = F_{cc} \cdot \left(\frac{h}{2} - \frac{0,8 \cdot x_{lim}}{2} \right) + F_{sc} \cdot \left(\frac{h}{2} - d_1 \right) - F_{st} \cdot \left(d - \frac{h}{2} \right), \qquad (4.1\text{-}171)$$

where: $f_{sc} \leq f_{yd}$.

At the point ⑤ of the interaction diagram of Figure 4.1-20, $N_{Ed} = N_{lim}$, $M_{Ed} = M_{lim}$, and $\sigma_{sc} = -f_{yd}$. When the design load $N_{Ed} > N_{lim}$, the section will be fail in compression, while if $N_{Ed} \leq N_{lim}$, where will be an initial tensile failure, with yielding of reinforcement A_{st}.

c) compression failure between points ① and ⑤ on Figure 4.1-19.

In this case $x > x_{lim}$ and $N_{Ed} > N_{lim}$. The change in slope at point ④ in Figure 4.1-19 occurs when:

$$\varepsilon_{sc} = \varepsilon_{sy}, \qquad (4.1\text{-}172)$$

and from the Equation (4.1-172):

$$x_{(c)} = \frac{0,0035 \cdot d_1}{\left(\varepsilon_{cu} - \varepsilon_{sy} \right)} = 2,63 \cdot d_1, \qquad (4.1\text{-}173)$$

for steel grade S500 and $\varepsilon_{cu} = 3,5$ ‰.

When $x < d$: $\sigma_{st} \leq f_{yd}$ and tensile;

When $x = d$: $\sigma_{st} = 0$, and

When $x \geq d$: $\sigma_{st} = f_{yd}$ and compressive.

When x become very large and section approaches a state of uniform axial compression, $\varepsilon_{st} = \varepsilon_{sc} = \varepsilon_{sy}$. At this stage (see Figure 4.1-25), the both layers of steel will have yielded and there will be zero moment of resistance M_{Rd} with symmetrical cross-section, so that:

$$N_{Rd,n} = \eta \cdot f_{cd} \cdot b \cdot h + f_{yd} \cdot \left(A_{st} + A_{sc} \right). \qquad (4.1\text{-}174)$$

At the stage, where the neutral axis coincides with the bottom of the cross-section, the strain diagram changes from that shown in Figure 4.1-23 to the alternative strain diagram shown in Figure 4.1-25.

To calculate $M_{Rd,i}$ and $N_{Rd,i}$ at this stage, corresponding to point ② in Figure 4.1-19, Equation (4.1-159) and Equation (4.1-160) should be used, taking the neutral axis depth equal to the overall section depth *h*.

4.1.2.3.2 Calculation of the required area of the reinforcement

(1) Rectangular cross-section with an asymmetric reinforcement arrangement

For the assymetric arrangement of the reinforcement in cross section, the area of reinforcement is determined from the equilibrium conditions (4.1-159)–(4.1-161), assuming that: $\xi_{eff} = \xi_{eff,lim}$ (where $\xi_{eff} = \frac{x_{eff}}{d} = \frac{\lambda \cdot x}{d}$). Hence:

$$A_{sc} = \frac{N_{Ed} \cdot e_{s1} - \xi_{eff} \cdot \left(1 - 0,5 \cdot \xi_{eff}\right) \cdot b \cdot d^2 \cdot \eta \cdot f_{cd}}{\left(d - d_1\right) \cdot f_{yd}}, \qquad 4.1\text{-}175)$$

where: e_{s1} is the eccentricity of the axial load, that is equal to $e_{tot} + 0,5 \cdot h - a_1$.

In case when obtained area of the compressive reinforcement $A_{sc} > 0$, required area of the reinforcement A_{st} can be calculated assuming that $\xi_{eff} = \xi_{eff,lim}$ as follows:

$$A_{st} = \left(\xi_{eff} \cdot d \cdot b \cdot \eta \cdot f_{cd} + A_{sc} \cdot f_{yd} - N_{Ed}\right) \cdot \frac{1}{f_{yd}}. \qquad (4.1\text{-}176)$$

If the area of the compressive reinforcement $A_{sc,prov} > A_{sc}$ (when $A_{sc} \leq 0$), the depth of the neutral axis ξ_{eff} have to be corrected, taking a new value of the depth of the neutral axis ξ_{eff} from Table 4.1-7 as a function of $a_{m,eff}$, calculated from the following equation:

$$a_{m,eff} = \frac{N_{Ed} \cdot e_{s1} - A_{sc,prov} \cdot \left(d - d_1\right) \cdot f_{yd}}{\eta \cdot f_{cd} \cdot b \cdot d^2}. \qquad 4.1\text{-}177)$$

Substituting into Equation (4.1-176), a new area of the reinforcement A_{st} have to be calculated. If the obtained area of the reinforcement is positive ($A_{st} > 0$), than the result of the calculation is considered as satisfactory. When obtained area of the reinforcement A_{st} is negative ($A_{st} < 0$), this means that there is a case of small eccentricity: $\xi_{eff} > \xi_{eff,lim}$ and $k_s < 1$ (see Figure 4.1-25). In the case of small eccentricity, the reinforcement area A_{st} will not be used in its entirety, and, therefore, it can be assumed that $A_{st,prov} > A_{st,min}$. Actual depth of the neutral axis $\xi_{eff} > \xi_{eff,lim}$ is calculated from the following moment equation:

$$N_{Ed} \cdot e_{s2} = S_{cc,eff} \cdot \eta \cdot f_{cd} - A_{st,prov} \cdot \left(d - d_1\right) \cdot k_s \cdot f_{yd}, \qquad (4.1\text{-}178)$$

where: $S_{cc,eff}$ is the moment of the resultant of the compressive stress in concrete about the centroid of the reinforcement A_{sc};

e_{s2} is the eccentricity of the axial load N_{Ed} and it is equal: $0,5 \cdot h - e_{tot} - a_2$.

The value of the depth of the neutral axis ξ_{eff} can be obtained by means of the Table 4.1-9, depending on the ratio ${a_1}/{d}$ and coefficient $a_{m,eff}$:

$$a_{m,eff} = \frac{N_{Ed} \cdot e_{s2}}{\eta \cdot f_{cd} \cdot b \cdot d^2}. \tag{4.1-179}$$

Table 4.1-9 – Coefficients $a_{m,eff}$ values for the rectangular section under compression (case of the snall eccentricities)

ξ_{eff}	Coefficient $a_{m,eff}$				
	Ratio a_1 / d				
	0,04	**0,08**	**0,12**	**0,16**	**0,20**
0,41	0,068	0,051	0,035	0,018	0,002
0,42	0,071	0,054	0,038	0,021	0,004
0,43	0,074	0,059	0,040	0,024	0,007
0,44	0,077	0,062	0,042	0,025	0,009
0,45	0,083	0,065	0,047	0,029	0,011
0,46	0,088	0,069	0,051	0,032	0,014
0,47	0,092	0,073	0,055	0,036	0,017
0,48	0,096	0,076	0,057	0,038	0,019
0,49	0,100	0,081	0,061	0,042	0,022
0,50	0,105	0,085	0,065	0,045	0,025
0,51	0,110	0,089	0,069	0,048	0,028
0,52	0,114	0,093	0,073	0,052	0,031
0,53	0,119	0,098	0,076	0,055	0,034
0,54	0,124	0,103	0,081	0,060	0,038
0,55	0,128	0,106	0,084	0,062	0,040
0,56	0,135	0,112	0,090	0,067	0,045
0,57	0,139	0,116	0,094	0,071	0,048
0,58	0,145	0,122	0,098	0,075	0,052
0,59	0,150	0,127	0,103	0,080	0,056
0,60	0,156	0,132	0,108	0,084	0,060
0,61	0,162	0,137	0,113	0,088	0,064
0,62	0,167	0,142	0,118	0,093	0,068
0,63	0,173	0,148	0,122	0,097	0,072
0,64	0,179	0,154	0,128	0,103	0,077
0,65	0,185	0,159	0,133	0,107	0,081
0,66	0,192	0,165	0,139	0,112	0,086
0,67	0,198	0,171	0,144	0,117	0,090
0,68	0,204	0,178	0,149	0,122	0,095
0,69	0,210	0,183	0,155	0,128	0,100
0,70	0,217	0,189	0,161	0,133	0,105
0,71	0,224	0,195	0,167	0,138	0,110
0,72	0,230	0,201	0,173	0,144	0,115
0,73	0,237	0,208	0,178	0,149	0,120
0,74	0,244	0,215	0,185	0,156	0,126
0,75	0,251	0,221	0,191	0,161	0,131
0,76	0,259	0,228	0,198	0,167	0,137
0,77	0,265	0,234	0,204	0,173	0,142
0,78	0,273	0,242	0,210	0,179	0,148
0,79	0,280	0,249	0,217	0,186	0,154
0,80	0,288	0,256	0,224	0,192	0,160
0,81	0,296	0,263	0,231	0,198	0,166
0,82	0,303	0,270	0,238	0,205	0,172

Table 4.1-9 (end)

0,83	0,311	0,278	0,244	0,211	0,178
0,84	0,319	0,286	0,252	0,219	0,185
0,85	0,327	0,293	0,259	0,225	0,191
0,86	0,336	0,301	0,267	0,232	0,198
0,87	0,343	0,308	0,273	0,239	0,204
0,88	0,352	0,317	0,281	0,246	0,211
0089	0,360	0,325	0,289	0,254	0,218
0,90	0,369	0,333	0,297	0,261	0,225
0,91	0,378	0,341	0,305	0,268	0,232
0,92	0,386	0,349	0,313	0,276	0,239
0,93	0,395	0,358	0,320	0,283	0,246
0,94	0,404	0,367	0,329	0,292	0,254
0,95	0,413	0,375	0,337	0,299	0,261
0,96	0,423	0,384	0,346	0,307	0,269
0,97	0,431	0,392	0,354	0,315	0,276
0,98	0,441	0,402	0,362	0,323	0,284
0,99	0,450	0,411	0,371	0,332	0,292
1,00	0,460	0,420	0,380	0,340	0,300
1,01	0,470	0,429	0,389	0,348	0,308
1,02	0,479	0,438	0,398	0,357	0,316
1,03	0,489	0,448	0,406	0,365	0,324
1,04	**0,500**	0,458	0,416	0,375	0,333
1,05		0,467	0,425	0,383	0,341
1,06		0,477	0,435	0,392	0,350
1,07		0,486	0,444	0,401	0,358
1,08		**0,497**	0,453	0,410	0,367
1,09			0,463	0,420	0,376
1,10			0,473	0,429	0,385
1,11			0,483	0,438	0,394
1,12			**0,493**	0,448	0,403
1,13				0,457	0,412
1,14				0,468	0,422
1,15				0,477	0,431
1,16				**0,487**	0,441
1,17					0,450
1,18					0,460
1,19					0,470
1,20					**0,480**

The required area of the reinforcement A_{sc} is calculated from the general Equation (4.1-175).

In case coefficient $a_{m,eff} \le a_{m,eff,max}$ ($a_{m,eff,max}$ is the maximum values from Table 4.1-9 for an assumed ratio of a_1/d), than obtained area of the reinforcement $A_{st} = A_{st,min}$.

In case coefficient $a_{m,eff} > a_{m,eff,max}$, required area of the reinforcement A_{sc} is calculated from the Equation (4.1-175) and A_{st} from the following equation:

$$A_{st} = \frac{N_{Ed} \cdot e_{s2} - 0,5 \cdot \eta \cdot f_{cd} \cdot b \cdot d^2}{(-1) \cdot f_{yd} \cdot (d - a_2)}. \qquad (4.1\text{-}180)$$

If the obtained from Equation (4.1-171) area of the reinforcement A_{sc} is negative, this means that the cross-sectional area is an excessive (applied forces balanced by the concrete compressive stresses resultant only) and $A_{sc} = A_{sc,min}$.

(2) Rectangular cross-section with symmetrical reinforcement arrangement

For this case ($A_{st} = A_{sc}$ and $a_1 = d_1$) depth of the neutral axis ξ_{eff} can be calculated as follows:

$$\xi_{eff} = \frac{N_{Ed}}{\eta \cdot f_{cd} \cdot b \cdot d}. \tag{4.1-181}$$

If the condition $\xi_{eff} < \xi_{eff,lim}$ is satisfied, the required area of the reinforcement is obtained from the following equation:

$$A_{st} = A_{sc} = \frac{N_{Ed} \cdot \left[e_{s1} - d \cdot \left(1 - 0,5 \cdot \xi_{eff}\right)\right]}{\left(d - d_1\right) \cdot f_{yd}}. \tag{4.1-182}$$

For very small value of the neutral axis depth $\xi_{eff} \leq 2 \cdot d_1 / d$, the required area of reinforcement is calculated:

$$A_{st} = A_{sc} = \frac{N_{Ed} \cdot e_{s2}}{\left(d - d_1\right) \cdot f_{yd}}. \tag{4.1-183}$$

If $\xi_{eff} > \xi_{eff,lim}$, the case of small eccentricity takes place, and actual depth of the neutral axis is calculated from the following equation:

$$\begin{gathered} \xi_{eff}^3 - \left(2 + \xi_{eff,lim}\right) \cdot \xi_{eff}^2 + \left[1 + \xi_{eff,lim} - \frac{a_2}{d} \cdot \left(1 - \xi_{eff,lim}\right) + A\right] \cdot \xi_{eff} - \\ - A \cdot \xi_{eff,lim} - B \cdot \left(1 - \xi_{eff,lim}\right) = 0, \end{gathered} \tag{4.1-184}$$

where: $A = \dfrac{2 \cdot N_{Ed} \cdot e_{s1}}{\eta \cdot f_{cd} \cdot b \cdot d^2}$; $B = \dfrac{N_{Ed} \cdot \left(d - a_2\right)}{\eta \cdot f_{cd} \cdot b \cdot d^2}$.

If the obtained value $\xi_{eff} > 1,0$, then for the further calculation it should be taken $\xi_{eff} = 1,0$ and the required area of the reinforcement is calculated as follows:

$$A_{st} = A_{sc} = \frac{N_{Ed} - \eta \cdot f_{cd} \cdot b \cdot d}{2 \cdot f_{yd}}. \tag{4.1-185}$$

If the obtained value $\xi_{eff} < 1,0$, then the required area of the reinforcement is equal:

$$A_{st} = A_{sc} = \frac{\left(N_{Ed} - \xi_{eff} \cdot \eta \cdot f_{cd} \cdot b \cdot d\right) \cdot \left(1 - \xi_{eff,lim}\right)}{2 \cdot \left(\xi_{eff} - \xi_{eff,lim}\right) \cdot f_{yd}}. \quad (4.1\text{-}186)$$

4.1.2.3.3 Cross-section resistance checking based on the rectangular compressive stress block

For a given dimensions of the rectangular cross-section of the compressed element and for the accepted reinforcement area $A_{st} = A_{sc}$, cross section resistance M_{Rd} and N_{Rd} can be calculated from the following equations:

$$M_{Rd} = \frac{\xi_{eff} \cdot \left(1 - 0,5 \cdot \xi_{eff}\right) \cdot \eta \cdot f_{cd} \cdot b \cdot d^2 + A_{sc} \cdot f_{yd} \cdot \left(d - d_1\right)}{e_{s1}}, \quad (4.1\text{-}187)$$

and:

$$N_{Rd} = \xi_{eff} \cdot \eta \cdot f_{cd} \cdot b \cdot d + A_{sc} \cdot f_{yd} - A_{st} \cdot k_s \cdot f_{yd}. \quad (4.1\text{-}188)$$

The value of the neutral axis depth ξ_{eff} can be calculated from the following equation:

$$\xi_{eff} = B + \sqrt{B^2 + 2 \cdot \left(\mu_{s1} \mp \mu_{s2}\right)}, \quad (4.1\text{-}189)$$

where: $B = 1 - \dfrac{e_{s1}}{d}$; $\mu_{s1} = \dfrac{A_{st} \cdot e_{s1} \cdot f_{yd}}{b \cdot d^2 \cdot \eta \cdot f_{cd}}$; $\mu_{s2} = \dfrac{A_{sc} \cdot e_{s2} \cdot f_{yd}}{b \cdot d^2 \cdot \eta \cdot f_{cd}}$.

The coefficient μ_{s2} in the Equation (4.1-189) should be taken with a «minus»-sign, if $e_{s1} > \left(d - d_1\right)$.

If the obtained from the Equation (4.1-185) value $\xi_{eff} > \xi_{eff,lim}$, $k_s \neq 1$ and:

$$\xi_{eff} = B - C + \sqrt{\left(B - C\right)^2 + 2 \cdot \left(C - \mu_{s1} \mp \mu_{s2}\right)}, \quad 4.1\text{-}190)$$

where: $C = \dfrac{2 \cdot \mu_{s1}}{1 - \xi_{eff,lim}}$.

In the obtained from Equation (4.1-189) value $\xi_{eff} > 1,0$, than in Equation (4.1-187) and Equation (4.1-188) $\xi_{eff} = 1,0$ and $k_s = -1$ should be taken.

4.1.2.4 Interaction diagram

As it was shown, a reinforced concrete column with specified amount of longitudinal steel reinforcement has different carrying capacities of a pair of N_{Ed} and M_{Ed} before its collapse depending on the eccentricity of the load. In general case, the interaction diagram has three distinct zones of failure (see Figure 4.1-19):

1) from point ① to just before point ⑤ is the zone of compression failure;
2) point ⑤ is the balanced failure;
3) from point ⑤ to point ⑥ is the zone of tension failure.

In the compression failure zone, small eccentricities produce failure of concrete in compression, while large eccentricities cause failure triggered by yielding of tension steel reinforcement.

In between, point ⑤ is the critical point at which both the failures of concrete in compression and steel in yielding occur simultaneously.

The interaction diagram further reveals that as the force N_{Ed} becomes larger, the section can carry smaller M_{Ed} before failing in the compression zone. The reverse is the case in the tension zone, where the moment carrying capacity (moment of resistance) $M_{Rd,u}$ increases with the axial load increasing $N_{Rd,u}$. In the compression failure zone, the failure occurs due to the over straining of concrete. The large axial force produces high compressive strain of concrete keeping smaller margin available for an additional compressive strain line to bending. On the other hand, in the tension failure zone, yielding of steel initiates failure. This tensile yield stress reduces with the additional compressive stress due to additional axial load. As a result, further moment can be applied till the combined stress of steel due to the axial force and increased moment M_{Rd} reaches the yield strength.

Therefore, the design of column with given N_{Ed} and M_{Ed} should be done following the three steps, as it is given below:

1) selection of a trial section with an assumed longitudinal steel;
2) construction of the interaction diagram of the selected trial column section by successive choices of the neutral axis depth from infinity (pure axial load) to a very small value (to be found by trial to get $N_{Ed} = 0$ for the pure bending).
3) checking of the given N_{Ed} and M_{Ed}, if they are within the diagram.

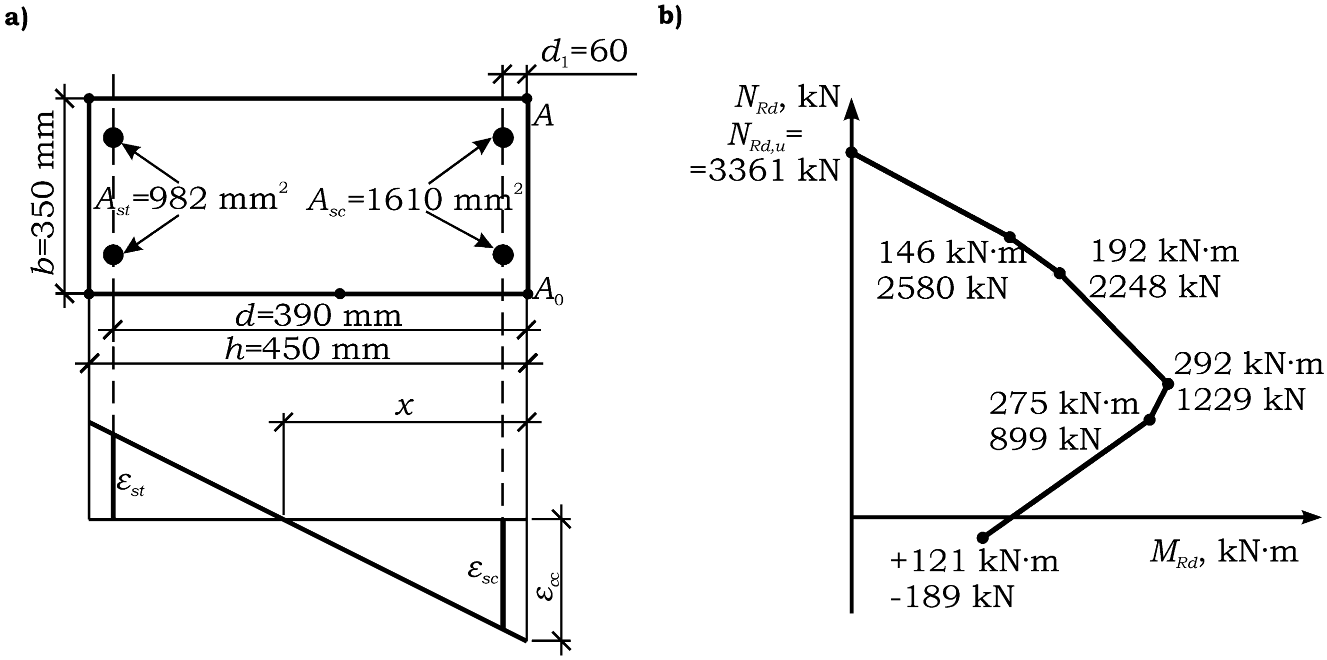
Such an «M_{Rd}–N_{Rd}» interaction diagram can be constructed for the any shape of the cross-section, which has an axis of symmetry by applying the basic equilibrium and strain compatibility equations with the «σ–ε» relations (stress-strain relations). This diagrams can be very useful for the design purpose.

Example to Section 4.1.2

Example 1. Example of the interaction diagram construction

Construct the interaction diagram for the cross-section shown in the Figure E 4.1-1 with $f_{ck} = 25$ N/mm² ($C\,{}^{20}\!/_{25}$) and $f_{yk} = 500$ N/mm² (S500). The bending causes maximum compression on the face adjacent to the steel area A_{sc}.

For symmetrical cross section, taking the moment about centre-line of the concrete cross section will give $M_{Rd} = 0$ with $N_{Ed} = N_{Rd,u}$ and both areas of steel at the yield stress. This is no longer true for the unsymmetrical steel areas as $F_{sc} \neq F_{st}$ at yield therefore, theoretically, moment should be calculated about an axis referred to as the «*plastic centroid*». The ultimate axial force $N_{Rd,u}$ acting through the plastic centroid causes a uniform strain across the section with compression yielding of all of the reinforcement and, thus, there is zero moment of resistance M_{Rd}. With uniform strain the neutral axis depth *x*, is at infinity.



a) – cross-section; b) – interaction diagram

Figure E 4.1-1 – Non-symmetrical cross-section « $M_{Rd} - N_{Rd}$ » interaction example

The location of the plastic centroid is determined by the taking moments of the all the stress resultants about an arbitrary axis such as *A-A* in Figure E 4.1-1, so that:

$$x_p = \frac{\sum\left(\frac{F_{cc}\cdot h}{2} + F_{sc}\cdot d_1 + F_{st}\cdot d\right)}{\sum\left(F_{cc}+F_{sc}+F_{st}\right)} N_{Rd,n} =$$

$$= \frac{\eta\cdot f_{cd}\cdot A_{cc}\cdot 450/2 + f_{yd}\cdot A_{sc}\cdot 60 + f_{yd}\cdot A_{st}\cdot 390}{\eta\cdot f_{cd}\cdot A_{cc} + f_{yd}\cdot A_{sc} + f_{yd}\cdot A_{st}} = \tag{E 4.1-1}$$

$$= \frac{0,8\cdot\frac{25}{1,5}\cdot\frac{450}{2}\cdot 350 + \frac{500}{1,15}\cdot\left(1610\cdot 60 + 982\cdot 390\right)}{0,8\cdot\frac{25}{1,5}\cdot 350\cdot 450 + \frac{500}{1,15}\cdot\left(1610+982\right)} =$$

$$= 212 \text{ mm from axis A - A.}$$

The fundamental equations for calculating points of the interaction diagram with varying the depth of the neutral axis are:

1) compatibility of strains (used in Table E 4.1-1, columns 2 and 3):

$$\varepsilon_{sc} = 0,0035\cdot\left(\frac{x-d_1}{x}\right);$$
$$\varepsilon_{st} = 0,0035\cdot\left(\frac{d-x}{x}\right), \tag{E 4.1-2}$$

or, when the neutral axis depth extends below the bottom of the section ($x > h$):

$$\varepsilon_{sc} = 0,002\cdot\frac{7\cdot\left(x-d^{/}\right)}{\left(7\cdot x - 3\cdot h\right)};$$
$$\varepsilon_{st} = 0,002\cdot\frac{7\cdot\left(x-d\right)}{\left(7\cdot x - 3\cdot h\right)}. \tag{E 4.1-3}$$

2) Stress-strain relation for the steel reinforcement as it is shown in Figure E 4.1-1 (Table E 4.1-1, columns 4 and 5):

$$\varepsilon_{st(sc)} \geq \varepsilon_{sy} \qquad f_{yd} = \frac{f_{yk}}{1,15};$$
$$\varepsilon_{st(sc)} < \varepsilon_{sy} \qquad \sigma_{st(sc)} = E_s\cdot\varepsilon_{st(sc)}. \tag{E 4.1-4}$$

3) Equilibrium equations (Table E 4.1-1, columns 6 and 7):

$$N_{Rd} = F_{cc} + F_{sc} + F_{st},$$
$$\text{for } \lambda\cdot x < \lambda\cdot h \quad N_{Rd} = \eta\cdot f_{cd}\cdot b\cdot 0,8x + \sigma_{sc}\cdot A_{sc} + \sigma_{st}\cdot A_{st}, \tag{E 4.1-5}$$
$$\text{for } \lambda\cdot x \geq \lambda\cdot h \quad N_{Rd} = \eta\cdot f_{cd}\cdot b\cdot h + \sigma_{sc}\cdot A_{sc} + \sigma_{st}\cdot A_{st}.$$

F_{st} is negative, when σ_{st} is a tensile stress.

Table E 4.1-1 – «M_{Rd}–N_{Rd}» interaction values for example [6]

x, мм	ε_{sc} , %	ε_{st} , %	σ_{sc} , [N/mm²]	σ_{st} , [N/mm²]	N_{Rd} , [kN]	M_{Rd} , [kN·m]
1	2	3	4	5	6	7
$d^{/}$	0	>2,17	0	f_{yd}	-189	121,0
2,63·d_1	2,17	>2,17	f_{yd}	f_{yd}	899	275
$x_{lim} = 0,617 \cdot d = $ $= 241$	>2,17	2,17	f_{yd}	-f_{yd}	1229	292
d=390	>2,17	0	f_{yd}	0	2248	192
h=450	>2,17	0,47	f_{yd}	93,3	2580	146
0	2,17	2,17	f_{yd}	f_{yd}	3361	0

These equations have been applied to provide the values in Table E 4.1-1 for a range values of x. Then «$M_{Rd}-N_{Rd}$» interaction diagram has been plotted in Figure E 4.1-1 b from the values in Table E 4.1-1 as a series of straight lines. Of course, M_{Rd} and N_{Rd} could have been calculated for the intermediate values of x to provide a more accurate curve, as it is presented in Figure E 4.1-1 b.

4.1.3 ANALYSIS OF SECOND ORDER EFFECTS WITH AXIAL LOAD

4.1.3.1 General requirements and definitions in accordance with EN 1992 [N3]

Column design is largely covered within the Section 5.8 and Section 6.1 of EN 1992 [N3]. Global second order effects are likely to occur in the structures with a flexible bracing system.

As it was shown in EN 1992 [N3], where second order effects are taken into account, equilibrium and resistance shall be verified in the ***deformed state***. Deformations shall be calculated taking into account the relevant effects of cracking, non-linear material properties and creep.

The structural behavior shall be considered in the direction in which deformations can occur, and biaxial bending shall be taken into account when necessary.

Uncertainties in geometry and position of axial loads shall be taken into account as additional first order effects based on the geometric imperfections. Second order effects may be ignored if they are less than 10 % of the corresponding first order effects.

Column design generally involves determining the slenderness ratio, λ, of the member. If it lies below a critical value, λ_{lim} , the column can be simply designed to resist axial actions and moment, obtained from an elastic analysis, but including the effect of geometrical imperfections. These are termed ***first order effects***.

In accordance with Section 5.8.1 of EN 1992 [N3]:

–***first order effects***: action effects calculated without consideration of the effect of structural deformations, but including geometrical imperfections. However, when the column slenderness exceeds the critical value, an additional (*second order*) moments caused by structural deformations can occur and must also be taken into account.

–***second order effects***: an additional action effects caused by structural deformations. In EN 1992 [N3], the slenderness ratio above which one columns are subjected to the second order effects has to be evaluated (see Section 4.1.3.2).

In this chapter (section) only the design of the most common types of columns found in building structures, namely braced columns, will be described.

–***braced members or systems***: structural members or subsystems, which in analysis and design are assumed not to contribute to overall horizontal stability of a structure;

–***bracing members or systems***: structural members or subsystems, which in analysis and design are assumed to contribute to the overall horizontal stability of a structure.

A column may be considered to be braced in a given plane if the bracing element or system (e.g. core or shear walls) is sufficiently stiff to resist all the lateral forces in that plan. Thus, braced columns are assumed to not contribute to the overall horizontal stability of a structure and as such are only designed to resist axial load and bending due to the vertical loading. The design of braced columns involves consideration of the following aspects, which are discussed individually below: slenderness ratio (λ); threshold slenderness (λ_{lim}); first order effects; second order effects (moments); reinforcement details.

4.1.3.2 Simplified criteria for second order effects

4.1.3.2.1 Slenderness criterion for isolated members

As it was noted above, the threshold slenderness value, λ_{lim}, is a key element of the design procedure as it provides a simple and convenient way of determining when to take into account of the first order effects only and when to include second order effects. The value of λ_{lim} is given by the following expression:

$$\lambda_{lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}, \tag{4.1-191}$$

where: $A = 1/\left(1 + 0,2 \cdot \varphi_{ef}\right)$, and in case φ_{ef} is not known, $A = 0,7$ may be used;

$B = \sqrt{1 + 2 \cdot \omega}$, and in case ω is not known, $B = 1,1$ may be used;

$C = 1,7 - r_m$, and in case r_m is not known, $C = 0,7$ may be used;

φ_{ef} is an effective creep ratio;

ω is a mechanical reinforcement ratio, that it is equal to $\left(A_s \cdot f_{yd}\right)/\left(A_c \cdot f_{cd}\right)$;

A_s is the total area of the longitudinal reinforcement;

$n = N_{Ed}/(A_c \cdot f_{cd})$ is a relative normal force;

$r_m = M_{0,1}/M_{0,2}$ is the end moment ratio, taking positive when moments produce tension on the same face (i.e. $C \le 1,7$);

$M_{0,1}$ and $M_{0,2}$ are the first order end moments, $|M_{0,2}| \ge |M_{0,1}|$.

The value of the creep coefficient, φ_{ef}, can be calculated using the guidance in EN 1992 [N3] (clause 5.8.4). However, in many cases the extra design effort required may not be justified and the recommended value of 0,7 for the factor A should be used.

Factor B depends upon the area of longitudinal steel, which will be unknown of the design stage. It would seem reasonable, therefore, to use the recommended value of 1,1, at least for the first iteration.

Factor C arguably has the largest influence on λ_{lim} and it is worthwhile calculating its value accurately rather than simply assuming it is equal to 0,7. Factor C gives an indication of the column's susceptibility to buckling under the action of applied moments. In the following cases, r_m, should be taken as 1,0 (i.e. $C = 0,7$): for the braced members, in which the first order moments arise only from or predominantly due to imperfections or transverse loading; for the unbraced members in general.

Thus, buckling is more likely where the end moments act in opposite senses as they will produce tension on the same face. Conversely, buckling is less likely when the end moments act in the same sense as the member will be in double curvature (see Figure 4.1-27).

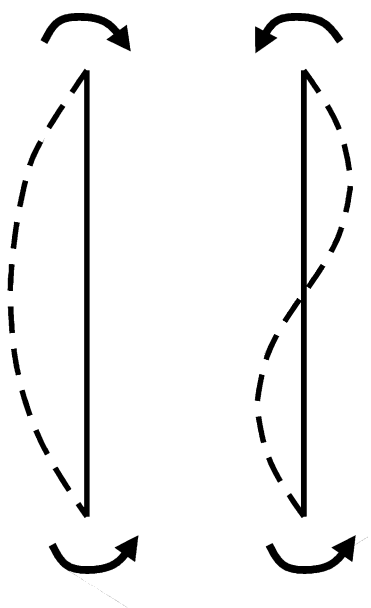


Figure 4.1-27 – Columns buckling modes

4.1.3.2.2 Slenderness and effective length of isolated members

Slenderness ratio, λ, is defined as follows:

$$\lambda = \frac{l_0}{i}, \tag{4.1-192}$$

where: l_0 is the effective length of the column;

i is the radius of gyration of the uncracked concrete section.

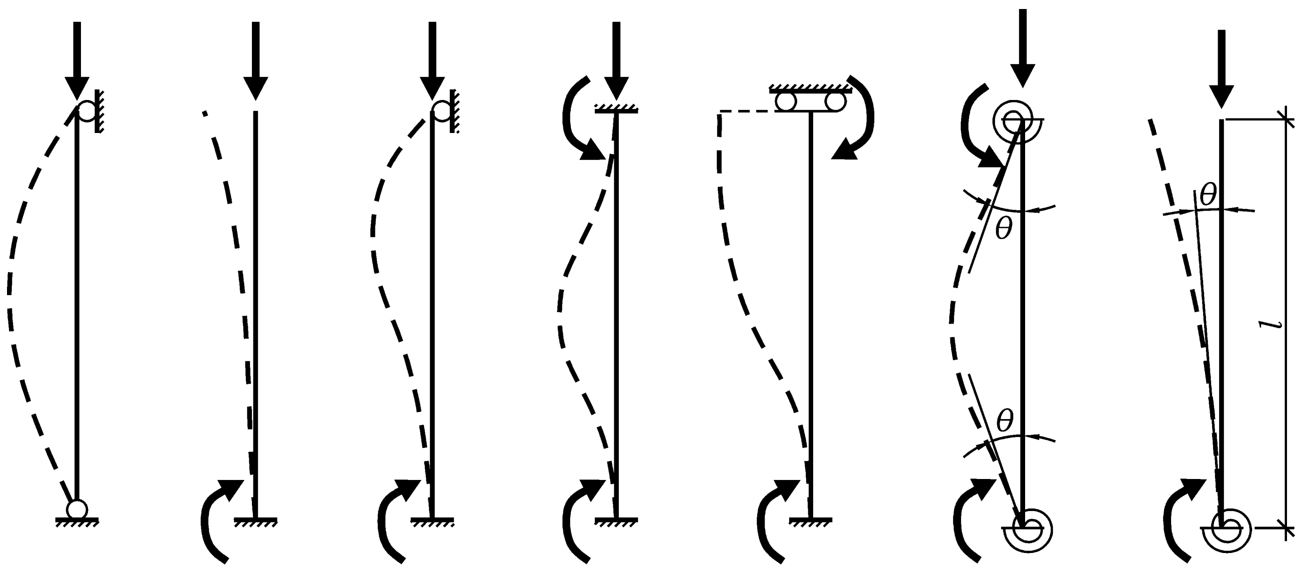
– ***effective length:*** a length used to take into account the shape of the deflection curve; it can also be defined as a buckling length, i.e. the length of the pin-ended column with the constant normal force, having the same cross section and buckling load as the actual member, and:

– ***buckling load:*** the load at which buckling occurs; for an isolated elastic members it is synonymous with the Euler load;

–***buckling:*** failure due to instability of a member or structure under the perfect axial compression and without transverse load.

«***Pure buckling***», as it is defined above, is not a relevant limit state in the real structures, due to an imperfections and transverse loads, but a nominal buckling load can be used as a parameter in some of the methods for the second order analysis.

Examples of the effective length for isolated members with a constant cross-section are given in Figure 4.1-28.



a) $l_0=l$; **b)** $l_0=2\cdot l$; **c)** $l_0=0,7\cdot l$; **d)** $l_0=l/2$; **e)** $l_0=l$; **f)** $l/2<l_0<l$; **g)** $l_0>2\cdot l$;

Figure 4.1-28 – Examples of the different buckling modes and corresponding effective length for isolated members (Figure 5.7 f, EN 1992 [N3])

Figure 5.7 f from EN 1992 [N3], reproduced here as a Figure 4.1-28, suggests that the effective length of braced member (l_0) can vary between half and full height of the member depending on the degree of the rotational restraint at column ends, i.e.:

$$l/2 < l_0 < l. \tag{4.1-193}$$

For the members in compression of a regular frames, the slenderness criterion should be checked with an effective length l_0 determined in the following way:

Braced members (see Figure 4.1-28 f):

$$l_0 = 0,5 \cdot l \sqrt{\left(1 + \frac{k_1}{0,45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0,45 + k_2}\right)}, \tag{4.1-194}$$

where: k_1 and k_2 are the relative flexibilities of the rotational restraints at the ends 1 and 2 respectively, in which:

$$k = (\theta / M) / (EI / l), \tag{4.1-195}$$

where: θ is the rotation of the restraining members for bending moment M (see Figure 4.1-28 f);

EI is the bending stiffness of the members in compression;

l is the clear height of the member in compression between end restraints.

Note, that in theory, $k = 0$ for the fully rigid rotational restraint and $k = \infty$ for the no restraint at all, i.e. pinned support. Since fully rigid restraint is rare in practice, EN 1992 [N3] recommends a minimum value of 0,1 for k_1 and k_2.

It is not an easy matter to determine the values of k_1 and k_2 in practice because:

a) the guidance in EN 1992 [N3] is somewhat ambiguous with regard to the effect of stiffness of columns attached to the column under the consideration, and:

b) the effect of cracking on the stiffness of the restraining member.

Unbraced members (see Figure 4.1-27 g):

$$l_0 = l \cdot \max\left\{\sqrt{1 + 10 \cdot \frac{k_1 \cdot k_2}{k_1 + k_2}}; \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right)\right\}. \tag{4.1-196}$$

In accordance with EN 1992 [N3], if an adjacent compression member (column) in a node is likely to contribute to the rotation at buckling, then (EI/l) in the definition of k should be replaced by $\left[(EI / l)_a + (EI / l)_b\right]$, a and b representing the compression member (column) above and below the node. In general case, in the definition of effective length, the stiffness of restraining members should include the effect of cracking, unless they can be shown to be uncracked in ***ULS***.

For the other cases than was presented above, e.g. members with varying normal force and/or cross-section, the criterion $\lambda \le \lambda_{lim}$ should be checked with an effective length based on the buckling load (calculated, for example, by a numerical method):

$$l_0 = \pi \cdot \sqrt{EI / N_B}, \tag{4.1-197}$$

where: EI is a representative bending stiffness;

N_B is a buckling load expressed in terms of this EI (in Expression (4.1-192), i should also corresponds to this EI).

The radius of gyration (i) is defined by:

$$i = \pi \cdot \sqrt{(I / A)}, \tag{4.1-198}$$

where: I is a moment of inertia of the uncracked concrete section;

A is an area of the uncracked concrete section.

4.1.3.3 Design of braced columns

Having determined the value of λ_{lim}, it is possible to design the column. The following sub-sections discuss the procedures recommended in EN 1992 [N3] for the design of the braced columns, when $\lambda \le \lambda_{lim}$ and $\lambda > \lambda_{lim}$.

4.1.3.3.1 Design of braced columns, when $\lambda \le \lambda_{lim}$

According to the EN 1992 [N3] (clause 5.8.3.1), in case, when slenderness, λ, is less than λ_{lim}, the column should be designed for the applied axial load (action), M_{Ed}, being numerical equal to the sum of the larger elastic end moment, $M_{0,2}$, plus any moment due to the geometric imperfection, $N_{Ed} \cdot e_i$, as follows:

$$M_{Ed} = M_{0,2} + N_{Ed} \cdot e_i, \tag{4.1-199}$$

where: e_i is the geometric imperfection, that is equal to $\theta_i \cdot \frac{l_0}{2}$, in the which one θ_i is the angle of inclination and can be taken as 1/200 for the isolated braced columns and l_0 is the effective length (see Section 4.1.3.2.2). The minimum design eccentricity, e_0, is $h/30$, but not less than 20 mm, where h is the depth of the cross-section.

Once N_{Ed} and M_{Ed} have been determined, the area of the longitudinal steel can be calculated by the strain compatibility using an iterative procedure [8]. However, this approach may not be practical for everyday design and, therefore, was produced a series of design charts (see Section 4.1.2.4), which can be used to determine the area of the longitudinal steel.

4.1.3.3.2 Design of braced columns, when $\lambda > \lambda_{lim}$. Methods of analysis

When $\lambda > \lambda_{lim}$., critical conditions may occur at the top, middle or bottom of the column.

According to EN 1992 [N3], the methods of analysis include a general method based on non-linear second order analysis and the following two simplified methods:

a) method based on the nominal stiffness;

b) method based on the nominal curvature.

As it was pointed in EN 1992 [N3], the nominal second order moments provided by the simplified methods a) and b) are sometimes greater than those corresponding to the instability. This is to ensure that the total moment is compatible with the cross-section resistance.

Method a) may be used for the both isolated members and the whole structures, if nominal stiffness values are estimated appropriately;

Method b) is mainly suitable for the isolated members.

However, with the realistic assumptions concerning the distribution of curvature, the method in this section can also be used for structures.

(1) General method

The general method is based on the non-linear analysis, including geometric non-linearity, i.e. second order effects. The general rules for the non-linear analysis is given in [8] should be applied. In the non-linear analysis, the stress-strain curves for concrete and steel are suitable for the overall analysis shall be used. The effect of the creep shall be taken into account.

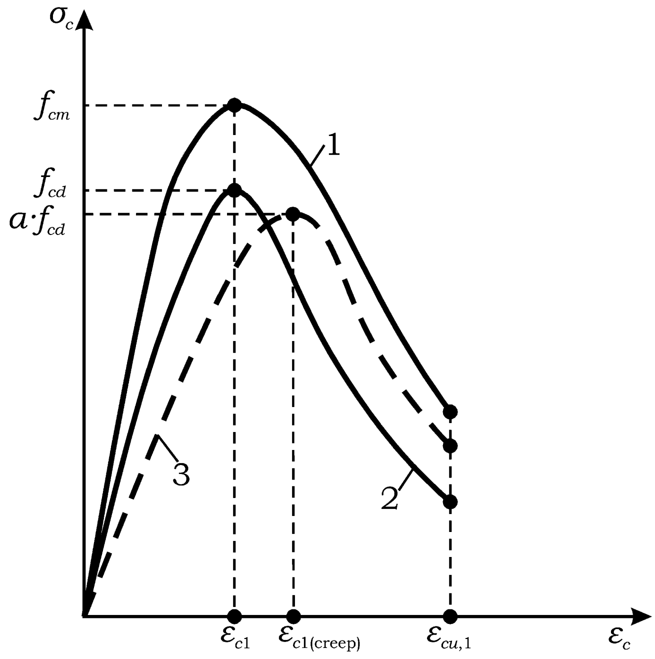
Stress-strain relationships for concrete and steel are given in the Chapter 3 may be used. With the stress-strain diagrams based on the design values, a design value of the ultimate load is obtained directly from the analysis. In Expression (4.1-195) and in the k-value, f_{cm} is then substituted by the design compressive strength f_{cd} and E_{cm} is substituted by:

$$E_{cd} = E_{cm}/\gamma_{CE}\,, \tag{4.1-200}$$

where: γ_{CE} is the partial factor, that is equal to 1,2 (the recommended value).

In absence of more refined models, creep may be taken into account by the multiplying all the strain values in the concrete «stress-strain» diagram (see Figure 4.1-29) with a factor $(1+\varphi_{ef})$, where φ_{ef} is the effective creep ratio according to EN 1992 [N3] (clause 5.8.4).

Normally, conditions of the equilibrium and strain compatibility are satisfied in a number of the cross-sections. A simplified alternative is to considered only the critical cross-section, and to assume the relevant variation of the curvature in between, e.g. similar to the first order moment or simplified in another appropriate way in accordance with [3].



1 – mean diagram; 2 – design diagram; 3 – transformed long-term diagram

Figure 4.1-29 – Stress-strain diagrams for concrete

(2) Methods based on nominal stiffness

General.

In a second order analysis based on the stiffness, nominal values of the flexural stiffness should be used, taking into account the effects of cracking, material non-linearity and creep on the overall behaviour. This also applies to adjacent members involved in the analysis, e.g. beams, slabs or foundations. Where the relevant, soil-structure interaction should be taken into account.

The resulting design moment is used for the design of the cross sections with respect to bending moment and axial force.

Nominal stiffness.

The following model may be used to estimate the nominal stiffness of slender compression members with arbitrary cross section:

$$EI = K_c \cdot E_{cd} \cdot I_c + K_s \cdot E_s \cdot I_s, \qquad (4.1\text{-}201)$$

where: E_{cd} is the design value of the modulus of elasticity of concrete ($E_{cd} = E_{cm}/\gamma_{CE}$);

I_c is the moment of inertia of the concrete cross-section;

E_s is the design value of the modulus of elasticity of reinforcement;

I_s is the second moment of area of reinforcement, about the centre of area of the concrete;

K_c is a factor for effects of cracking, creep, etc.;

K_s is a factor for contribution of reinforcement.

The following factors may be used in Expression (4.1-201), provided $\rho \geq 0{,}002$:

$$\begin{cases} K_s = 1; \\ K_c = k_1 \cdot k_2 / \left(1 + \varphi_{ef}\right), \end{cases} \tag{4.1-202}$$

where: ρ is the geometric reinforcement ratio, that is equal to A_s/A_c;

A_s is the total area of reinforcement;

A_c is the area of concrete cross-section;

φ_{ef} is the effective creep ratio;

k_1 is a factor which depends on concrete strength class, Expression (4.1-203);

k_2 is a factor which depends on axial force and slenderness, Expression (4.1-204).

$$k_1 = \sqrt{f_{ck} / 20}, \qquad 4.1\text{-}203)$$

$$k_2 = n \cdot \frac{\lambda}{170} \leq 0,20, \tag{4.1-204}$$

where: n is the relative axial force, that is equal to $N_{Ed}/(\, A_c \cdot f_{cd})$;

λ is the slenderness ratio.

If the slenderness ratio λ is not defined, k_2 may be taken as follows:

$$k_2 = n \cdot 0,30 \leq 0,20. \tag{4.1-205}$$

As a simplified alternative, provided $\rho_l \geq 0,001$, the following factors may be used in Expression (4.1-202): $K_s = 0$; $K_c = 0,3 / \left(1 + 0,5 \cdot \varphi_{ef}\right)$.

In statically indeterminate structures, unfavourable effects of cracking in the adjacent members should be taken into account. Expressions (4.1-201)–(4.1.205) are not generally applicable to such a members. Partial cracking and tension stiffening may be taken into account, e.g. according to EN 1992 [N3] (cl. 7.4.3). However, as a simplification, fully cracked sections may be assumed. The stiffness should be based on an effective concrete modulus:

$$E_{cd,eff} = E_{cd} \big/ \left(1 + \varphi_{ef}\right), \tag{4.1-206}$$

where: E_{cd} is the design value of the modulus of elasticity;

φ_{ef} is the effective creep ratio; the same value as for columns may be used.

Moment magnification factor.

The total design moment, including second order moment, may be expressed as a magnification of the bending moments resulting from a first order analysis, namely:

$$M_{Ed} = M_{0,Ed} \cdot \left[1 + \frac{\beta}{(N_B / N_{Ed}) - 1} \right], \tag{4.1-207}$$

where: $M_{0,Ed}$ is the first order moment;

β is a factor, which depends on distribution of the first and second order moments;

N_{Ed} is the design value of axial load;

N_B is the buckling load based on the nominal stiffness.

For isolated members with constant cross section and axial load, the second order moment may normally be assumed to have a sine-shaped distribution. Then:

$$\beta = \pi^2 / c_0, \tag{4.1-208}$$

where: c_0 is a coefficient which depends on the distribution of the first order moment (for instance, $c_0 = 8$ for a constant first order moment; $c_0 = 9{,}6$ for a parabolic and $c_0 = 12{,}0$ for a symmetric triangular distribution, etc.).

For members without transverse load, differing first order end moments M_{01} and M_{02} may be replaced by an equivalent constant first order moments M_{0e} (see Expression (4.1-211)). Consistent with the assumption of a constant first order moment, $c_0 = 8{,}0$ should be used. The value $c_0 = 8{,}0$ also applies to members bent in double curvature. It should be noted that in some cases, depending on slenderness and axial force, the end moment(s) can be greater than the magnified.

Where cases of isolated members or members without transverse load are not applicable, $\beta = 1$ is normally a reasonable simplification. Expression (4.1-207) can then be reduced to:

$$M_{Ed} = \frac{M_{0,Ed}}{1 - (N_{Ed} / N_B)}. \tag{4.1-209}$$

(3) Method based on the nominal curvature

General.

In accordance with EN 1992 [N3] (cl. 5.8.8), this method is primarily suitable for isolated members with constant normal force and a defined effective length l_0 (see Section 4.1.3.2.2). The method gives a nominal second order moment based on deflection, which in its turn is based on the effective length and an estimated maximum curvature.

The resulting design moment is used for the design of the cross sections with respect to bending moment and axial force according to Section 4.1.2.

Bending moments.

The design moment is following:

$$M_{Ed} = M_{0,Ed} + M_2, \tag{4.1-210}$$

where: $M_{0,Ed}$ is the first order moment, including the effect of imperfections;

M_2 is the nominal second order moment.

The maximum value of M_{Ed} is given by the distributions of $M_{0,Ed}$ and M_2, the latter may be taken as a parabolic or sinusoidal over the effective length.

For statically indeterminate members moment $M_{0,Ed}$ is determined for the actual boundary conditions, whereas M_2 will depend on the boundary conditions via the effective length.

For members without loads applied between their ends, differing first order end moments M_{01} and M_{02} may be replaced by an equivalent first order end moment M_{0e} (moment including the effect of imperfections at the about mid-height of the column):

$$M_{0e} = 0,6 \cdot M_{02} + 0,4 \cdot M_{01} \geq 0,4 \cdot M_{02}, \tag{4.1-211}$$

where: M_{01} and M_{02} are the first order end moments including the effect of the imperfections acting on the column. Moments M_{01} and M_{02} should have the same sign if they give a tension of the same side, otherwise opposite sign. Furthermore, $|M_{02}| \geq |M_{01}|$ (M_{02} is numerically larger of the elastic end moment acting on the column).

The nominal second order moment M_2 acting on the column is given by the following expression:

$$M_2 = N_{Ed} \cdot e_2, \tag{4.1-212}$$

where: N_{Ed} is the design value of the axial force;

e_2 is the deflection equal to $(1/r) \cdot l_0^2 / c$;

$1/r$ is the curvature;

l_0 is the effective length;

c is a factor depending on the curvature distribution.

For the constant cross section, $c = 10$ ($\approx \pi^2$) is normally used. If the first order moment is constant, a lower value should be considered ($c = 8$ is a lower limit, corresponding to the constant total moment).

It should be noted, that the value π^2 corresponds to a sinusoidal curvature distribution. The value for the constant curvature is $c = 8$. Note, that c depends on the distribution of the total curvature, whereas c_0 depends on the curvature corresponding to the first order moment only.

Curvature.

For members with constant symmetrical cross sections (including reinforcement), the following may be used:

$$\left(1/r\right)=k_r\cdot k_\varphi\cdot\frac{1}{r_0},\qquad(4.1\text{-}213)$$

where: k_r is a correction factor depending on the axial force (load);

k_φ is a factor for the concrete creep taking into account.

$$\left(1/r_0\right)=\frac{\left(f_{yd}/E_s\right)}{0{,}45\cdot d},\qquad(4.1\text{-}214)$$

where: d is the effective depth.

If all the reinforcement is not concentrated on the opposite sides, but the part of it is distributed parallel to the plane of bending, *d*, is defined as follows:

$$d=h_2+i_s,\qquad 4.1\text{-}215)$$

where: i_s is the radius of gyration of the total reinforcement area.

Correction factor k_r from the Expression (4.1-213) should be taken as follows:

$$k_r=\left(n_u-n\right)/\left(n_u-n_{bal}\right)\leq 1,\qquad(4.1\text{-}216)$$

where: $n=\dfrac{N_{Ed}}{A_c\cdot f_{cd}}$, is a relative axial force;

N_{Ed} is the design value of axial force;

$n_u=1+\omega$, where $\omega=\dfrac{A_s\cdot f_{yd}}{A_c\cdot f_{cd}}$;

n_{bal} is the value of η at maximum moment resistance and the value 0,4 may be used;

A_s is the total area of reinforcement;

A_c is the area of concrete cross section.

The effect of creep should be taken into account by the following factor:

$$k_\varphi=1+\beta_{eff}\geq 1,\qquad(4.1\text{-}217)$$

where: φ_{eff} is the effective creep ratio.

$$\beta=0{,}35+f_{ck}/200-\lambda/250,\qquad(4.1\text{-}218)$$

where: λ is the slenderness ratio.

Once, N_{Ed} and M_{Ed} are known, the area of longitudinal steel can be evaluated using an appropriate procedures in accordance with Section 4.1.2 or column design charts.

4.1.3.4 Biaxial bending

The general method, described in Section 4.1.3.2 may also be used for biaxial bending. The following provisions apply when simplified methods are used. Special care should be taken to identify the section along the member with the critical combination of moments.

Separate design in the each principal direction, disregarding biaxial bending, may be made as a first step. Imperfections need to be taken into account only in the direction where they will have the most unfavourable effect.

No further check is necessary if the slenderness ratios satisfy the following two conditions:

$$\lambda_y / \lambda_z \leq 2, \text{ and } \lambda_z / \lambda_y \leq 2, \qquad 4.1\text{-}219)$$

and the relative eccentricities e_y / h_{eq} and e_z / b_{eq} (see Figure 4.1-30) satisfy one the following conditions:

$$\frac{e_y / h_{eq}}{e_z / b_{eq}} \leq 0{,}2, \text{ or } \frac{e_z / b_{eq}}{e_y / h_{eq}} \leq 0{,}2, \qquad (4.1\text{-}220)$$

where: b, h are the width and the depth of the cross-section;

$b_{eq} = i_y \cdot \sqrt{12}$, and $h_{eq} = i_z \cdot \sqrt{12}$ for an equivalent rectangular section; .

λ_y, λ_z are the slenderness ratios l_0 / i with respect to y- and z-axis respectively;

i_y, i_z are the radii of gyration with respect to y- and z-axis respectively;

$e_z = M_{Ed,y} / N_{Ed}$ is the eccentricity along z-axis;

$e_y = M_{Ed,z} / N_{Ed}$ is the eccentricity along y-axis;

$M_{Ed,y}$ is the design moment about y-axis, including second order moment;

$M_{Ed,z}$ is the design moment about z-axis, including second order moment;

N_{Ed} is the design value of the axial load in the respective load combination.

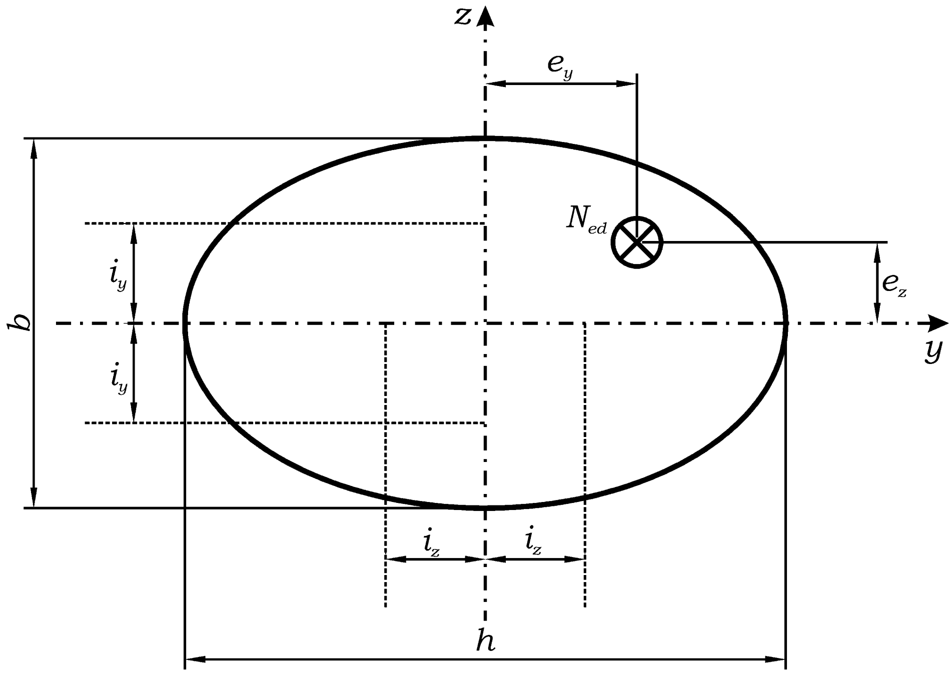


Figure 4.1-30 – Definition of the eccentricities e_y and e_z (Figure 5.8 from EN 1992 [N3])

If the condition of Expression (4.1-219) and Expression (4.1-220) is not fulfilled, biaxial bending should be taken into account including the second order effects in the each direction. In the absence of an accurate cross section design for biaxial bending, the following simplified criterion may be used:

$$\left(\frac{M_{Ed,z}}{M_{Rd,z}}\right)^a + \left(\frac{M_{Ed,y}}{M_{Rd,y}}\right)^a \leq 1,0, \tag{4.1-221}$$

where: $M_{Ed,z/y}$ is the design moment around the respective axis, including a second order moment;

$M_{Rd,z/y}$ is the moment resistance in the respective direction;

a is the exponent: for circular and elliptical sections $a = 2$; for rectangular cross sections: $N_{Ed} / N_{Rd} = 0,1 \rightarrow a = 1,0$; $N_{Ed} / N_{Rd} = 0,7 \rightarrow a = 1,5$; $N_{Ed} / N_{Rd} = 1,0 \rightarrow a = 2,0$ with linear interpolation for intermediate values;

N_{Ed} is the design value of axial force;

$N_{Rd} = A_c \cdot f_{cd} + A_s \cdot f_{yd}$ is the design axial resistance of section (A_c is the gross area of the concrete cross-section; A_s is the area of longitudinal reinforcement).

Examples to section 4.1.3

Example 1. Column supporting an axial load and uni-axial bending

An internal column in a multi-storey building is subjected to an ultimate axial load (N_{Ed}) of 1600 kN and bending moment (M_{Ed}) of 60 kN·m including effect of imperfections. Design the column cross-section assuming $f_{ck} = 30\ \mathrm{N/mm^2}$, $f_{yk} = 500\ \mathrm{N/mm^2}$ and $\Delta c_{ck} = 5\ \mathrm{mm}$.

Cross-section.

Since the design bending moment is relatively small, use equation to size the column:

$$n_u = 1 + \omega = 1 + A_s \cdot f_{yd} / A_c \cdot f_{cd}. \qquad \text{(E 4.1-6)}$$

Clause 9.5.2 of EN 1992 [N3] stipulates that that the percentage of longitudinal reinforcement, A_s, should generally lie within the following limits:

$$\text{greater of } \frac{0{,}10 N_{Ed}}{f_{yd}} \text{ and } 0{,}002 A_c < A_s < 0{,}04 A_c. \qquad \text{(E 4.1-7)}$$

Assuming that the percentage of reinforcement is equal to, say, 2 %, gives: $A_{sc} = 0{,}02 \cdot A_c$.

Substituting this into the above equation gives: $\dfrac{1.6 \cdot 10^6}{A_c \cdot (0.85 \cdot 30 / 1.5)} = 1 + \dfrac{0.02 \cdot A_c \cdot (500/1.15)}{A_c \cdot (0.85 \cdot 30 / 1.5)}$. Hence, $A_c = 62267\,\mathrm{mm^2}$. For a square column $b = n = \sqrt{62267} = 250\,\mathrm{mm}$. Therefore a 300 mm square column is suitable.

Longitudinal steel. Design moment, M_{Ed}.

Minimum eccentricity, $e_0 = \dfrac{h}{30} = \dfrac{300}{30} = 10\,\mathrm{mm} \geq 20\,\mathrm{mm}$.

Minimum design moment, $e_0 \cdot N_{Ed} = 20 \cdot 10^{-3} \cdot 1.6 \cdot 10^3 = 32\ \mathrm{kN \cdot m} < M$.

Hence, $M_{Ed} = M = 60\ \mathrm{kN \cdot m}$ assuming $\lambda < \lambda_{lim}$.

Design chart.

Minimum cover to links for exposure class XC1, $c_{min,dur} = 15\,\mathrm{mm}$.

Assuming diameter of longitudinal bars $\text{Ø} = 25\,\mathrm{mm}$, minimum cover to main steel for bond, $c_{min,b} = 25\ \mathrm{mm}$ and the nominal cover, $c_{nom} = c_{min,b} + \Delta c_{dev} = 25 + 5 = 30\ \mathrm{mm}$.

Assuming diameter of links, $\text{Ø}' = 8\,\mathrm{mm} \Rightarrow$ minimum cover to links $c_{nom} - \text{Ø}' - \Delta c_{dev} = 30 - 8 - 5 = 17\ \mathrm{mm} > c_{\min,dur} = 15\ \mathrm{mm}$.

Therefore, $d_2 = 30 + 25/2 = 42{,}5\ \mathrm{mm}$, $\dfrac{d_2}{h} = \dfrac{42.5}{300} = 0{,}142$.

Round up to 0,15 and use chart from the Figure 4.1-26.

Longitudinal steel area.

$$\frac{N_{Ed}}{b \cdot h \cdot f_{ck}} = \frac{1.6 \cdot 10^6}{300 \cdot 300 \cdot 30} = 0{,}593;$$

$$\frac{M_{Ed}}{b \cdot h^2 \cdot f_{ck}} = \frac{60 \cdot 10^6}{300 \cdot 300^2 \cdot 30} = 0{,}074;$$

$$\frac{A_s \cdot f_{yk}}{b \cdot h \cdot f_{ck}} = \frac{A_s \cdot 500}{300 \cdot 300 \cdot 30} \approx 0.28 \text{ (see Figure 4.1-25)},$$

$$\Rightarrow A_s = 1512 \text{ mm}^2.$$

Provide 4Ø25 S500 (1960 mm²).

$A_{sc} / A_c = 1960/(300 \cdot 300) = 2{,}2\ \%$ (acceptable).

Links.

Diameter of links is the greater of: 6 mm; $\frac{1}{4} \cdot Ø = \frac{1}{4} \cdot 25 = 6{,}25$ mm.

Spacing of links should not exceed the lesser of: $20 \cdot Ø = 20 \cdot 25 = 500$ mm; the least dimension of column, that is equal to 300 mm; 400 mm.

Therefore, provide Ø8 links at 300 mm centres.

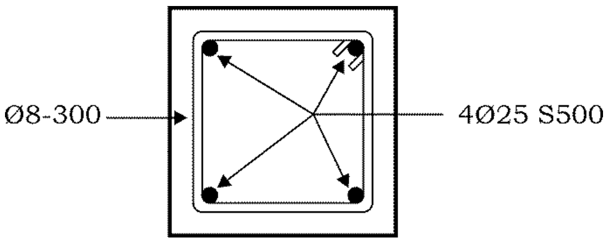


Figure E 4.1-6 – Classification of a column in accordance with EN 1992 [N3]

Determine if column GH shown in Figure E 4.1-7 should be designed for first or second order effects assuming that it resists the design loads and moments in b). Assume the structure is non-sway and $f_{ck} = 25 \text{ N/mm}^2$.

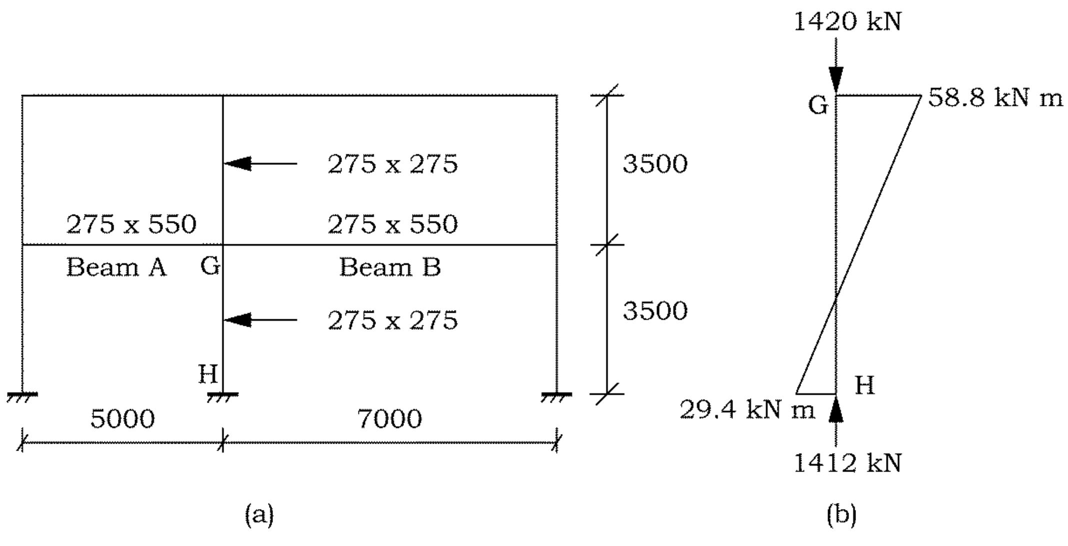


Figure E 4.1-7 – Input data for the worked example Example 1

Slenderness ration of column GH.

Effective height.

$$k_{beam\,A} = 2 \cdot (EI/L)_A = 2E \cdot \left(\frac{275 \cdot 550^3}{12 \cdot 5 \cdot 10^3} \right) = 1{,}525 \cdot 10^6\, E;$$

$$k_{beam\,B} = 2\cdot(EI/L)_B = 2E\cdot\left(\frac{275\cdot550^3}{12\cdot7\cdot10^3}\right) = 1{,}089\cdot10^6\,E;$$

$$k_{column\,GH} = (EI/L)_{GH} = E\cdot\left(\frac{275\cdot275^3}{12\cdot3{,}5\cdot10^3}\right) = 1{,}362\cdot10^5\,E;$$

$$k_G = \frac{k_{column\,GH}}{k_{beamA}+k_{beamB}} = \frac{1{,}362\cdot10^5\,E}{1{,}525\cdot10^6\,E+1{,}089\cdot10^6\,E} = 0{,}052 \geq 0{,}1.$$

$k_H = 0{,}1$ (since column is assumed to be fully fixed at the base).

$$l_0 = 0{,}5\cdot l\cdot\sqrt{\left(1+\frac{k_1}{0{,}45+k_1}\right)\cdot\left(1+\frac{k_2}{0{,}45+k_2}\right)};$$

$$l_0 = 0{,}5\cdot3500\cdot\sqrt{\left(1+\frac{0{,}1}{0{,}45+0{,}1}\right)\cdot\left(1+\frac{0{,}1}{0{,}45+0{,}1}\right)} = 0{,}59\cdot l = 2068\text{ mm}.$$

Radius of gyration.

Radius of gyration, i, is given by:

$$i = \sqrt{(I/A)} = \sqrt{[(b\cdot h^3/12)/(b\cdot h)]} = h/\sqrt{12} = 275/\sqrt{12} = 79{,}4\text{ mm}.$$

Slenderness ratio.

Slenderness ratio, λ, is given by:

$$\lambda = \frac{l_0}{i} = \frac{2067}{79{,}4} = 26.$$

Critical slenderness ratio, λ_{lim}.

$A = 0{,}7$; $B = 1{,}1$; $C = 1{,}7 - r_m = 1{,}7-(M_{01}/M_{02}) = 1{,}7-(-29{,}4/58{,}8) = 2{,}2$;

$n = N_{Ed}/A_c\cdot f_{cd} = 1402\cdot10^3/275\cdot275\cdot(0{,}85/1{,}5)\cdot25 = 1{,}31$;

$\lambda_{\text{lim}} = 20\cdot A\cdot B\cdot C/\sqrt{n} = 20\cdot0{,}7\cdot1{,}1\cdot2{,}2/\sqrt{1{,}31} = 29{,}6$;

Since $\lambda < \lambda_{lim}$, only the first order effects are needed to be considered.

Example 2. Classification of a column in accordance with EN 1992 [N3]

Determine if column PQ should be designed for second order effects assuming it the design loads and moment shown in Figure E 4.1-8. Further assume the structure is non-sway and $f_{ck} = 25\text{ N/mm}^2$.

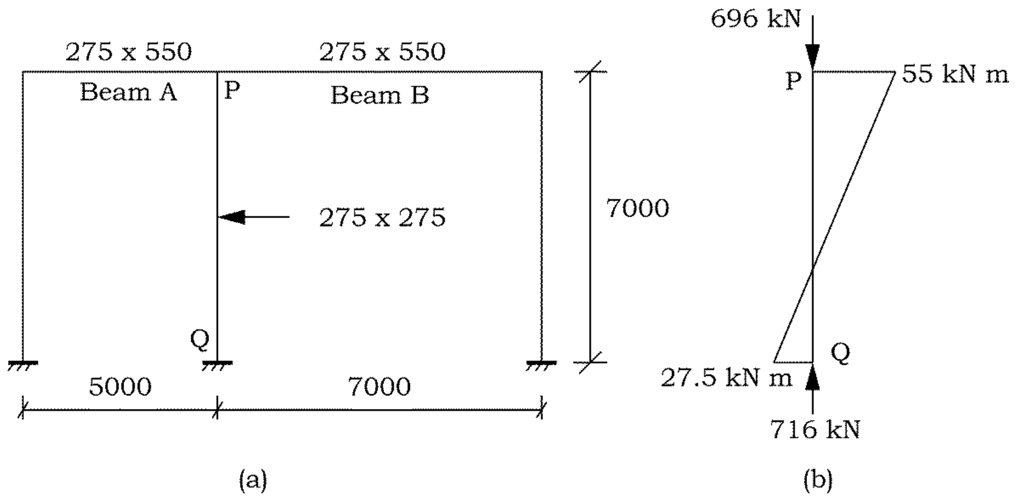


Figure E 4.1-8 – Input data for the worked Example 2

Slenderness ration of column PQ.
Effective height.

$$k_{beam\,A} = 2\cdot(EI/L)_A = 2E\cdot\left(\frac{275\cdot 550^3}{12\cdot 5\cdot 10^3}\right) = 1{,}525\cdot 10^6\,E;$$

$$k_{beam\,B} = 2\cdot(EI/L)_B = 2E\cdot\left(\frac{275\cdot 550^3}{12\cdot 7\cdot 10^3}\right) = 1{,}089\cdot 10^6\,E;$$

$$k_{column\,PQ} = (EI/L)_{PQ} = E\cdot\left(\frac{275\cdot 275^3}{12\cdot 7\cdot 10^3}\right) = 6{,}8085\cdot 10^4\,E;$$

$$k_P = \frac{k_{column\,PQ}}{k_{beamA}+k_{beamB}} = \frac{6{,}8085\cdot 10^4\,E}{1{,}525\cdot 10^6\,E + 1{,}089\cdot 10^6\,E} = 0{,}026 \ge 0{,}1.$$

$k_Q = 0{,}1$ (since column is assumed to be fully fixed at the base).

$$l_0 = 0{,}5\cdot l\cdot\sqrt{\left(1+\frac{k_1}{0{,}45+k_1}\right)\cdot\left(1+\frac{k_2}{0{,}45+k_2}\right)};$$

$$l_0 = 0{,}5\cdot 7000\cdot\sqrt{\left(1+\frac{0{,}1}{0{,}45+0{,}1}\right)\cdot\left(1+\frac{0{,}1}{0{,}45+0{,}1}\right)} = 0{,}59\cdot l = 4136\text{ mm}.$$

Radius of gyration.

Radius of gyration, i, is given by:

$$i = \sqrt{(I/A)} = \sqrt{[(b\cdot h^3/12)/(b\cdot h)]} = h/\sqrt{12} = 275/\sqrt{12} = 79{,}4\text{ mm.}$$

Slenderness ratio.

Slenderness ratio, λ, is given by:

$$\lambda = \frac{l_0}{i} = \frac{4136}{79{,}4} = 52.$$

Critical slenderness ratio, $\boldsymbol{\lambda_{lim}}$.

$A = 0{,}7$; $B = 1{,}1$; $C = 1{,}7 - r_m = 1{,}7 - (M_{01}/M_{02}) = 1{,}7 - (-27{,}5/55) = 2{,}2$;

$n = N_{Ed}/A_c\cdot f_{cd} = 696\cdot 10^3/275\cdot 275\cdot 0{,}567\cdot 25 = 0{,}6493$;

$\lambda_{lim} = 20\cdot A\cdot B\cdot C/\sqrt{n} = 20\cdot 0{,}7\cdot 1{,}1\cdot 2{,}2/\sqrt{0{,}6493} = 42$;

Since $\lambda > \lambda_{lim}$ column will need to be designed for second order moments.

Example 3. Column design: $\boldsymbol{\lambda < \lambda_{lim}}$; $\boldsymbol{\lambda > \lambda_{lim}}$.

Design the columns in Example 1 and Example 2. Assume the effective creep coefficient, φ_{ef}, is 0,87 and $\Delta c_{dev} = 5$ mm.

Column GH.
Longitudinal steel. Design moment, $\boldsymbol{M_{Ed}}$.

$$e_i = \left(\theta_i\cdot\frac{l_0}{2}\right) = \left(\frac{1}{200}\right)\cdot\left(\frac{2068}{2}\right) = 5{,}2\text{ mm.}$$

Minimum eccentricity, $e_0 = \left(\dfrac{h}{30}\right) = \left(\dfrac{275}{30}\right) = 9{,}2\text{ mm} \ge 20\text{ mm}$;

Minimum design moment, $e_0\cdot N_{Ed} = 20\cdot 10^{-3}\cdot 1402 = 28\text{ kN}\cdot\text{m}$;

First order end moment, $M_{02} = M + e_i\cdot N_{Ed} = 58{,}8 + 5{,}2\cdot 10^{-3}\cdot 1402 = 66{,}1\text{ kN}\cdot\text{m}$;

Hence, design moment, $M_{Ed} = 66{,}1\ \text{kN}\cdot\text{m}$.

Longitudinal steel area.

Assume: diameter of longitudinal steel, $\varnothing = 32\ \text{mm}$; diameter of links, $\varnothing^{/} = 8\ \text{mm}$; minimum cover for durability, $c_{\min,dur} = 25\ \text{mm} >$ $> c_{\min,b} = \varnothing - \varnothing^{/} = 32 - 8 = 24\ \text{mm}$ $\Rightarrow$ nominal cover to reinforcement, $c_{nom} = c + \Delta c_{dev} = 25 + 5 = 30\ \text{mm}$.

Therefore, $d_2 = \varnothing / 2 + \varnothing^{/} + c_{nom} = 32/2 + 8 + 30 = 54\ \text{mm}$; $\dfrac{d_2}{h} = \dfrac{54}{275} = 0{,}196$.

Use graph with $d_2 / h = 0{,}2$ (see Figure 4.1-26):

$$\frac{N_2}{b\cdot h\cdot f_{ck}} = \frac{1{,}402\cdot 10^6}{275\cdot 275\cdot 25} = 0{,}742;$$

$$\frac{M_{Ed}}{b\cdot h^2\cdot f_{ck}} = \frac{66{,}1\cdot 10^6}{275\cdot 275^2\cdot 25} = 0{,}127;$$

$$\frac{A_s\cdot f_{yk}}{b\cdot h\cdot f_{ck}} = \frac{A_s\cdot 500}{275\cdot 275\cdot 25} \approx 0{,}7 \text{ (see Figure 4.1-26)},$$

$$\Rightarrow A_s = 2647\ \text{mm}^2.$$

Provide 4Ø32 S500 (3220 mm^2).

Links.

Diameter of links is the greater of: 6 mm; $\dfrac{1}{4}\cdot\varnothing = \dfrac{1}{4}\cdot 32 = 8\ \text{mm}$.

Spacing of links should not exceed the lesser of: $20\cdot\varnothing = 20\cdot 32 = 640\ \text{mm}$; the least dimension of column, that is equal to 275 mm; 400 mm.

Therefore, provide Ø8 at 275 mm centers.

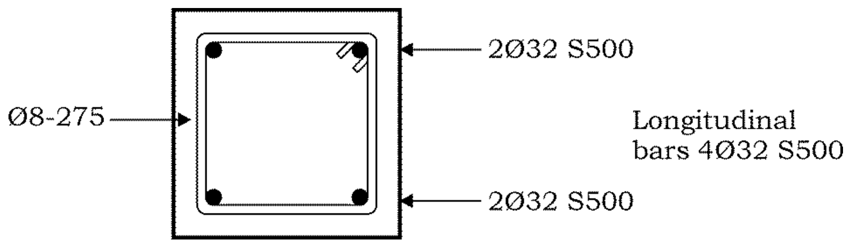


Figure E 4.1-9 – Column cross-section with the reinforcement arrangement

Column PQ.

First order end moments, M_{01}, M_{02}.

$$e_i = \left(\theta_i\cdot\frac{l_0}{2}\right) = \left(\frac{1}{200}\right)\cdot\left(\frac{4136}{2}\right) = 10{,}34\ \text{mm};$$

$$e_i\cdot N_{Ed} = 10{,}34\cdot 10^{-3}\cdot 696 = 7{,}2\ \text{kN}\cdot\text{m};$$

$$M_{01} = M_Q + e_i\cdot N_{Ed} = -27{,}5 + 7{,}2 = -20{,}3\ \text{kN}\cdot\text{m};$$

$$M_{02} = M_P + e_i\cdot N_{Ed} = 55 + 7{,}2 = 62{,}2\ \text{kN}\cdot\text{m};$$

Equivalent first order moment, M_{0Ed}.

$$M_{0Ed} = M_{0e} = (0{,}6\cdot M_{02} + 0{,}4\cdot M_{01}) \geq 0{,}4\cdot M_{0.2} = 0{,}4\cdot 62{,}2 = 24{,}9\ \text{kN}\cdot\text{m};$$

$$M_{0Ed} = M_{0e} = (0{,}6\cdot 62{,}2 + 0{,}4\cdot(-20{,}3)) = 29{,}2\ \text{kN}\cdot\text{m} \geq 0{,}4\cdot M_{0.2} = 24{,}9\ \text{kN}\cdot\text{m};$$

Nominal second order moment, M_2.

Assume: diameter of longitudinal steel, $Ø = 20$ mm; diameter of links, $Ø^{/} = 8$ mm; minimum cover for durability, $c_{\min,dur} = 30$ mm $>$ $c_{\min,b} = Ø - Ø^{/} = 20 - 8 = 12$ mm $\Rightarrow$ nominal cover to reinforcement, $c_{nom} = c + \Delta c_{dev} = 30 + 5 = 35$ mm.

Thus:

$$d = h - (Ø/2 + Ø^{/} + c_{nom}) = 275 - (20/2 + 8 + 35) = 222 \text{ mm};$$

$$\frac{1}{r_0} = \frac{\varepsilon_{yd}}{0,45 \cdot d} = \frac{(500/1,15)/200 \cdot 10^3}{0,45 \cdot 222} = 2,176 \cdot 10^{-5};$$

$$\beta = \frac{\varepsilon_{yd}}{0,45 \cdot d} = 0,35 + f_{ck}/200 - \lambda/150 = 0,35 + 25/200 - 52/150 = 0,1283;$$

$$K_{\varphi} = 1 + \beta \cdot \varphi_{ef} = 1 + 0,1283 \cdot 0,87 = 1,111 \geq 1,0.$$

Assume $K_r = 0,8$:

$$\frac{1}{r} = K_r \cdot K_{\varphi} \cdot \left(\frac{1}{r_0}\right) = 0,8 \cdot 1,111 \cdot 2,176 \cdot 10^{-5} = 1,934 \cdot 10^{-5};$$

$$e_2 = \left(\frac{1}{r}\right) \cdot l_0^2 / 10 = 1,934 \cdot 10^{-5} \cdot 4136^2 / 10 = 33,1 \text{ mm};$$

$$M_2 = N_{Ed} \cdot e_2 = 696 \cdot 33.1 \cdot 10^{-3} = 23 \text{ kN} \cdot \text{m}.$$

Design moment, M_{Ed}.

$$M_{Ed} = \text{maximum of } \{M_{0Ed} + M_2; M_{02}; M_{01} + 0,5 \cdot M_2\};$$

$M_{Ed} =$ maximum of $\{29,2 + 23 = 52,2$ kN$\cdot$m; $62,2$ kN$\cdot$m; $-20,3 + 0,5 \cdot (-23) =$ $= 31,8$ kN$\cdot$m$\}$.

Longitudinal steel area.

$$d_2 = Ø/2 + Ø^{/} + c_{nom} = 20/2 + 8 + 35 = 53 \text{ mm};$$

$$\frac{d_2}{h} = \frac{53}{275} = 0,193.$$

Use graph with $d_2/h = 0,2$ (see Figure 4.1-25):

$$\frac{N_{Ed}}{b \cdot h \cdot f_{ck}} = \frac{696 \cdot 10^3}{275 \cdot 275 \cdot 25} = 0,368;$$

$$\frac{M_{Ed}}{b \cdot h^2 \cdot f_{ck}} = \frac{62.2 \cdot 10^6}{275 \cdot 275^2 \cdot 25} = 0,12;$$

$$\frac{A_s \cdot f_{yk}}{b \cdot h \cdot f_{ck}} = \frac{A_s \cdot 500}{275 \cdot 275 \cdot 25} = 0,3 \ \Rightarrow A_s = 1134 \text{mm}^2.$$

Provide 4Ø20 S500 (1260 mm²).

Checking of the assumed value of K_r.

$$n_u = 1 + A_s \cdot f_{yd} / A_c \cdot f_{cd} = 1 + 1260 \cdot (0,87 \cdot 500)/(275 \cdot 275 \cdot 0,567 \cdot 25) = 1,51;$$

$$n = N_{Ed} / A_c \cdot f_{cd} = 696 \cdot 10^3 / (275 \cdot 275 \cdot (0,85/1,5) \cdot 25) = 0,65;$$

$$K_r = \frac{n_u - n}{n_u - n_{bal}} = \frac{1,51 - 0,65}{1,51 - 0,4} = 0,775.$$

Therefore assumed value is acceptable.

Links.

Diameter of links is the greater of: 6 mm; $\frac{1}{4}\cdot Ø=\frac{1}{4}\cdot 20=5$ mm.

Spacing of links should not exceed the lesser of: $20\cdot Ø=20\cdot 20=400$ mm; the least dimension of column is equal to 275 mm; 400 mm.

Therefore, provide Ø6 links at 275 mm centres.

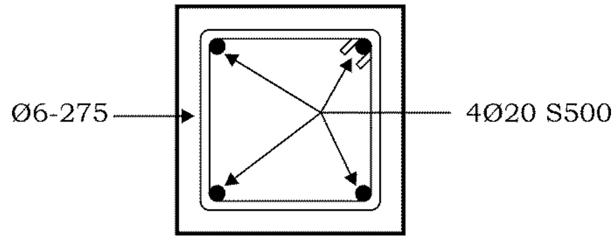


Figure E 4.1-10 – Column cross-section with the reinforcement arrangement

Example 4. Column subjected to combined axial load and biaxial bending [7]

Check the column (see Figure E 4.1-11) using the procedure in EN 1992 [N3].

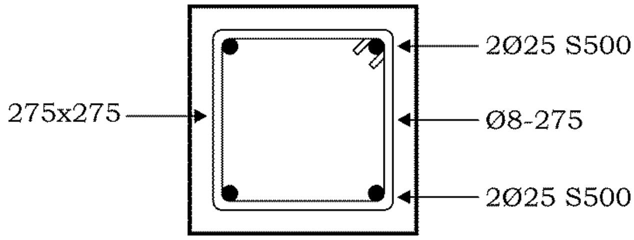


Figure E 4.1-11 – Column cross-section with the reinforcement arrangement

Relative eccentricities.

The column is subject to an axial load, $N_{Ed}=1250$ kN, a moment about the major axis (y-y), $M_{Edy}=35$ kN·m and a moment about the minor axis (z-z), $M_{Edz}=25$ kN·m.

$$e_y=\frac{M_{Edy}}{N_{Ed}}=\frac{35\cdot 10^6}{1250\cdot 10^3}=28\text{ mm};$$

$$e_z=\frac{M_{Edz}}{N_{Ed}}=\frac{25\cdot 10^6}{1250\cdot 10^3}=20\text{ mm};$$

$$(e_y/h)\div(e_z/b)=(28/275)\div(20/275)=1,4>0,2.$$

Therefore, check column for bi-axial bending.

Design ultimate moment of resistance, M_{Rd}.

$$d_2=35+8+25/2=55,5\text{ mm and } d_2/h=55,5/275=0,2;$$

$$\frac{A_s\cdot f_{yk}}{b\cdot h\cdot f_{ck}}=\frac{1960\cdot 500}{275\cdot 275\cdot 25}=0,518;$$

$$\frac{N_{Ed}}{b \cdot h \cdot f_{ck}} = \frac{1250 \cdot 10^3}{275 \cdot 275 \cdot 25} = 0,66;$$

$$\frac{M_{Rd}}{b \cdot h^2 \cdot f_{ck}} \approx 0{,}105 \text{ (see Figure 4.1-25) } \Rightarrow M_{Rd} = 54,6 \text{ kN} \cdot \text{m}.$$

Exponent, a.

$$N_{Rd} = A_c \cdot f_{cd} + A_s \cdot f_{yd} = [275^2 \cdot (0,85 \cdot 25 / 1,5) + 1960 \cdot (500 / 1,15)] \cdot 10^{-3} =$$
$$= 1071,35 + 852{,}17 = 1923,5 \text{ kN};$$

$$\frac{N_{Ed}}{N_{Rd}} = \frac{1250}{1923,5} = 0,65.$$

From linear interpolation between $a = 1$, when $\frac{N_{Ed}}{N_{Rd}} = 0,1$, and $a = 1,5$, when $\frac{N_{Ed}}{N_{Rd}} = 0,7$, the following value of $a \approx 1,46$ is obtained.

Resistance check.

$$\left(\frac{M_{Edz}}{M_{Rdz}}\right)^a + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^a \leq 1 = \left(\frac{25 \cdot 10^6}{54,6 \cdot 10^6}\right)^{1,46} + \left(\frac{35 \cdot 10^6}{54,6 \cdot 10^6}\right)^{1,46} = 0,32 + 0,52 = 0,84 < 1.$$

4.2 SHEAR

4.2.1 SHEAR CAPACITY OF RC-BEAMS

4.2.1.1 Formation of diagonal cracks due to shear

The so-called shear failure is one of the failure mode of RC structural element of which the mechanism is much different from flexural failure. In actual RC-structures, there is a combination of forces as shear forces and flexural moment, axial force, torsional moment and their failure modes are very complicated.

The shear failure follows a formation of diagonal cracks. It is brittle failure compared with flexure tension failure. Therefore, in the case of design involving the ductility of structures such as seismic design, this type of failure has to be avoided.

4.2.1.1.1 Principle tensile stress in an elastic beam

To illustrate different type of shear failure, let's consider a beam that is loaded by a four point bending test (see Figure 4.2-1). Between the support and the point load, a shear force and a moment is presented. The moment decreases linearly over the distance from the point load to the support.

In the case of simply supported beam subjected to two-point concentrate loading (see Figure 4.2-1), the moment and shear distribution is such that the moment is constant in the mid-span and in two side spans, shear force is constant. These two side spans are called «***shear span***». For the elastic beam, the flexural stress σ, shear stress ν and the principal strains and stresses are determined according to the beam theory.

Since concrete material is weak in tension, the magnitude and direction of principal tensile stresses are important.

Figure 4.2-1 c represents the distribution of principal stresses across the span of homogeneous concrete beam. The direction of the principal compressive stresses takes the form of an arch, while the tensile stresses have a curve of a catenary and suspended chain. Towards mid-span, where shear is low and the bending stresses are dominant, the direction of the stresses tends to be parallel to the beam axis. Near the supports, where the shearing forces are greater, the principal stresses become inclined.

The tensile stresses due to shear are liable to cause diagonal cracking of the concrete near to the support so that shear reinforcement must be provided.

As it was shown in Figure 4.2-1 c, at the location of zero shear stress, i.e. the extreme tension fiber, the principal tensile stress takes the near horizontal direction. At the point of zero normal stress σ, i.e., the neutral axis, the principal tensile stress is equal to shear stress, and its direction is 45° with respect to member axis (see Figure 4.2-1 b and c).

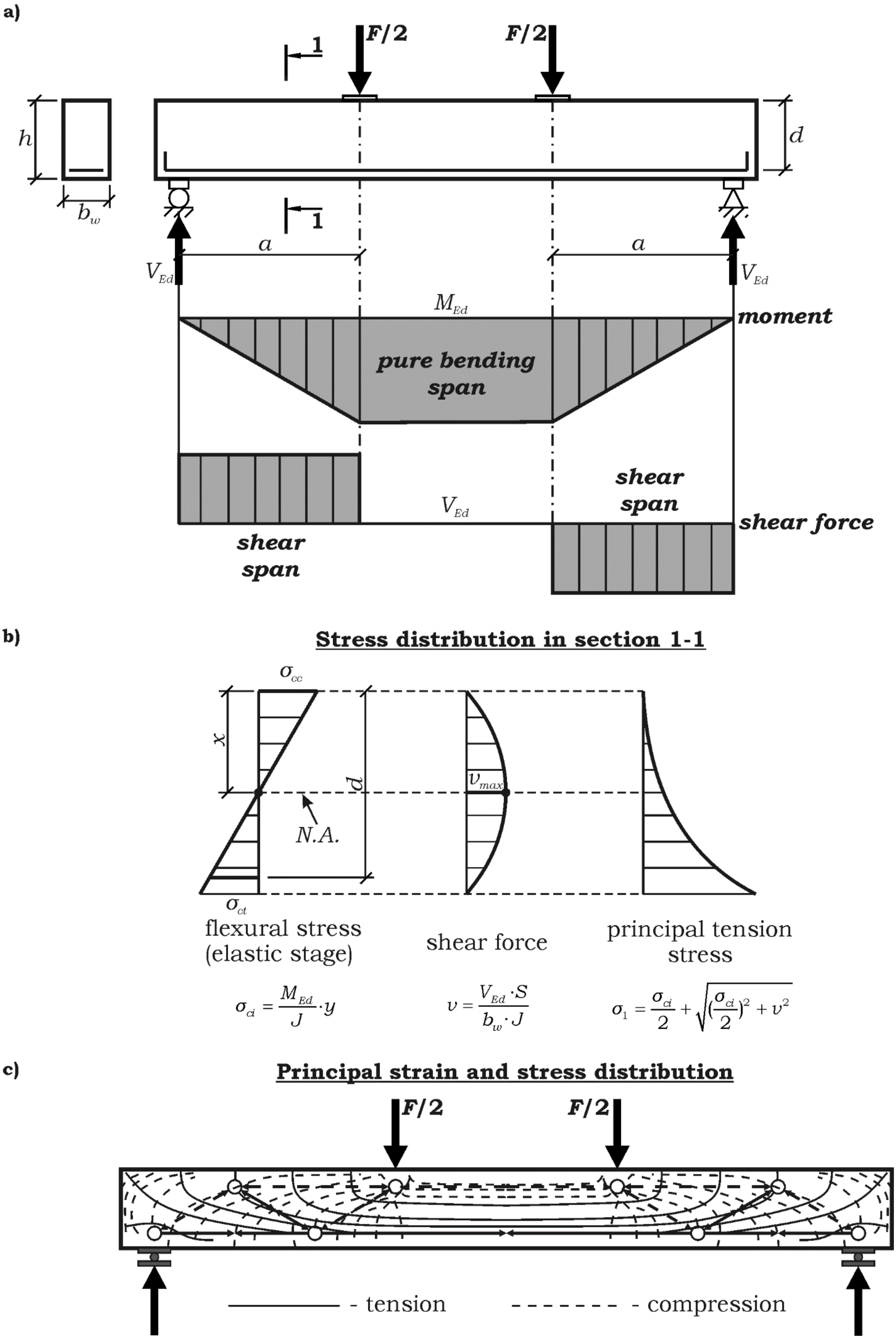


Figure 4.2-1 – Stress conditions in the elastic beam subjected to moment and shear force

4.2.1.1.2 Types of the cracks, modes of shear failure and crack patterns

As was shown earlier, before cracking, RC-beams can be considered as an elastic body. Hence, the maximum principal tensile stress occurs at the extreme tension fiber within the mid-span, and its direction is parallel to the member axis. As this principal tensile stress (strain) increases and exceeding the tensile strength of concrete, cracks occurs in the direction perpendicular to the direction of principal tension stress. These cracks are called ***«flexural cracks»*** (see Section 4.1).

After the flexural crack is formed, a RC-beam is no longer considered to be an elastic body. However, since the tensile force is carried by longitudinal reinforcement, the state of stress even after flexural cracking is still similar to that of the principal stress of an elastic beam.

When applied load is increased, the flexural crack propagates to the compression zone of the cracked section. Also in both of side spans, the formation of cracks occurs with an inclination with respect to the member axis. These cracks, in general case, are called ***«diagonal cracks»*** or ***«shear cracks»***.

The types and formation of cracks depends on the span-to-depth ratio of the beam and loading. These variables influence the moment and shear along the length of the beam.

As was shown, in general case, for simply supported beam under uniformly distributed load (in case without prestressing), three types of cracks are identified (see Figure 4.2-2):

1) ***flexural cracks:*** these cracks form at the bottom near the midspan and propagate upwards;

2) ***web-shear cracks:*** these cracks form in the web of the beam near the neutral axis (where proved to be the highest shear stress region) close to the support and propagate inclined to the beam axis;

3) ***flexural-shear cracks:*** these cracks form at the bottom due to flexure and propagate due to both flexure and shear.

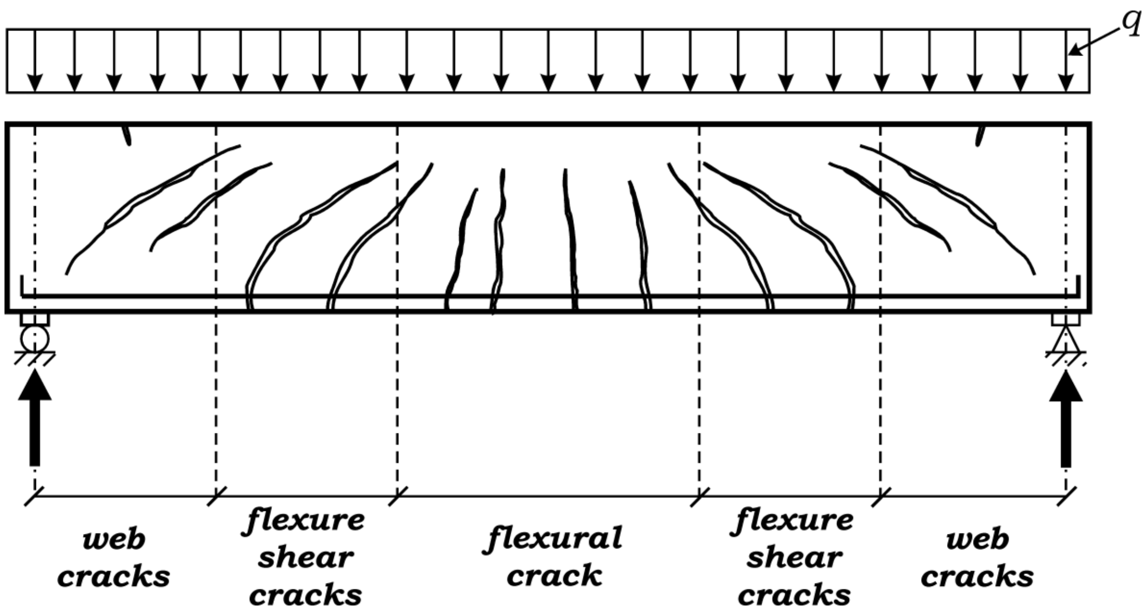


Figure 4.2-2 – Types of the cracks in reinforced beam

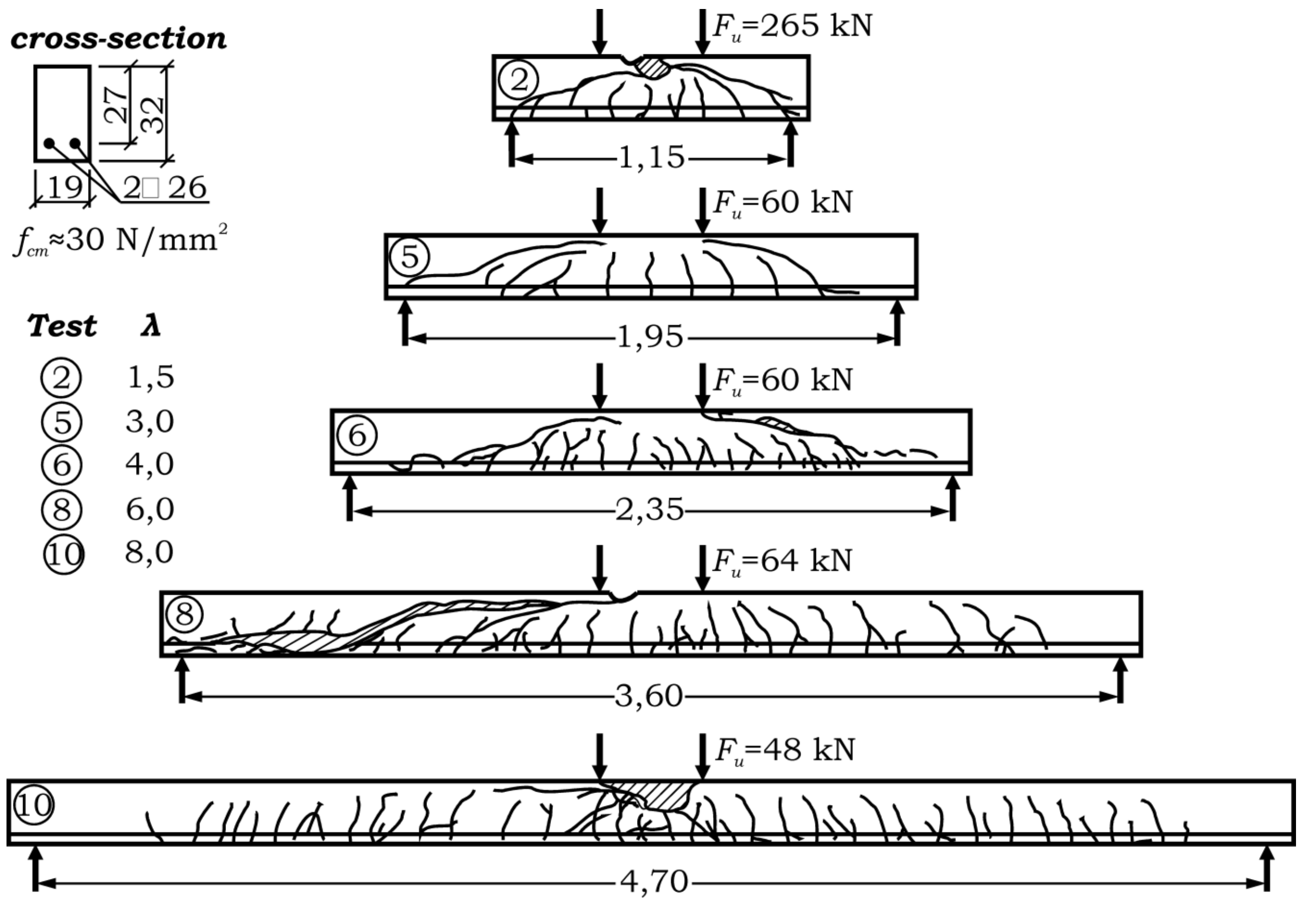


Figure 4.2-3 – Types of the cracks failure (crack patterns) depending on the span-to-depth ratio of the beam and loading [1]

As was shown in [14], for beam with low span-to-depth ratio or inadequate shear reinforcement, the failure can be due to shear.

A failure due to shear is sudden as compared to a failure due to flexure.

The occurrence of a mode of failure depends on the span-to-depth ratio, loading, cross-section of the beam, amount and anchorage of reinforcement. The modes of failure are explained next:

1) *Flexural shear failure.*

Flexural shear failure is a mechanisms in which flexure induced crack grow at an angle into the web (see Figure 4.2-4 a).

Therefore instead of cracks perpendicular to the longitudinal reinforcement, a crack rotation will occur. Increasing the load leads to larger crack and eventually causes the beam to fail do to this mechanisms.

In general flexural shear is caused by a standard load situation, with a normal ratio between the flexural and shear stresses. The use of transverse reinforcement such as stirrups will present the propagation of crack into the web.

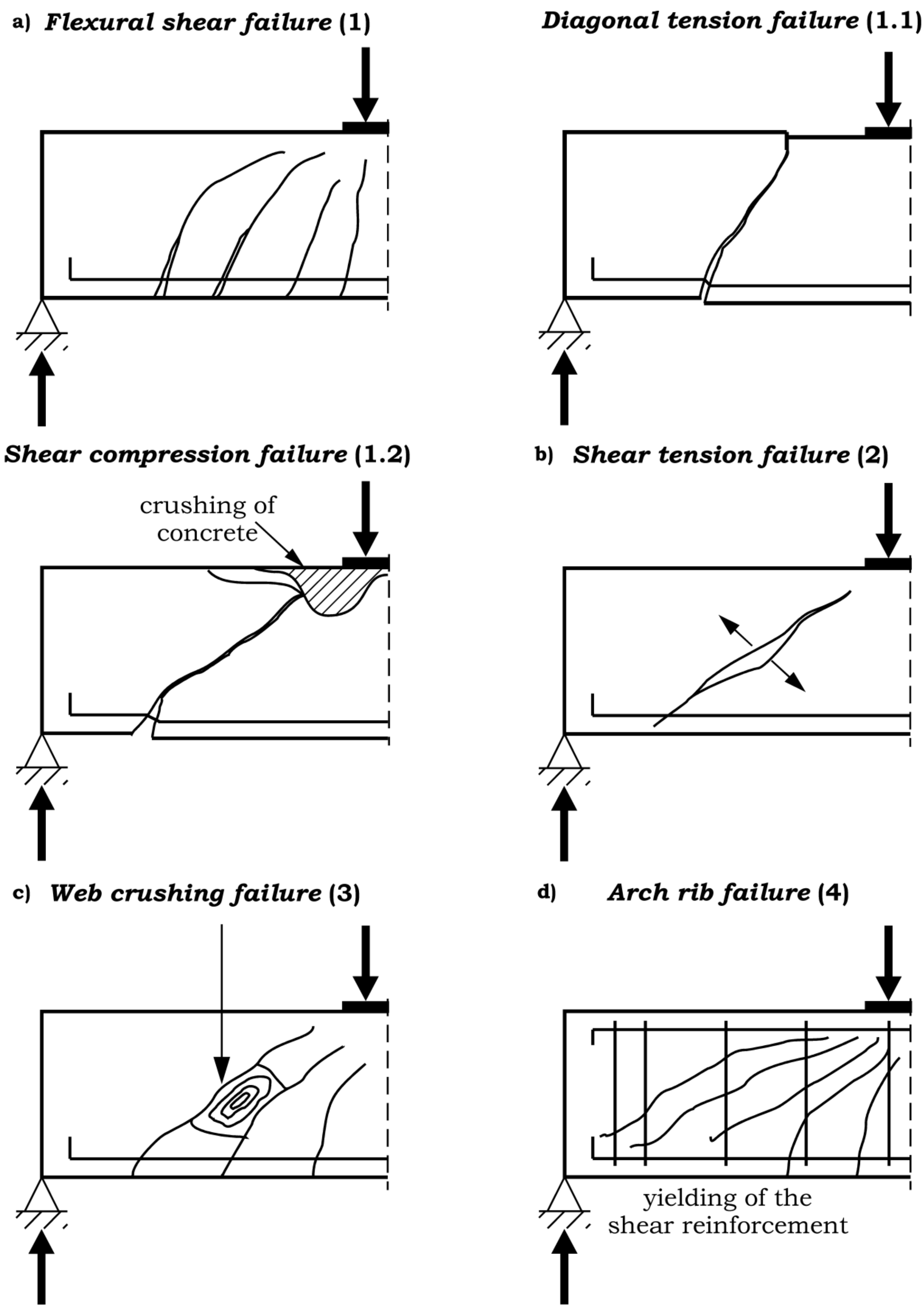


Figure 4.2-4 – Shear failure modes

1.1) *Diagonal tension failure.*

In this mode, an inclined crack propagates rapidly due to inadequate shear reinforcement (see Figure 4.2-4 a).

When diagonal crack occurs, the tensile force carried by concrete is released, and if reinforcement effective in the direction of principal tensile stress is not provided, the RC-beam fails suddenly under so-called «***diagonal tension failure***» mode.

Diagonal tension failure usually occurs in concrete members with low amount of stirrups and longitudinal reinforcement. For concrete members with low amount of web reinforcement but adequate longitudinal reinforcement ratio to form a compression zone, shear cracks may easily initiate from former flexural cracks but do not pass through the compression zone.

Yielding of the shear reinforcement is a mechanisms that is to be expected if the structural design of the beam is correct. In this case the stirrups will yield before failure occurs. The failure will be accompanied by a considerable deformations, implying that the failure mechanisms provides warning before reaching the moment of failure.

This type of failure has a smeared crack pattern, with a small number of dominating cracks. There are no typical load situation in which only this type of failure occurs. This type of mechanisms is dependent on the type of structure and how it has been designed.

1.2) *Shear compression failure.*

There is crushing of the concrete near the compression flange above the tip of the inclined crack (see Figure 4.2-4 a).

A RC-beam can resist increasing loads after the diagonal crack formation. The stress state becomes like a compression arch formed by diagonal cracks. In this case, the beam fails when this arch crushes under diagonal compression. This type of failure mode depends largely on the shear-effective depth ratio (a/d).

2) *Shear tension failure.*

Shear tension failure is a mechanisms in which a diagonal crack occurs due to tension component of the principal stress. When we consider the strut and tie analogy, this failure can be considered as failure of the compression diagonal due to a biaxial tension-compression state.

The main crack occurs at a varying angle of approximately 30° to 45° to the longitudinal axis, and is characterized by a sudden (brittle) development into the web of the beam (Figure 4.2-4 b). This means that this type of shear fracture can be considered to be a failure type without warning. In general this type of failure occurs for beam structures that have a high level of shear stress and a relative low level of flexural stress. For example, beam regions near supports are sensitive to this type of fracture.

In some publication term «***shear tension failure***» is defined as a failure due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars (see Figure 4.2-4 b).

3) *Web crushing failure.*

The concrete in the web crushes due to inadequate web thickness (see Figure 4.2-4 c).

Web crushing occur when a structure has a high shear reinforcement ratio and a small amount of web surface. This implies that if a beam is reinforced with a large amount of stirrups, the compression diagonal may fail due to crushing of the concrete, before the stirrups have the possibility of reaching their yield strength. In principle this mechanisms is the equivalent of failure of the compression zone in heavily reinforced structure loaded in flexure. There are no typical loading situations where only this type of failure occurs. However, this failure does tend to occurs in T-beam with large flanges and small web dimensions.

4) *Arch rib failure:* for deep beams, the web may buckle and subsequently crush (see Figure 4.2-4 d).

For case of so-called deep beams, i.e., where the shear span-effective depth ratio is very small $a / d \leq 1{,}0$, the tied-arch shear resisting mechanism is formed as a compression strut joining the loading and support points, and this failure mode, sometimes is called ***«deep-beam failure»***.

It can be summarized, that the concrete itself can resist shear by a combination of the un-cracked concrete in the compression zone, the dowelling action of the bending reinforcement and aggregate interlock across tension crack but, because concrete is weak in tension, the shear reinforcement is designed to resist all tensile stresses caused by the shear forces.

Even where the shear forces are small near the centre of span of a beam a minimum amount of shear reinforcement in the form of links must be provided in order to form a cage supporting the longitudinal reinforcement and to resist any tensile stresses due to factors such as thermal movement and shrinkage of the concrete.

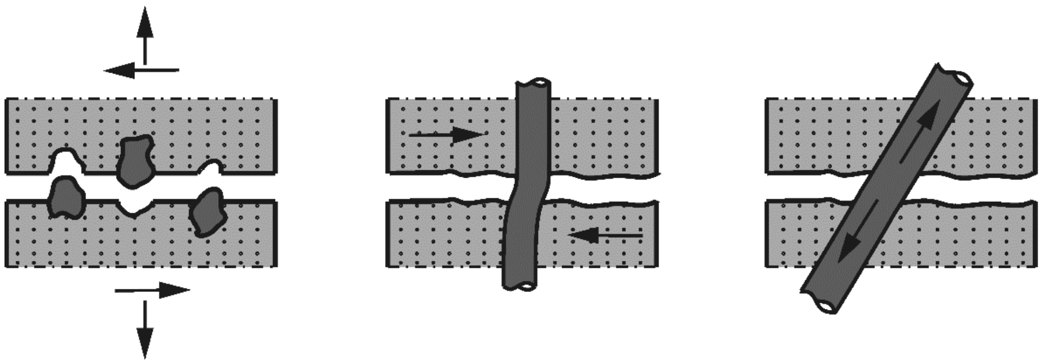
The actual behaviour of reinforced concrete in shear is complex, and difficult to analyze theoretically, but by applying the results from many experimental investigations, reasonable simplified procedures for analyses and design have been developed.

The objective of design for shear is to avoid shear failure. The beam should fail in flexure at its ultimate flexural strength. Hence, each mode of failure is addressed in the design for shear. The design involves not only the design of the stirrups, but also limiting the average shear stress in concrete, providing adequate thickness of the web and adequate development length of the longitudinal bars.

4.2.1.2 Shear transfer in cracked concrete

The transfer of shear force in cracked reinforced concrete is characterized by number complex phenomena, consisting of: 1) aggregate interlock; 2) axial steel stress; 3) residual tensile stresses across the crack. These mechanisms are strongly dependent on the state of stress, the opening of the crack, and the restraint conditions.

The shear transfer capacity is also strongly dependent on the interaction between the mentioned transfer mechanisms. When shear stresses arise across a crack surface, a displacement (slip) tangential to the crack face occurs and the crack surfaces tend to separate. The reinforcing bars provide resistance against the separation of this crack face via the dowel mechanism and the axial steel stress. These mechanisms cause a strain in the steel and a decreased bond action, this than permits a crack to increase in width. The amount of reinforcement is therefore of large influence in the containment of the crack face. The shear transfer mechanisms described are depicted in Figure 4.2-5 and are elaborated on in the following section.



a) b) c)

a) – aggregate interlock; b) – dowel action; c) – axial steel stress

Figure 4.2-5 – Shear transfer mechanisms in cracked concrete

Aggregate interlock. In normal strength concrete, the strength of the aggregate material will exceed the strength of the cement matrix material. Therefore cracking in concrete will commonly occur through the matrix and the bond zone between the matrix and the aggregate, as is depicted in Figure 4.2-5 a. Because the protruding aggregate particles on the crack face are lager than the crack width, the crack plane is considered to be rough. Therefore the crack plane provides resistance against slip, and is capable of transmitting shear force.

This principle is called aggregate interlock. The magnitude of the aggregate interlock mechanism is dependent of the width of the crack. A lager crack width means a reduction of aggregate interlock because of the decrease of contact area between the aggregate particles. Another parameter that influences the aggregate interlock mechanism is the aggregate size itself; smaller particles will provide a smoother crack plane and therefore less friction. When considering high strength concrete, the aggregate interlock contribution becomes even less, due to smoother crack faces. This is caused by the fact that the crack will not only propagate through the cement matrix, but also through the aggregate particles.

Dowel action. A dowel is a reinforcement bar that is loaded by transverse force. The mechanism of dowel action is based upon the behavior of the bar and surrounding concrete. The dowel action consist of two components, namely: bending action and shear action of the reinforcing bar. The contribution of dowel action to the shear resistance is a function of the amount of concrete cover of the longitudinal bars and the degree to which the vertical displacements of those bars at the inclined crack are restrained by transverse reinforcement (see Figure 4.2-5 b).

Axial steel stress. Reinforcing bars generally cross cracks at different angles, this is particularly the case for transverse shear reinforcement. The component of the steel stress normal to the crack plane provides a contribution to the transfer of stresses across a crack. The magnitude of this force is strongly dependent on the amount of reinforcement and the bond properties. In members with shear reinforcement a large portion of the shear is carried by the shear reinforcement after diagonal cracking has occurs. Next to the contribution to the shear capacity, shear reinforcement also provides a level of restraint against the growth of inclined cracks and thus helps to ensure a more ductile behaviour (see Figure 4.2-5 c).

Residual stresses. When cracks are formed in concrete, the concrete still has the ability to transfer tensile stresses across the crack face.

These so-called residual stresses are present until the crack width becomes too large. This behaviour is described by the strain softening diagram as was discussed in Chapter 3.

Shear stress in the compression zone. Shear stresses that are present in the compression zone of the concrete, contribute to the shear resistance in a concrete member. The magnitude of that shear resistance is limited by the depth of the compression zone. Therefore, in relative slender beams without axial compression, the shear contribution becomes relatively small, due to the minimal height of the compression zone.

4.2.1.3 Internal forces in a beam without stirrups

The forces transferring shear across an inclined crack in a beam without stirrups are shown in Figure 4.2-6.

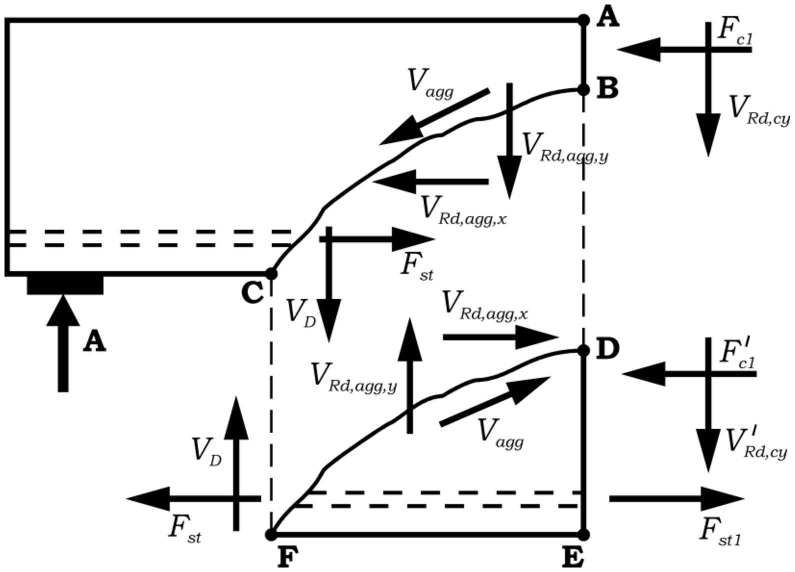


Figure 4.2-6 – Internal forces in a cracked beam without stirrups (see Figure 6-13 from [15])

As was shown in [15] shear is transferred across line *A-B-C* by $V_{Rd,cy}$, the shear in the compression zone, by $V_{Rd,agg,y}$, the vertical component of the shear transferred across the crack by interlock of the aggregate particles on the two faces of the crack, and by V_d, the dowel action of the longitudinal reinforcement. Immediately after inclined cracking, as much 40 to 60 % of the total shear carried by V_d and $V_{Rd,cy}$ together.

As the crack widens, V_{agg} decreases, increasing the fraction of shear resisted by $V_{Rd,cy}$ and V_d. The dowel shear, V_d, leads to splitting crack in the concrete along the reinforcement.

When this crack occurs, V_d drops, approaching zero. When V_{agg} and V_d disappear, so $V_{Rd,cy}$ and $V_{c1}^{/}$, with the result that all the shear and compression are transmitted in the depth *AB* above the crack [15]. At this point in the loading, the section *A-B* is too shallow to resist the compression forces needed to equilibrium. As a result, this region crushes or buckles upward.

The shear failure of slender beam without stirrups is sudden and dramatic.

4.2.1.4 Behaviour of beams with web reinforcement by [15]

The forces in the beam with stirrups and an inclined crack are shown in Figure 4.2-7. The loading history of such beam is shown qualitatively in Figure 4.2-8.

Inclined cracking causes the shear strength of the beams to drop below the flexural capacity, as shown in Figure 4.2-8. The purpose of web reinforcement is to ensure that the full flexural capacity can be developed.

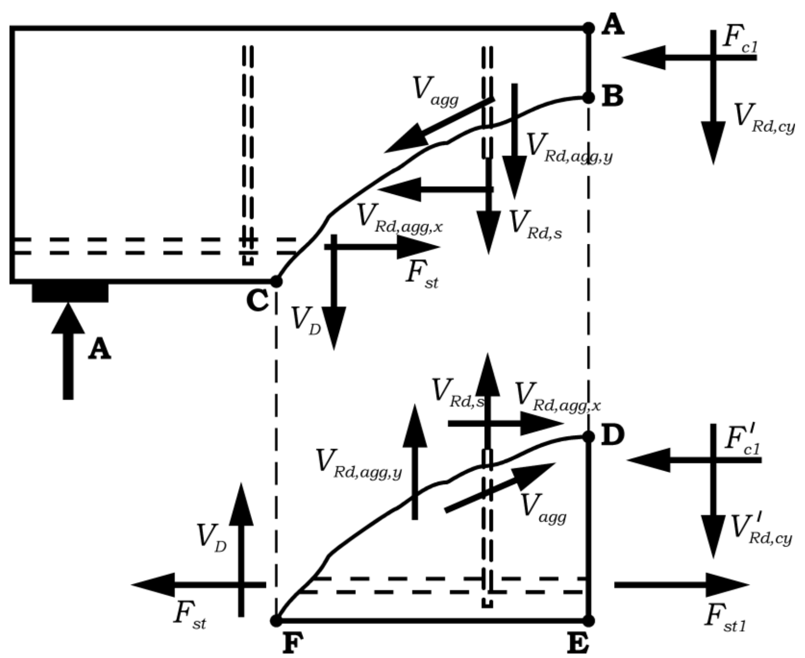


Figure 4.2-7 – Internal forces in a cracked beam with stirrups (see Figure 6-17 from [15])

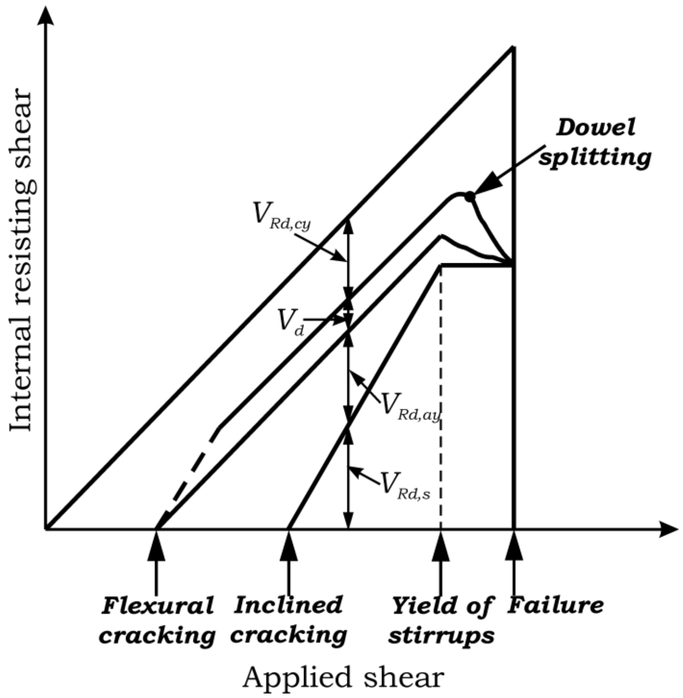


Figure 4.2-8 – Distribution of internal shears in a beam with web reinforcement (see Figure 6-18 [15])

Prior to inclined cracking, the strain in the stirrups is equal to the corresponding strain of the concrete. Because concrete cracks at very small strain, the stress in the stirrups prior to inclined cracking will not exceed 20 to 30 MPa. Thus, stirrups do not prevent inclined crack from forming; they come into play after the cracks have formed. The shear transferred by tension in the stirrups, $V_{Rd,s}$, does not disappear when crack opens wider, so there will always be a compression force $F_{c1}^{/}$ and $V_{Rd,cy}^{/}$ acting on the part of the beam below the crack. As a result, F_{st2} will be less than F_{st1}, the difference depending on the amount of the reinforcement. The force F_{st2} will, however, be larger than flexural tension $F_{st} = M_{Ed} / z$ based on the moment at C. Components of the internal shear resistance must equal to the applied shear, indicated by the upper 45° line. Prior to flexural cracking, all the shear is carried by the uncracked concrete. Between flexural and inclined cracking, the external shear is resisted by $V_{Rd,cy}$, $V_{agg,y}$ and V_d. Eventually, the stirrups crossing the crack yield, and $V_{Rd,s}$ stay constant for higher applied shears. Once the stirrups yield, the inclined crack opens more rapidly. As the inclined crack widens, $V_{agg,y}$ decreases further, forcing V_d and $V_{Rd,cy}$ to increase at an accelerated rate, until either a splitting (dowel) failure occurs, compression zone crushes due to combined shear and compression, or web crushes. Each component of this process except $V_{Rd,s}$ has a brittle load-deflection response. As a result, it is difficult to quantify the contributions of $V_{Rd,cy}$, V_d and $V_{agg,y}$. In design, they are lumped together as $V_{Rd,c}$, referred to somewhat incorrectly as *shear carried by the concrete*. Thus, shear resistance V_{Rd} is assumed to be:

$$V_R = V_{Rd,c} + V_{Rd,s} \tag{4.2-1}$$

4.2.2 TRUSS ANALOGY BY RITTER AND MÖRSH AND ITS MODIFICATION

Science the early days of reinforced concrete the so-called classical truss analogy developed by Ritter and Mörsh was proposed for shear design of reinforced concrete members [11, 12].

In 1899 and 1902, respectively, the Swiss engineer Ritter and German engineer Mörsh independently published papers proposing the truss analogy (see Figure 4.2-9) for the design of reinforced beams for shear.

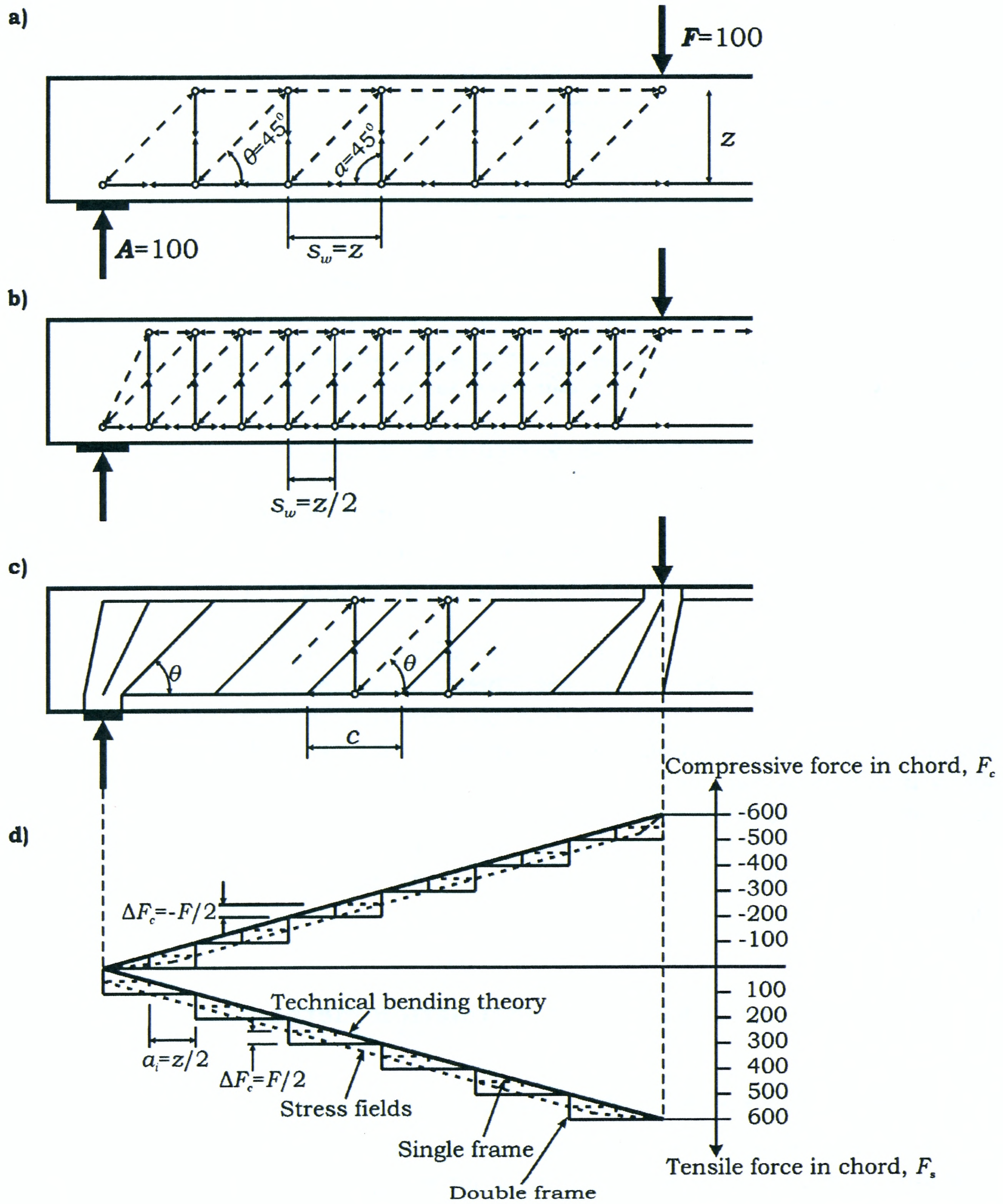
Mörsh analyzed the angle of inclination on two simply supported T-beams subjected to increasing uniformly distributed load. He concluded that the shear cracking angle, θ, was variable and not mathematically defined. Consequently, Mörsh derived an equation for the required amount of shear reinforcement assuming $\theta = 45^{\circ}$.

This model consists of a tension ties (longitudinal reinforcement bars), a top compressive chord, vertical or inclined tension ties between 45° and 90° (stirrups), and 45° inclined concrete compression struts.

The truss is referred as the plastic truss model depending on plasticity in the nodes leading to a system, which is statically determinate. In this model the shear resisting force is given only by the shear reinforcement. The classical truss analogy is based on a truss model with parallel chords and web members connected by means of pin joints, where the concrete compressive struts are inclined at 45° with respect to the longitudinal axis of the beam while the shear reinforcement represents the tensile web members (see Figure 4.2-9).

As was pointed in [14], according to Zilch and Zehetmaier [1], when the shear reinforcement (stirrups) is placed closely to each other the simple truss becomes a statically indeterminate truss (see Figure 4.2-9). Generally, the truss model may be considered as a statically determined simple truss composed of resultant forces from parallel tension and compression stress fields with pinned joints (see Figure 4.2-9).

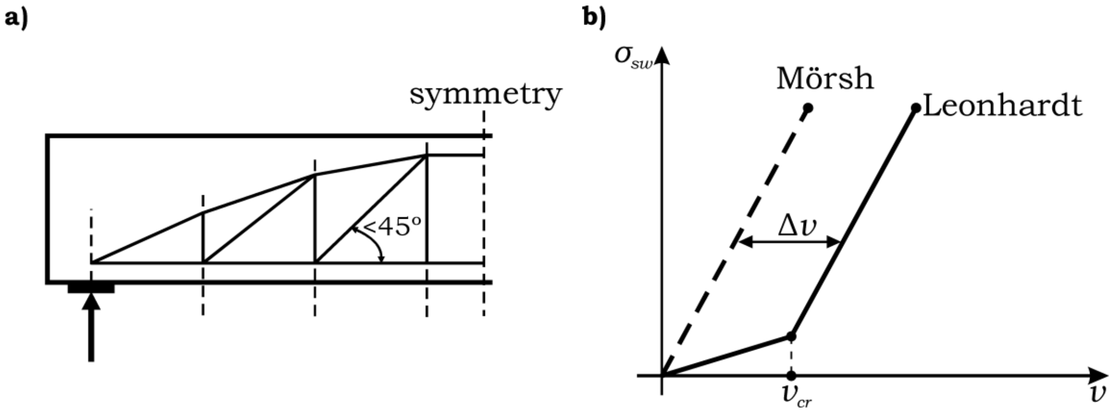
In 1960's Leonhardt and Walter [10] did numerous beam tests called the «Stuttgarter Schubversuche» at the University of Stuttgart, Germany. Based on these tests series, they came to conclusion that Mörsh's truss analogy was too conservative and had to be modified. It was stated, the stresses in shear reinforcement were considerably lower than those predicted by the truss analogy model. This is due to the contribution of other components to the shear carrying mechanism, among which the most significant are: contribution of concrete in the compression zone, aggregate interlock along inclined cracks and dowel action of the longitudinal reinforcement crossing the crack. Authors concluded that the actual top compressive chord should be inclined and that the angle θ between the compression strut and the x-axis of the member is often less than 45° and greatly depends on the shape of the member cross-section.



a) – double frame; b) – single frame; c) – stress field mode; d) – chord forces

Figure 4.2-9 – Mörsh's truss analogy model [11]

Consequently, the forces in the tension ties (shear reinforcement) are reduced but the forces in the tension chord (longitudinal reinforcement) are increased (see Figure 4.2-10).



a) – modified truss action; b) – relation « $\sigma_{sw} - \upsilon$ »

Figure 4.2-10 – Modified truss action simply supported reinforced beam (a) and more realistic curve on the shear stress versus stress in the stirrups by Leonhardt (b) [10]

Reinforced concrete beams subjected to shear have been traditionally designed using one of the two following methods:

1) truss model with 45° compressive strut inclination angle and directly included the so-called concrete contribution (correct term) ΔV_c (see Figure 4.2-10 b). This method is the so-called standard method according to [11] and also referred as the extended Mörsh analogy.

2) truss model with variable compressive strut inclination angle lower than 45°. This method, based on plasticity theory, is accepted in the current European Codes [N3]. The truss model with variable compressive strut inclination angle is a logical extension of the application of the ***strut and tie models***, which have already been included in various Codes [N3, N5], on standard structural members. This method fits in the consistent approach of structural analysis, design and detailing of reinforced concrete structures and members, such as beams, columns, plates, deep beams, corbels, beam-column joints etc., subjected to bending, shear and axial forces, torsion and punching shear.

4.2.2.1 The variable strut inclination method

4.2.2.1.1 Basic equations

Consider a reinforced concrete beam with stirrups uniformly spaced at distance s, subjected to a design shear force V_{Ed}. The resulting strut angle is θ. Assuming the height of the analogous truss is z (lever arm), the number of links per strut is equal $\frac{z \cdot \cot\theta}{s}$. The truss model of the reinforced concrete beam is shown in Figure 4.2-11.

The notations in Figure 4.2-11 are as follows:

$V_{Ed}, N_{Ed}, M_{Ed,s}$ are the design values of the cross-sectional forces: shear force, axial force and bending moment with respect to the centroid of the tensile reinforcement, respectively;

F_{swd} and F_{cwd} are the tensile and compressive web member forces respectively;

F_{sd} and F_{cd} are the tensile and compressive chord member forces respectively;

θ is the angle of inclination of the concrete compressive strut with respect to the longitudinal beam axis;

α is the angle of inclination of the tensile web member (shear reinforcement) with respect to the longitudinal beam axis.

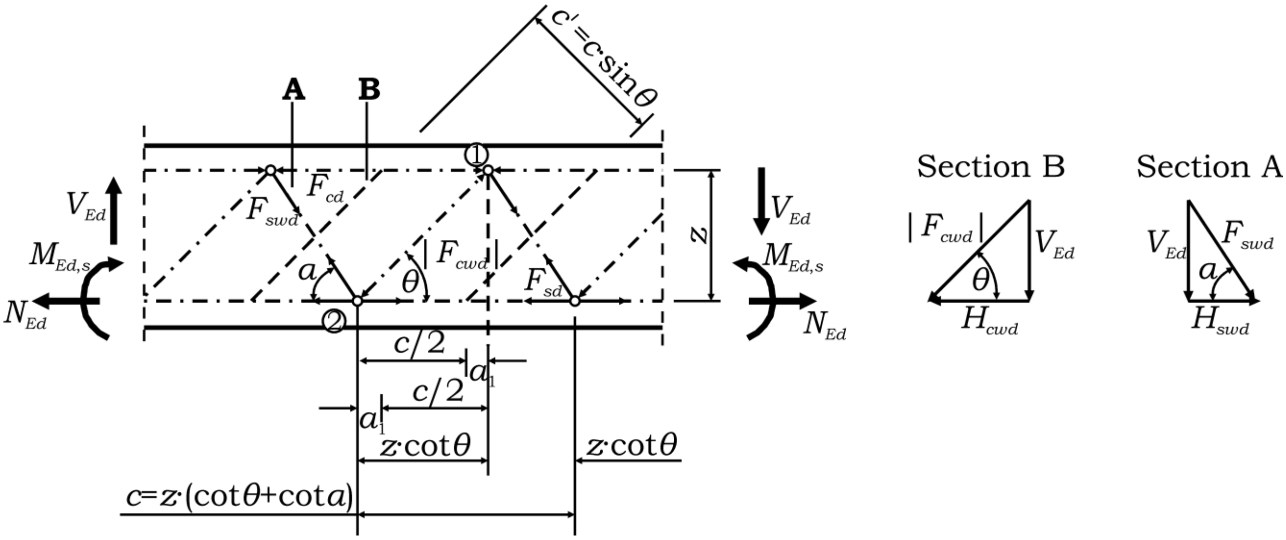


Figure 4.2-11 – Forces of the truss model [1]

Compressive stress in concrete σ_{cwd} as well as the force F_{cwd} in the compressive strut in Figure 4.2-11 should be taken with a negative sign ($\sigma_{cwd} < 0$ and $F_{cwd} < 0$). Actually, stress in shear reinforcement A_{sw} is defined as the web member forces F_{cwd} and F_{swd} distributed per area with width b_w and length c' and c, respectively:

$$c = z\left(cot\theta + cot\alpha\right), \tag{4.2-2}$$

$$c' = c \cdot \sin\theta = z \cdot \sin\theta\left(\cot\theta + \cot\alpha\right). \tag{4.2-3}$$

Assuming that compressive struts represent the resultant of the inclined stress field, we get:

$$\left|\sigma_{cwd}\right| = \frac{\left|F_{cwd}\right|}{b_w \cdot c'} = \frac{V_{Ed}}{b_w \cdot z \cdot \sin^2\theta\left(\cot\theta + \cot\alpha\right)} = \frac{V_{Ed}\left(1 + \cot^2\theta\right)}{b_w \cdot z\left(\cot\theta + \cot\alpha\right)}, \tag{4.2-4}$$

and:

$$\sigma_{swd} = \frac{F_{swd}}{a_{sw} \cdot c} = \frac{V_{Ed}}{a_{sw} \cdot z \cdot \sin\alpha\left(\cot\theta + \cot\alpha\right)}, \tag{4.2-5}$$

where: b_w is the minimum beam web width;

z is the lever arm (distance between the centroid of compressive and tensile chord);

$a_{sw} = \frac{A_{sw}}{s}$ is the cross-sectional area of the shear reinforcement with longitudinal spacing s.

From Figure 4.2-11 it can be seen that a concrete struts are b_w width and $z \cdot \sin\theta \cdot (\cot\theta + \cot a)$ deep. Hence, the compressive capacity of each strut, F_{cwd}, is given by:

$$|F_{cwd}| = |\sigma_{cwd}| \cdot z \cdot \sin\theta \left(\cot\theta + \cot a\right). \tag{4.2-6}$$

Shear failure of the strut will not be occur provided F_{cwd} equal or exceeds the design shear force acting on the strut equal to $\frac{V_{Ed}}{\sin\theta}$, i.e.:

$$\frac{V_{Ed}}{\sin\theta} \leq |F_{cwd}| = |\sigma_{cwd}| \cdot z \cdot \sin\theta \cdot (\cot\theta + \cot a). \tag{4.2-7}$$

Replacing σ_{cwd} in Equation (4.2-7) with effective compressive strength of concrete cracked in shear $f_{cwd} = v_1 \cdot f_{cd}$, along with $V_{Ed} = V_{Rd,max}$, we obtain the maximum design shear resistance of the member controlled (limited) by crushing of compression struts, $V_{Rd,max}$:

$$V_{Rd,max} = |F_{cwd}| \cdot \sin\theta = b_w \cdot z \cdot f_{cwd} \cdot \frac{\left(\cot\theta + \cot a\right)}{\left(1 + \cot^2\theta\right)}. \tag{4.2-8}$$

For case $a = 90^\circ$, $\cot a = 0$:

$$V_{Rd,max} = \frac{b_w \cdot z \cdot f_{cwd} \cdot \cot\theta}{\left(1 + \cot^2\theta\right)}. \tag{4.2-9}$$

Using the identity $\cos^2\theta + \sin^2\theta = 1$ gives:

$$V_{Rd,max} = \frac{b_w \cdot z \cdot f_{cwd} \cdot \cos\theta \cdot \sin\theta}{\left(\cos^2\theta + \sin^2\theta\right)}. \tag{4.2-10}$$

Dividing top and bottom by $\cos\theta \cdot \sin\theta$ and simplifying gives:

$$V_{Rd,max} = b_w \cdot z \cdot f_{cwd} \cdot \frac{\cos\theta \cdot \sin\theta \Big/ \cos\theta \cdot \sin\theta}{\left(\cos^2\theta + \sin^2\theta\right) \Big/ \cos\theta \cdot \sin\theta}, \tag{4.2-11}$$

and:

$$V_{Rd,max} = \upsilon_1 \cdot f_{cd} \cdot b_w \cdot z \cdot \frac{1}{(\cot\theta + \cot a)}, \qquad (4.2\text{-}12)$$

where (in accordance with clause 6.2.3 (2) from EN 1992 [N3]):

$z = 0{,}9 \cdot d$;

$f_{cd} = \frac{a_{cc} \cdot f_{ck}}{\gamma_c} = \frac{0{,}8 \cdot f_{ck}}{1{,}5}$ for $f_{ck} \leq 50\ \frac{\mathrm{N}}{\mathrm{mm}^2}$;

Note: $a_{cc} = 1{,}0$ may be used here;

$\upsilon_1 = 0{,}6\left(1 - {f_{ck}}/{250}\right)$ for $f_{ck} \leq 50\ \frac{\mathrm{N}}{\mathrm{mm}^2}$;

θ is the angle between concrete strut and axis beam (see Figure 4.2-11).

Using the identity: $\frac{1}{(\cot\theta + \cot a)} = \cos\theta \cdot \sin\theta = 0{,}5 \cdot \sin 2\theta$, Expression (4.2-12) can be transposed to make the concrete strut angle the subject of the formula as follows:

$$\theta = 0{,}5 \cdot \sin^{-1}\left[\frac{V_{Rd,max} / b_w \cdot d}{0{,}153 \cdot f_{ck} \cdot \left(1 - {f_{ck}}/{250}\right)}\right], \qquad (4.2\text{-}13)$$

Expression (4.2-13) can be used to calculation the minimum strut angle for given value of applied shear by equating $V_{Rd,max} = V_{Ed}$, subjected to the condition that $\cot\theta$ lies between 1,0 and 2,5.

Replacing σ_{swd} in Equation (4.2-5) with design yield strength of shear reinforcement f_{ywd} and $V_{Ed} = V_{Rd,s}$, we obtain design shear resistance on member, limited by yielding of tensile reinforcement $V_{Rd,s}$:

$$V_{Rd,s} = F_{swd} \cdot \sin a = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot (\cot\theta + \cot a) \cdot \sin a, \qquad (4.2\text{-}14)$$

It will be noted from Equation (4.2-14) that the smaller angle θ is, the greater is the shear capacity based on shear reinforcement. However, the shear capacity based on the crushing strength of the struts, given by Equation (4.2-12), decreases with decreasing values of θ below 45°. Hence, the maximum capacity corresponds to the situation where the capacity based on the shear reinforcement just equals the capacity based on the strength of the strut. This implies that the actual conditions at failure may be established by using Equation (4.2-13) to estimate the value of θ for which $V_{Ed} = V_{Rd,max}$, and then using this value of θ to obtain the required amount of shear reinforcement. It should be noted that $V_{Rd,max}$ reaches a maximum value when $\cot\theta = 1{,}0$, and hence values of θ greater than 45° will not only occur if other factors constrain the failure to occur at such an angle.

The result of combining the strength defined by the crushing strength of the strut and the limitations applied to the strut angle is illustrated in Figure 4.2-12.

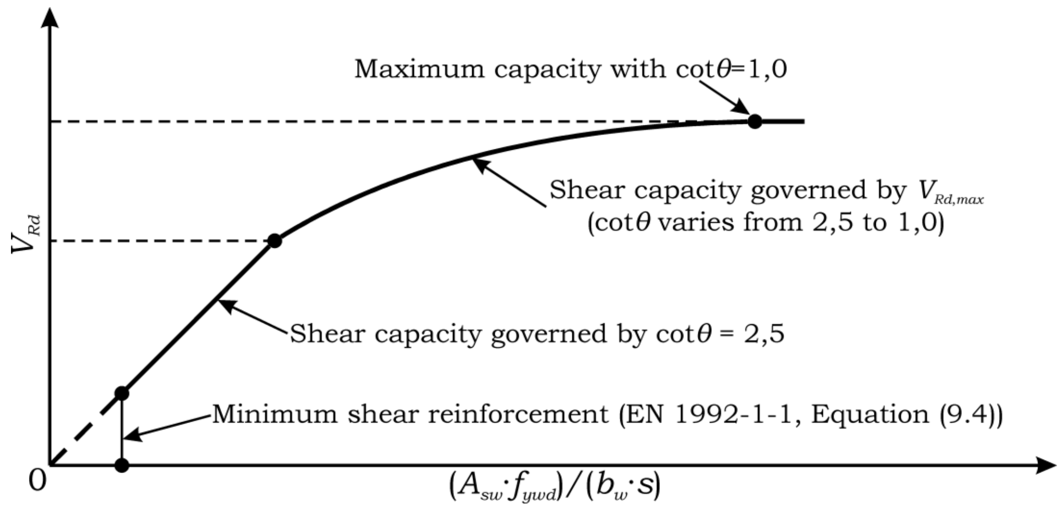


Figure 4.2-12 – Relationship between the design shear and amount of shear reinforcement (see Figure 6.4 from [8])

Term $V_{Rd,max}$ also denotes the maximum shear capacity which a member can sustain for a chosen angle θ, provided that the shear force is carried by an appropriate shear reinforcement, i.e. $V_{Rd,max} = V_{Rd,s}$ and that an additional longitudinal tensile reinforcement to resist ΔF_{st} is provided.

From Figure 4.2-11 it can be seen that for equilibrium a tensile force $\frac{V_{Ed}}{\tan\theta}$ also acts on the section. Assuming that half of this force acts in the bottom chord, and additional tensile force present in the longitudinal reinforcement, ΔF_{sd}, is given by (see Figure 4.2.11) the following expressions:

$$H_{cwd} = |F_{cwd}| \cdot \cos\theta = V_{Ed} \cdot \cot\theta; \tag{4.2-15}$$

$$H_{swd} = F_{swd} \cdot \cos a = V_{Ed} \cdot \cot a; \tag{4.2-16}$$

$$H_{wd} = H_{cwd} - H_{swd} = V_{Ed} \cdot (\cot\theta - \cot a); \tag{4.2-17}$$

$$\Delta F_{sd} = \frac{H_{wd}}{2} = \frac{V_{Ed}}{2}(\cot\theta - \cot a). \tag{4.2-18}$$

The presence of this additional longitudinal force is responsible for the shift rule, necessitating that longitudinal tension steel extend further in the span than required for bending alone. Also since the tensile force in the longitudinal steel due to bending is $\frac{M_{Ed,s}}{z}$, the total force in the tensile reinforcement, F_{sd}, in general case is given by:

$$F_{sd} = \left(\frac{M_{Ed,s}}{z} + N_{Ed} \right) + \frac{V_{Ed}}{2} \cdot (\cot\theta - \cot a), \tag{4.2-19}$$

and force F_{cd} in compressed concrete:

$$F_{cd} = -\frac{M_{Ed,s}}{z} + \frac{V_{Ed}}{2} \cdot (\cot\theta - \cot a). \tag{4.2-20}$$

From the analysis of the truss model in Figure 4.2-11 it can be noticed that there are three equilibrium equations (1, 2, 3), but four unknowns: concrete stress σ_{cwd}, additional tensile force in reinforcement ΔF_{sd}, stress in the shear reinforcement σ_{swd} and inclination angle of the compressive strut θ.

4.2.2.1.2 General design requirements in accordance with EN 1992 [N3]

In accordance with EN 1992 [N3] (clause 6.2.1 (1)P) for verification of the shear resistance the following symbols are defined:

$V_{Rd,c}$ is the design shear resistance of the member without shear reinforcement;

$V_{Rd,s}$ is the design value of the shear force which can be sustained by the yielding shear reinforcement;

$V_{Rd,max}$ is the design value of the maximum shear force which can be sustained by the member, limited by crushing of the compressive struts.

Based on the code EN 1992 [N3] the following requirement can be formulated:

1) In region of the member where $V_{Ed} \leq V_{Rd,c}$ no calculated shear reinforcement is necessary (where: V_{Ed} is the design shear force is section considered);

2) When, on the basis of the design shear calculation, no shear reinforcement is required, minimum shear reinforcement should nevertheless be provided according to EN 1992 [N3]. The minimum shear reinforcement ratio, ρ_{sw}, is obtained from the following expression:

$$\rho_{sw} = \frac{A_{sw}}{b_w \cdot s \cdot sin\alpha} \geq \rho_{sw,min}, \tag{4.2-21}$$

where: A_{sw} is the area of shear reinforcement within length;

s is the spacing of the shear reinforcement. EN 1992 [N3] recommends that the maximum longitudinal spacing of the shear reinforcement (s_{max}) should be not exceed $0,75 \cdot d \cdot (1 + \cot a)$. For vertical stirrups $\sin a = 1,0$ $(a = 90°)$.

$\rho_{sw,min} = \dfrac{0,08 \cdot \sqrt{f_{ck}}}{f_{yd}}$ is the minimum area of shear reinforcement.

The minimum shear reinforcement may be omitted in members such as slabs (solids, ribbed or hollow core slabs) where transverse redistribution of loads is possible. Minimum shear reinforcement may also be omitted in members of minor importance (e.g. lintels with span ≤2 m) which do not contribute significantly to the overall resistance and stability of the structure.

3) In region where $V_{Ed} > V_{Rd,c}$, sufficient shear reinforcement should be provided in order that $V_{Ed} \le V_{Rd}$;

4) The longitudinal tension reinforcement should be able to resist the additional tensile force ΔF_{sd} caused by shear;

5) For members subjected to predominantly uniformly distributed loading the design shear force need not to be checked at distance less than *d* from face of the support. Any shear reinforcement required should continue to the support. In addition, it should be verified that the shear at the support does not exceed $V_{Rd,max}$ in accordance with Equation (4.2-12).

(1) Members not required design shear reinforcement

The concrete sections that do not require shear reinforcement are mainly lightly loaded floor slabs and pad foundations. Beams are generally more heavily loaded and have a smaller cross-section so that they nearly always require shear reinforcement. Even lightly loaded beams are required to have a minimum amount of shear links. The only exceptions to this are very minor beams such as short span, lightly loaded lintels over windows and doors.

Where shear forces are small the concrete section on its own may have sufficient shear capacity ($V_{Rd,c}$) to resist the ultimate shear force (V_{Ed}) resulting from the worth combination of actions on the structure, although in most cases a nominal or minimum amount of shear reinforcement will usually be provided.

In those sections where $V_{Ed} \le V_{Rd,c}$ then no calculated shear reinforcement is required.

Members without shear reinforcement are represented by two mechanical models which are arch and/or «sprengwerk» with a tension tie (see Figure 4.2-13). The shear mechanism are the shear strength of the arch or strut provided by the concrete and dowel reaction of the longitudinal bars.

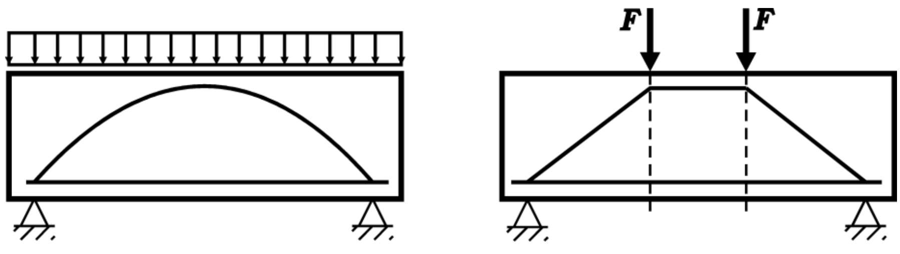


Figure 4.2-13 – Arch models for the beam without shear reinforcement

The design value for the shear resistance $V_{Rd,c}$ is given by the following expression:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d, \qquad (4.2\text{-}22)$$

with a minimum of:

$$V_{Rd,c} = \left[v_{min} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d, \qquad (4.2\text{-}23)$$

where: f_{ck} is in MPa;

$k = 1 + \sqrt{\dfrac{200}{d}} \le 2{,}0$ with d in mm;

$\rho_l = \dfrac{A_{sl}}{b_w \cdot d} \le 0{,}02$;

A_{sl} is the area of tensile reinforcement, which extends $\ge (l_{bd} + d)$ beyond the section considered (see Figure 4.2-14);

b_w is the smallest width of the cross-section in the tensile area [mm]

$\sigma_{cp} = N_{Ed} / A_c < 0{,}2 \cdot f_{cd}$ [MPa];

N_{Ed} is the axial force in the cross-section due to loading or prestressing [in N] ($N_{Ed} > 0$ for compression). The influence of imposed deformations on N_{Ed} may be ignored;

A_c is the area of concrete cross-section [mm²];

$V_{Rd,c}$ is in [N].

Note: *The values of $C_{Rd,c}$, v_{min} and k_1 for use in a Country may be found in its National Annex to EN 1992 [N3]. The recommended value for $C_{Rd,c}$ is $0{,}18/\gamma_c$, $k_1 = 0{,}15$ that for v_{min} is given by following expression: $v_{min} = 0{,}035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$.*

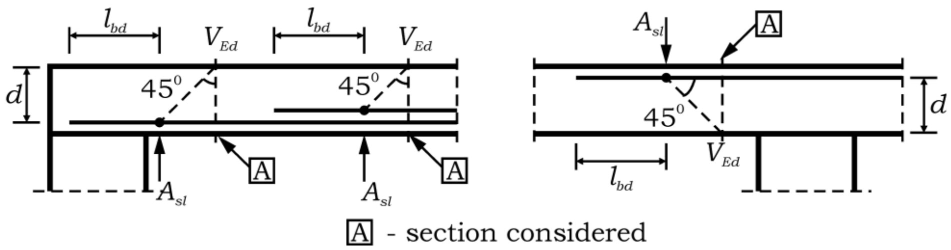


Figure 4.2-14 – Definition of A_{sl} in Expression (4.2-22) (see Figure 6.3 from EN 1992 [N3])

Expression (4.2-22) is empirical. If $V_{Ed} \le V_{Rd,c}$ no shear reinforcement is required, except, possibly, in beams where it is normal to provide a minimum amount of shear links.

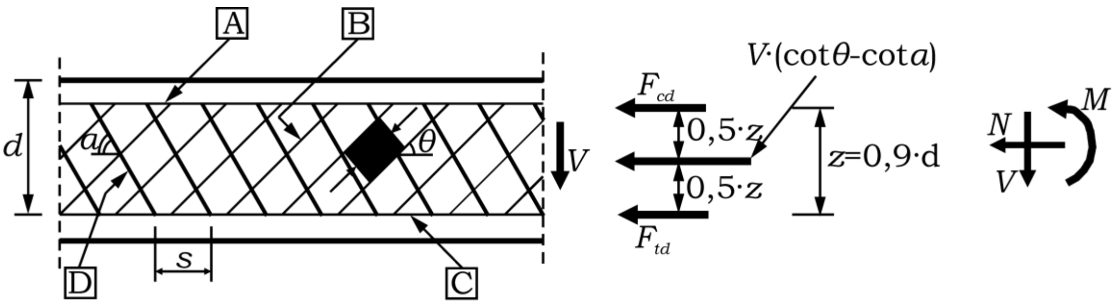
However, if $V_{Ed} > V_{Rd,c}$, shear failure may occur as a result of either compressive failure of the diagonal concrete strut or diagonal tension failure of the member.

(2) Members required design shear reinforcement

The design of members with shear reinforcement is based on a truss model (see Figure 4.2-15). The angle θ should be limited. Limiting values for the angle Θ of the inclined struts in the web are given in EN 1992 [N3] (clause 6.2.3(2)). According to EN 1992 [N3], the strut angle (and hence the strut capacity) is not unique but can vary between $\cot\theta = 2{,}5$ (i.e. $\theta = 21{,}8°$) and $\cot\theta = 1{,}0$ (i.e. $\theta = 45°$), depending on the value of the applied shear.

In Figure 4.2-15 the following notations are shown:

α is the angle between shear reinforcement and the beam axis perpendicular to the shear force (measured positive as shown in Figure 4.2-15);
θ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force;
F_{td} is the design value of the tensile force in the longitudinal reinforcement;
F_{cd} is the design value of the concrete compression force in the direction of the longitudinal member axis;
b_w is the minimum width between tension and compression chords;
z is the inner lever arm, for a member with constant depth, corresponding to the bending moment in the element under consideration. In the shear analysis of reinforced concrete without axial force, the approximate value $z = 0{,}9 \cdot d$ may normally be used.



A - compression chord; B - struts; C - tensile chord; D - shear reinforcement.

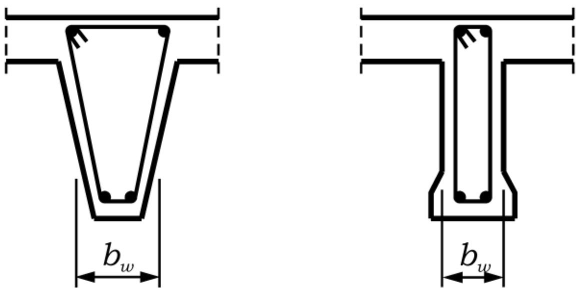


Figure 4.2-15 – Truss model and notation for shear reinforced members (see Figure 6.5 from EN 1992 [N3])

In accordance with EN 1992 [N3] (clause 6.2.3), the design procedure for non-prestressed members is based on the following equation, which was derived in clause 4.2.2.1.

1. For members with vertical shear reinforcement, the shear resistance, V_{Rd} is the smaller value of:

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot\theta, \tag{4.2-25}$$

and

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / \left(\cot\theta + \tan\theta\right), \tag{4.2-26}$$

where: A_{sw} is the cross-sectional area of the shear reinforcement;

s is the spacing of the stirrups;

f_{ywd} is the design yield strength of the shear reinforcement;

υ_1 is a strength reduction factor for concrete cracked in shear. If the design stress of the shear reinforcement is below f_{yk}, value υ_1 may be taken as:

$$\text{for } f_{ck} \le 60 \text{ MPa}, \ \upsilon_1 = 0,6, \tag{4.2-27}$$

$$\text{for } f_{ck} > 60 \text{ MPa}, \upsilon_1 = 0,9 - \frac{f_{ck}}{200} > 0,5; \tag{4.2-28}$$

α_{cw} is a coefficient taking account of the state of the stress in the compression chord, in case the non-prestressed structures $\alpha_{cw} = 1,0$.

It should be noted, that if Expression (4.2-27) is used, the value of f_{ywd} should be reduced to $0,8 \cdot f_{ywd}$ in Expression (4.2-28).

The maximum effective cross-section area of the shear reinforcement, $A_{sw,max}$, for $\cot\theta = 1,0$ is given by:

$$\frac{A_{sw,max} \cdot f_{ywd}}{b_w \cdot s} \le \frac{1}{2} \cdot \alpha_{cw} \cdot \upsilon_1 \cdot f_{cd}. \tag{4.2-29}$$

2. For members with inclined shear reinforcement, the shear resistance is the smaller value of:

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot (\cot\theta + \tan\theta) \cdot \sin\alpha. \tag{4.2-30}$$

and:

$$V_{Rd,max} = \frac{\alpha_{cw} \cdot b_w \cdot z \cdot \upsilon_1 \cdot f_{cd} \cdot (\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)}. \tag{4.2-31}$$

The maximum effective shear reinforcement, $A_{sw,max}$, for $\cot\theta = 1,0$ follows from:

$$\frac{A_{sw,max} \cdot f_{ywd}}{b_w \cdot s} \le \frac{1}{2} \cdot \frac{\alpha_{cw} \cdot \upsilon_1 \cdot f_{cd}}{\sin\alpha}. \tag{4.2-32}$$

In regions where there is no discontinuity of V_{Ed} (e.g. for uniformly loading applied at the top) the shear reinforcement in any length increment $l = z \cdot \cot\theta$ may be calculated using the smallest value of V_{Ed} in the increment.

The additional tensile force, ΔF_{td}, in the longitudinal reinforcement due to shear V_{Ed} may be calculated from:

$$\Delta F_{td} = 0,5 \cdot V_{Ed} \cdot (\cot\theta - \cot\alpha), \tag{4.2-33}$$

and $(M_{Ed} / z) + \Delta F_{td}$ should be taken not greater than $M_{Ed,max} / z$, where $M_{Ed,max}$ is the maximum moment along the beam.

For members with loads applied on the upper side within a distance $0,5 \cdot d \le a_v \le 2,0 \cdot d$ the contribution of this load to the shear force V_{Ed} may be reduced by $\beta = a_v / 2 \cdot d$.

The shear force V_{Ed}, calculated in this way, should satisfy the following condition:

$$V_{Ed} \le A_{sw} \cdot f_{ywd} \cdot \sin\alpha, \tag{4.2-34}$$

where: $A_{sw} \cdot f_{ywd}$ is the resistance of the shear reinforcement crossing the inclined shear crack between the loaded areas (see Figure 4.2-16). Only the shear reinforcement within the central $0,75 \cdot a_v$ should be taken into account. The reduction by β should only be applied for calculating the shear reinforcement. It is only valid provided that the longitudinal reinforcement is fully anchored at the support.

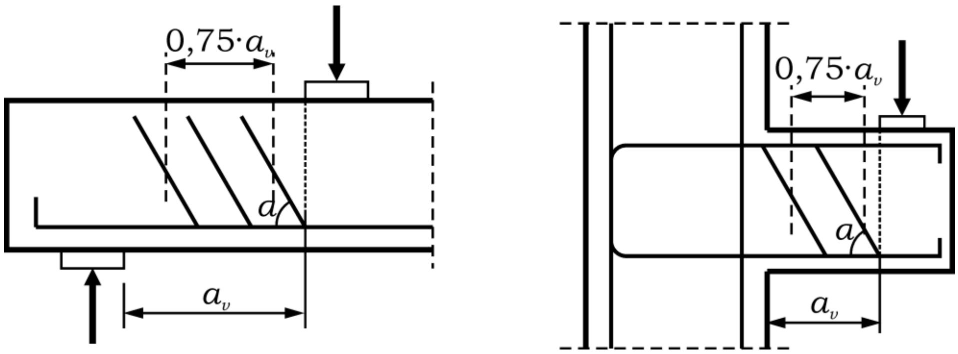


Figure 4.2-16 – Shear reinforcement in short shear spans with direct strut action (see Figure 6.6 from EN 1992 [N3])

For $a_v \le 0,5 \cdot d$ the value $a_v = 0,5 \cdot d$ should be used.

The value V_{Ed} calculated without reduction by β, should however always be less than $V_{Rd,max}$, see Expression (4.2-26).

(3) Shear between web and flanges

The shear strength of the flange may be calculated by considering the flange as a system of compressive struts combined with ties in the form of tensile reinforcement.

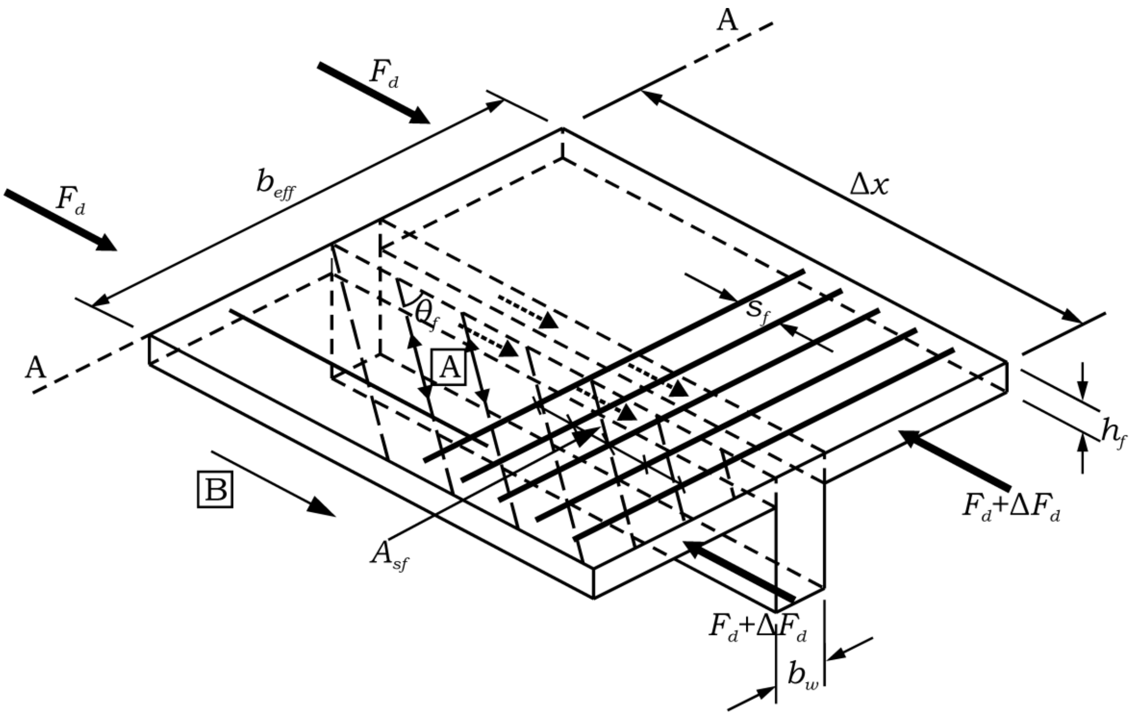
The longitudinal shear stress, V_{Ed}, at the junction between one side of a flange and the web is determined by the change of the normal (longitudinal) force in the part of the flange considered, according to:

$$V_{Ed} = \frac{\Delta F_d}{h_f \cdot \Delta x}, \tag{4.2-35}$$

where: h_f is the thickness of flange at the junctions;

Δx is the length under consideration, see Figure (4.2-17);

ΔF_d is the change of the normal force in the flange over the length Δx.



A - compressive struts B - longitudinal bar anchored beyond this projected point

Figure 4.2-17 – Notations for the connection between flange and web (see Figure 6.7 from EN 1992 [N3])

The maximum value that may be assumed for Δx is half the distance between the section where the moment is 0 and the section where the moment is maximum. Where point loads are applied, the length Δx should not exceed the distance between point loads.

The transverse reinforcement per unit length A_{sf} / s_f may be determined as follows:

$$\frac{A_{sf} \cdot f_{yd}}{s_f} \geq \frac{V_{Ed} \cdot h_f}{\cot\theta_f}. \tag{4.2-36}$$

To prevent crushing of the compression struts in the flange, the following condition should be satisfied:

$$V_{Ed} \le \nu \cdot f_{cd} \cdot \sin\theta_f \cdot \cos\theta_f. \qquad (4.2\text{-}37)$$

The recommended values in the absence of more rigorous calculation are: $1{,}0 \le \cot\theta_f \le 2{,}0$ for compression flanges ($45^\circ \ge \theta_f \ge 26{,}5^\circ$); $1{,}0 \le \cot\theta_f \le 1{,}25$ for tension flanges ($45^\circ \ge \theta_f \ge 38{,}6^\circ$).

In the case of combined shear between the flange and the web, and transverse bending, the area of steel should be the greater than that given by Expression (4.2-36) or half that given by Expression (4.2-37) plus that required for transverse bending.

If V_{Ed} is less than or equal to $0{,}4 \cdot f_{ctd}$ no extra reinforcement above that for flexure is required.

Longitudinal tension reinforcement in the flange should be anchored beyond the strut required to transmit the force back to the web at the section where this reinforcement is required (see section (A–A) in Figure 4.2-17).

Design procedure can be summarized as follows:

1. Calculate the design shear force, V_{Ed};
2. Determinate the shear resistance of the member without shear reinforcement, $V_{Rd,c}$;
3. Check, if $V_{Ed} \le V_{Rd,c}$, shear reinforcement can be omitted except in beams where a minimum area of shear reinforcement must be provided;
4. If $V_{Ed} > V_{Rd,c}$ all the shear force must be resisted by the shear reinforcement. Provided $V_{Ed} \le V_{Rd,max}$, the area of shear reinforcement can be determined using Expression (4.2-25). $V_{Rd,max}$ is estimated from the Equation (4.2-26) assuming initially $\cot\theta = 2{,}5$. However, if the result is $V_{Ed} > V_{Rd,max}$ a larger strut angle may be used. The maximum allowable angle θ value is 45° (i.e. $\cot\theta = 1{,}0$). The minimum value of the strut angle for a given design shear force V_{Ed}, can be determined from Equation (4.2-26) and used, in turn, to calculate the area of shear reinforcement from Equation (4.2-25).
5. If the strut angle exceeds $\theta = 45^\circ$, however, a deeper concrete section or higher concrete strength must be provided and steps (2) to (4) repeated.

Examples to section 4.2.2

Example 1. Design of shear reinforcement for a beam

Design the shear reinforcement for the free supported beam shown in Figure E 4.2-1.

Concrete C25/30 ($f_{ck} = 25\ \mathrm{N/mm^2}$), steel S500 ($f_{yk} = 500\ \mathrm{N/mm^2}$).

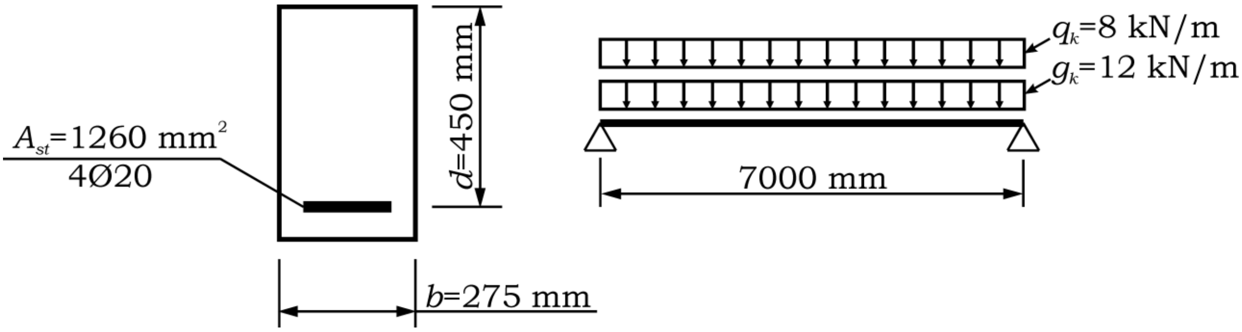


Figure E 4.2-1 – Input data for worked Example 1

Ultimate design load.

$P = 1,35 \cdot g_k + 1,5 \cdot q_k = 1,35 \cdot 12 + 1,5 \cdot 8 = 28,2\ \text{kN/m}.$

Design shear force, V_{Ed}.

This should be determined at distance d from fall of the support but for simplicity has been calculated at the centre of supports:

$V_{Ed} = 0,5 \cdot (P \cdot l) = 0,5 \cdot 28,2 \cdot 7 = 98,7\ \text{kN}.$

Shear resistance of concrete, $V_{Rd,c}$.

$$f_{ck} = 25\ N/\text{mm}^2;\ C_{Rd,c} = \frac{0,18}{\gamma_c} = \frac{0,18}{1,5} = 0,12\ \text{N/mm}^2;$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{450}} = 1,67 < 2,0\ .$$

Assuming all the tension reinforcement is taken onto supports and anchored:

$$\rho_l = \frac{A_{st}}{b_w \cdot d} = \frac{1260}{275 \cdot 450} = 0,0102 < 0,02;\ \sigma_{cp} = 0;$$

$$\upsilon_{min} = 0,035k^{3/2} \cdot f_{ck}^{1/2} = 0,035 \cdot 1,67^{3/2} \cdot 25^{1/2} = 0,378\ \text{N/mm}^2;$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d = \left[0,12 \cdot 1,67 \cdot (100 \cdot 0,0102 \cdot 25)^{1/3}\right] \times$$

$$\times 275 \cdot 450 = 72994\ \text{N} \geq (\nu_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d = 0,378 \cdot 275 \cdot 450 = 46778\ \text{N}.$$

Since $V_{Rd,c} \leq V_{Ed}$, shear reinforcement must be provided.

Compression capacity of compression strut, $V_{Rd,max}$:

Assuming, that $\theta = 21,8^\circ$:

$$\upsilon_1 = 0,6 \cdot (1 - f_{ck}/250) = 0,6 \cdot (1 - 25/250) = 0,54;$$

$$f_{cd} = \frac{a_{cc} \cdot f_{ck}}{\gamma_c} = \frac{0,85 \cdot 25}{1,5} = 14,2\ \text{N/mm}^2.$$

Note: $a_{cc} = 1,0$ *may be used.*

$$V_{Rd,max} = a_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot f_{cd} / (\cot\theta + \tan\theta) =$$

$$= \left[1 \cdot 275 \cdot (0,9 \cdot 450) \cdot 0,54 \cdot 14,2 / (2,5 + 0,4)\right] \cdot 10^{-3} = 294,5\ \text{kN} > V_{Ed} = 98,7\ \text{kN}.$$

Diameter and spacing of links.

(1) Where $V_{Ed} < V_{Rd,c}$, provide minimum shear reinforcement, $\rho_{w,min}$, according to:

$$\rho_{w,min} = \frac{0,08 \cdot \sqrt{f_{ck}}}{f_{yk}} = \frac{0,08 \cdot \sqrt{25}}{500} = 8 \cdot 10^{-4};$$

$\rho_{w,min} = \dfrac{A_{sw}}{s \cdot b_w \cdot \sin\alpha} \cdot \dfrac{A_{sw}}{s} = 8 \cdot 10^{-4} \cdot 275 \cdot 1 = 0,22$ mm (assuming the use of vertical links).

Maximum spacing of links, s_{max}, is:

$s_{max} = 0,75 \cdot d = 0,75 \cdot 450 = 338$ mm.

Provide Ø8 S500 at 300 mm centers and $A_{sw} / s = 0,335$ mm.

(2) Where $V_{Ed} > V_{Rd,c}$, provide shear reinforcement according to:

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot\theta = 98700 \text{ N},$$

and: $$\frac{A_{sw}}{s} = \frac{98700}{(0,9 \cdot 450) \cdot (500/1,15) \cdot 2.5} = 0,224\ mm.$$

Hence, provide Ø8 S500 at 300mm centers ($A_{sw} / s = 0,335$) throughout.

Example 2. Design of shear reinforcement at beam support

Design the shear reinforcement for the beam shown in Figure E 4.2-2, assuming it resist an ultimate design shear force at distance *d* from the face of support of $F_{Ed} = 450\ kN$.

Concrete C25/30 ($f_{ck} = 25 \text{ N/mm}^2$), steel S500 ($f_{yk} = 500 \text{ N/mm}^2$).

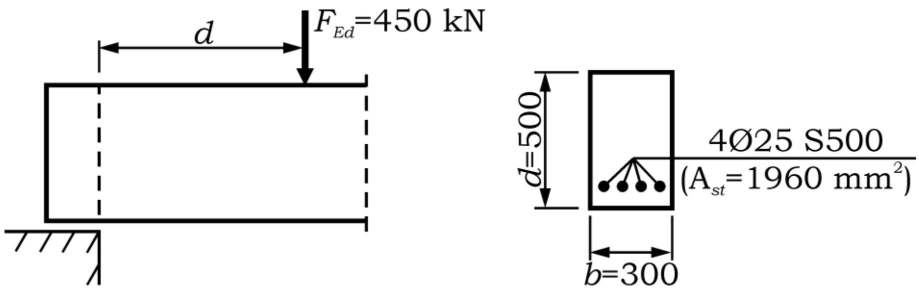


Figure E 4.2-2 – Input data for worked Example 2

Shear resistance of concrete, $V_{Rd,c}$.

$$f_{ck} = 25\ N/\text{mm}^2;\ C_{Rd,c} = \frac{0,18}{\gamma_c} = \frac{0,18}{1,5} = 0,12 \text{ N/mm}^2;$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{500}} = 1,63 < 2,0;\ \rho_l = \frac{A_{st}}{b_w \cdot d} = \frac{1960}{300 \cdot 500} = 0,013 < 0,02;$$

$$\sigma_{cp} = 0;\ v_{min} = 0,035 \cdot k^{3/2} \cdot f_{ck}^{1/2} = 0,035 \cdot 1,63^{3/2} \cdot 25^{1/2} = 0,364 \text{ N/mm}^2;$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d = \left[0,12 \cdot 1,63 \cdot (100 \cdot 0,013 \cdot 25)^{1/3}\right] \times$$

$$\times 300 \cdot 500 \cdot 10^{-3} = 93,6 \text{ kN} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d = 0,364 \cdot 300 \cdot 500 \cdot 10^{-3} = 54 \text{ kN}$$

Since $V_{Rd,c} < V_{Ed} = 450$ kN, shear reinforcement must be provided.

Checking of the compression capacity of compression strut, $V_{Rd,max}$.

Assuming, that $\theta = 21,8°$:

$$\upsilon_1 = 0,6 \cdot (1 - f_{ck} / 250) = 0,6 \cdot (1 - 25 / 250) = 0,54\,;$$

$$f_{cd} = \frac{a_{cc} \cdot f_{ck}}{\gamma_c} = \frac{0,85 \cdot 25}{1,5} = 14,2\ \mathrm{N/mm^2}.$$

Note: $a_{cc} = 1,0$ *may be used.*

$$V_{Rd,max} = a_{cw} \cdot b_w \cdot z \cdot \upsilon_1 \cdot f_{cd} / (\cot\theta + \tan\theta) =$$
$$= \left[1 \cdot 300 \cdot (0,9 \cdot 500) \cdot 0,54 \cdot 14,2 / (2,5 + 0,4)\right] \cdot 10^{-3} = 357\ \mathrm{kN}.$$

Since $V_{Rd,max} < V_{Ed}$, strut angle $\theta > 21,8^{\circ}$.

From Equation (4.2-13):

$$\theta = 0,5 \cdot \sin^{-1}\left(\frac{V_{Rd,max} / b_w \cdot d}{0,153 \cdot f_{ck} \cdot (1 - f_{ck} / 250)}\right) = 0,5 \cdot \sin^{-1}\left(\frac{450 \cdot 10^{-3} / 300 \cdot 500}{0,153 \cdot 25 \cdot (1 - 25 / 250)}\right) = 30,3^{\circ}.$$

Diameter and spacing of links.

Provide shear reinforcement according to:

$V_{Rd,s} = \dfrac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot\theta = 450\ \mathrm{kN}$, and so:

$$\frac{A_{sw}}{s} = \frac{450 \cdot 10^3}{(0,9 \cdot 500) \cdot (500 / 1,15) \cdot \cot(30,3^{\circ})} = 1,344\ \mathrm{mm}.$$

Maximum spacing of links, s_{max}, is:

$$s_{max} = 0,75 \cdot d = 0,75 \cdot 500 = 375\ mm.$$

Therefore, Ø12 S500 at 150mm centers ($A_{sw} / s = 1,507$) would be suitable.

Note: *If longitudinal reinforcement is fully anchored at support and* $V_{Ed} = 450\ \mathrm{kN} \le 0,5 \cdot b_w \cdot d \cdot \upsilon \cdot f_{cd} = 0,5 \cdot 300 \cdot 500 \cdot 0,54 \cdot 14,2 \cdot 10^{-3} = 575\ \mathrm{kN}$, *shear force may be reduced to* $(a_v / 2 \cdot d) \cdot 450 = 225\ \mathrm{kN}$. *In this case* $\theta = 21,8^{\circ}$ *is good result and shear reinforcement is reduced by over 50 %.*

4.2.3 SHEAR IN ACCORDANCE WITH *fib* MODEL CODE 2010 (MC 2010) [N5]

4.2.3.1 General

As it was shown in [17], the ***fib*** Model Code 2010 [N5] comprises a mechanical based set of shear design process that is intended to offer for the engineer flexibility in selecting a balance between complexity and accuracy for new structural design and for evaluating or verification of existing structures as well.

The equations of the proposed in ***fib*** MC 2010 [N5] design methods was derived from the ***Modified Compression Field Theory (MCFT)*** and assume that the member contains well-detailed reinforcement in at least the longitudinal direction [14].

The ***Compression Field Theory (CFT)*** was firstly developed by Collins and Mitchell and co-workers, for the analyses of beam under combined torsion, shear, flexure. The theory provided a conceptual model for the behaviour of cracked reinforced concrete under *two-dimensional stress states*, following essentially a smeared, rotating crack idealization. Formulations satisfying conditions of equilibrium and compatibility in continuum were based on average values of stress and strain in component materials. It was assumed that the direction of the principal stresses coincided with direction of the principal strains. Concrete in compression was modeled using the parabolic curve; concrete in tension was assumed to carry no stress after cracking.

To develop more accurate constitutive relations for cracked concrete, a new testing facility was developed and utilized in an extensive experimental investigations.

Based on the results of these initial tests, the ***Modified Compression Field Theory (MCFT)*** was developed [18]. The refinements introduced by the MCFT related to:

1) strain softening concrete in compression, due to the action of transverse tensile strain;

2) tension stiffening effects in cracked concrete in tension, due to continued presence of tensile stresses in concrete between cracks;

3) the transfer of stresses across cracks (i.e. the need to consider local stress conditions at crack surfaces). These effects were embodied in the analytical model by a new set of constitutive relations.

The new code provisions contains four ***«Levels of Approximations» (LOA)***. As shown in ***fib*** MC 2010 [N5], ***Level I*** provides the simplest calculation whereas this is the most conventional method among others. ***Level II*** is a balanced model in complexity and accuracy, while ***Level III*** is most accurate and general approximation but needs more complex computation than that of the other Levels. ***Level IV*** is a further option, which can be used in nonlinear finite element analysis or generalized stress-field approaches.

4.2.3.2 Design shear force and shear resistance

The design model for resistance to beam in shear is shown in Figure 4.2-18.

The shear resistance of web or slab will be determined according to:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \geq V_{Ed}\,, \qquad (4.2\text{-}38)$$

where: V_{Rd} is the design shear resistance, which included both the design shear resistance attributed to the concrete ($V_{Rd,c}$), and the design shear resistance provided by the shear reinforcement ($V_{Rd,s}$);

V_{Ed} is design value of shear force.

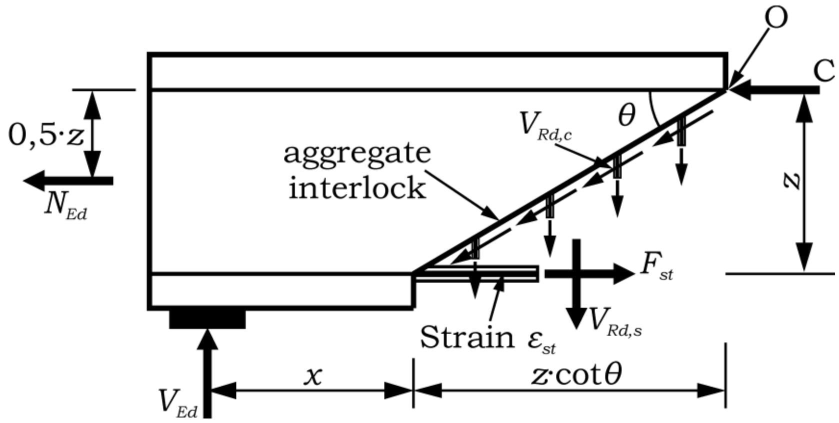


Figure 4.2-18 – Forces in the web of a beam

In general, for determining the design shear force, a location at z from the face of support (see Figure 4.2-19), where $z = 0,9 \cdot d$, discontinuities of geometry or transverse applied force are used. Other control sections may be required, for example, near points of curtailment of reinforcement.

Sections located closer to support or the applied force than the control section may be designed for the same shear force as that computed at the control section provided that the respective force introduce compression into member.

In the design for shear in webs and slabs, the effects of axial tension due to creep, shrinkage and thermal effects in restrained members should be considered wherever applicable.

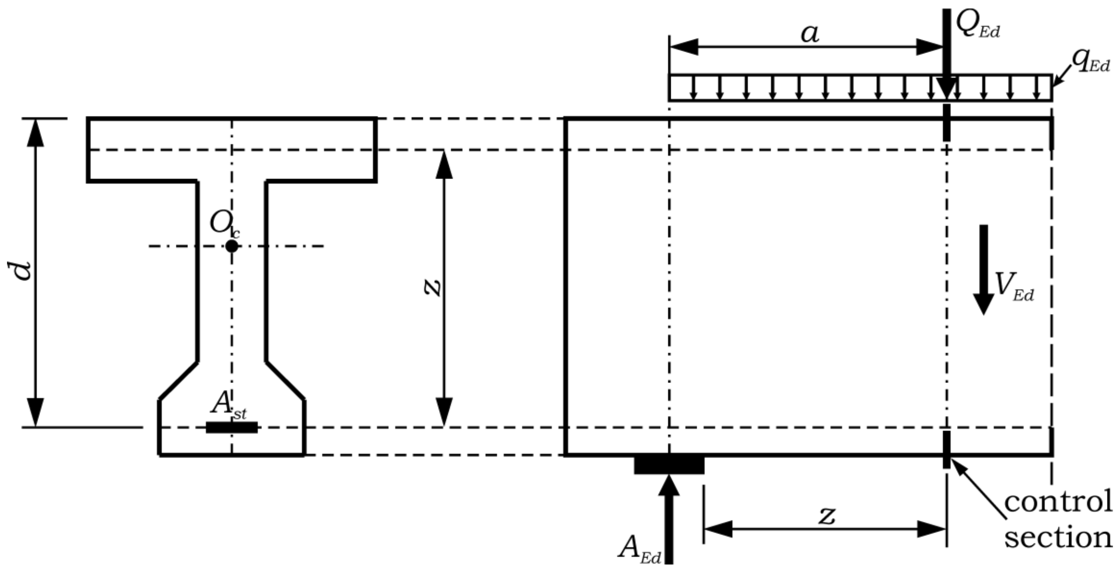


Figure 4.2-19 – Definition of control section for sectional design
(see Figure 7.3-5 from *fib* MC 2010 [N5])

4.2.3.3 Levels of approximation

In accordance with ***fib*** MC 2010 [N5] in determining the shear resistance of a member different levels of approximation may be regarded. The levels differ in the complexity of the applied methods and the accuracy of the results.

Level I Approximation: In general, this level of approximation may be used for the conception or the design of a new structure;

Level II Approximation: This level of approximation is appropriate for the design of a new structure as well as for a general or brief assessment of an existing structure;

Level III Approximation (and higher): A level III (and higher) approximation may be used for the design of a member in a complex loading state or a more elaborate assessment of a structure.

There can be different levels of approximation for each design case and location in a structure; they do not necessarily need to correspond to each other as different portions of a structure will justify different levels of precision and design effort.

4.2.3.4 Design equations for region cracked in bending

4.2.3.4.1 General

The design shear resistance of web and slab should be determined as:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \leq V_{Rd,max}, \qquad (4.2\text{-}39)$$

and, value of V_{Rd} cannot exceed the crushing capacity of concrete calculated as:

$$V_{Rd,max} = k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot z \cdot \frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta}, \qquad (4.2\text{-}40)$$

where: θ denotes the selected inclination of the compressive stresses resultant;

α is the inclination of the stirrups relative to the beam axis (see Figure 4.2-20).

According to ***fib*** MC 2010 [N5], for Level I and Level III Approximation, a value $\theta_{min} \leq \theta \leq 45^\circ$ should be inserted in Equation (4.2-40).

In member that contain a percentage of shear reinforcement of, $\rho_w \geq \rho_{w,min} = 0{,}08 \cdot \left(\sqrt{f_{ck}} / f_{yk}\right)$, the design shear resistance provided by the stirrups may be calculated as follows:

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \left(\cot\theta + \cot\alpha\right) \cdot \sin\alpha. \qquad (4.2\text{-}41)$$

The design shear resistance attributed to the concrete can be taken as:

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot z, \qquad (4.2\text{-}42)$$

where the value of $\sqrt{f_{ck}}$ should not be taken as greater than 8 MPa.

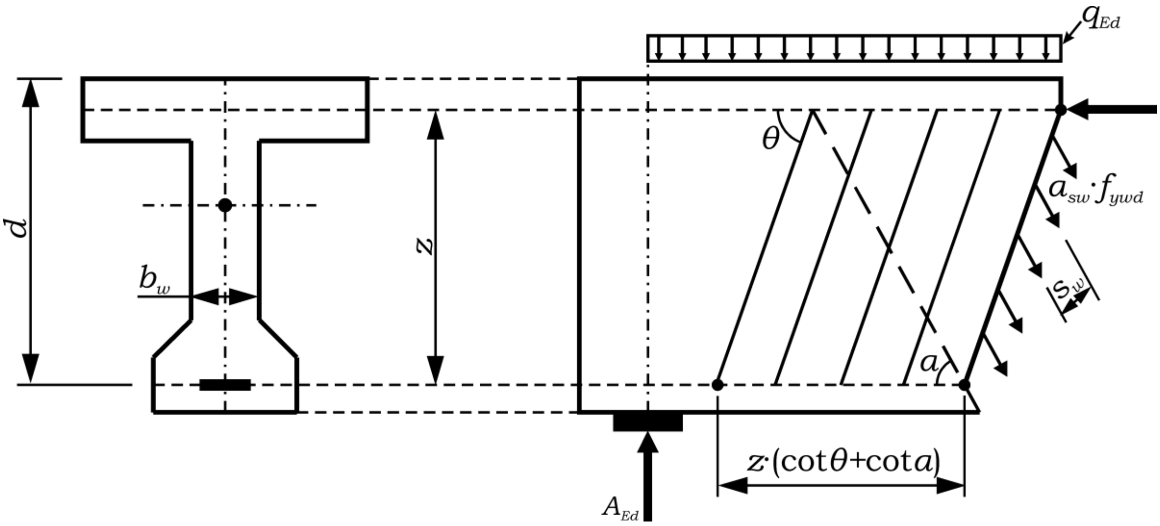


Figure 4.2-20 – Geometry and definition in accordance with *fib* MC 2010 [N5]

According to the proposed model the design shear resistance of concrete contribution ($V_{Rd,c}$) affected by the concrete compressive strength (f_{ck}), the width of the web (b_w) the effective depth of the section (d), the partial factor (γ_c) according design situation, and the *first parameter* of the model k_v, defined by the level of approximation and indicates the ability of the web to resist aggregate interlock stresses which provide the concrete contribution to shear strength.

In Equation (4.2-42) the concrete strength reduction factor k_c is taking into consideration the effect of cracked concrete can be calculated according to the level of approximation.

The design shear resistance provided by shear reinforcement ($V_{Rd,s}$) is generally defined by the amount of shear reinforcement (A_{sw} / s) and the strength properties of reinforcement used ($f_{ywd} = f_{ywk} / \gamma_s$).

The longitudinal reinforcement at the section of interest must be able to resist the additional force due to shear of:

$$\Delta F_{td} = 0{,}5 \cdot V_{Ed} \cdot (\cot\theta - \cot a) + 0{,}5 \cdot V_{Rd,c} \cdot (\cot\theta + \cot a). \qquad (4.2\text{-}43)$$

Note, that the total demand on longitudinal reinforcement need not exceed the demand at the maximum moment location due to moment alone.

It must be noted, that in the proposed design model angle θ defined by the different level of approximation – is the *second parameter* of the model and indicates the angle of principal compressive stress in web.

4.2.3.4.2 Level I Approximation

As was shown, Level I Approximation is suitable for pre-design stage (conceptual design) or initial sizing of structural members, where a conservative calculation methods is acceptable. It may be used for the efficient design of most common members with or without shear reinforcement.

These Level I Approximation for members with no significant axial loads, where $f_{ck} \leq 70\ \dfrac{\mathrm{N}}{\mathrm{mm}^2}$, $f_{yk} \leq 600\ \dfrac{\mathrm{N}}{\mathrm{mm}^2}$, cast with an aggregate size d_g of at least 10 mm, design shear resistance of reinforced concrete cross section – ***with no shear reinforcement*** can be determined by the following equation:

$$V_{Rd} = V_{Rd,c} = 0,9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d., \tag{4.2-44}$$

The design shear resistance of reinforced concrete cross section with shear reinforcement can be calculated as – neglecting the concrete contribution:

$$V_{Rd} = V_{Rd,s} \leq V_{Rd,max}(\theta_{min}). \tag{4.2-45}$$

The following values of the coefficients k_c and k_v can be inserted in Equation (4.2-40) and Equation (4.2-44):

$$k_c = 0,5 \cdot \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 0,5, \left(f_{ck} \text{ in MPa}\right); \tag{4.2-46}$$

$$k_v = \begin{cases} \dfrac{200}{(1000 + 1,3 \cdot z)} \leq 0,15 \ \mathit{if}\ \rho_w = 0; \\ 0,15 \ \mathit{if}\ \rho_w \geq 0,08 \sqrt{f_{ck}} / f_{yk}. \end{cases} \tag{4.2-47}$$

As it was noted in ***fib*** MC 2010 [N5], if there is no (considerable) sufficient axial force the inclination of the fictional compressed strut (θ) according to the proposal to be at least (30°). The crushing capacity of the concrete at that angle ($\theta_{min} = 30^\circ$) of the principal compressive stress can be calculated by the use of $k_\varepsilon = 0,55$. Results of this level of approximation can only be accepted if the longitudinal strain $\varepsilon_x = \left(\dfrac{M_{Ed}}{0,9 \cdot d} + V_{Ed}\right) / 2 \cdot E_s \cdot A_s$ calculated at the middle of the effective depth ($\sim d/2$) is not exceed of $\varepsilon_{x,lim} \cong 1$ ‰, i.e. in case of B500 steel quality the tensioned steel bars are in elastic state ($\varepsilon_s \cong 2 \cdot \varepsilon_x = 2$ ‰ $\leq f_{yk} / E_s$):

$$V_{Rd} = V_{Rd,s} = 0,9 \cdot \frac{A_{sw}}{s} \cdot f_{ywd} \cdot d \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \leq V_{Rd,max}(\theta_{min}). \tag{4.2-48}$$

4.2.3.4.3 Level II Approximation

Level II Approximation comes from variable angle stress field approach. The model is applicable to members with a minimum amount of stirrups ($\rho_w \geq \rho_{w,min}$) reinforcement. It is based on an inclination of the compression stresses which can be chosen within the following limits:

$$20^{\circ} + 10000 \cdot \varepsilon_x \leq \theta \leq 45^{\circ}, \tag{4.2-49}$$

where: ε_x represents the longitudinal strain at the mid-depth of the member as shown in Figure 4.2-21 and may be chosen as 0,001 for preliminary design. If required, ε_x should be calculated with help of plane section analysis (ignoring tension stiffening), but should not be taken as less than – 0,0002.

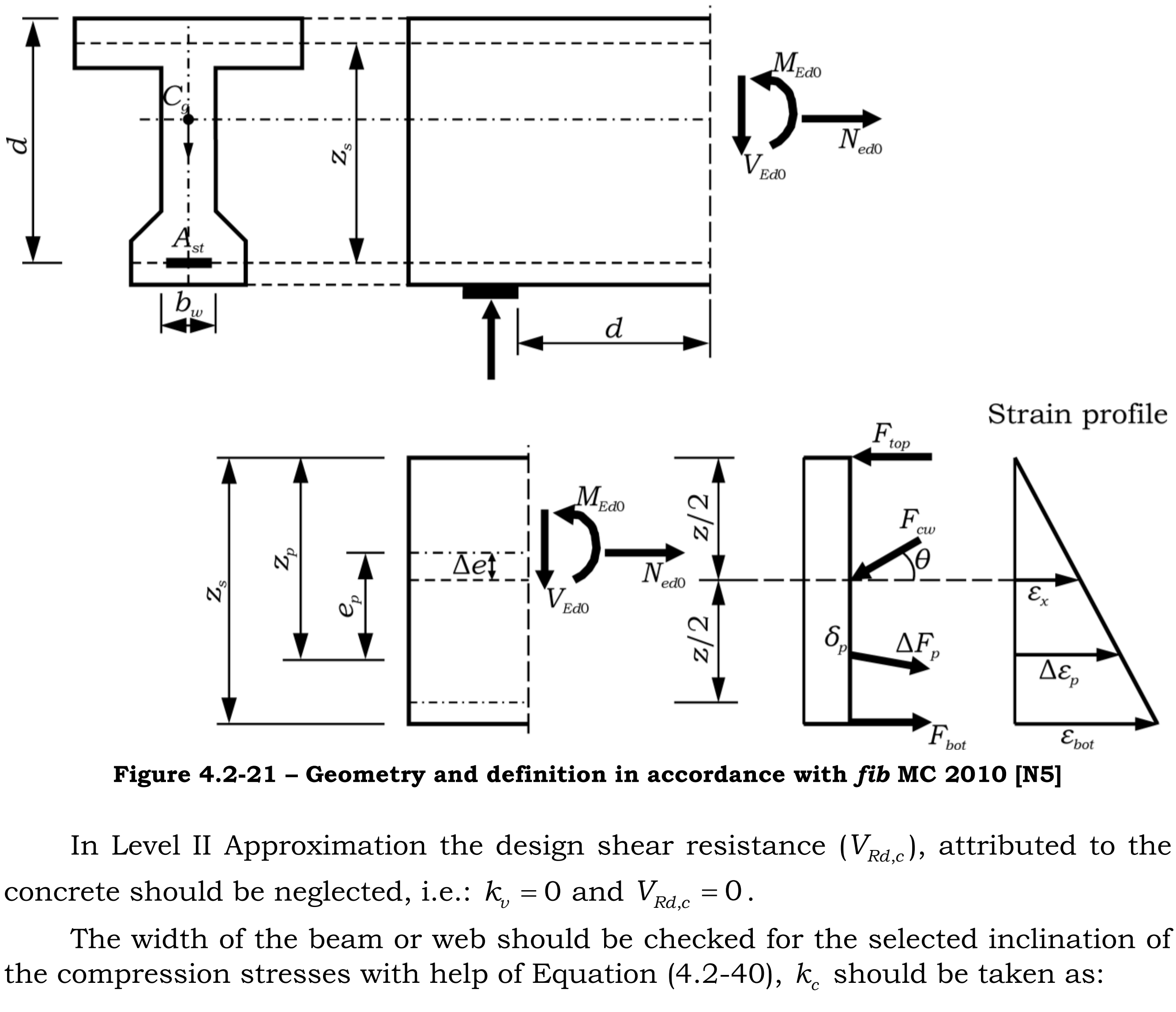


Figure 4.2-21 – Geometry and definition in accordance with *fib* MC 2010 [N5]

In Level II Approximation the design shear resistance ($V_{Rd,c}$), attributed to the concrete should be neglected, i.e.: $k_v = 0$ and $V_{Rd,c} = 0$.

The width of the beam or web should be checked for the selected inclination of the compression stresses with help of Equation (4.2-40), k_c should be taken as:

$$k_c = k_\varepsilon \cdot \eta_{fc} = k_\varepsilon \cdot \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 0,55. \tag{4.2-50}$$

where: $k_\varepsilon = 1/(1{,}2 + 55 \cdot \varepsilon_1) \leq 0{,}6$ and $\varepsilon_1 = \varepsilon_x + \left(\varepsilon_x + 2 \cdot 10^{-3}\right) \cdot \cot^2\theta$ can be adopted for calculation of the k_c.

In addition of the validity of the material properties specified at the Level of Approximation I, with a more accurate determination of the axial strain (ε_x) defined at the middle of the effective depth, the design shear resistance of a reinforced concrete cross-section with *no shear reinforcement* can be written at the Level of Approximation II:

$$V_{Rd} = V_{Rd,c} = 0{,}9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d = \frac{0{,}36}{1000 + 0{,}9 \cdot k_{dg} \cdot d} \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} b_w \cdot d, \qquad (4.2\text{-}51)$$

where: $k_{dg} = 1$, if $d_g = d_{max} \geq 1{,}6$ mm;

$k_v = 520/\left(1000 + 0{,}9 \cdot k_{dg} \cdot d\right)$, if $\varepsilon_x \leq 0$, and $k_v = 95/\left(1000 + 0{,}9 \cdot k_{dg} \cdot d\right)$, if $\varepsilon_x \geq 3$ ‰.

When shear reinforcement is applied, the shear resistance of a reinforced concrete can be calculated – neglecting also the concrete contribution as:

$$V_{Rd} = V_{Rd,s} = 0{,}9 \cdot \frac{A_{sw}}{s} \cdot f_{ywd} \cdot d \cdot \left(\cot\theta + \cot a\right) \cdot \sin a \leq V_{Rd,max}\,, \qquad (4.2\text{-}52)$$

where the minimum angle of the principal compressive stress in the web can be calculated on a more precise way than in case of LOA I by the use of $\theta_{min} = 20^\circ + 10^4 \cdot \varepsilon_x$, while the maximum shear capacity defined by the failure of the compressive concrete strut:

$$V_{Rd,max}\left(\theta_{min}\right) = 0{,}9 \cdot k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{min} + \cot a}{1 + \cot^2\theta_{min}}, \qquad (4.2\text{-}53)$$

where: k_c – coefficient according to Equation (4.2-50).

4.2.3.4.4 Level III Approximation

Level III Approximation represents a general form of sectional shear equations applicable to beams as well slabs and any amount of shear reinforcement.

The angle θ may be assumed as follows:

$$\theta_{min} = 20^\circ + 10^4 \cdot \varepsilon_x\,, \qquad (4.2\text{-}54)$$

where the variable ε_x represents the average longitudinal strain at the mid-depth of the member, and should be taken as:

$$\varepsilon_x = \frac{M_{Ed}/z + V_{Ed} + 0{,}5 \cdot N_{Ed} - A_p \cdot f_{po}}{2 \cdot \left(E_s \cdot A_s + E_p \cdot A_p\right)}. \qquad (4.2\text{-}55)$$

At Level of Approximation III, ***fib*** Model Code 2010 [N5] provides opportunity to take into account shear force contributed by the concrete.

When $V_{Rd} \le V_{Rd,max}\left(\theta_{min}\right)$, the design shear capacity of reinforced cross-section applying also shear reinforcement according to the Level III can be calculated as:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \le V_{Rd,max}\left(\theta_{min}\right), \tag{4.2-56}$$

where:

$$V_{Rd,c} = 0,9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d = \frac{0,36}{\left(1+1500 \cdot \varepsilon_x\right)} \cdot \left(1 - \frac{V_{Ed}}{V_{Rd,max}\left(\theta_{min}\right)}\right) \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d, \tag{4.2-57}$$

$$V_{Rd,s} = 0,9 \cdot \frac{A_{sw}}{s} \cdot f_{ywd} \cdot d \cdot \left(\cot\theta + \cot a\right) \cdot \sin a, \tag{4.2-58}$$

$$V_{Rd,max}\left(\theta_{min}\right) = 0,9 \cdot k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{min} + \cot a}{1 + \cot^2\theta_{min}}, \tag{4.2-59}$$

Equation (4.2-57) and Equation (4.2-59) can be used with:

$$k_c = k_\varepsilon \cdot \eta_{fc} = k_\varepsilon \cdot \left(\frac{30}{f_{ck}}\right)^{1/3}; \tag{4.2-60}$$

$$k_v = \begin{cases} \dfrac{0,40}{\left(1+1500 \cdot \varepsilon_x\right)} \cdot \dfrac{1300}{\left(1000 + 0,7 \cdot k_{dg} \cdot z\right)} & if \;\; \rho_w = 0; \\ \dfrac{0,40}{\left(1+1500 \cdot \varepsilon_x\right)} & if \;\; \rho_w \ge 0,08 \cdot \sqrt{f_{ck}} \,/\, f_{yk}, \end{cases} \tag{4.2-61}$$

where: $k_{dg} = \dfrac{48}{16 + d_g} \ge 1,15$, in which d_g is aggregate diameter.

In using from Equation (4.2-57) to Equation (4.2-59), the following conditions apply:

- V_{Ed} and M_{Ed} should be taken less than $V_{Ed} \cdot z$;

- In calculating A_s and A_p the area of bars or tendons which are terminated less than their development length from the section under consideration should be reduced in proportion to their lack of full development;

- If the value of ε_x calculated from Equation (4.2-55) is negative it should be taken as zero or the value should be recalculated with the denominator of the equation replaced by $2 \cdot \left(E_s \cdot A_s + E_p \cdot A_p + E_c \cdot A_{ct}\right)$, however ε_x should not be taken as less than -0,0002;

- For section closer than z to the face of the support, the value of ε_x calculated at z from the face of the support may be used in evaluating shear resistance;
- If the axial tension is large enough to crack the flexural compression face of the section, the resulting increase in ε_x should be taken into account. In lieu of more accurate calculations, the value calculated from the Equation (4.2-55) should be doubled;
- It is permissible to determine θ and k_v using a value of ε_x which is greater than that calculated from the Equation (4.2-55), however ε_x should not be taken greater than 0,003;
- For concrete strengths that exceed 70 MPa, the aggregate size should be taken as zero as aggregate particles tend to fracture at cracking and are less able to contribute to crack roughness. To avoid a discontinuity, as concrete strengths vary from 64 to 70 MPa, the effective aggregate size can be linearly reduced to zero.

Example to Section 4.2.3

Example 1. Calculation of the shear resistance of RC-beam according to *fib* MC 2010

Concrete C30/37 ($f_{ck} = 25\ \mathrm{N/mm^2}$), $\gamma_c = 1{,}5$.

Steel S500 ($f_{yk} = 550\ \mathrm{N/mm^2}$), $\gamma_s = 1{,}15$, $s = 120$ mm.

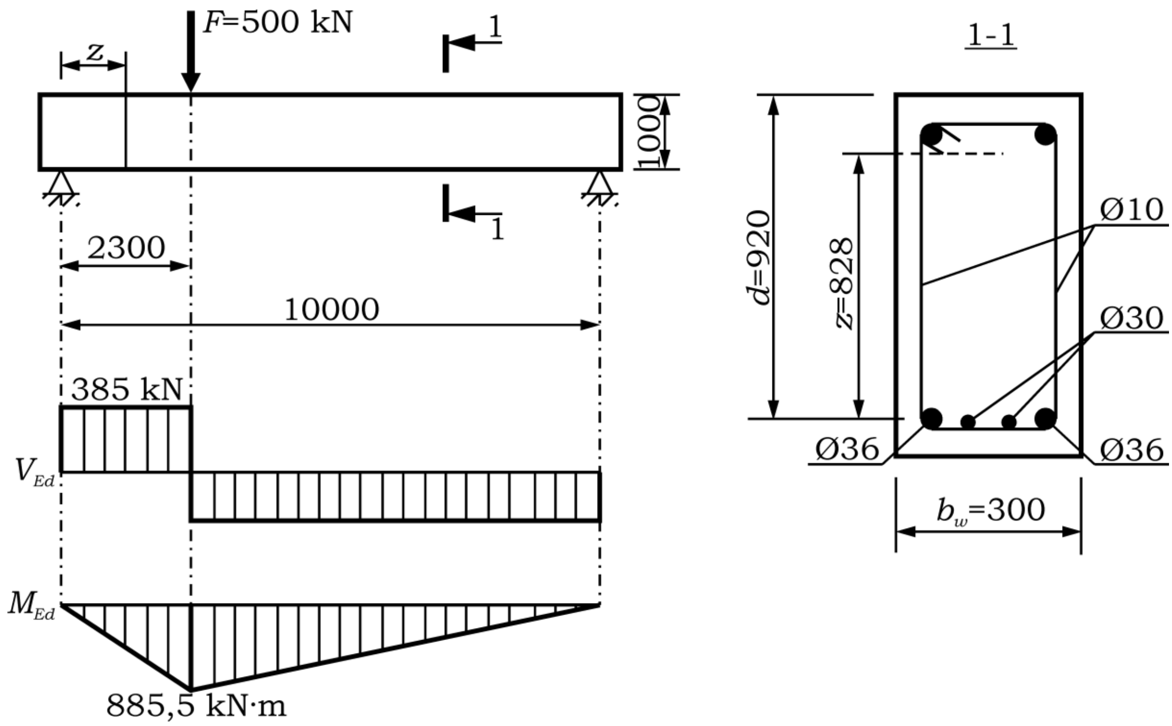


Figure E 4.2-1 – Input data for worked Example 1

Firstly, to calculate the resistance of the RC member, it is necessary to check if it can to carry the forces without shear reinforcement. To verify if this is possible, according to ***fib*** MC2010 it is necessary to check $V_{Rd,c}$, in this case in the Level III approximation:

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot z, \tag{E 4.2-1}$$

where: $k_v = \frac{0,40}{(1+1500 \cdot \varepsilon_x)} \cdot \frac{1300}{(1000+0,7 \cdot k_{dg} \cdot z)}$, taking into account that:

$\varepsilon_x = \frac{1}{2 \cdot E_s \cdot A_s} \cdot \left(\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed} \cdot \left(\frac{1}{2} \mp \frac{\Delta \cdot e}{z} \right) \right) = 6,16 \cdot 10^{-4}$; $k_{dg} = \frac{48}{16+d_g} = \frac{48}{32} = 1,5 \geq 1,15$

and $k_v = \frac{0,40}{(1+1500 \cdot 6,6 \cdot 10^{-4})} \cdot \frac{1300}{(1000+0,7 \cdot 1,15 \cdot 828)} = 0,16$.

Thus, substituting in Equation (E 4.2-1):

$V_{Rd,c} = 0,16 \cdot \frac{\sqrt{30}}{1,5} \cdot 300 \cdot 828 = 145,12 \text{ kN}$, and as: $V_{Rd,c} = 145,12 \text{ kN} < V_{Ed} = 385 \text{ kN}$, the RC-member needs to be calculated with shear reinforcement according to the same regulation. But, in this case, Level III of approximation is used and the below formula is necessary to be implemented:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \leq V_{Rd,max}(\theta_{min}), \tag{E4.2-2}$$

where:

$$V_{Rd,max}(\theta_{min}) = k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot z \cdot \sin\theta_{min} \cdot \cos\theta_{min}. \tag{E4.2-3}$$

Taking in account, that: $b_w = 300$ mm; $k_c = k_\varepsilon \cdot \eta_{fc} = 0,55$; where: $\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} = \left(\frac{30}{30}\right)^{1/3} = 1,0$; $k_\varepsilon = 1/(1,2+55 \cdot \varepsilon_1) = 1/(1,2+55 \cdot 0,0085) = 0,55 < 0,65$; $\varepsilon_x = 0,000616$; $\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 2 \cdot 10^{-3}) \cdot \cot^2\theta = 0,0085$; $\theta_{min} = 20^\circ + 10^4 \cdot \varepsilon_x = 26,16^\circ$.

Substituting in Equation (E 4.2-3): $V_{Rd,max}(\theta_{min}) = 1074,16 \text{ kN}$, and:

$V_{Rd,s} = \frac{A_{sw}}{s_w} \cdot f_{ywd} \cdot z \cdot \cot\theta = \frac{2\pi \cdot 5^2}{120} \cdot 828 \cdot 478,26 \cdot 2,03 = 1051,7 \text{ kN}$.

Thus:

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot z, \tag{E 4.2-4}$$

where: $k_v = \frac{0,40}{(1+1500 \cdot \varepsilon_x)} \cdot \left(1 - \frac{V_{Ed}}{V_{Rd,max}(\theta_{min})}\right) = 0,13$.

And substituting according to Equation (E 4.2-4):

$V_{Rd,c} = 0,13 \cdot \frac{\sqrt{30}}{1,5} \cdot 300 \cdot 828 = 120,98$ kN, thus: $V_{Rd} = 1051,7 + 120,98 = 1172,68$ kN.

$$V_{Rd} \le V_{Rd,max}. \tag{E 4.2-5}$$

$V_{Rd} = 1172,68 \text{ kN} > V_{Rd,max}(\theta_{min}) = 1074,16 \text{ kN}$.

It can be seen, that Inequality (E 4.2-5) is not satisfied. So, we must to change initial value of the θ in the range $\theta_{min} \le \theta \le 45°$ and repeat calculation again.

4.3 PUNCHING SHEAR

4.3.1 GENERAL CONSIDERATION

Punching shear is a local shear failure around a concentrated load on a slab. The most common situations where punching shear has to be considered is the region immediately surrounding a column in a flat slab or where a column is supported on a pad footing or foundation raft.

Punching shear failure may be considered to be shear failures rotated around the loaded area so as to give a failure surface which has the form a truncated cone. This is illustrated in Figure 4.3-1. The «***critical section***» for shear failure in a beam is transformed into a «***basic control perimeter***» when punching shear is considered. This conversion of the problem from a basically two-dimensional one to a three-dimensional problem does not change the basic phenomenon as described in Section 4.2, through there are a number of practical points which need further consideration.

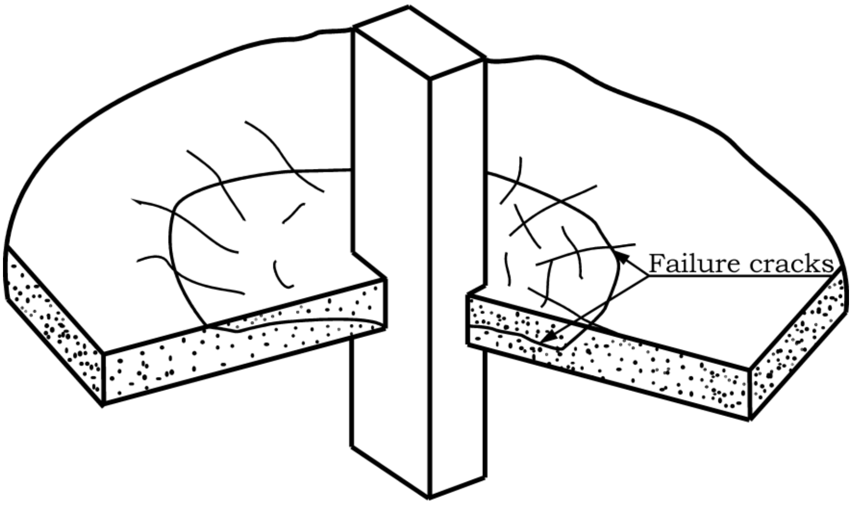


Figure 4.3-1 – Schematic illustration of a punching shear failure

In flat slab, punching shear failures normally develop around supported areas (columns, capitals, walls). In other cases (e.g. foundation slabs, transfer slabs, deck slabs of bridges) punching failures can also develop around loaded areas.

4.3.2 PUNCHING SHEAR FAILURE MODES IN FLAT SLABS

As was shown in [8] design of slabs with punching shear reinforcement typically considers several potential failure modes, see Figure 4.3-2:

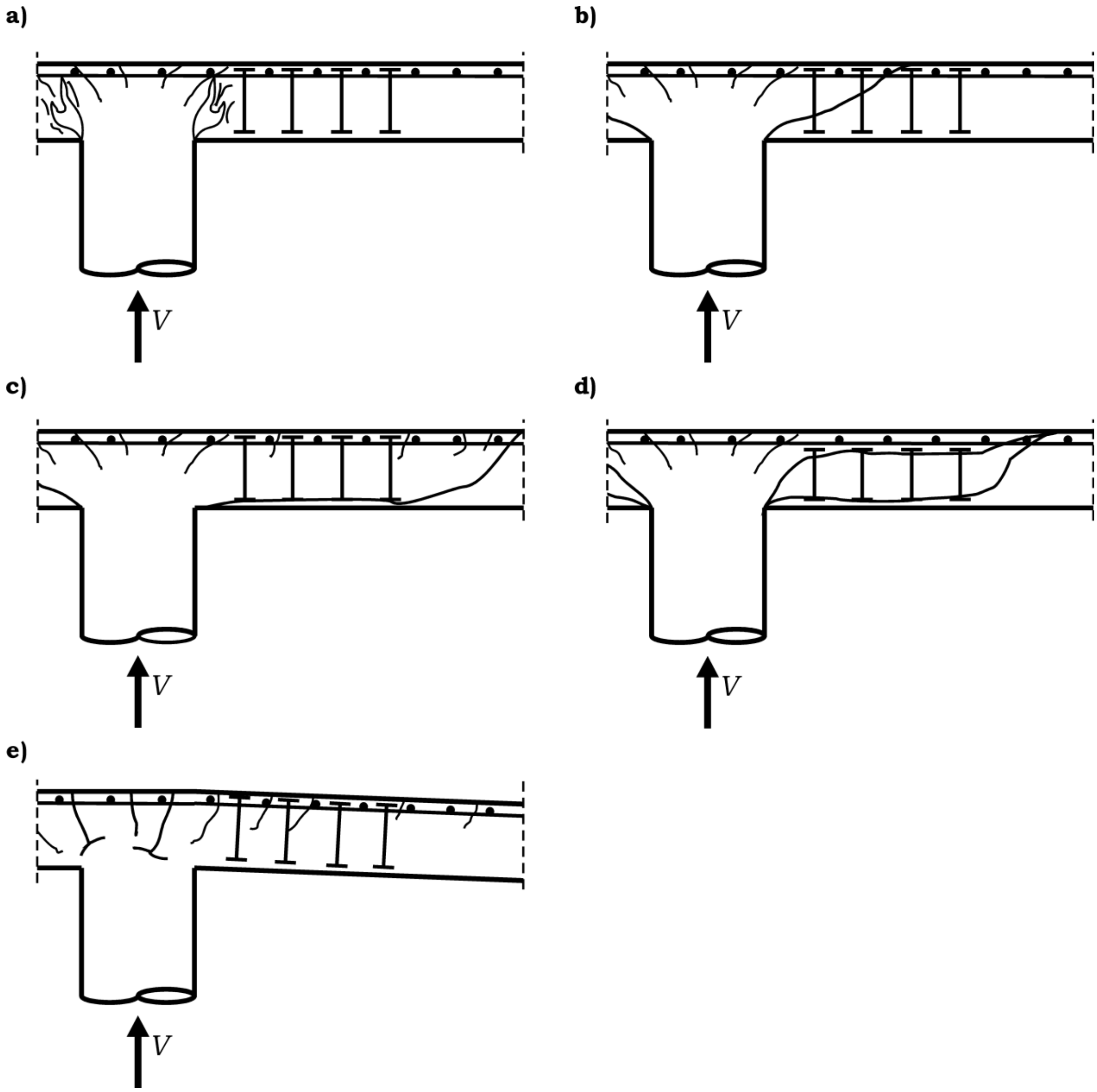
a) ***Crushing of compression struts*** (see Figure 4.3-2 a). This failure mode becomes governing for high amounts of bending and transverse reinforcement, where large compressive stresses develop in the concrete near the column region. Crushing of concrete struts limits thus the maximum strength that can be provided by a shear reinforcing system. This is instrumental for design as it determines the applicability of such systems with respect to the effective depth of the slab and size of support region.

b) ***Punching within the shear-reinforced zone*** (see Figure 4.3-2 b). Such failure develops for moderate or low amounts of shear reinforcement, when a shear crack localizes the strains within the shear-reinforced zone. Shear strength is thus governed by the contribution of concrete and of the transverse reinforcement. For design, this failure mode is used to determine the amount of shear reinforcement to be arranged.

c) ***Punching outside the shear-reinforced zone*** (see Figure 4.3-2 c). This failure mode may be governing when the shear-reinforced zone extends over a small region. Check of this failure mode is typically performed in design to determine the extent of the slab to be shear reinforced.

d) ***Delamination of concrete core*** (see Figure 4.3-2 d). When the shear reinforcement is not enclosing the flexural reinforcement, delamination of the concrete core may occur. This leads to a rather ductile failure mode but with limited strength and with loss of development on the flexural reinforcement. Typical detailing provided in codes of practice avoids the use of shear reinforcement systems leading to such failure mode.

e) ***Flexural yielding*** (see Figure 4.3-2 e). Slabs with low flexural reinforcement ratios and with sufficient transverse reinforcement can fail by development of a flexural plastic mechanism. Bending strength and not punching shear strength is thus governing for the strength of the slab.



a) – crushing of concrete struts; b) – punching within the shear-reinforced zone;
c) – punching outside the shear-reinforced zone; d) – delamination;
e) – flexural yielding

Figure 4.3-2 – Failure modes in flat slabs

4.3.3 BASIC CONTROL PERIMETER

The starting point for design for punching shear is the definition of the critical perimeter. This has greater importance than the selection of the critical section in a beam because, as perimeters closer to the loaded area are considered, the length of the perimeter rapidly gets shorter, and hence the shear force per unit length of the perimeter rapidly increases.

Observation of failures shows that the outer perimeter of punching failure takes the general form sketched in Figure 4.3-3. For this reason, EN 1992 [N3] proposed the idealized form shown in Figure 4.3-4.

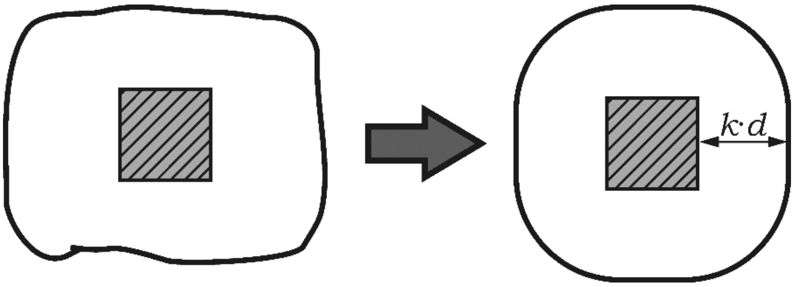


Figure 4.3-3 – Rationalization of failure perimeter [8]

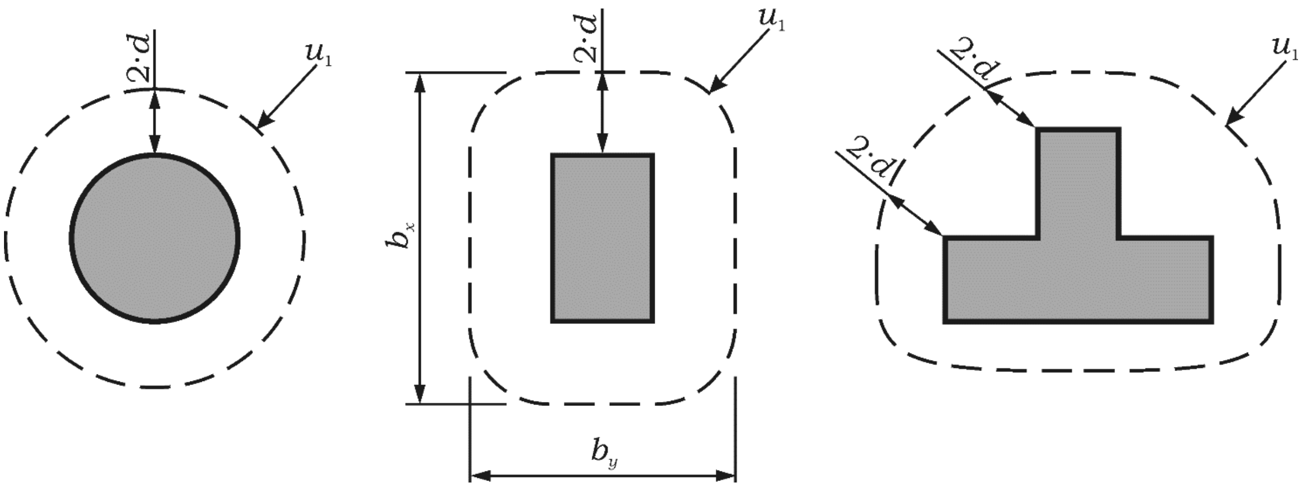


Figure 4.3-4 – Typical basic control perimeters around loaded areas (see Figure 6.13 from EN 1992 [N3])

Having chosen a basic form for the perimeter, it is next necessary to consider the distance from the loaded area at which it should be located. As it was shown in [8], in drafting EN 1992 [N3], it was decided that the shear strength should be given by the same formula for punching shear as is used for shear in beams. Having decided on this and on the shape of the perimeter, the distance of the perimeter from loaded area can be established from test data. This led to a value of $2{\cdot}d$.

So, in accordance with EN 1992 [N3], the basic control perimeter u_1 may normally be taken to be at distance $2 \cdot d$ from the loaded area and should be constructed so, as to minimize its length (see Figure 4.3-4).

The effective depth of the slab is assumed constant and may normally be taken as:

$$d_{eff} = \frac{(d_y + d_z)}{2}, \tag{4.3-1}$$

where: d_y and d_z are the effective depths of the reinforcement in two orthogonal directions.

Control perimeters at a distance less than $2 \cdot d$ should be considered where the concentrated force is opposed by a high pressure (e.g. soil pressure on a base), or by the effects of load or reaction within a distance $2 \cdot d$ of the periphery of area of application of the force.

For loaded areas situated near openings, if the shortest distance between the perimeter of the loaded area and the edge of the opening does not exceed $6 \cdot d$, that part of the control perimeter contained between two tangents drawn to the outline of the opening from the centre of the loaded area considered to be ineffective (see Figure 4.3-5).

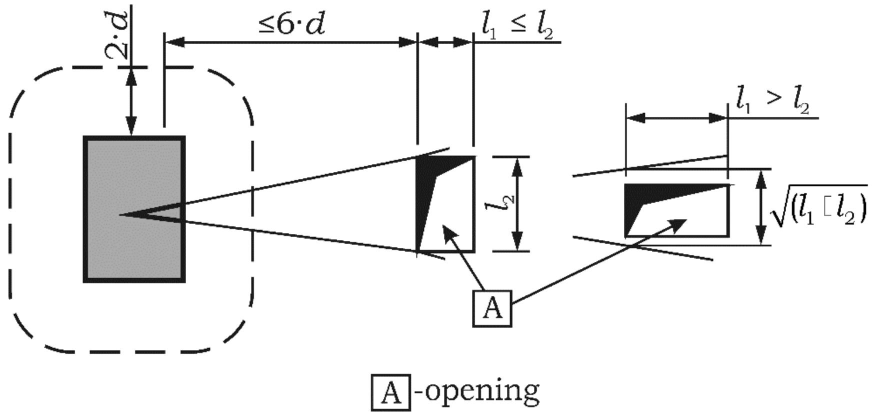


Figure 4.3-5 – Control perimeters near an opening (see Figure 6.14 from EN 1992 [N3])

For a loaded area situated near an edge or a corner, the control perimeter should be taken as shown in Figure 4.3-6, if this gives a perimeter (excluding the unsupported edges) smaller than that obtained from Figure 4.3-4 and Figure 4.3-5 above.

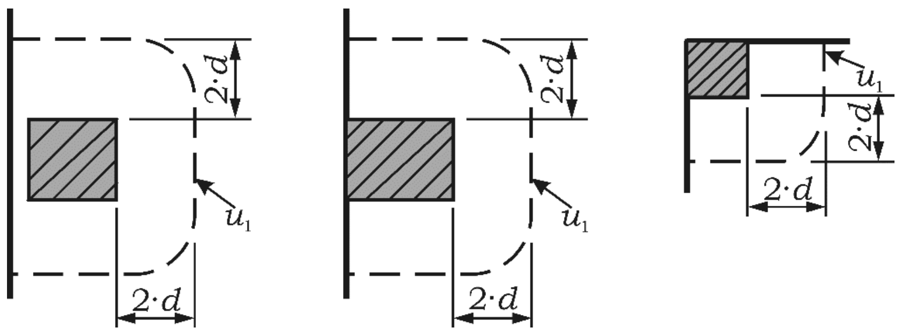


Figure 4.3-6 – Basic control perimeters for loaded areas close to or at edge or corner (see Figure 6.15 from EN 1992 [N3])

The control section is that which follows the control perimeter and extends over the effective depth d. For slabs of constant depth, control section is perpendicular to the middle plane of the slab. For slabs or footings of variable depth other than step footings, the effective depth may be assumed to be the depth at the perimeter of the loaded area as shown in Figure 4.3-7.

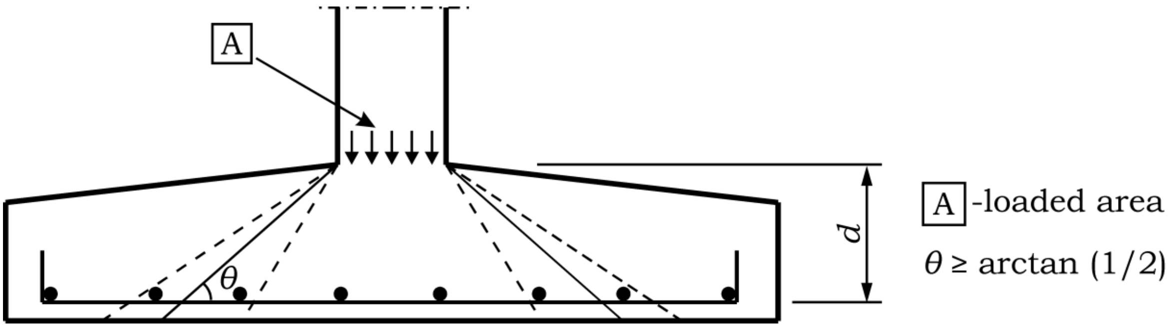


Figure 4.3-7 – Depth of control section in a footing with variable depth (see Figure 6.16 from EN 1992 [N3])

Further perimeters u_i, inside and outside the basic control area should have the same shape as the basic control perimeter.

For slabs with circular column heads for which $l_H < 2 \cdot h_H$ (see Figure 4.3-8) a check of the punching shear stresses is only required on the control section outside the column head. The distance of this section from the centroid of the column r_{cont} may be taken as:

$$r_{cont} = 2 \cdot d + l_H + 0,5 \cdot c, \tag{4.3-2}$$

where: l_H is the distance from the column face to the edge of the column head;

c is the diameter of a circular column.

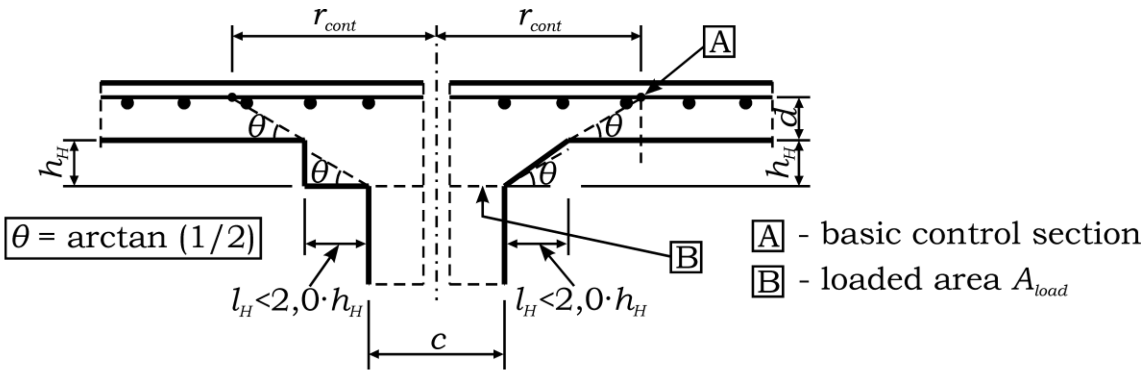


Figure 4.3-8 – Slab with enlarged column head where $l_H < 2,0 \cdot h_H$ (see Figure 6.17 from EN 1992 [N3])

For a rectangular column with a rectangular head with $l_H < 2,0 \cdot h_H$ (see Figure 4.3-8) dimensions l_1 and l_2 ($(l_1 = c_1 + 2 \cdot l_{H1},\ l_2 = c_2 + 2 \cdot l_{H2},\ l_1 \le l_2)$), the value r_{cont} may be taken as the lesser of:

$$r_{cont} = 2 \cdot d + 0,56 \cdot \sqrt{l_1 \cdot l_2}, \tag{4.3-3}$$

and:

$$r_{cont} = 2 \cdot d + 0,69 \cdot l_1. \tag{4.3-4}$$

For slabs with enlarged column heads where $l_H > 2{,}0 \cdot h_H$ (see Figure 4.3-9) control sections both within the head and in the slab should be checked.

For circular columns the distances from the centroid of the column to the control sections in Figure 4.3-9 may be taken as:

$$r_{cont,ext} = l_H + 2 \cdot d + 0{,}5 \cdot c; \tag{4.3-5}$$

$$r_{cont,\text{int}} = 2 \cdot (d + h_H) + 0{,}5 \cdot c. \tag{4.3-6}$$

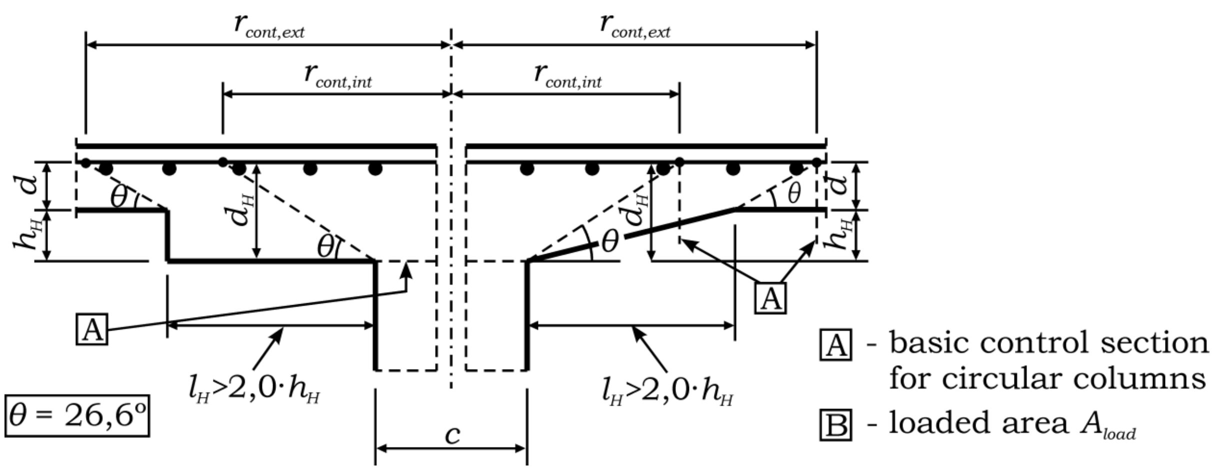


Figure 4.3-9 – Slab with enlarged column head where $l_H \geq 2 \cdot h_H$ (see Figure 6.18 from EN 1992 [N3])

4.3.4 PUNCHING SHEAR CALCULATION

4.3.4.1 General

The design procedure for punching shear is based on checks at the face of column and at the basic control perimeter u_1. If shear reinforcement is required a further perimeter $u_{out,ef}$ (see Figure 4.3-13) should be found where shear reinforcement is no longer required. The following design shear stresses (MPa) along the control sections, are defined:

$V_{Rd,c}$ is the design value of the punching shear resistance of slab without punching shear reinforcement along the control section considered.

$V_{Rd,cs}$ is the design value of the punching shear resistance of slab with punching shear reinforcement along the control section considered.

$V_{Rd,\max}$ - is the design value of the maximum punching shear resistance along the control section considered.

The following checks should be carried out:

a) At the column perimeter, or the perimeter of the loaded area, the maximum punching shear stress should not be exceeded: $V_{Ed} \leq V_{Rd,max}$;

b) Punching shear reinforcement is not necessary if: $V_{Ed} \leq V_{Rd,c}$.

c) Where V_{Ed} exceeds the value $V_{Rd,c}$ for the control section considered, punching shear reinforcement should be provided according to EN 1992 [N3].

It is usual to assume in design that the distribution of shear force around critical perimeter is uniform. In fact, this is untrue, particularly in the case slab-column connection where there is a moment transfer between the slab and the column. In such cases, a rigorous analysis would show that the distribution of shear varied markedly around the perimeter and was accompanied by torsional moments. Extensive experimental work shows that punching shear strength can be significantly reduced where substantial moment transfer occurs. This implies that punching shear is not entirely plastic phenomenon, and the shear at the ultimate limit state cannot be fully redistributed around the perimeter. A way of dealing with this in design is to increase the design shear force by a factor which is a function of the geometry of critical perimeter and the moment transferred. The provisions in EN 1992 [N3] introduce: a multiplier, β, to increase the average shear stress around the perimeter such that:

$$V_{Ed} = \beta \cdot \frac{V_{Ed}}{u_i \cdot d}, \tag{4.3-7}$$

where: d is the mean effective depth of the slab, which may be taken as $(d_y + d_z)/2$, where d_y, d_z are the effective depths in the y and z directions respectively of the control section;

u_i - is the length of the control perimeter being considered;

β is the shear multiplier, taking account of moment transfer.

The principle behind the definition of β is that the effect of transferring a moment between the slab and a column can be modeled by considering a distribution of shear around the control perimeter considered such it provides a moment equal to the moment transferred.

The distribution of shear assumed is sketched in Figure 4.3-10, which is redrawn after Figure 6.19 in EN 1992 [N3].

It will be seen that the magnitude of distributed shear force is a function of the moment transferred, the distance of the perimeter from the loaded area and the shape of the loaded area. It will be seen that:

$$\frac{\beta \cdot V_{Ed}}{u_i} = \frac{V_{Ed}}{u_i} + \Delta\upsilon, \tag{4.3-8}$$

hence:

$$\beta = 1 + \frac{\Delta\upsilon}{(V_{Ed} / u_i)} = 1 + \frac{\Delta\upsilon \cdot u_i}{V_{Ed}}. \tag{4.3-9}$$

In principle, Δv can be calculated from simple static, since the moment transferred between the slab and the column, ΔM_{Ed}, must be equal to the moment produced by the shear Δv distributed around the perimeter, u_i.

This may be written as follows (see Figure 4.3-10):

$$\Delta M_{Ed} = 4 \cdot \Delta v \cdot \left[\frac{c_1}{2} \cdot \frac{c_1}{4} + c_2 \cdot \left(\frac{c_1}{2} + 2 \cdot d \right) + 2 \cdot d \cdot \frac{\pi}{2} \cdot \left(\frac{c_1}{2} + 2 \cdot d \cdot \frac{2}{\pi} \right) \right] =$$

$$= \Delta v \cdot \left(\frac{c_1^2}{2} + c_1 \cdot c_2 + 4 \cdot c_2 \cdot d + 16 \cdot d^2 + 2 \cdot \pi \cdot d \cdot c_1 \right) = w_1 \cdot \Delta v.$$

Hence, $\Delta v = \Delta M_{Ed} / w_1$.

Substituting for Δv into equation for β gives:

$$\beta = 1 + \frac{\Delta M_{Ed} \cdot u_i}{w_1 \cdot V_{Ed}}. \tag{4.3-10}$$

It is found that a further correction factor is required to adjust the value of β for the aspect ratio of the column section. The equation in the code is thus:

$$\beta = 1 + k \cdot (\Delta M_{Ed} / V_{Ed}) \cdot (u_i / w_i). \tag{4.3-11}$$

It should be noticed that $\Delta M_{Ed} / V_{Ed}$ is actually the eccentricity of the concentrated load relative to the centroid of the loaded area. In some equations, EN 1992 [N3] uses «e» for this rather than $\Delta M_{Ed} / V_{Ed}$. It should also be noted that EN 1992 [N3] uses the term ΔM_{Ed} for the moment transferred between the slab and the column. This seems a possible cause of confusion, since the moment transferred is not the moment in the slab but rather the difference in moment between one side of the column and the other. For this reason, ΔM_{Ed} has been used here for the moment transferred.

Table 4.3-1 – Values of *k* for rectangular loaded areas

c_1 / c_2	≤ 0,5	1,0	2,0	≥ 3,0
k	0,45	0,60	0,70	0,80

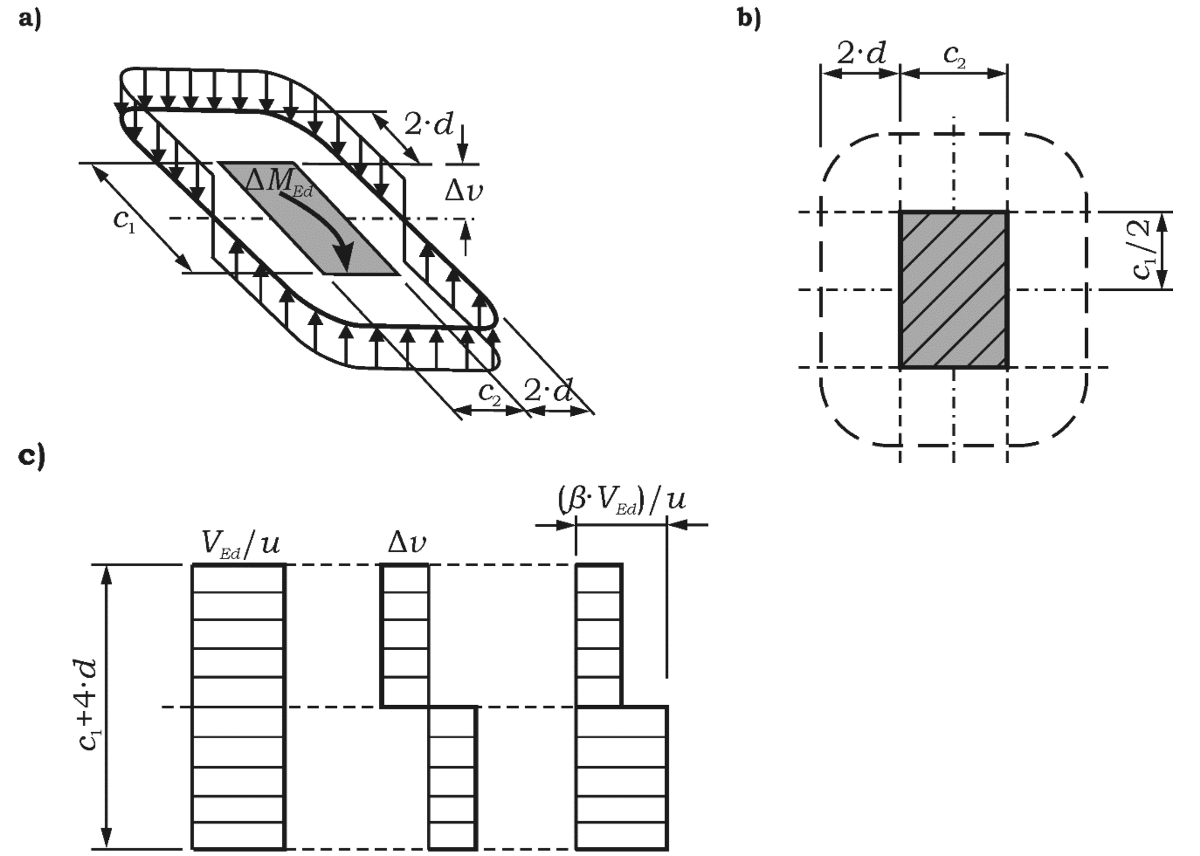


Figure 4.3-10 – Shear distribution due to an unbalanced moment at a slab-internal column connection (see Figure 6.19 from EN 1992 [N3])

The derivation of the equation for β for rectangular internal column has been derived in order to show the basic logic of the system. There are many other possible configurations of column and appropriate values are given in Table 4.3-2.

Table 4.3-2 – Values of the punching shear enhancement factor for various types of column (see Table 6.6 from EN 1992 [N3])

Case	Value for β
Internal rectangular column, uniaxial bending.	$\beta = 1 + k \cdot (\Delta M_{Ed} / V_{Ed}) \cdot (u_i / W_1)$; $W_1 = c_1^2 / 2 + c_1 \cdot c_2 + 4 \cdot c_2 \cdot d + 16 \cdot d^2 + 2 \cdot \pi \cdot d \cdot c_1$. Values of k from Table 4.3-1.
Internal rectangular column, biaxial bending.	$1 + 1,8 \cdot \sqrt{\{[\Delta M_{Edy} / (c_z + 4 \cdot d)]^2 + [\Delta M_{Edz} / (c_y + 4 \cdot d)]^2\}}$, ΔM_{Edy} and ΔM_{Edz} are respectively the moments transferred in the y and z directions while c_y and c_z are respectively the section dimensions in the y and z directions.
Rectangular edge column; axis of bending parallel to the slab edge, eccentricity is towards the interior.	$\beta = u_1 / u_1^*$ (i.e. shear is assumed uniformly distributed over perimeter u_1^* as defined in Figure 6.20 a of EN 1992 [N3], see Figure 4.3-11 a).
Rectangular edge column; bending about both axes. Eccentricity perpendicular to the slab edge is towards the exterior.	$\beta = 1 + k \cdot (\Delta M_{Ed} / V_{Ed}) \cdot (u_i / W_1)$. W_1 calculated by taking moments about the centroid of the control perimeter, u_1. Values of k from Table 4.3-1.

Table 4.3-2 (end)

Rectangular edge column; bending about both axes. Eccentricity perpendicular to the slab edge is towards the interior.	$\beta = u_1 / u_1^* + k \cdot (\Delta M_{Ed,par} / V_{Ed}) \cdot (u_1 / W_1)$; $W_1 = c_2^2 / 4 + c_1 \cdot c_2 + 4 \cdot c_1 \cdot d + 8 \cdot d^2 + \pi \cdot d \cdot c_2$. $\Delta M_{Ed,par}$ is the moment transfer about an axis perpendicular to the slab edge. Value of k is determined from Table 4.3-1 with c_1 / c_2 put equal to $0,5 \cdot (c_1 / c_2)$. c_1 is the section dimension perpendicular to the slab edge. c_2 is the dimension parallel to the slab edge.
Rectangular corner column, eccentricity is towards the interior.	$\beta = u_1 / u_1^*$ (i.e. punching force is considered uniformly distributed along perimeter u_1^* in Figure 6.20 b of EN 1992 [N3], see Figure 4.3-11 b).
Rectangular corner column, eccentricity is towards the exterior.	$\beta = 1 + k \cdot (\Delta M_{Ed} / V_{Ed}) \cdot (u_i / W_1)$; $W_1 = c_1^2 / 2 + c_1 \cdot c_2 + 4 \cdot c_2 \cdot d + 16 \cdot d^2 + 2 \cdot \pi \cdot d \cdot c_1$. Values of k from Table 4.3-1.
Interior circular column.	$\beta = 1 + 0,6 \cdot \pi \cdot (\Delta M_{Ed} / V_{Ed}) / (D + 4 \cdot d)$. D is the diameter of the column.
Circular edge or corner columns.	No information given.

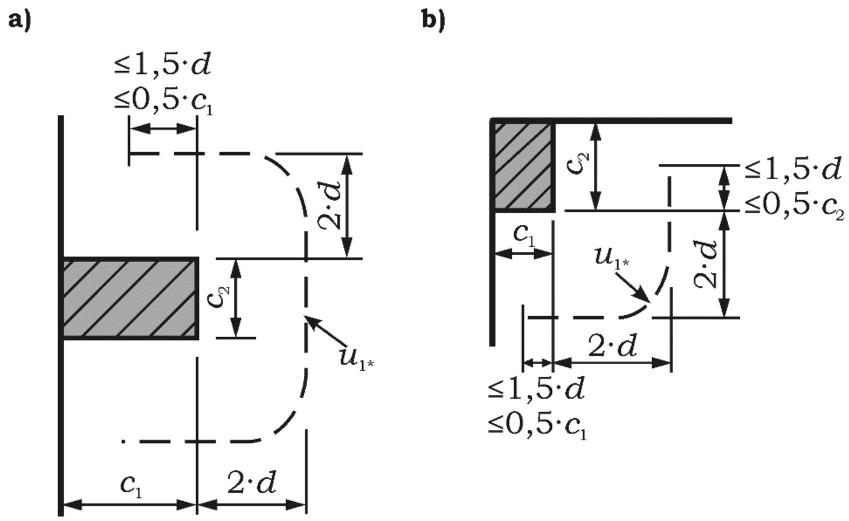


Figure 4.3-11 – Reduced basic control perimeter u_{1*} (see Figure 6.20 from EN 1992 [N3])

For structures where the lateral stability does not depend on frame action between the slabs and the columns, and where the adjacent spans do not differ in length by more than 25 %, approximation values for β may be used.

Recommended values are given in Figure 4.3-12.

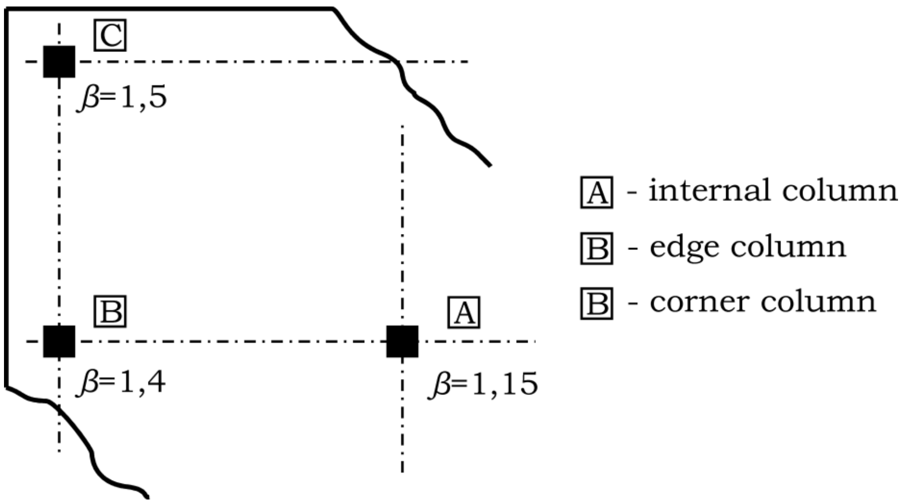


Figure 4.3-12 – Recommended values for β (see Figure 4.21 from EN 1992 [N3])

4.3.4.2 Punching shear resistance of slabs and columns bases without shear reinforcement

The punching shear resistance of a slab should be assessed for the basic control perimeter according to Section 4.3.3. The design punching shear resistance [MPa] may be calculated as follows:

$$v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \geq (v_{\min} + k_1 \cdot \sigma_{cp}), \tag{4.3-12}$$

where: f_{ck} is in MPa;

$k = 1 + \sqrt{\dfrac{200}{d}} \leq 2,0$, ($d$ in mm);

$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0,02$;

$\rho_{ly} \cdot \rho_{lz}$ relate to the bonded tension steel in y- and z- directions respectively. The values ρ_{ly} and ρ_{lz} should be calculated as mean values taking into account a slab width equal to the column width plus $3{\cdot}d$ each side.

$\sigma_{cp} = (\sigma_{cy} + \sigma_{cz})/2$, where: σ_{cy}, σ_{cz} are the normal concrete stresses in the critical section in y- and z- directions (MPa, positive if compression): $\sigma_{cy} = \dfrac{N_{Ed,y}}{A_{cy}}$ and $\sigma_{cz} = \dfrac{N_{Ed,z}}{A_{cz}}$. $N_{Ed,y}$, $N_{Ed,z}$ are the longitudinal forces across the full bay for internal columns and the longitudinal forces across the control section for edge columns. The force may be from a load or prestressing action, and A_c is the area of concrete according to the definition of N_{Ed}.

Note: *The values of $C_{Rd,c}$, v_{min} and k_1 for use in a Country may be found in its National Annex to EN 1992 [N3]. The recommended value for $C_{Rd,c}$ is $0,18/\gamma_c$, $k_1 = 0,1$ for v_{min} is given by Expression 4.2-27N from EN 1992 [N3].*

The punching resistance of column bases should be verified at control perimeters within $2 \cdot d$ from the periphery of the column.

For concentric loading the net applied force is equal to:

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed}, \tag{4.3-13}$$

where: V_{Ed} is the applied shear force;

ΔV_{Ed} is the net upward force within the control perimeter considered, i.e. upward pressure from soil minus self weight of base.

$$\upsilon_{Ed} = V_{Ed,red} / u \cdot d; \tag{4.3-14}$$

$$\upsilon_{Rd} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} \cdot 2 \cdot d / a \geq \upsilon_{\min} \cdot 2 \cdot d / a, \tag{4.3-15}$$

where: a is the distance from the periphery of the column to the control perimeter considered.

For eccentric loading:

$$\upsilon_{Ed} = \frac{V_{Ed,red}}{u \cdot d} \cdot \left[1 + k \cdot \frac{M_{Ed} \cdot u}{V_{Ed,red} \cdot W} \right], \tag{4.3-16}$$

where: k is defined in Section 4.3.4.1 (see Table 4.3-1) as appropriate, and W is similar to W_1, but for perimeter u.

4.3.4.3 Punching shear resistance of slabs and columns bases with shear reinforcement

Where shear reinforcement is required it should be calculated in accordance with following expression:

$$\upsilon_{Rd,cs} = 0,75 \cdot \upsilon_{Rd,c} + 1,5 \cdot (d / s_r) \cdot A_{sw} \cdot f_{ywd,ef} \cdot (1 / (u_1 \cdot d)) \cdot \sin a, \tag{4.3-17}$$

where: A_{sw} is the area of one perimeter of shear reinforcement around the column [mm^2];

s_r is the radial spacing of perimeters of shear reinforcement [mm];

$f_{ywd,ef}$ is the effective design strength of the punching shear reinforcement, according to $f_{ywd,ef} = 250 + 0,25 \cdot d \leq f_{ywd}$ [MPa];

d is the mean of the effective depths in the orthogonal directions [mm];

a is the angle between the shear reinforcement and the plane of slab.

If a single line of bent-down bars is provided, then the ratio d / s_r in Expression (4.3-17) may be given the value 0,67.

Detailing requirements for punching shear reinforcement are given in EN 1992 [N3].

Adjacent to the column the punching shear resistance is limited to a maximum of:

$$v_{Ed} = \frac{\beta \cdot V_{Ed}}{u_0 \cdot d} \leq v_{Rd,max}, \tag{4.3-18}$$

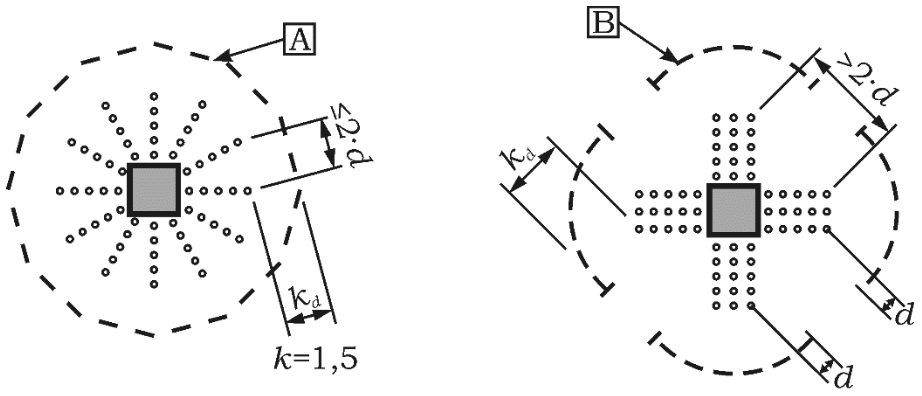
where: u_0 for an interior column is equal to the enclosing minimum periphery [mm]: for an edge column $u_0 = c_2 + 3 \cdot d \leq c_2 + 2 \cdot c_1$; for a corner column $u_0 = 3 \cdot d \leq c_1 + c_2$, where $c_1 + c_2$ are the column dimensions as it is shown in Figure 4.3-11.

Note: *The value of $v_{Rd,max}$ for use in a Country may be found in its National Annex to EN 1992 [N3]. The recommended value is $0,4 \cdot v \cdot f_{cd}$, where v is given in Expression (4.2-30).*

The control perimeter at which shear reinforcement is not required, u_{out}, (or $u_{out,ef}$, see Figure 4.3-13) should be calculated from following expression:

$$u_{out,ef} = \beta \cdot V_{Ed} / (v_{Rd,c} \cdot d). \tag{4.3-19}$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than $k \cdot d$ within u_{out} (or $u_{out,ef}$, see Figure 4.3-13). The recommended value is equal to 1,5.



A - Perimeter u_{out} B - Perimeter $u_{out,eff}$

Figure 4.3-13 – Control perimeters at internal columns (see Figure 6.22 from EN 1992 [N3])

It will be noted that the approach to the design of shear reinforcement is in principle different to that used in beams where the concrete is assumed not to contribute to the shear strength of a section reinforced in shear. Here, a contribution from the concrete is assumed but, unlike the general international consensus, the concrete contribution is reduced from the full while $\cot\theta$ is effectively assumed to be 1,5 rather than the more common 1,0 [8]. The formula has to be seen to be basically empirical, and one cannot been avoid an impression that all the parameters have been changed slightly from previous formulae just for the sake of being different.

It should be noted, that the concepts of a perimeter of shear reinforcement and the spacing of the perimeters are awkward, and will have to be interpreted for each particular case. A perimeter of reinforcement is presumably reinforcement provided on a perimeter parallel to the control perimeter u_1 but lying inside it.

Having found that the shear reinforcement is required on the control perimeter at a distance of $z \cdot d$ from face of the loaded area (first control perimeter), the control perimeter beyond which shear reinforcement is no longer required is found from:

$$u_{out,eff} = \frac{\beta \cdot V_{Ed}}{v_{Rd,c} \cdot d}. \qquad (4.3\text{-}20)$$

Having calculated $u_{out,eff}$ the distance of this perimeter from the loaded area may easily be calculated. While failure will not take place on this perimeter, it can occur on any perimeter within this (even, in theory, 1 mm inside it!).

The failure is predicted to occur over a distance, measured radially inwards from the perimeter, equal to $2 \cdot d$. Shear reinforcement for this failure must, therefore, be within this zone between the perimeter and a parallel perimeter $2 \cdot d$ inside it. Reinforcement close to either boundary of this zone is unlikely to be effective, and therefore a margin is needed on either edge. On the inner edge, reinforcement should not be closer than $0,3 \cdot d$ to the loaded area, which constitutes the inner boundary of the failure zone for the first control perimeter. The maximum radial spacing of shear reinforcement is $0,75 \cdot d$, and at least two sets of reinforcement are required. This would suggest, for the first control perimeter, that reinforcement could reasonably be provided at distance of $0,5 \cdot d$ and $1,25 \cdot d$ from the loaded area (column face), leaving a gap of $0,75 \cdot d$ between the outermost reinforcement and the first control perimeter.

It is suggested that this procedure is generalized into the following rules presented in [8].

Considering any possible perimeter, u_n, lying between $u_{out,eff}$ and the first control perimeter, shear reinforcement should be arranged so that a total area of shear reinforcement equal to $A_{sw,tot}$ lies within a zone between a perimeter situated $0,3 \cdot d$ and a perimeter situated $1,7 \cdot d$ inside u_n.

The radial spacing of the reinforcement should not exceed $0,75 \cdot d$, and the circumferential spacing should not exceed $1,5 \cdot d$ for the reinforcement for the first control perimeter or $2 \cdot d$ for subsequent perimeters. $A_{sw,tot}$ is given by the following expression:

$$A_{sw,tot} \geq u_n \cdot d \cdot (v_{Ed} - 0,75 \cdot v_{Rd,c}) / f_{ywd,eff}. \qquad (4.3\text{-}21)$$

This procedure is illustrated in Figure 4.3-14.

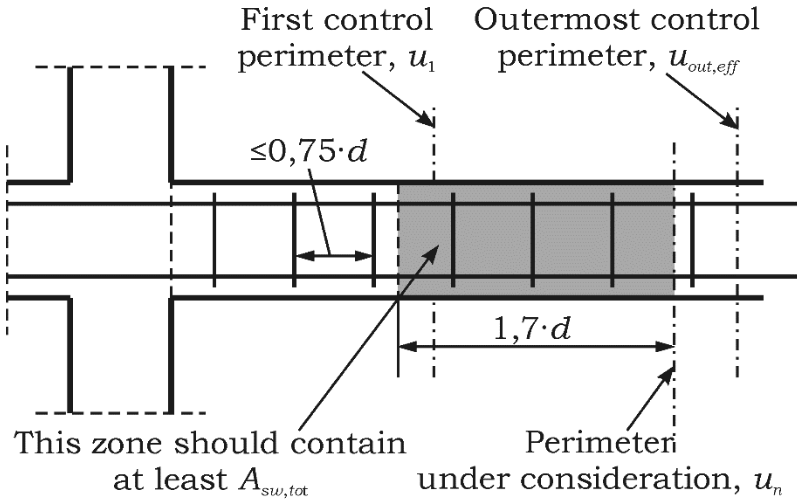


Figure 4.3-14 – Arrangement of punching reinforcement (see Figure 6.15 from [8])

Examples to Section 4.3

Example 1. Lightly loaded slab-column connection

Check the punching shear strength of the slab around an internal column supporting a 225 mm thick flat slab having 6 m spans in both directions. The column is 300x400 mm, and the design shear force established from of the slab, which has 6,5 m spans in both directions and supports a design ultimate load of 9 kN/m^2 , is 400 kN. Design of the slab for flexure gave an average value for the reinforcement ratio as 0,0077. The characteristic concrete strength is 30 N/mm^2. Bending is about the major axis of the column only, and the moment transferred between the slab and the columns is 45 kN·m.

Assuming 20 mm cover and 12 mm diameter bars give the average effective depth as: $225\text{-}20\text{-}12 = 193$ mm.

Table 4.8 of EN 1992 [N3] gives the basic design shear strength as 0,34 N/mm^2 for 30 N/mm^2 concrete. The critical perimeter is equal to: $2\cdot(300+400)+193\cdot 4\cdot 3{,}1416 = 3825$ mm.

To find the design effective shear stress it is now necessary to calculate β. For an internal rectangular column, Table 4.3-2 gives: $W_1 = c_1^2/2 + c_1\cdot c_2 + 4\cdot c_2\cdot d + 16\cdot d^2 + 2\cdot\pi\cdot d\cdot c_1$.

For the slab and column dimensions considered, this gives:
$W_1 = 400^2/2 + 400\cdot 300 + 4\cdot 300\cdot 193 + 16\cdot 193^2 + 2\cdot\pi\cdot 193\cdot 400 = 1512{,}6\cdot 10^3$;
$c_1/c_2 = 400/300 = 1{,}333$.

From Table 4.3-1, $k = 0{,}63$. This gives:
$\beta = 1 + 0{,}63\cdot 45\cdot 10^6\cdot 3825/(400\cdot 1000\cdot 1512{,}6\cdot 1000) = 1{,}17$.

The effective design shear stress is following:
$\upsilon_{Ed} = 400\cdot 1000\cdot 1{,}17/(3825\cdot 193) = 0{,}633$ N/mm^2.

The depth factor, k, for a slab with an effective depth less than 200 mm is 2,0. Equation (4.3-12) gives: $\upsilon_{Rd,c} = 0{,}18/1{,}5\cdot 2{,}0\cdot(100\cdot 0{,}0077\cdot 30)^{1/3} = 0{,}684$ N/mm^2.

This exceeds 0,633 N/mm², hence, no shear reinforcement is needed.

Example 2. Heavily loaded slab-column connection required shear reinforcement

The slab-column connection considered in Example E4.3-1 will now be designed for an increased design shear force of 600 kN. All other factor will be assumed to be as in Example 1.

In calculating v_{Ed}, W_1 and c_1 / c_2 will remain as in Example 1, but β will change because the eccentricity of the load will reduce. Value of β can now be calculated as follows: $\beta = 1 + 0,63 \cdot 45 \cdot 10^6 \cdot 3825 / (600 \cdot 1000 \cdot 1512,6 \cdot 1000) = 1,12$.

Value of v_{Ed} is now given by the following expression:
$v_{Ed} = 600 \cdot 1000 \cdot 1,12 / (3825 \cdot 193) = 0,91$ N/mm².

As before, $v_{Rd,c} = 0,684$, thus, punching shear reinforcement is required.

It is convenient at this stage to calculate u_{outer} to establish the extent of punching reinforcement: $u_{outer} = 1,12 \cdot 600 \cdot 1000 / (0,684 \cdot 193) = 5090$ mm.

This can easily be calculated to be a perimeter at a distance of $(5090 - 1400) / 2 \cdot \pi = 587$ mm or $3,04 \cdot d$ from the column face.

The area of reinforcement required across the failure zone can be calculated for the first control perimeter and for u_{outer}. At any point in between these, the required area can be found by linear interpolation. For the first control perimeter, the total area of reinforcement that must be provided within the perimeter is given by:
$A_{s,tot} = 3825 \cdot 193 \cdot (0,91 - 0,75 \cdot 0,684) / 435 = 674$ mm².

At the outer perimeter, the total area is given by:
$A_{s,tot} = 5090 \cdot 193 \cdot (0,91 - 0,75 \cdot 0,684) / 435 = 896$ mm².

Figure E 4.3-1.1 shows the required steel area for perimeters within this area.

Assuming that the first perimeter of reinforcement is provided at 125 mm from the column face and then at 150 mm centres as far as necessary, the length of each perimeter can be calculated, and, hence, from the maximum spacing rules, the minimum number of bars which should be provided. For example, the length of a perimeter 125 mm from the column face is 2185 mm. The maximum permissible spacing around the perimeter is $1,5 \cdot d = 290$ mm. To meet this requirement, 2185/290=8 bars are required. Trial and error suggests that 8 mm diameter bars, which are the smallest bars generally available, should be used on all perimeters. Table E 4.3-1 gives the steel areas supplied on each perimeter in this case.

The areas in Table E 4.3-1 together with the assumption that reinforcement is only effective for a particular failure zone when it is at least $0,3 \cdot d$ from the inner adge of the failure zone permits the total reinforcement provided for any outer perimeter of a failure zone to be calculated. This is plotted in Figure E 4.3-1. It will be seen that adequate reinforcement is provided at all perimeters and that all the detailing rules are obeyed. It will also be seen that the amount of reinforcement provided is actually defined by the minimum circumferential and radial bar spacing rules rather than the required strength. This seems likely to be generally the case. In particular, the radial maximum spacing of $0,75 \cdot d$ ensures that, in many place, these are, unavoidably, three perimeters of reinforcement within the failure zone in many situations.

Table E 4.3-1 – Calculations for each perimeter of reinforcement in Example 2 [8]

	Perimeter of reinforcement		
	1	2	3
Distance from column face (mm)	125	250	375
(Distance from column face)/d	0,65·d	1,3·d	1,9
Perimeter (mm)	2185	2970	3756
Maximum spacing	290	290	290
Number of bars	8	10	13
Bar diameter	8	8	8
Area on perimeter	402	603	553

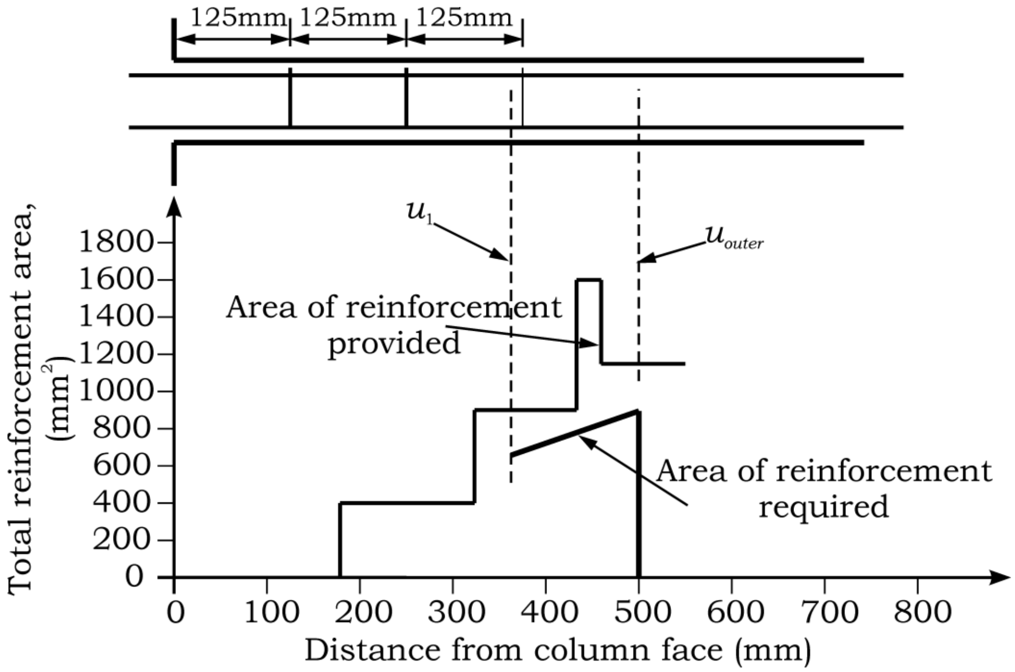


Figure E 4.3-1 – Provision of shear reinforcement in Example E 4.3-2 (Example 6.3 from [8])

4.3.5 PUNCHING SHEAR CALCULATION IN ACCORDANCE WITH *fib* MC 2010 [N5]

4.3.5.1 General

As was stated shown in ***fib*** MC 2010 [N5], punching shear is a failure mode that may potentially develop with limited deformation capacity. In such cases (brittle failure) the effects of imposed deformations (temperature, creep, shrinkage, sefflements, etc.) should be taken into account in the design. The influence of imposed deformations can, however, be neglected if sufficient deformation capacity is provided in accordance with ***fib*** MC 2010 [N5]. Some suggested strategies for increasing the deformation capacity are:

- choice of a sufficient large support region and depth of slab in combination with fair ratios of bending reinforcement;
- use of punching shear reinforcement.

In flat slabs, safety against punching shear is particularly significant as punching of a slab around one column can propagate to adjacent columns leading to complete collapse of the structure. In order to avoid progressive collapses, one of the following strategies should be adopted:

- increase of the deformation capacity at failure (see above) to allow internal forces redistribution, or
- arrangement of appropriate integrity reinforcement for slabs with limited deformation capacity.

4.3.5.2 Design shear force, shear-resisting effective depth and control perimeter

Range of application.

Punching shear can result from a concentrated load or reaction applied over a relatively small area. The rules for design presented here after apply to punching of flat slabs or foundation slabs.

As described in ***fib*** MC 2010 [N5], different levels of approximation may be regarded. ***fib*** MC 2010 has four level of design of which Level 1 to Level 3 are intended for design and Level 4 for assessment of the existing structures.

Design shear force.

The design shear force with respect to punching (V_{Ed}) is calculated as the sum of design shear forces acting on a ***basic control perimeter*** (b_1).

Basic control perimeter (b₁).

It may normally be taken to be at a distance $0{,}5 \cdot d_V$ from support region or loaded area (see Figure 4.3-15) and should be constructed so as to minimize its length. The length of the control perimeter is limited by slab edges (see Figure 4.3-15 d).

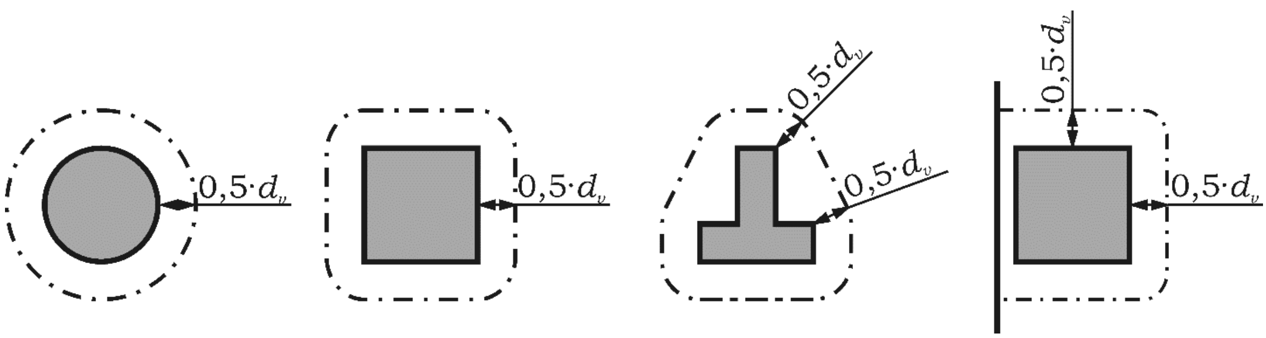


Figure 4.3-15 – Basic control perimeter around column (see Figure 7.3-19 from *fib* MC 2010 [N5])

The effective depth of the slab (d_V) shall account for the effective level of the support region, see Figure 4.3-16.

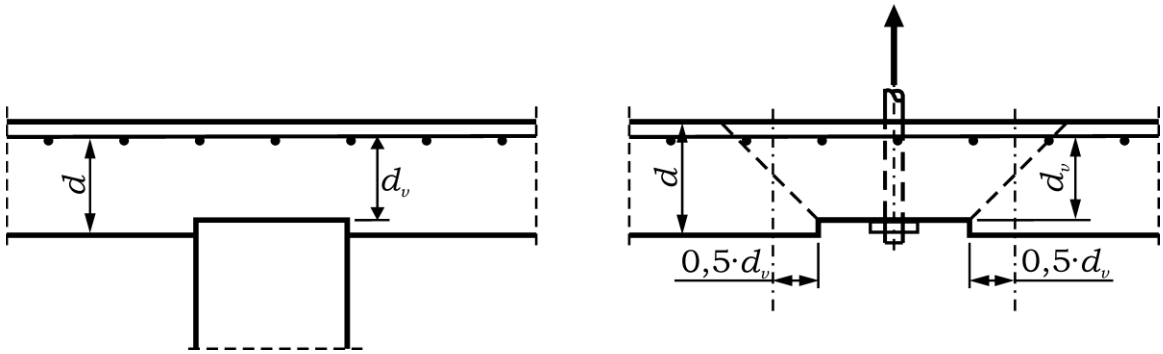


Figure 4.3-16 – Effective depth of the slab accounting for support area penetration (d_V) and effective depth for bending calculation (*d*) (see Figure 7.3-20 from *fib* MC 2010 [N5])

In the case of slabs of non-uniform thickness control section at a greater distance from the support area may be govern for punching shear resistance (see Figure 4.3-17).

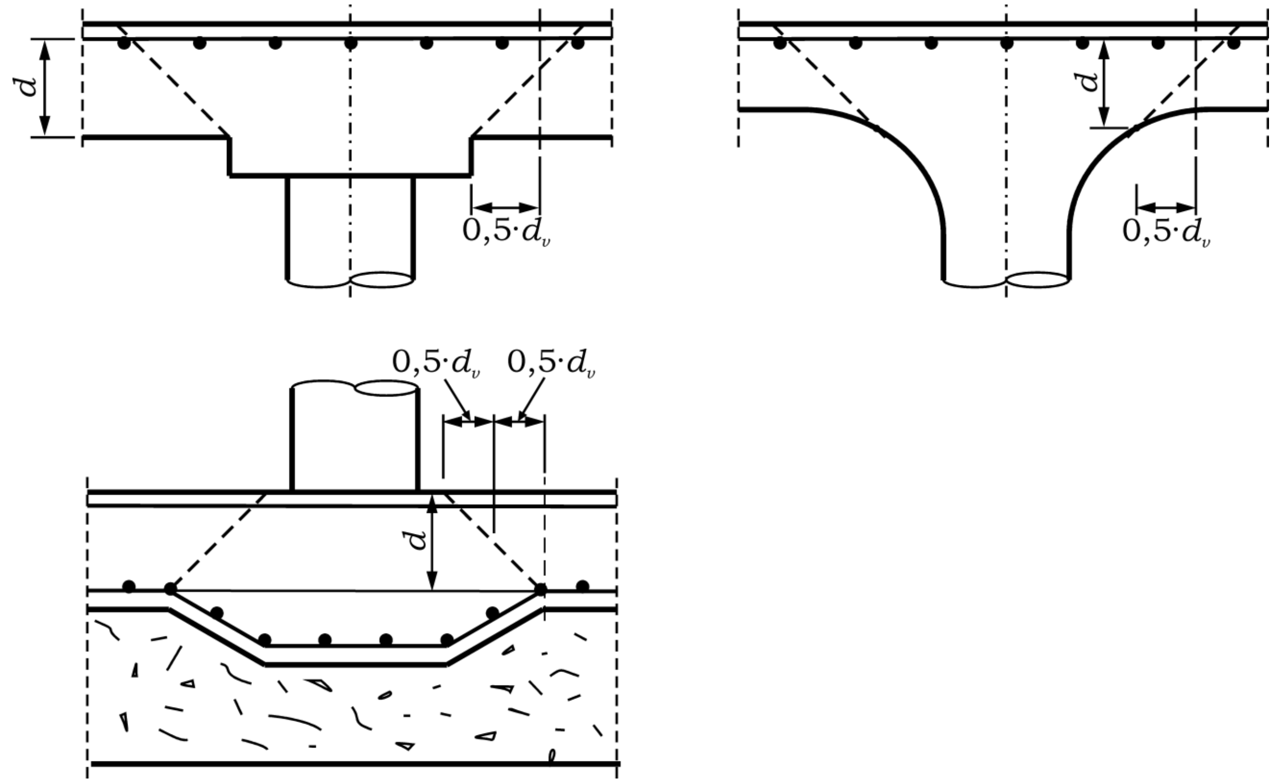


Figure 4.3-17 – Choice of potentially governing control section (see Figure 7.3-21 from *fib* MC 2010 [N5])

For flat slabs and footing, the design shear force is equal to the value of the column reaction minus the sum of the actions applied inside the basic control perimeter (such as gravity loads, earth pressure of footings and deviation force of prestressing cables), as it was shown in case of application of the EN 1992 [N3] provisions.

For walls and long columns, the design shear force may be calculated using the wall reaction in a distance $1,5 \cdot d_V$ from the edges (see Figure 4.3-18).

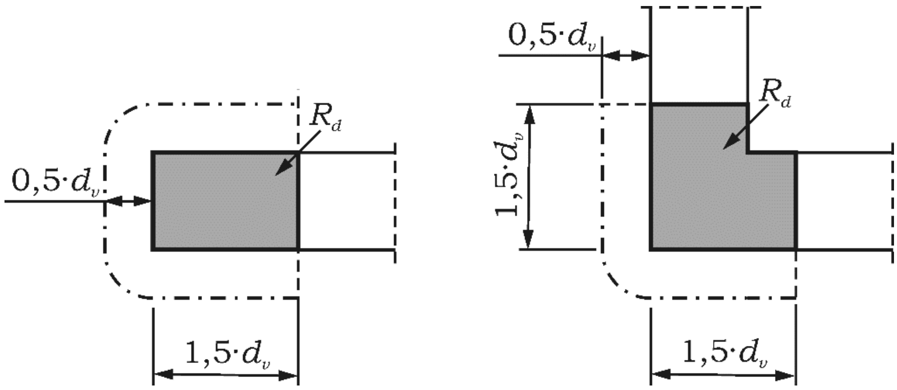


Figure 4.3-18 – Basic control perimeter for walls and long columns

Shear-resisting control perimeter (b_0).

The shear-resisting control perimeter accounts for the non-uniform distribution of the shear along the basic control perimeter.

The general procedure for calculation of b_0 is suggested when significant concentrated load ($\geq 0,2 \cdot V_{Ed}$) are applied near support regions (closer then $3 \cdot d$ of the border of the support region) or for highly asymmetrical slabs.

For the calculation of the punching shear resistance, a shear-resisting *control perimeter* b_0 is used. For a general case, perimeter b_0 can be obtained on the basis of shear fields as:

$$b_0 = \frac{V_{Ed}}{V_{perp,d,max}}, \tag{4.3-22}$$

where: $V_{perp,d,max}$ is the maximum value of the projection of the shear force perpendicular to the *basic control perimeter*.

In accordance with ***fib*** MC 2010 [N5], a non-uniform distribution of the shear forces may result due to:

1. Concentrations of the shear forces at the corner of large supported areas. This effect can approximately taken into account by reducing the basic control perimeter ($b_{1,red}$) assuming that the length of its straight segments does not exceed $3 \cdot d_V$ for each edge (Figure 4.3-19 a);

2. Geometrical and statical discontinuities of the slab. In the presence of opening and insert, the basic control perimeter ($b_{1,red}$) is to be reduced according to the rules of Figures 4.3-19 b and Figure 4.3-20;

3. Concentration of the shear force due to moment transfer between the slab and supported area. This effect can approximately be taken into account by multiplying the length of the reduced basic control perimeter ($b_{1,red}$) by the coefficient of eccentricity (k_e): $b_0 = k_e \cdot b_{1,red}$.

In presence of column to slab moments, concentrations on the shear field shall be accounted for by reducing the control perimeter by factor:

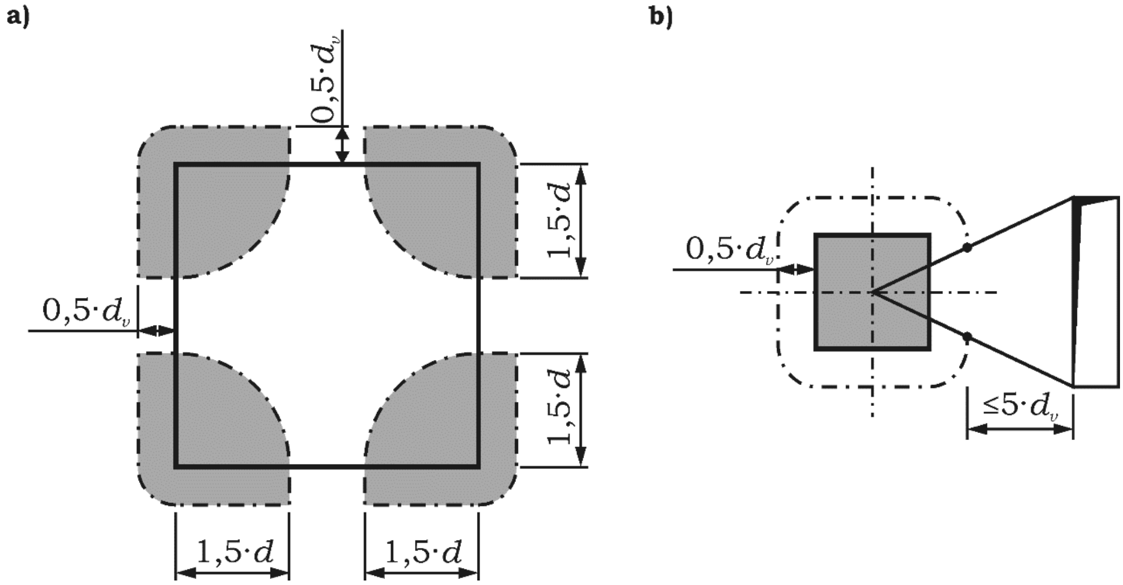
$$k_e = \frac{1}{1 + (e/b)}, \tag{4.3-23}$$

where: $e = \left|\frac{M_{Ed}}{N_{Ed}}\right|$ is the load eccentricity;

b is the diameter of the circle with the same surface as the support region.

For structures where the lateral stability does not depend on frame action between the slabs and the columns and where the adjacent spans do not differ in length by more than 25 %, the following values may be adopted for coefficient k_e: 0,90 for inner columns; 0,70 for edge columns; 0,65 for corner columns.

4. Presence of significant loads near the supported area. In cases where significant concentrated loads ($\geq 0,2 \cdot V_{Ed}$) are applied near the supported area (close than $3 \cdot d_V$ from edge of the supported area) the general procedure for calculating b_0 should be used, refer to Equation (4.3-22).



a) – long columns; b) – presence of openings

Figure 4.3-19 – Reduction of control perimeter (see Figure 7.3-24 from *fib* MC 2010 [N5])

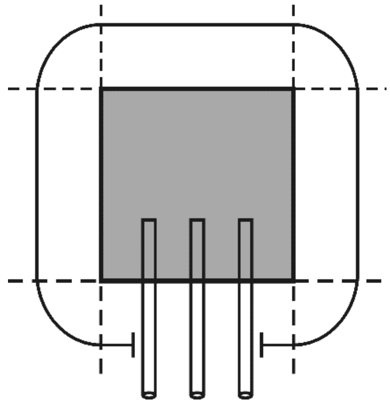


Figure 4.3-20 – Reduction of control perimeter in presence of pipes or inserts (see Figure 7.3-23 from *fib* MC 2010 [N5])

Cast-in pipes, pipe bundles or slabs inserts, where the distance from the supported area is less than $5 \cdot d_V$ shall be placed perpendicular to the control perimeter (see Figure 4.3-19) and the length of the shear-resisting control perimeter shall be reduced accordingly.

4.3.5.3 Punching shear resistance calculation

4.3.5.3.1 Design equations

The punching shear resistance is calculated as:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \geq V_{Ed}. \tag{4.3-24}$$

The design shear resistance attributed to the concrete may be taken as follows:

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_0 \cdot d_V, \text{ with } f_{ck} \text{ in [MPa]}. \tag{4.3-25}$$

The parameter k_ψ depends on the rotation of the slab around the support region and it is calculated as:

$$k_\psi = \frac{1}{1{,}5 + 0{,}9 \cdot \psi \cdot d \cdot k_{dg}} \leq 0{,}6, \tag{4.3-26}$$

where: d is the mean value (in [mm]) of the (flexural) effective depth in x and y directions.

Provided that the size of the maximum aggregate particles, d_g, is not less than 16 mm, k_{dg} can be taken as $k_{dg} = 1{,}0$. This is evidence that the punching shear resistance is influenced by the maximum size of the aggregate (d_g). If concrete with a maximum aggregate smaller than $d_g = 16$ mm is used, the value of k_{dg} is assessed as:

$$k_{dg} = \frac{32}{16 + d_g}, \tag{4.3-27}$$

where: d_g is the maximum aggregate size.

For aggregate sizes larger than 16 mm, Equation (4.3-27) may also be used.

For high strength and light-weight concrete, the aggregate particles may be broken, and value d_g should be the assumed to be 0.

The parameter ψ refers to the rotation of the slab around the support region outside the critical shear crack (see Figure 4.3-21).

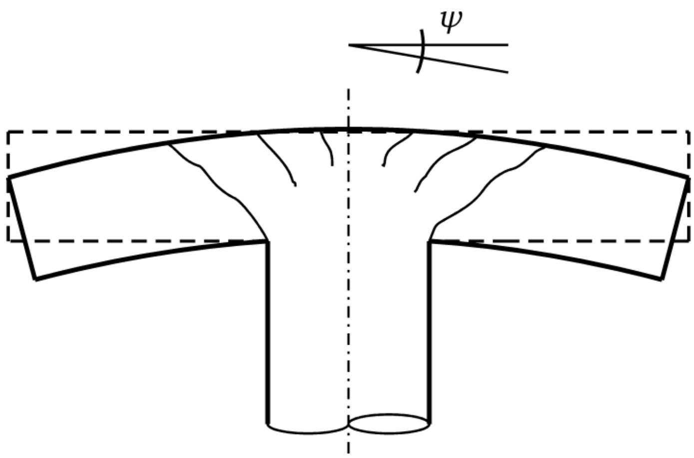


Figure 4.3-21 – Rotation (ψ) of slab around the support region (see Figure 7.3-25 from *fib* MC 2010 [N5])

The design shear resistance provided by the stirrups (or other type of the transverse reinforcement) may be calculated as:

$$V_{Rd,s} = \sum A_{sw} \cdot k_e \cdot \sigma_{sw} \cdot \sin a, \tag{4.3-28}$$

where: $\sum A_{sw}$ is the sum of the cross-sectional area of all shear reinforcement suitably anchored, or developed, and intersected by the potential failure surface (conical surface with angle 45°) within the zone bounded by $0{,}35 \cdot d_V$, and d_V from the border of the support region (Figure 4.3-22).

The angle a is taken with respect to the reference surface of the slab and σ_{sw} is the stress that can be mobilized in the shear reinforcement and is taken as:

$$\sigma_{sw} = \frac{E_s \cdot \psi}{6} \cdot (1 + \frac{f_{bd}}{f_{ywd}} \cdot \frac{\varnothing}{\varnothing_w}) \le f_{ywd}, \tag{4.3-29}$$

where: $\varnothing_w$ is the shear reinforcement diameter;

f_{bd} is the bond strength which is taken as 3,0 MPa in accordance with ***fib*** MC 2010 [N5].

In order to ensure sufficient deformation capacity at failure, the design of slabs with punching shear reinforcement requires a minimum amount of transverse reinforcement such that:

$$\sum A_{sw} \cdot k_e \cdot f_{ywd} \ge 0{,}5 \cdot V_{Ed}. \tag{4.3-30}$$

The maximum punching shear resistance (for the case where transverse reinforcement is provide) is limited by crushing of concrete struts near the support region such that:

$$V_{Rd,\max} = k_{sys} \cdot k_{\psi} \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_0 \cdot d_V \le \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_0 \cdot d_V, \tag{4.3-31}$$

where the coefficient k_{sys} accounts for the performance of punching shear reinforcing system. In the absence of other data, for a reinforcement detailed a value $k_{sys} = 2,0$ can be adopted. Other values may be used for coefficient k_{sys} provided that they are experimentally verified. In accordance with ***fib*** MC 2010 [N5] the coefficient k_{sys} is taken as 2,4 for stirrups and 2,8 for studs provided that radial spacing to the first perimeter of shear reinforcement from the column face is less or equal to $0,5 \cdot d_V$ and spacing of successive perimeters of shear reinforcement is less than $0,6 \cdot d_V$.

The spacing of vertical legs of shear reinforcement around perimeter should not exceed $3 \cdot d_V$ where the part of perimeter is assumed to contribute to the shear capacity.

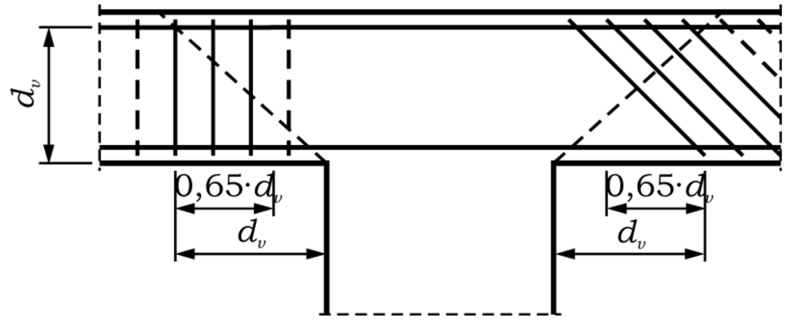


Figure 4.3-22 – Shear reinforcement activated at failure (see Figure 7.3-26 from *fib* MC 2010 [N5])

4.3.5.3.2 Calculation of rotation around the support region

Level I of approximation. For regular flat slabs designed according to an elastic analysis and without significant redistribution of internal forces, a safe estimate of the rotation of a slab at failure is:

$$\psi = 1,5 \cdot \frac{r_s}{d} \cdot \frac{f_{yd}}{E_s}, \tag{4.3-31}$$

where: r_s indicates the position where the radial bending moment is zero with respect to the column axis.

Level II of approximation. In cases where significant bending moment redistribution is considered in the design of the bending reinforcement, the slab rotation can be calculated as:

$$\psi = 1,5 \cdot \frac{r_s}{d} \cdot \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed}}{m_{Rd}}\right)^{1,5}, \tag{4.3-32}$$

where: m_{Ed} is the average bending moment per unit length in the support strip of the column, which is assumed to be of width $1{,}5 \cdot r_s$, where $r_s = 0{,}226 \cdot L_x$ or $0{,}226 \cdot L_y$ (for the considered direction);

m_{Rd} is the design average flexural strength per unit length in the support strip (for the considered direction).

It can be seen, that the same value for r_s as that for Level I of approximation can be adopted $(r_s = 0{,}226 \cdot L_x$ or $0{,}226 \cdot L_y)$.

The width of the support strip for calculation of the design average flexural strength is:

$$b_s = 1{,}5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} \leq L_{\min}, \tag{4.3-33}$$

where close to slab edge, the width of the strip is limited to b_s according to Figure 4.3-23.

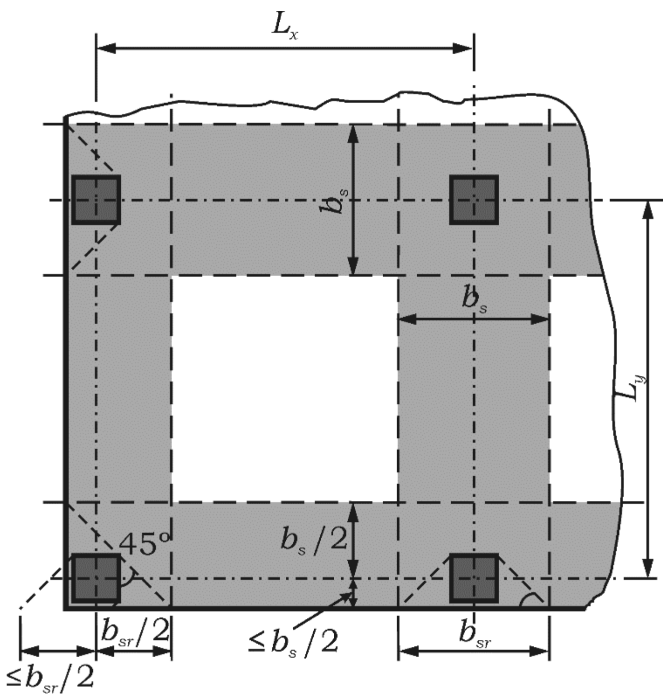


Figure 4.3-23 – Support strip dimensions (see Figure 7.3-27 from *fib* MC 2010 [N5])

The average bending moment acting in the support strip (m_{Ed}) in general case can be approximated for each reinforcement direction and support type as:

- for inner columns (top reinforcement in each direction):

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{2 \cdot b_s} \right). \tag{4.3-34}$$

- for edge columns: m_{Ed} is equal to:

a) when calculations are made considering m_{Rd} calculated for the smallest of the upper and lower reinforcement perpendicular to the edge:

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right). \quad (4.3\text{-}35)$$

b) when calculations are made considering m_{Rd} calculated with the upper reinforcement parallel to the edge:

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{2 b_s} \right) \geq \frac{V_{Ed}}{4}. \quad (4.3\text{-}36)$$

- for corner columns (tension reinforcement in each direction):

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \geq \frac{V_{Ed}}{2}. \quad (4.3\text{-}37)$$

The rotation has to be calculated along the two main directions of the reinforcement.

Level III of approximation. Level III of approximation is recommended for irregular slabs or for flat slabs where the ratio of the span length (L_x / L_y) is not between 0,5 and 2,0. The width of the support strip can be calculated as in Level II of approximation taken $r_{s,x}$ and $r_{s,y}$ as the maximum value in the direction being investigated. For edge or corner columns the following minimum value of r_s has to be considered: $r_s \geq 0,67 \cdot b_{sr}$.

The coefficient 1,5 in Equation (4.3-31) and Equation (4.3-32) can be replaced by 1,2 if:

- r_s is calculated for the flat slab using linear elastic (uncracked) model;
- m_{Ed} is calculated from a linear elastic (uncracked) model as the average value of bending moment in the support strip.

Level IV of approximation. The rotation angle ψ can be calculated on the basis of a non-linear analysis of the structure and with full account of cracking, tension-stiffening effects, yielding of the reinforcement and any other non-linear actions relevant to providing an accurate assessment of structure.

Analytical or numerical techniques (for example, finite elements, finite differences, etc.) may be used for Level IV approximation. This level of approximation is in principle only suggested for the case of assessment of complex existing structures.

Parameter m_{Ed} has to be calculated consistently with the method used for determining the flexural reinforcement and is to be determined at the edge of the supported area maximizing m_{Ed} (see Figure 4.3-24).

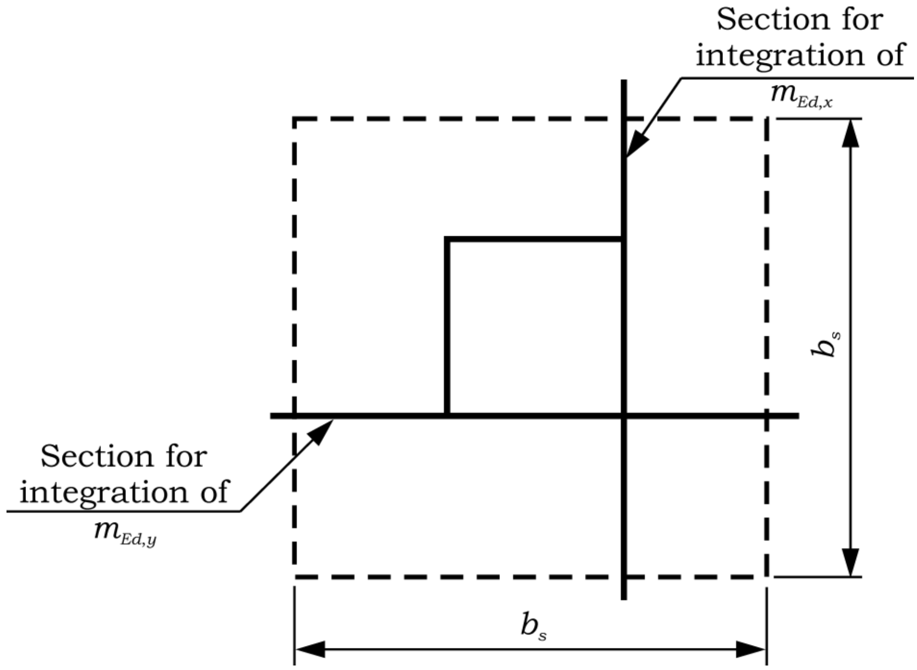
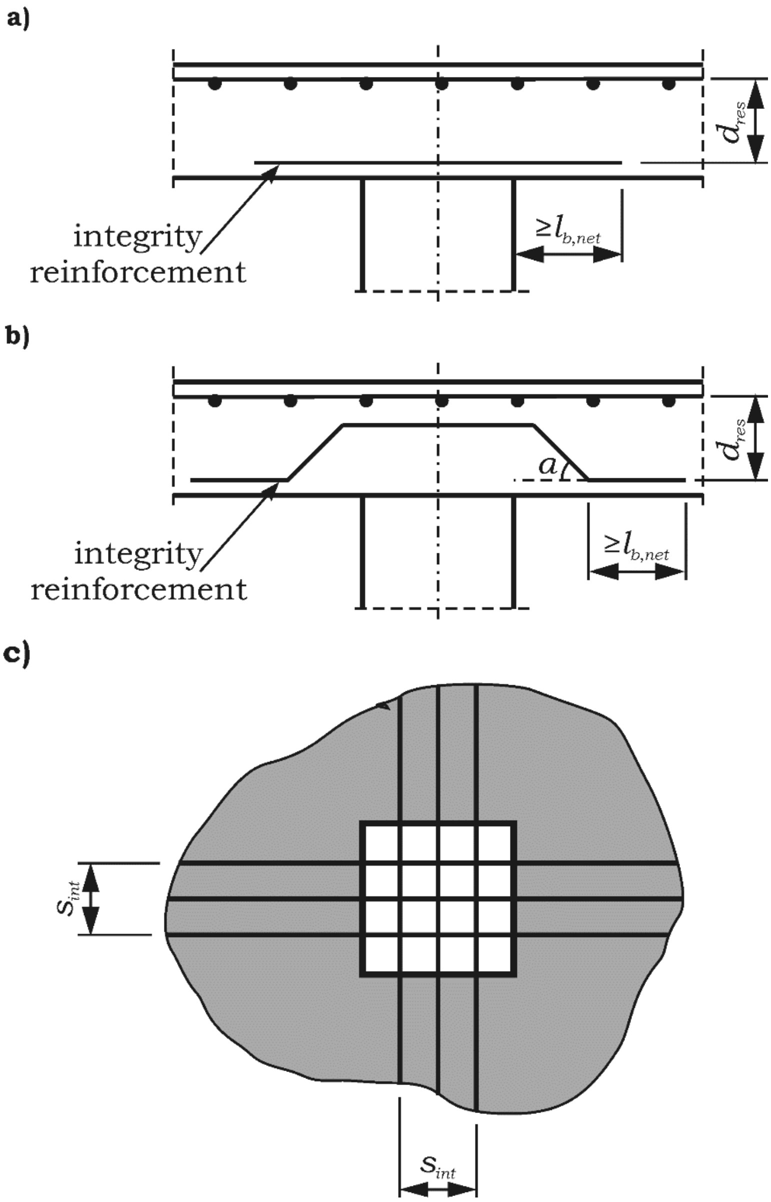


Figure 4.3-24 – Example of sections for integration of support strip moments (see Figure 7.3-31 from *fib* MC 2010 [N5])

4.3.5.4 Integrity reinforcement

The design shear for calculation of the integrity reinforcement can be calculated on the basic of an accidental design situation where progressive collapse has to be avoided. Slabs without shear reinforcement, or with insufficient deformation capacity, must be provided with integrity reinforcement (see Figure 4.3-25).



a) – straight bars; b) – bent-up; c) – slab plan

Figure 4.3-25 – Integrity reinforcement (see Figure 7.3-34 from *fib* MC 2010 [N5])

The resistance provided after punching by the integrity reinforcement can be calculated as follows:

$$V_{Rd,\mathrm{int}} = \sum A_s \cdot f_{yd} \cdot \left(\frac{f_t}{f_y} \right)_k \cdot \sin \alpha_{ult} \leq \frac{0,5 \cdot \sqrt{f_{ck}}}{\gamma_c} \cdot d_{res} \cdot b_{\mathrm{int}}, \tag{4.3-38}$$

where: A_s refers to the sum of the cross-sections of all reinforcement suitably developed beyond the supported area on the compression side of the slab or to well-anchored bent-up bars;

f_{yd} is the design yield strength of the integrity bars; the ratio $(f_t / f_y)_k$ and parameter ε_{uk} are defined in accordance with EN 1992 [N3] and depends on the ductility class of the reinforcement (see Chapter 3);

α_{ult} is the angle of the integrity bar with respect to the slab plan at failure (after development of plastic deformations in the post – punching regime), as it is shown in Table 4.3-3.

α is the angle of the integrity bars with respect to the slab plan (before punching occurs);

d_{res} is the distance between centroid of the flexural reinforcement ratio and the centroid of the integrity reinforcement (see Figure 4.3-25 a and Figure 4.3-25 b);

b_{int} is the control perimeter activated by the integrity reinforcement after punching. It can be calculated as:

$$b_{int} = \sum (s_{int} + \frac{\pi}{2} \cdot d_{res}), \qquad (4.3\text{-}39)$$

where the summation refers to the groups of bars activated at the edge of the supported area and s_{int} is equal to the width of the group of bars (refer to Figure 4.3-25).

Table 4.3-3 – Values of the angle of the integrity bar α_{ult}

α_{ult}	**Type of integrity reinforcement**
0°	Straight bars, class of ductility A
20°	Straight bars, class of ductility B
25°	Straight bars, class of ductility C or D
$\alpha \leq 40°$	Inclined or bent-up bars, class of ductility B, C or D

CHAPTER 5

SERVICEABILITY LIMIT STATES (SLS)

5.1 GENERAL PROVISIONS

In general case, EN 1992 [N3] deal in some detail with three common serviceability limit states. There are: limitation of stresses; control of cracking; control of deflections.

It was pointed in [3, 8], that other limit states (such as vibration) may be of importance in particular structures, but are not considered in EN 1992 [N3].

As it was show in [3, 8] design for any limit state requires the definition of four quantities. There are:

1) definition of appropriate loading and methods of analysis so that the design load effects can be established;

2) definition of the material properties to be assumed in the verification;

3) definition of criteria defining the limit of satisfactory performance;

4) definition of suitable methods by which performance may be predicted.

It should be noted that in most cases it will not be necessary to carry out explicit calculations for the serviceability limit states, as simple, «deemed to satisfy» procedures are given in the code for dealing with all three of the limit states covered. This approach is acceptable because serviceability is intrinsically less critical than the ultimate limit states, and major calculation effort is not justified.

For example, if the structure is wrongly designed and the strength is ever 1 % below the imposed loads, the structure collapse (theoretically!). By comparison, if the crack width turn out to be 0,33 mm instead of 0,3 mm, nothing more serious than some grumbling from the owner and a cosmetic repair is likely to result [8].

5.1.1 ASSESSMENT OF DESIGN ACTION EFFECTS

For serviceability limit states verification, EN 1990 [N1] defines three combinations of actions (action effects) which may need to be considered. These are:

– characteristic combination:

$$\sum_j G_{k.j}"+"Q_{k.1}"+"\sum_{i>1}\psi_0 \cdot Q_{k,j}; \qquad (5.1\text{-}1)$$

– frequent combination:

$$\sum_j G_{k,j}"+"Q_{k,1}"+"\sum_{i>1}\psi_{2,i} \cdot Q_{k,i}; \qquad (5.1\text{-}2)$$

– quasi-permanent combination:

$$\sum_j G_{k,j}"+"\sum_i \psi_{2,i} \cdot Q_{k,i}; \qquad (5.1\text{-}3)$$

It will be understood from these formulae that the partial factor on the loads is always 1,0 for serviceability limit states. For buildings, the following simplifications which may be used for the characteristic or frequent combinations:

– where there is only one variable action:

$$G_{k,j}"+"Q_k; \qquad (5.1\text{-}4)$$

– where there are two or more variable actions:

$$G_{k,j}"+"0{,}9\sum Q_{k,i}. \qquad (5.1\text{-}5)$$

Analysis may be elastic (without any redistribution). This analysis may be normally based on the stiffness of the uncracked section. However, if it is suspected that the cracking may have a significant unfavourable effect on the performance, then a more realistic analysis taking into account of the cracking should be used. This possibility may be ignored for normal building structures.

5.1.2 MATERIAL PROPERTIES

In accordance with EN 1990 [N1] the partial factors applied to material properties should generally be 1,0. The properties of materials which are normally significant in serviceability calculations are reinforcement modulus of elasticity, modulus of elasticity, creep coefficient, shrinkage strain and tensile strength of concrete.

5.1.3 MODULUS OF ELASTICITY

For ordinary reinforcement modulus of elasticity may be taken as 200 kN/mm^2.
The elastic modulus of concrete varies with other factors than just the strength (e.g. aggregate type), and, if an accurate prediction of serviceability conditions is required, it will be necessary to establish the value of E_{cm} by test on the type of concrete actually being used (in accordance with specification requirements EN 206 [N4]). In general case, values of the modulus of elasticity are given in Table 3.1-3.

5.1.4 CREEP COEFFICIENT AND FREE SHRINKAGE STRAIN

As it was shown in Chapter 3, the creep coefficient, which is defined as the ratio of the creep deformation to the instantaneous elastic deformation, depends upon many factors, the most significant of which are the age of loading, the time under load, the relative humidity, the section geometry, the concrete strength and the type of cement. As with the modulus of elasticity, the only way of determining the creep performance of concrete with any real reliability is to obtain data from creep tests on the concrete actually being used. Full equations for the prediction of the creep coefficient are given in EN 1992 [N3] (Annex A).
Free shrinkage strain depends upon the same basic variables as creep, and equations for its prediction are given in EN 1992 [N3] (Annex A). Simplified approach to assess time-dependent parameters for the concrete are given in Chapter 3.

5.1.5 TENSILE STRENGTH OF CONCRETE

Table 3.1-3 gives the mean tensile strength and the upper and lower characteristic tensile strengths as a function of the characteristic compressive strength of the concrete.
In calculation of stresses and deflections, cross-section should be assumed to be uncracked provided that the flexural tensile stress does not exceed $f_{ct,eff}$. The value of $f_{ct,eff}$ may be taken as f_{ctm} or $f_{ctm,fe}$ provided that the calculation for minimum tension reinforcement is also based on the same value. For the purposes of calculating crack widths and tension stiffening f_{ctm} should be used.

5.2 CONTROL OF CRACKING

5.2.1 GENERAL CONSIDERATIONS

As it was shown in [8], members subjected to loading generally exhibit a series of distributed cracks.

Cracks is normal in reinforced concrete structures subject to bending, shear, torsion or tension resulting from either direct loading or restraint or imposed deformations.

Cracks can be usually observed on concrete structures in service. Cracks have significant influence on serviceability, durability, aesthetics and force transfer. Cracking of concrete (related to its limited tensile deformation capacity) is usually expected under tensile stresses.

The actual width of cracks in reinforced concrete structures will vary between wide limits and cannot be precisely estimated, thus the limiting requirements to be satisfied is that probability of the maximum width exceeding a satisfactory value is small.

5.2.2 CAUSES OF CRACKING

There are many possible causes of cracking and only a few of these lead to cracks that can be controlled by measures taken during the design. The following are the more common causes [8]:

– ***Plastic shrinkage or plastic settlement.*** These are phenomena which occur within the first few hours after casting while the concrete is still in a plastic state. The likelihood of cracks being caused by these phenomena depends upon the bleeding rate of the mix and the evaporation rate. The resulting cracks may be large: up to 2mm is not common.

– ***Corrosion.*** Rust occupies a greater volume than the metal from which it is formed. Its formation therefore causes internal pressures to build up around the bar surface, which will lead to the formation of cracks running along the line of the corroding bars and eventually, spalling of the concrete.

– ***Expansive chemical reactions within the concrete.*** Expansive reactions occurring at the concrete surface tend to lead to scaling of the concrete rather than cracking; however, some reactions, such as the alkali-silica reaction, occur within the body of the concrete and can lead to large surface cracks.

– ***Restrained deformations,*** such as shrinkage or temperature movements.

– ***Loading.***

Of this list, only the last two causes can be treated by the designer. They are probably the two least serious causes of cracking.

5.2.3 CRACK WIDTH LIMITS

There are many reasons for wishing to limit the width of cracks to a relatively low value. Among the most commonly cited reasons are to avoid or limit:

– possible corrosion damage to the reinforcement due to deleterious substances penetrating to the reinforcement down the cracks;

– leakage through cracks, this is commonly a critical design consideration in water retaining structure;

– an unsightly appearance.

All of the above reasons have been researched to some degree, but no clear definition of permissible crack width has emerged from any of these studies.

The following provides a very brief summary of the results from these studies as it was presented in [3, 8].

Cracking and corrosion. This is the most extensively researched area. Summaries of the findings have been published by a number of authors. The development of corrosion is a two-phase process. In fresh concrete the reinforcement is protected from corrosion by the alkaline nature of the concrete. This protection can be destroyed by two mechanisms: carbonation of the concrete to the surface of the reinforcement or ingress of chlorides. Cracks will lead to a local acceleration of both processes by permitting more rapid ingress of either carbon dioxide or chlorides to the surface of the reinforcement. Once the protection provided by the concrete has been destroyed, corrosion can start if the environmental conditions are right. The period from construction up till the initiation of corrosion is usually referred to as the «initiation phase», while the period after the initiation of corrosion is usually referred to as ‘active phase’. The length of the initiation phase is likely to be influenced by crack width. However, this period is likely to be short at a crack, and some corrosion can usually be found on the bar surface where a crack reaches a bar after as little as 2 years even with very small cracks. It is found, however, that this initial corrosion does not develop in cases where the cracks are small or where the bars intersect the cracks. Quite possibly the corrosion products block the cracks and inhibit further corrosion. A more serious situation exists where a crack runs along the line of a bar. There is limited evidence to suggest that, in this case, sustained corrosion may develop in salty environments where the crack width exceeds about 0,3 mm.

Though less research has been done on the relation between cracking and corrosion in pre-stressed concrete members, it is generally believed that the risks posed by cracks are greater, and therefore more stringent criteria should be imposed. For this reason, EN 1992 [N3] does not permit cracks to penetrate to the pre-stressing tendons where the member is exposed to aggressive environments.

Leakage. Only very limited research has been carried out so far into this problem, and this has not led to any agreed basis for crack width limits. Practical experience has suggested that cracks of less than 0,2 mm width which pass right through a section will leak somewhat initially but will quickly seal themselves. This problem is not specifically considered in EN 1992 [N3] as liquid-retaining structures are covered in EN 1992 [N3].

Appearance. Limited studies suggest that noticeable cracks in structural members' cause concern to the occupants of structures, and it is therefore advisable to keep cracks below a width that will not generally be noticed by a casual observer. On a smoothly finished concrete surface, it appears that cracks are unlikely to lead to complaint if the maximum width is kept below 0,4mm. Clearly, larger widths may be used on rougher forms of surface or where the cracking cannot be seen. This is mentioned in the note to Table 5.2-1.

In accordance with EN 1992 [N3] cracking shall be limited to an extent that will not impair the proper functioning or durability of the structure or cause its appearance to be unacceptable. A limiting value, w_{max}, for the calculated crack width, w_k, taking into account the proposed function and nature of the structure and the costs of limiting cracking should be established.

The recommended values for relevant exposure classes are given in Table 5.2-1.

Table 5.2-1 – Recommended values of w_{max} (mm) (Table 7.1 N from EN 1992 [N3])

Exposure class	**Reinforced members and prestressed members with unbonded tendons**	**Prestressed members with bonded tendons**
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4	0,2
XC2, XC3, XC4,	0,3	0,2
XD1, XD2, XD3, XS1, XS2, XS3		Decompression

Notes: *1. For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to give generally acceptable appearance. In the absence of appearance conditions this limit may be relaxed.*

2. For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

As it is stated in EN 1992 [N3] in the absence of specific requirements (e.g. water-tightness), it may be assumed that limiting the calculated crack widths to the values of w_{max} are given in Table 5.2-1, under quasi-permanent combination of loads, will generally be satisfactory for reinforced concrete members in buildings with respect to appearance and durability.

Flexural cracking is generally controlled by providing a minimum area of tension reinforcement and limiting bar spacings or limiting bar side. If calculations to estimate maximum crack widths are performed, they are based on the quasi-permanent combination of loads (action effect) and an effective modulus of elasticity of the concrete should be used to allow for creep effects.

5.2.4 MECHANISM OF CRACKING

Mechanism of flexural cracking can be illustrated by considering behaviour of a concrete member zones, subjected to a uniform moment.

A length of beam as shown in Figure 5.2-1 will initially behave elastically throughout, as an applied bending moment M_{Ek} is increased.

When ultimate tensile strain for the concrete is reached, a crack will form and the adjacent tensile done will no longer be acted on by direct tension force. The formation of this crack lead to a local redistribution of stresses within section. At the crack, all tensile force will be transferred to the reinforcement and the stress in concrete immediately adjacent to the crack must clearly be zero (see Figure 5.2-1). The curvature of the beam, however, causes further direct tension stresses to develop at same distance from the original crack to maintain equilibrium. So, with increasing of the distance from the crack, force is transferred by bond from the reinforcement to concrete until, at some distance, s_0, from crack, the stress distribution within the section remains unchanged from what it was before the crack formed. This in turn causes further cracks to form and process continues until the distance, does not permit sufficient tensile stresses to develop and cause further cracking.

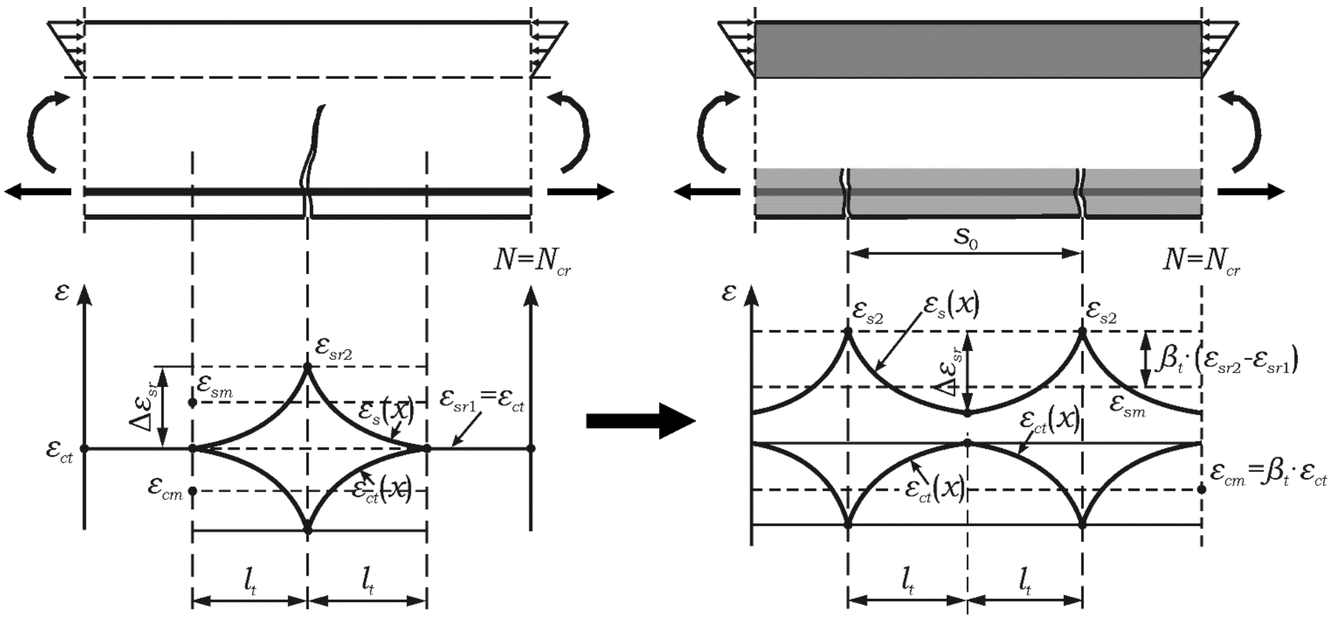


Figure 5.2-1 – Mechanism of crack formation

These initial cracks are called «***primary cracks***», and the average spacing in a region of constant moment are largely independent of reinforcement detailing.

This local redistribution of forces in the region of the crack is accompanied by an extension of the member. This extension, plus a minor shortening of the concrete which has been relieved of the tensile crack it was surrounding, is accommodated in the crack. The crack then opens up to a finite width immediately on its formation.

The formation of the crack and the resulting extension of the member also reduced the stiffness of the member.

As further load is applied, a second crack will form at the next weakest section, though it will not form within s_0 of the first crack since the stresses within the region will have been reduced by formation of the first crack.

Tensile stresses in the concrete surrounding reinforcing bars are caused by bond as the strain in the reinforcement increases. These stresses increase with distance from the primary cracks and may eventually cause further cracks to form approximately mid-way between the primary cracks.

This action may continue with increasing bending moment until the bond between concrete and steel is incapable of developing sufficient tension in the concrete to cause further cracking in the length between existing cracks. Since the development of the tensile stresses is caused directly by the presence of reinforcing bars, the spacing of cracks will be influenced by the spacing of the reinforcement. If bars are sufficiently close for their «zones of influence» to overlap, then secondary cracks will join up across the member while otherwise they will form only adjacent to individual bars.

Further increases in loading will lead to the formation of further cracks until, eventually, there is no remaining area of the member surface which is not within s_0 of previously formed crack.

The formation of each crack will lead to a reduction in the member stiffness. After all the cracks have formed, further loading will result in a widening of the existing cracks but no new cracks formation. Stresses in the concrete will be relieved by limited bond-slip near the crack faces and by the formation of internal cracks. This process leads to further reduction of stiffness, but clearly, the stiffness cannot reduce to below that of the bare reinforcement.

According to EN 1992 [N3] (clause 6.4.2) the average crack spacing in a flexural member depends in part on the efficiency of bond, the diameter of reinforcing bar used and quantity and location of the reinforcement in relation to the tensile face of the section.

5.2.5 DERIVATION OF CRACK PREDICTION FORMULAE

The development of formulae for the prediction of crack widths given in EN 1992 [N3] (clause 7.3.4) follows from the description of the cracking phenomenon given above.

In general case, the crack width can be calculated from following expression based on «bond-slip» theory:

$$w = 2 \cdot l_t \cdot \int_0^{l_t} [\varepsilon_s(x) - \varepsilon_{ct}(x)]dx. \tag{5.2-1}$$

If it is assumed that all the extension occurring when a crack forms is accommodated in that crack, then, when all the cracks have formed, the crack width will be given by following relationship, which is simply a statement of compatibility:

$$w = s_{rm} \cdot \varepsilon_m, \tag{5.2-2}$$

where: w is crack width;

s_{rm} is the average crack spacing;

ε_m is the average strain.

The average strain can be more rigorously stated to be equal to the strain in the reinforcement, taking into account of tension stiffening, ε_{sm}, less the average strain in concrete at the surface, ε_{cm}.

Since, in design, it is a maximum width of crack which is required rather than the average, the final formulae given in EN 1992 [N3] is following:

$$w_k = s_{r,max} \cdot \left(\varepsilon_{sm} - \varepsilon_{cm}\right). \tag{5.2-3}$$

Since no crack can form within l_t of an existing crack, this defines the minimum spacing of the cracks. The maximum spacing is $2 \cdot l_t$, since if a spacing existed wider than this, a further crack could form. It follows that the average crack spacing will lie between l_t and $2 \cdot l_t$. It is frequently assumed to be $1{,}5 \cdot l_t$.

The distance l_t, and hence s_{rm}, depends on the rate at which stress can be transferred from the reinforcement, which is carrying all the force at crack, to the concrete. This transfer is effected by bond stresses on the bar surface. If the bond stress is assumed to be constant along the length l_t and that the stress will just reach the tensile strength of concrete at a distance l_t from crack, then:

$$\tau_{sm} \cdot \pi \cdot \varnothing \cdot l_t = f_{ctm} \cdot A_{c,eff} \cdot \left(1 - \rho_{p,eff}\right); \tag{5.2-4}$$

$$l_t = \frac{f_{ctm} \cdot A_{c,eff} \cdot \left(1 - \rho_{p,eff}\right)}{\tau_{sm} \cdot \pi \cdot \varnothing}.$$

Maximum spacing between cracks $s_{r,max} = 2l_t$:

$$\begin{aligned} s_{r,max} &= 2 \cdot \frac{f_{ctm} \cdot A_{c,eff} \cdot \left(1 - \rho_{p,eff}\right)}{\tau_{sm} \cdot \pi \cdot \varnothing} = \frac{f_{ctm} \cdot A_{c,eff} \cdot \left(1 - \rho_{p,eff}\right) \cdot \varnothing}{2 \cdot \tau_{sm} \cdot A_s} = \frac{f_{ctm} \cdot \left(1 - \rho_{p,eff}\right) \cdot \varnothing}{2 \cdot \tau_{sm} \cdot \rho_{p,eff}} = \\ &= 0{,}5 \cdot \frac{f_{ctm} \cdot \varnothing}{2 \cdot \tau_{sm} \cdot \rho_{p,eff}} = 0{,}5 \cdot k \cdot \frac{\varnothing}{\rho_{p,eff}}, \end{aligned} \tag{5.2-5}$$

where: τ_{sm} is the bond stress;

$A_{c,eff}$ is the effective area of concrete in tensile zone (effective tension area);

f_{ctm} is the averge tensile strength of concrete;

$\varnothing$ is the bar diameter.

This is the oldest form of relationship for prediction of crack spacings. More recent studies have shown that the cover also a significant influence, and that a better agreement with test result s_{rm} is obtained from an equation of the form:

$$s_{rm} = k \cdot c + 0,25 \cdot k_1 \cdot \frac{\varnothing}{\rho}, \tag{5.2-6}$$

where: c is the concrete cover.

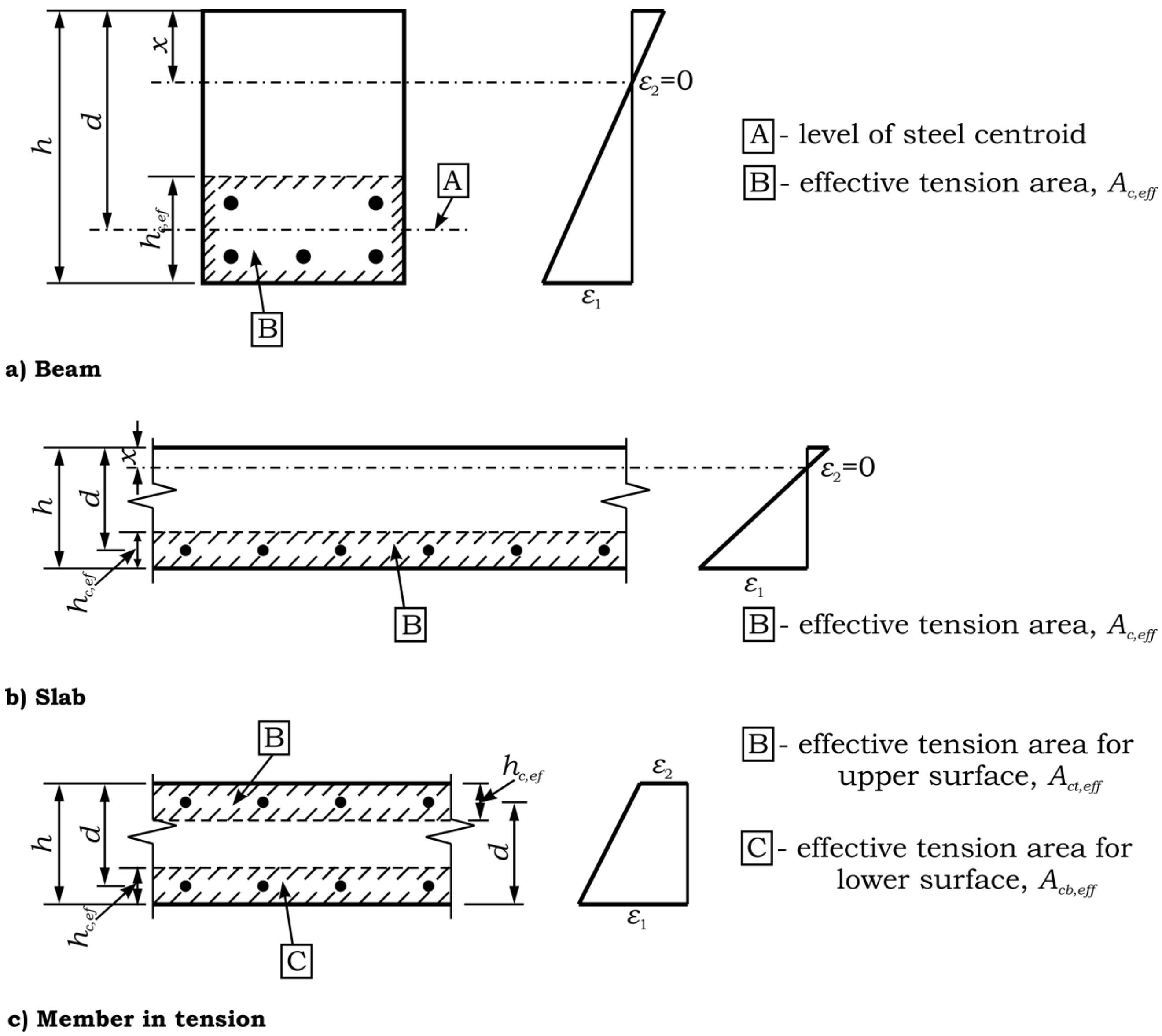


Figure 5.2-2 – Effective tension area (typical cases) (Figure 7.1 from EN 1992 [N3])

This formula has been derived for members subjected to pure tension. In order to be able to apply it to bending, it is necessary to introduce a further coefficient, k_2, and to define an effective reinforcement ratio, ρ_{eff}. These modifications take account of the different form of stress distribution within tension done and the fact that only part of section is in tension. Coefficients k_2 and ρ_{eff} can be derived empirically from tests. The resulting formula is following:

$$s_{rm} = 2 \cdot c + 0,5 \cdot k_1 \cdot k_2 \cdot \frac{\varnothing}{\rho_{eff}}, \tag{5.2-7}$$

where: k_1 is a coefficient taking into account of the bond propties of reinforcement. A value of 0,8 is taken for high bond bars and 1,6 for smooth bars.

k_2 is a coefficient depending on the form of the stress distribution. A value 0,5 is taken for bending and 1 – for pure tension. Intermediate values can be obtained from:

$$k_2 = \frac{(\varepsilon_1 + \varepsilon_2)}{2 \cdot \varepsilon_1}, \tag{5.2-8}$$

where: ε_1 and ε_2 are, respectively, the greater and lesser tensile strains at the faces of the member;

ρ_{eff} is the effective reinforced ration, where A_s is the area of tension reinforcement contained within the effective are of concrete in tension $A_{c,eff}$ (see Figure 5.2-2).

In design, it is not the average crack width which is required but a value which is unlikely to be exceeded. EN 1992 [N3] uses the characteristic crack width, which is defined as a width with a 5 % probability of exceedance. It is found experimentally that a reasonable estimate of characteristic width is obtained if the maximum crack spacing is assumed to be 1,7 times the average value. In EN 1992 [N3], therefore, the maximum spacing are used.

In situations where loaded reinforcement is fixed at reasonably close centres within the tension zone (spacing $\leq 5 \cdot (c + \varnothing/2)$), the maximum final crack spacing may be calculated from following expression (see Figure 5.2-3):

$$s_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot \frac{\varnothing}{\rho_{p,eff}}, \tag{5.2-9}$$

where: $\varnothing$ is the bar diameter. Where a mixture of bar diameters is used in a section, an equivalent diameter, $\varnothing_{eq}$, should be used. For section with n_1 bars $\varnothing$ diameter $\varnothing_1$ and n_2 bars of diameter $\varnothing_2$, the following expression should be used:

$$\varnothing_{eq} = \frac{n_1 \cdot \varnothing_1^2 + n_2 \cdot \varnothing_2^2}{n_1 \cdot \varnothing_1 + n_2 \cdot \varnothing_2}, \tag{5.2-10}$$

c is the cover to the longitudinal reinforcement

k_1 is a coefficient which takes account of the bond properties of the bonded reinforcement: it is equal to 0,8 for high bond bars, and it is equal to 1,6 for bars with an effectively plain surface (e.g. prestressing tendons);

k_2 is a coefficient, which takes into account of the distribution of strain: it is equal to 0,5 for bending, and it is equal to 1,0 for pure tension.

For cases of eccentric tension or for local area, intermediate values of k_2 should be used which may be calculated from the relation:

$$k_2 = (\varepsilon_1 + \varepsilon_2)/(2 \cdot \varepsilon_1), \tag{5.2-11}$$

where ε_1 is the greater and ε_2 is the lesser tensile strain at the boundaries of the section considered, assessed on the basis of a cracked section.

The values of k_3 and k_4 may be found in National annex to EN 1992 [N3]. The recommended values: $k_3 = 3{,}4$, and $k_4 = 0{,}425$.

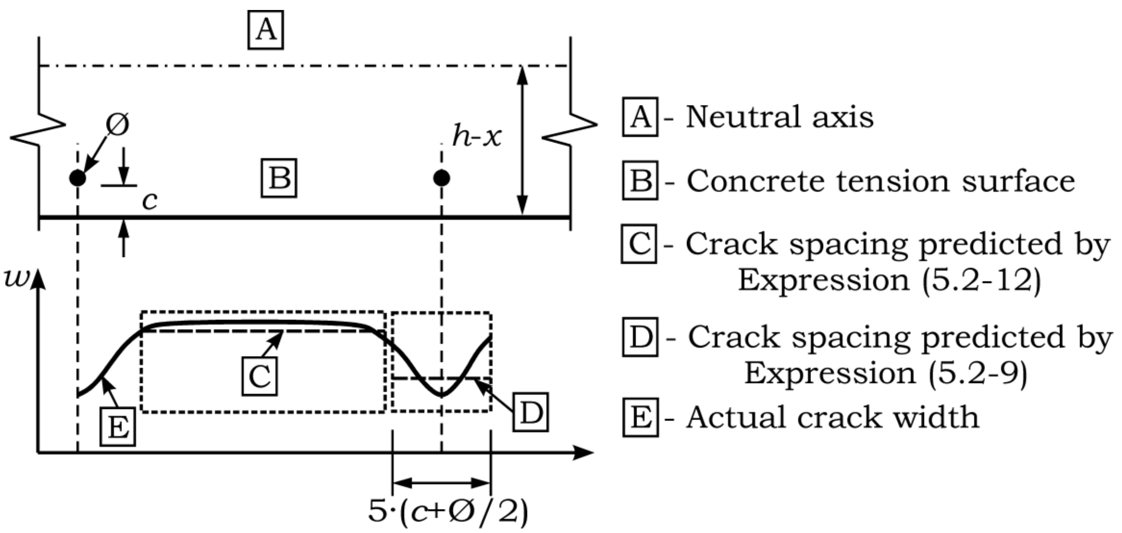


Figure 5.2-3 – Crack width, *w*, at concrete surface relative to distance from bar (Figure 7.2 from EN 1992 [N3])

Where the spacing of the bonded reinforcement exceeds $5 \cdot (c + \varnothing/2)$ (see Figure 5.2-3), an upper bound to the crack width may be found by assuming a maximum crack spacing:

$$s_{r,max} = 1{,}3 \cdot (h - x). \tag{5.2-12}$$

The other parameter in the crack width equation is the average strain $(\varepsilon_{sm} - \varepsilon_{cm})$. This is obtained from following procedure.

For the section at a distance l_t from crack the following equilibrium condition can be written (see Figure 5.2-4):

$$\varepsilon_{ct} \cdot E_{cm} \cdot A_{c,eff} = E_s \cdot A_s \cdot (\varepsilon_{sr2} - \varepsilon_{sr1}) = E_s \cdot A_s \cdot \Delta\varepsilon_{sr,max}. \tag{5.2-13}$$

For any section along l_t:

$$\Delta\varepsilon_{sr}(x)\cdot E_s\cdot A_s=\varepsilon_{ct}(x)\cdot E_{cm}\cdot A_c. \tag{5.2-14}$$

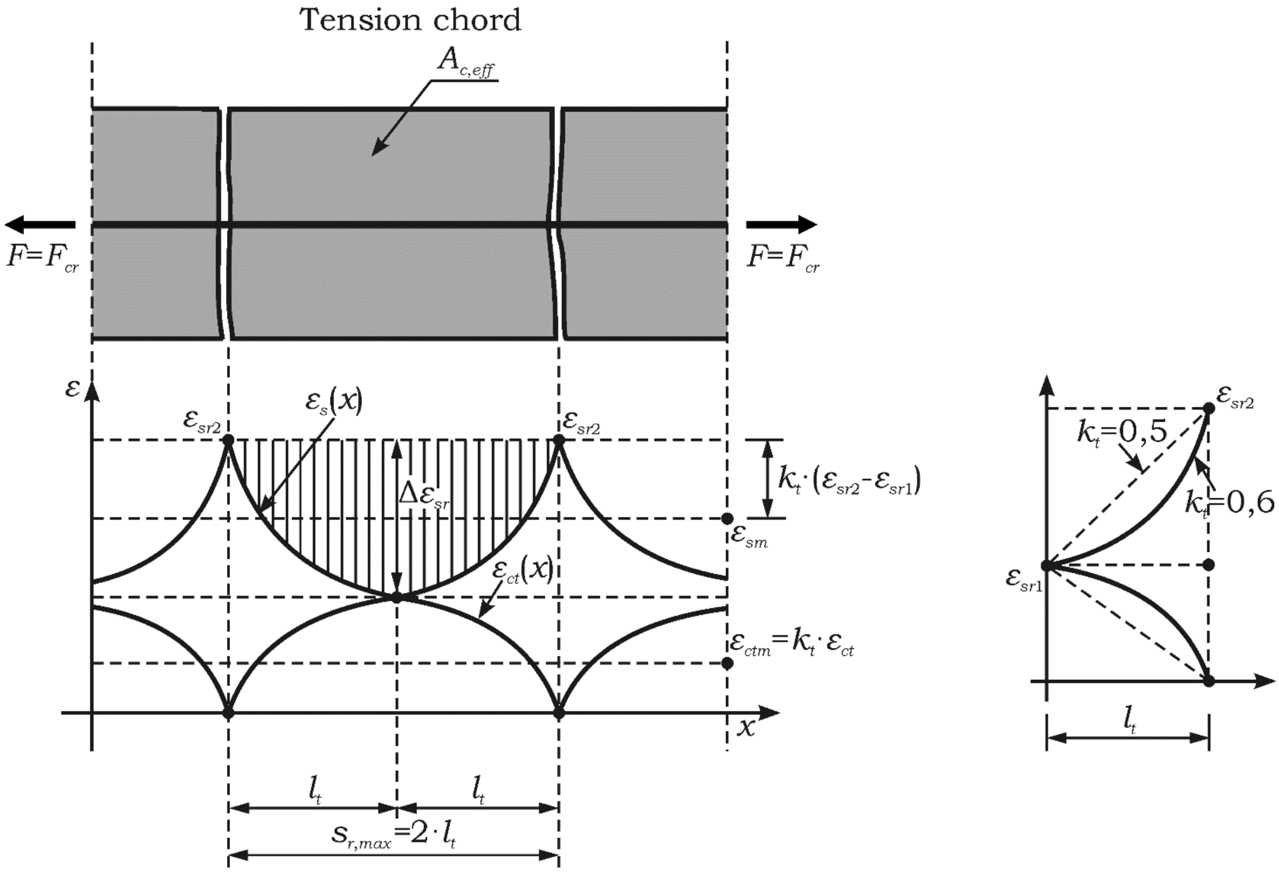


Figure 5.2-4 – For cracking width prediction

Average tensile strain in reinforcement and tensile strain in concrete can be expressed as follows:

$$\varepsilon_{cm}=\frac{1}{l_t}\cdot\int_0^{l_t}\varepsilon_{ct}(x)dx=k_t\cdot\varepsilon_{ct}; \tag{5.2-15}$$

$$\varepsilon_{sm}=\frac{1}{l_t}\cdot\int_0^{l_t}\varepsilon_s(x)dx=\frac{1}{l_t}\cdot\int_0^{l_t}\left(\varepsilon_{sr2}-\Delta\varepsilon_{sr}(x)\right)dx. \tag{5.2-16}$$

Solving Equation (5.2-14) and Substituting in Equation (5.2-16):

$$\varepsilon_{sm}=\frac{1}{l_t}\cdot\int_0^{l_t}\varepsilon_{sr2}dx-\frac{1}{l_t}\cdot\int_0^{l_t}\Delta\varepsilon_{sr}(x)dx=\varepsilon_{sr2}-\frac{E_c\cdot A_c}{E_s\cdot A_s}\cdot k_t\cdot\varepsilon_{ct}. \tag{5.2-17}$$

Substituting Equation (5.2-13) in Equation (5.2-17) given:

$$\varepsilon_{sm} = \varepsilon_{sr2} - k_t \cdot (\varepsilon_{sr2} - \varepsilon_{sr1}). \qquad (5.2\text{-}18)$$

Average strain (ε_{sm} – ε_{cm}) is obtained from the following equation:

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \varepsilon_{sr2} - k_t \cdot (\varepsilon_{sr2} - \varepsilon_{sr1}) - k_t \cdot \varepsilon_{ct} = \varepsilon_{cr2} - k_t \cdot \left(\frac{1}{\rho_{p,eff}} - 1 \right) \cdot \frac{f_{ctm}}{E_s} - k_t \cdot \frac{f_{ctm}}{E_c} = \\ = \frac{\sigma_{s2}}{E_s} - k_t \cdot \frac{f_{ctm}}{\rho_{p,eff} \cdot E_s} \cdot (1 + a_e \cdot \rho_{p,eff}). \qquad (5.2\text{-}19)$$

The final formulae given in EN 1992 [N3] is following:

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\sigma_s - k_t \cdot f_{ct,eff} / \rho_{p,eff} \cdot (1 + a_e \cdot \rho_{p,eff})}{E_s} \geq 0,6 \cdot \frac{\sigma_s}{E_s}, \qquad (5.2\text{-}20)$$

where: σ_s is the stress in the tension reinforcement assumed a cracked section;

a_e is the ratio E_s / E_{cm};

$\rho_{p,eff} = A_s / A_{c,eff}$;

k_t is a factor dependent on the duration of the load: it is equal to 0,6 for short term loading, and it is equal to 0,4 for long term loading;

$A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement of the depth, $h_{c,eff}$, where $h_{c,eff}$ is the lesser of $2,5 \cdot (h - d)$; $(h - x)/3$ and $h/2$ (see Figure 5.2-2).

Example to Section 5.2

Example 1. Calculation of flexural crack width

Calculate the design flexural crack widths for the beam shown in Figure 5.2-1 when subject to a quasi-permanent moment $M_{E,k} = 650$ kN·m. The concrete is class C25/30, and the reinforcement is high bond with a total cross-section area $A_s = 3770$ mm^2.

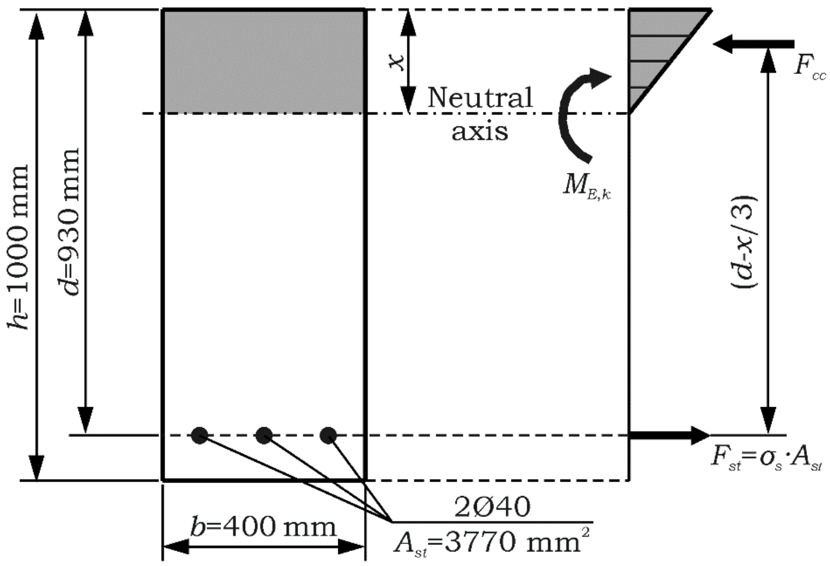


Figure E 5.2-1 – Crack width calculation example

Calculate the main strain, ε_{sm}.

From Table 3.1-3: $E_{cm} = 31$ GPa for the concrete class C25/30. From Figure 3.1-7, assuming loading at 28 days with indoor exposure, the creep coefficient (because $2 \cdot A_c / n = 2 \cdot 1000 \cdot 400 / 2800 = 285$), and, hence, the effective modulus is given as follows: $E_{c,eff} = \dfrac{E_{cm}}{(1+\varphi)} = \dfrac{31}{(1+2{,}63)} = 8{,}54$ GPa .

Calculate the neutral axis depth of the cracked section, x.

Taking moment about the neutral axis:

$$b \cdot x \cdot \frac{x}{2} = a_e \cdot A_s \cdot (d-x);$$

$400 \cdot \dfrac{x^2}{2} = \dfrac{200}{8{,}54} \cdot 3770 \cdot (930 - x)$, which has the solution $x = 457$ mm.

Calculate the stress in the tension steel, σ_s.

Taking moments about the level of the compressive force in the concrete:

$$\sigma_s = \frac{M_{E,k}}{(d - x/3) \cdot A_s} = \frac{650 \cdot 10^6}{(930 - 457/3) \cdot 3770} = 222 \, \frac{\text{N}}{\text{mm}^2}.$$

Calculate ($\varepsilon_{sm} - \varepsilon_{cm}$).

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot f_{ct,eff} / \rho_{p,eff} \cdot (1 + a_e \cdot \rho_{p,eff})}{E_s} = \frac{222 - 0{,}4 \cdot 2{,}6 / 0{,}0539 \cdot (1 + 6{,}45 \cdot 0{,}0539)}{200 \cdot 10^3} =$$

$$= \frac{222 - 19{,}97}{200 \cdot 10^3} = 0{,}001 \geq 0{,}6 \cdot \frac{\sigma_s}{E_s} = 0{,}6 \frac{222}{200 \cdot 10^3} = 0{,}00067,$$

where: $k_t = 0{,}4$, assuming long-term loading; $f_{ct,eff} = f_{ctm} = 2{,}6$ N/mm² (from Table 3.1-3); $a_e = \dfrac{E_s}{E_{cm}} = \dfrac{200}{31} = 6{,}45$; $\rho_{p,eff} = \dfrac{A_s}{A_{c,eff}} = \dfrac{3770}{2{,}5 \cdot (1000 - 930) \cdot 400} = 0{,}0539$;

Calculate the maximum crack spacing, $s_{r,max}$.

$$s_{r,max} = 3{,}4 \cdot c + 0{,}425 \cdot k_1 \cdot k_2 \cdot \varnothing / \rho_{p,eff} = 3{,}4 \cdot 50 + \frac{0{,}425 \cdot 0{,}8 \cdot 0{,}5 \cdot 40}{0{,}0539} = 296 \text{ mm}.$$

where: $1000 - 930 - /40 = 2 \quad 50$, cover to main bars; $k_1 = 0,8$ for ribbed bars; $k_2 = 0,5$ for flexure; $\varnothing = 40$ mm – bar diameter.

The maximum crack spacing $s_{r,max} = 296$ mm, that is less than $5 \cdot (c + \varnothing/2) = 350$ mm.

Calculate crack width, w_k.

$w_k = 0,001 \cdot 296 = 0,30$ mm, which just satisfied the recommended limit.

5.2.6 CONTROL OF CRACKING WITHOUT DIRECT CALCULATION

For reinforced slabs in buildings subjected to bending without significant axial tension, specific measures to control cracking are not necessary where the overall depth does not exceed 200 mm and the detailing provisions in accordance EN 1992 [N3] (clause 9.3) have been applied.

Table 5.2-2 – Maximum bar diameters $\varnothing_s$ for crack control (Table 7.2N from EN 1992 [N3])

Steel stress [MPa]	Maximum bar size [mm]		
	$w_k = 0,4$ mm	$w_k = 0,3$ mm	$w_k = 0,2$ mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Notes: *1. The values in the table are based on the following assumptions: c = 25 mm; $f_{ct,eff} = 2,9$ MPa; $h_{cr} = 0,5 \cdot h$; $(h - d) = 0,1 \cdot h$; $k_1 = 0,8$; $k_2 = 0,5$; $k_c = 0,4$; $k = 1,0$; $k_t = 0,4$; $k_4 = 1,0$.*

2. Under the relevant combinations of actions.

The rules given in EN 1992 [N3] (clause 7.7.3) may be presented in a tabular form by restricting the bar diameter or spacing as a simplification.

Where the minimum reinforcement given by section $A_{s,min}$ is provided, crack widths are unlikely to be excessive if:

– for cracking caused dominantly by restraint, the bar sizes given in Table 5.2-2 are not exceeded where the steel stress is the value obtained immediately after cracking;

– for cracks caused mainly by loading, either the provisions of Table 5.2-2 or the provisions of Table 5.2-3 are compiled with. The steel stress should be calculated on the basis of a cracked section under the relevant combination of actions.

Table 5.2-3 – Maximum bar spacing for crack control (Table 7.3N from EN 1992 [N3])

Steel stress [MPa]	Maximum bar spacing [mm]		
	$w_k = 0{,}4$ mm	$w_k = 0{,}3$ mm	$w_k = 0{,}2$ mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-
For notes see Table 5.2-2.			

The maximum bar diameter should be modified as follows:

– bending (at least part of section in compression):

$$\varnothing_s = \varnothing_s^* \cdot \left(f_{ct,eff} / 2{,}9 \right) \cdot \frac{k_c \cdot h_{cr}}{2 \cdot (h - d)}; \tag{5.2-21}$$

– tension (uniform axial tension):

$$\varnothing_s = \varnothing_s^* \cdot \left(f_{ct,eff} / 2{,}9 \right) \cdot h_{cr} / \left(8 \cdot (h - d) \right). \tag{5.2-22}$$

where: $\varnothing_s$ is the adjusted maximum bar diameter;

$\varnothing_s^*$ is the maximum bar size given in the Table 5.2-2;

h is the overall depth of the section;

h_{cr} is the depth of the tensile zone immediately prior to cracking, considering the characteristic values of prestress and axial forces under the quasi-permanent combination of actions;

d is the effective depth to the centroid of the outer layer of reinforcement.

Where all the section is under tension ($h - d$) is the minimum distance from the centroid of the layer of reinforcement to the face of the concrete (consider each face where the bar is not placed symmetrically).

Beams with a total depth of 1000 mm or more, where the main reinforcement is concentrated in only a small portion of the depth should be provided with additional skin reinforcement to control cracking on the side faces of the beam. This reinforcement should be evenly distributed between the level of the tension steel and the neutral axis and should be located within the links. The area of the skin reinforcement should not be less than the amount obtained from the Equation (5.2-23), taking k as 0,5 and σ_s as f_{yk}. The spacing and size of suitable bars may be obtained from EN 1992 [N3] (clause 7.3.4) or a suitable simplification assuming pure tension and a steel stress of half the value assessed from the main tension reinforcement.

It should be noted that there are particular risks of large cracks occurring in sections where there are sudden changes of stress, e.g.: at changes of section; near concentrated loads; positions, where bars are curtailed; area of high bond stress, particular at the ends of laps.

Care should be taken at such areas to minimise the stress changes wherever possible. However, the rules for crack control given above will normally ensure adequate control at these points provided that the rules for detailing reinforcement given in Sections 6.

5.2.7 MINIMUM REINFORCEMENT AREAS

In accordance with EN 1992 [N3] (clause p.7.3.2), if crack control is required, a minimum amount of bonded reinforcement is required to control cracking in areas where tension is expected. The amount may be estimated from equilibrium between the tensile forces in concrete just before cracking and the tensile force in reinforcement at yielding or at a lower stress if necessary to limit the crack width.

Unless a more rigorous calculation shows lesser areas to be adequate, the required minimum areas of reinforcement may be calculated as follows. In profiled cross sections like T-beams and box girders, minimum reinforcement should be determined for the individual parts of the section (webs, flanges).

$$A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}, \tag{5.2-23}$$

where: $A_{s,min}$ is the minimum area of reinforcing steel within the tensile zone

A_{ct} is the area of concrete within tensile zone. The tensile zone is that part of the section which is calculated to be in tension just before formation of the first crack.

σ_s is the absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack. This may be taken as the yield strength of the reinforcement, f_{yk}. A lower value may, however, be needed to satisfy the crack width limits according to the maximum bar size or spacing (see clause 7.3.3 (2) from EN 1992 [N3])

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur: $f_{ct,eff} = f_{ctm}$ or lower, ($f_{ctm}(t)$), if cracking is expected earlier than 28 days.

k is the coefficient which allows for the effect of nonuniform selfequilibrating stresses, which lead to a reduction of restraint forces. It is equal to 1,0 for webs with h≤300mm or flanges with widths less than 300 mm, and it is equal to 0,65 for webs with h≥800mm or flanges with widths greater than 800mm intermediate values may be interpolated.

k_c is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm. It is equal to 1,0 for pure tension, and for bending or bending combined with axial forces:

– for rectangular sections and webs of box sections and T-sections:

$$k_c = 0,4 \cdot \left[1 - \frac{\sigma_c}{k_1 \cdot \left(h / h^* \right) \cdot f_{ct,eff}} \right] \leq 1; \qquad (5.2\text{-}24)$$

– for flanges of box sections and T-sections:

$$k_c = 0,9 \cdot \frac{F_{cr}}{A_{ct} \cdot f_{ct,eff}} \geq 0,5; \qquad (5.2\text{-}25)$$

where: σ_c is the mean stress of the concrete acting on the part of the section under consideration, and it is equal to:

$$\sigma_c = \frac{N_{Ed}}{b \cdot h} \qquad (5.2\text{-}26)$$

N_{Ed} is the axial force at the serviceability limit state acting on the part of the cross-section under consideration (compressive force positive). N_{Ed} should be determined considering the characteristic values of prestress and axial forcer the relevant combination of actions

h^* is equal to h for h<1,0 m, and equal to 1,0 m for h≥1,0 m.

k_1 is a coefficient considering the effects of axial forces on the stress distribution. It is equal to 1,5 if N_{Ed} is a compressive force, and it is equal to $\left(2 \cdot h^*\right) / \left(3 \cdot h\right)$ if N_{Ed} is a tensile force.

F_{cr} is the absolute value of the tensile force within the flange immediately prior to cracking due to cracking moment calculated with $f_{ct,eff}$.

5.3 DEFLECTION CONTROL

5.3.1 GENERAL CONSIDERATIONS

In accordance with EN 1992 [N3] (clause p.7.4.1), the deformation of a member or structure shall not be such that it adversely effects its proper functioning or appearance.

Appropriate limiting values of deflection taking into account the nature of the structure, of the finishes, partitions and fixings and upon the function of the structure should be established.

As was it shown in [8] deformations should not exceed those that can be accommodated by other connected elements such as partitions, glazing, cladding services or finishes. In some cases limitation may be required to ensure the proper functioning of machinery or apparatus supported by the structure, or to avoid ponding on flat roofs.

The limiting deflections given in EN 1992 [N3] are derived from ISO 4356 [9] and should generally result in satisfactory performance of buildings public buildings such as dwellings, offices, public buildings or factories. Care should be taken to ensure that the limits are appropriate for the particular structure considered and that there are no special requirements. Further information on deflections and limiting values may be obtained from ISO 4356 [9].

The appearance and general utility of the structure could be impaired when the calculated sag of a beam, slab or cantilever subjected to quasi-permanent load exceeds span/250. The sag is assessed relative to the supports. Pre-camber may be used to compensate for some or all of the deflection but any upward deflection incorporated in the formwork should not generally exceed span/250.

Deflections that could damage adjacent parts of the structure should be limited. For the deflection after construction, span/500 is normally an appropriate limit for quasi permanent loads. Other limits may be considered, depending on the sensitivity of adjacent parts.

The limit state of deformation may be checked by either:

— by limiting the span/depth ratio, according to section, or

— by comparing a calculated deflection, according to Section 5.3.2.3, with a limit value

The actual deformations may differ from the estimated values, particularly if the values of applied moments are close to the cracking moment. The differences will depend on the dispersion of the material properties, on the environmental conditions, on the load history, on the restraints at the supports ground conditions, etc.

5.3.2 CALCULATION OF DEFLECTION

The code EN 1992 [N3] suggests that deflection should be calculated under the action of quasi-permanent load combination in accordance with EN 1990 [N1], assuming this loading to be of long-term duration. Hence, the total loading to be taken in the calculation will be the permanent load plus a portion of the variable (imposed) load as in following expression:

$$\sum_j G_{k,j} + \sum_{i=1}^{n} \psi_{2,i} \cdot Q_{k,i}, \tag{5.3-1}$$

where: $G_{k,j}$ is a permanent load;

$Q_{k,i}$ is a variable (imposed) load.

This is the reasonable assumption as deflection will be affected by long-term effects such as concrete creep, while not all of the variable load is likely to be long-term and hence will not contribute to the creep effects.

Lateral deflection must not be ignored, especially on tall slender structures, and limitation in these must be judged by the engineer. It is important to realise that there are many factors which may have significant effects on deflections, and are difficult to allow for. Thus any calculated values must be regarded as an estimate only.

The most important of these factors are:

1) support restraint must be estimated on the basis of simplified assumptions, will have varying degrees of accuracy (boarding conditions uncertainty);

2) the precise loading cannot be predicted and errors in permanent loading may have a significant effect;

3) a cracked member will behave differently one that is uncracked this may be a problem in lightly reinforced members where the working load may be near to the cracking limit;

4) the effects of floor screed, finishes and portions are very difficult to assess, frequently these are neglected despite their *stiffening effect*.

The method adopted by EN 1992 [N3] is based on the calculation of curvature of sections subjected to the appropriate moments with allowance for creep and shrinkage effects where necessary. Deflections are then calculated from these curvatures. A rigorous approach to deflection is to calculate the curvature at the interval along the span and then use numerical integration techniques to estimate the critical deflections taking into account the fact that some sections along the span will be cracked under load and other, in region of lesser moment, will be un-cracked. Such an approach is rarely justified and the approach adopted below, based on EC2, assumes that acceptably accurate to calculate the curvature of the beam or slab based on both the cracked and un-cracked sections and then to use an *average* value in estimating the final deflection using standard deflection formulae or single numerical integration based on elastic theory.

5.3.2.1 Calculation of curvature

Curvature under the action of the quasi-permanent load combination should be calculated based on both cracked and un-cracked sections. An estimate of an average value of curvature can then be obtained using the formulae:

$$1/r = \xi \cdot \left(1/r\right)_{cr} + (1-\xi) \cdot \left(1/r\right)_{uc}, \tag{5.3-2}$$

where: $1/r$ is an average curvature;

$\left(1/r\right)_{uc}, \left(1/r\right)_{cr}$ are the values of curvature calculated for the uncracked case and cracked case respectively;

ξ is a coefficient given by $\left[1 - \beta \cdot \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2\right]$ allowing for tension stiffening effect (TSE);

β is the load duration factor (1 – for a single short-term load; 0,5 – for sustained load or cyclic loading);

σ_{sr} is the stress in the tension steel calculated on the basis of a cracked section under the loading that will just cause cracking at the section being considered;

σ_s is the stress in the tension steel for the cracked concrete section.

In calculating ξ, the ratio (σ_{sr}/σ_s) can more conveniently be replaced by (M_{cr}/M_{Ek}), where M_{cr} is the moment that will just cause cracking of the section and M_{Ek} is the design moment for the calculation of curvature and defection.

In order to calculate the average curvature, separate calculations have to be carried out for both the cracked uncracked cases.

5.3.2.1.1 Uncracked section

The assume classic strain and stress distribution for an uncracked section is shown in Figure 5.3-1.

For a given moment, M_{Ek}, and from elastic bending theory, the curvature of the section, $\left(1/r\right)_{uc}$, is given by:

$$\left(1/r\right)_{uc} = \frac{M_{Ek}}{E_{c,eff} \cdot I_{uc}}, \tag{5.3-3}$$

where: $E_{c,eff}$ is effective elastic modulus of the concrete allowing for creep effects;

I_{uc} is the second moment of area of the uncracked section.

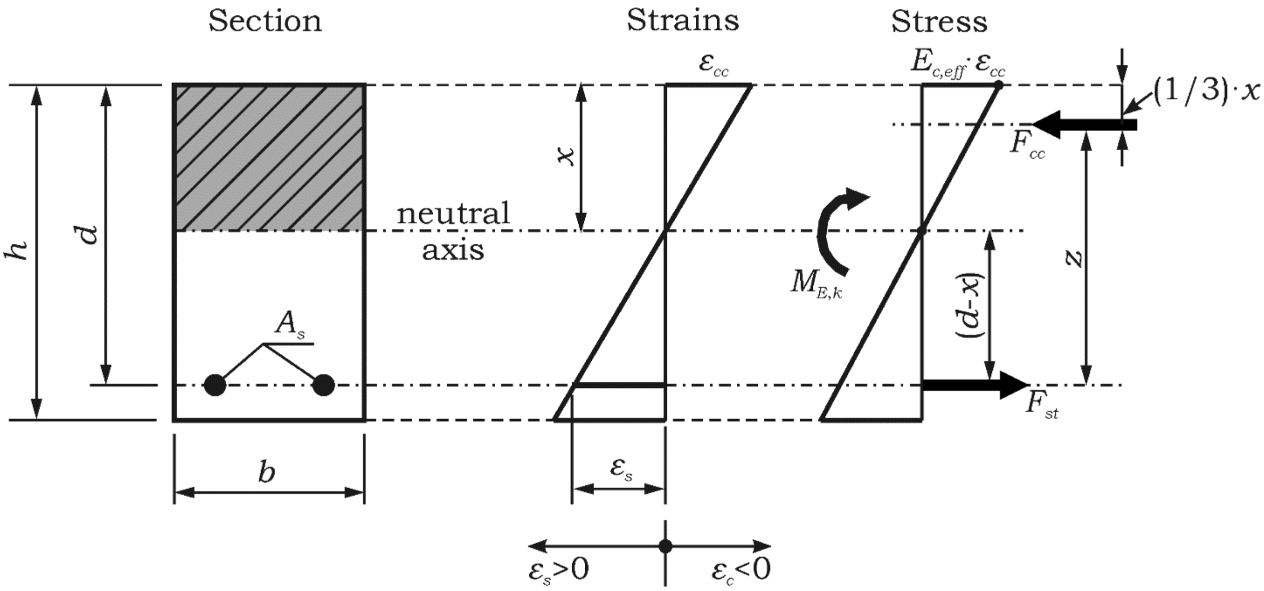


Figure 5.3-1 – Uncracked section: strain and stress distribution

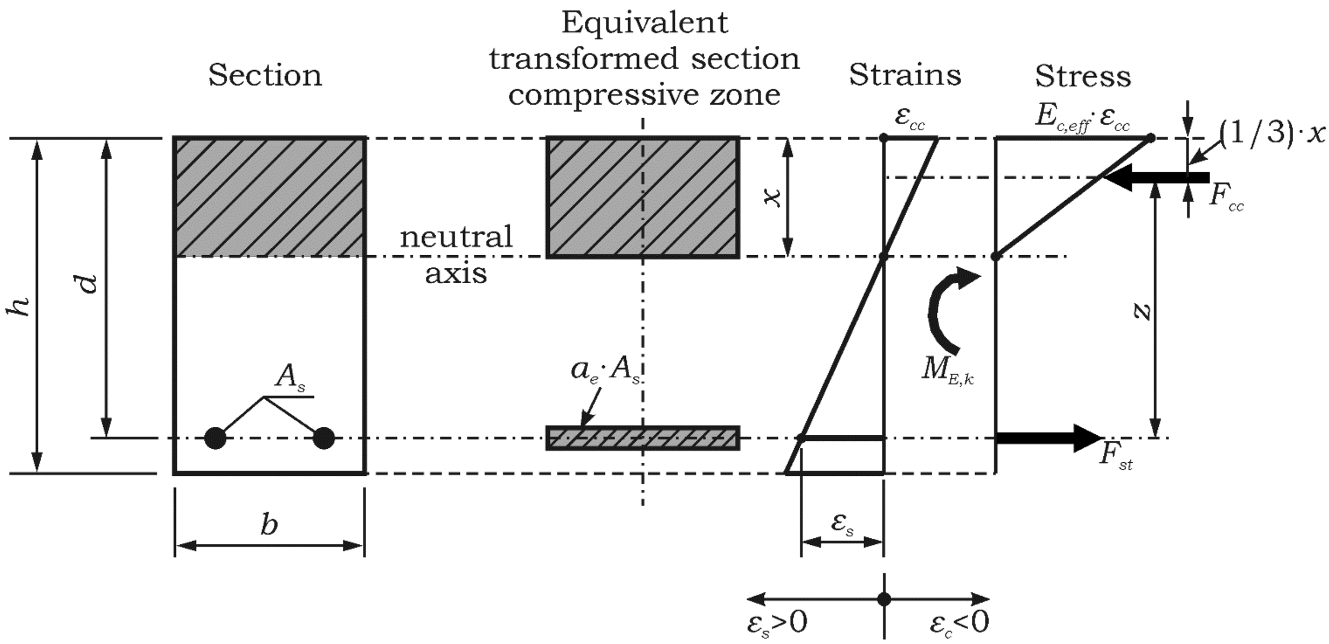


Figure 5.3-2 – Cracked section – strain and stress distribution

5.3.2.1.2 Cracked section

The assumed elastic strain and stress distribution for a cracked section is shown in Figure 5.3-2.

Moments of area can be taken to establish the neutral-axis depth directly. The second moment of area of the cracked section can be than by taking second moment of area about the neutral axis:

$$I_{cr} = \frac{b \cdot x^3}{3} + a_e \cdot A_s \cdot (d - x)^2, \tag{5.3-4}$$

where: a_e – is the modular ratio equal to the ratio of the elastic modulus of the reinforcement to that of the concrete; $\left(a_e = E_s / E_{cm}\right)$.

For a given moment, M_{Ek}, and from elastic berding theory, the curvature of the cracked section, $\left(1/r\right)_{cr}$, is therefore given by the following expression:

$$\left(1/r\right)_{cr} = \frac{M_{Ek}}{E_{c,eff} \cdot I_{cr}} \tag{5.3-5}$$

5.3.2.1.3 Creep and shrinkage effects

Creep

The effect of creep will be to increase deflections with time and thus should be allowed for in the calculations by using an effective modulus, $E_{c,eff}$, using the equation:

$$E_{c,eff} = \frac{E_{cm}}{\left[1+\varphi\left(\infty, t_0\right)\right]}, \tag{5.3-6}$$

where: $\varphi\left(\infty, t_0\right)$ is a creep coefficient equal to ratio of creep strain to initial elastic strain (see Chapter 3).

Shrinkage

The effect of shrinkage of the concrete will be to increase the curvature and hence the deflection of the beam or slab. The curvature due to shrinkage can be calculated using the equation:

$$\left(1/r\right)_{cs} = \left(\varepsilon_{cs} \cdot a_e \cdot s\right) / I, \tag{5.3-7}$$

where: $\left(1/r\right)_{cs}$ is the shrinkage curvature;

ε_{cs} is free shrinkage strain (see Chapter 3);

I is the second moment of area of section (cracked or uncracked as appropriate);

a_e is effective modular ration $\left(E_s / E_{c,eff}\right)$.

5.3.2.2 Caclulation of the defection from curvature

The total curvature can be determined by adding the shrinkage curvature due to the quasi-permanent loads, having made allowance for creep effects.

The deflection of the beam can be calculated from the total curvature using elastic bending theory which for the small deflections is based on the expression:

$$M_x = E \cdot I \cdot \frac{d^2 y}{dx^2}, \tag{5.3-8}$$

where: M_x is the bending moment at a section distance x from the origin as shown in Figure 5.3-3.

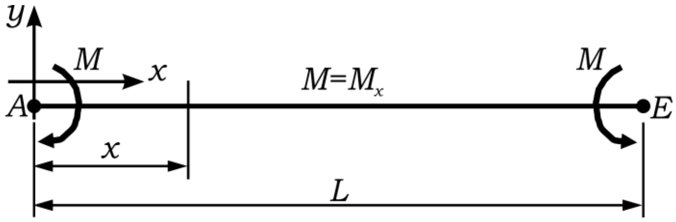


Figure 5.3-3 – Pin-ended beam subject to a constant moment *M*

For small deflections the term $\frac{d^2 y}{dx^2}$ approximately equals the curvature which is the reciprocal of the radius of the curvature. Double integration of Equation (5.3-8) will yield an expression for deflection. This may be illustrated by considering the case of a pin-ended beam subjected to constant moment *M* throughout its length, so that $M_x = M$ (see Figure 5.3-3):

$$E \cdot I \cdot \frac{d^2 y}{dx^2} = M, \tag{5.3-9}$$

therefore:

$$E \cdot I \cdot \frac{dy}{dx} = M \cdot x + C, \tag{5.3-10}$$

but if the slope is zero at mid-span where $x = L/2$, then:

$$C = -\frac{M \cdot L}{2},$$

and:

$$E \cdot I \cdot \frac{dy}{dx} = M \cdot x - \frac{M \cdot L}{2}, \tag{5.3-11}$$

Integrating again gives:

$$E \cdot I_y = \frac{M \cdot x^2}{2} - \frac{M \cdot L \cdot x}{2} + D, \tag{5.3-12}$$

but at support A, when x=0, y=0. Hence: $D=0$ and $y=\frac{M}{E\cdot I}\left(\frac{x^2}{2}-\frac{L\cdot x}{2}\right)$ at any section. The maximum deflection in this case will occur at a mid-span, where $x=L/2$, in which case:

$$y_{max}=-\frac{M\cdot L^2}{E\cdot I\cdot 8}, \tag{5.3-13}$$

but since at any cracked section $\frac{M}{E\cdot I}=\frac{1}{r}$, the maximum deflection may be expressed as $y_{max}=-\frac{1}{8}\cdot L^2\cdot\left(\frac{1}{r}\right)$.

In general case, the bending moment distribution along a member will not be constant, but will be a function of x. The basic form of result will however be the same, and the deflection may be expressed as:

$$a_{max}=K\cdot\left(\frac{1}{r}\right)\cdot L^2, \tag{5.3-14}$$

where: K is a constant, the value of which depends on the distribution of bending moments in the member;

L is the effective span

$\left(\frac{1}{r}\right)$ is the mid-span curvature for beams, or the support curvature for cantilevers.

Typical values of K are given in Table 5.3-1 for various common shapes of bending-moment diagrams. If the loading is complex, then a value of K must be estimated for the complete load since summing deflections of simpler components will yield incorrect result.

Table 5.3-1 – Values of *K* for various bending moment diagrams

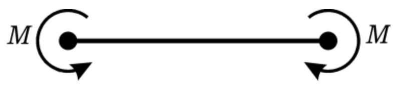
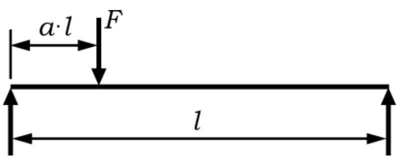
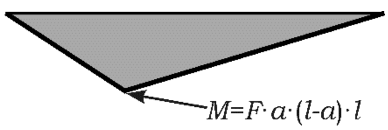
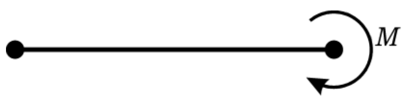
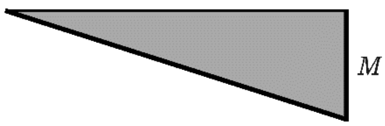
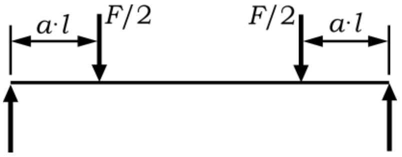
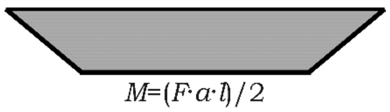
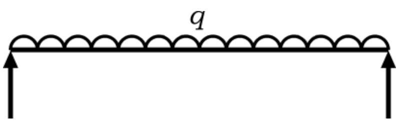
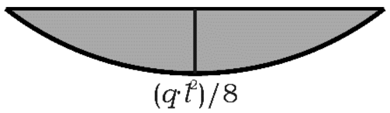
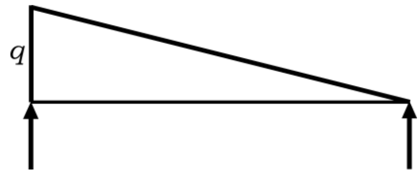
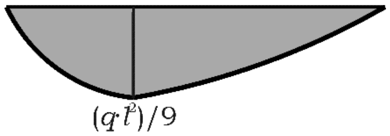
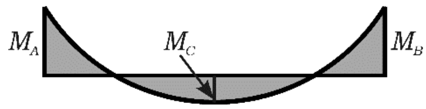
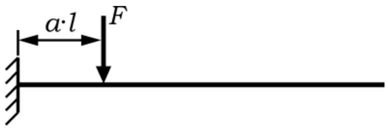
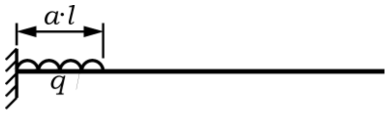
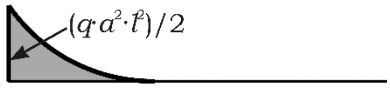
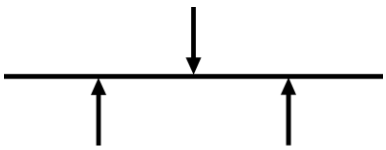
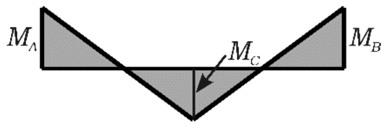
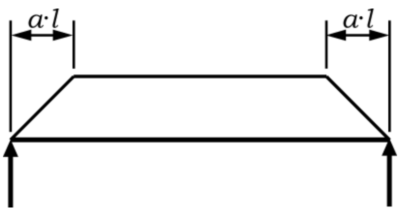
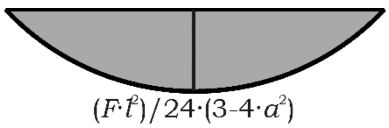
Loading	Bending moment diagram	K
		0,125
		$\frac{3-4\cdot a^2}{48\cdot(1-a)}$, if $a=\frac{1}{2}$ $K=\frac{1}{12}$
		0,0625

Table 5.3-1 (end)

		$0{,}125-\frac{a^2}{6}$
		0,104
		0,102
		$K=0{,}104\cdot\left(1-\frac{\beta}{10}\right),\ \beta=\frac{M_A+M_B}{M_C}$
		end deflection $=\frac{a\cdot(3-a)}{6}$, load at end $K=0{,}333$
		$\frac{a(4-a)}{12}$, if $a=1$ $K=0{,}25$
		$K=0{,}083\left(1-\frac{\beta}{4}\right),\ \beta=\frac{M_A+M_B}{M_C}$
		$\frac{1}{80}\cdot\frac{\left(5-4\cdot a^2\right)^2}{3-4\cdot a^2}$

Although the deviation has been on the basis of an uncracked sections, the final expression is in form that will deal with cracked section simply by the substitution of appropriate curvature.

Since the expression involves the square of the span, it is important that the true effective span as defined at Chapter 2 is used particularly in the case of cantilevers. Deflections of cantilevers may also be increased by rotation of supporting member; and this must be taken into account when the supporting structure is fairly flexible.

Example to Section 5.3

Example 1. Calculation of deflection

Estimate the long-term deflection of the beam shown in Figure 5.3-1 it spans 9,5 meters and is designed to carry uniformly distributed load giving rise to a quasi-permanent moment of $M_{E,k} = 200$ kN·m. It is constructed with class C25/30 concrete, is made of normal aggregates and the construction props are removed at 28 day.

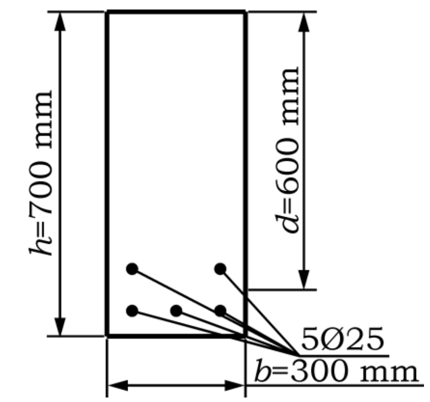


Figure E 5.3-1 – Deflection calculation example

Calculate curvature due to uncracked section.

From Equation (5.3-3):

$\left(1/r\right)_{uc} = \dfrac{M_{E,k}}{E_{c,eff} \cdot I_{uc}} = \dfrac{200 \cdot 10^6}{8{,}15 \cdot 10^3 \cdot \left[\left(300 \cdot 700^3\right)/12\right]} = 2{,}86 \cdot 10^{-6} \dfrac{1}{\text{mm}}$, where from Table 3.1-3: $E_{cm} = 31 \text{kN}/\text{mm}^2$; from Figure 3.1-7, assuming loading at 28 days with indoor exposure, the creep coefficient $\varphi \approx 2{,}8$, because $2 \cdot A_c / u = \left[2 \cdot (700 \cdot 300)\right]/2000 = 210$, and, hence, the effective modulus is given by the following expression: $E_{c.eff} = \dfrac{E_{cm}}{(1+\varphi)} = \dfrac{31}{(1+2{,}8)} = 8{,}15 \dfrac{\text{kN}}{\text{mm}^2}$.

Note that in the above calculation I_{uc} has been calculated on the basic of the gross concrete sectional area ignoring the contribution of the reinforcement.

To calculate the curvature of the cracked section the I value of the transformed concrete section must be calculated.

Calculate the neutral axis position.

Taking area moments about the neutral axis:

$$\frac{b \cdot x^2}{2} = a_e \cdot A_s \cdot (d - x);$$

$\dfrac{300 \cdot x^2}{2} = \dfrac{200}{8{,}15} \cdot 2450 \cdot (600 - x)$, which has the solution: $x = 329$ mm.

Calculate the second moment of area of cracked section.

$$I_{cr}=\frac{b\cdot x^3}{3}+a_e\cdot A_s\cdot(d-x)^2=\frac{300\cdot 329^3}{3}+\frac{200}{8{,}15}\cdot 2450(600-329)^2=7976\cdot 10^6\ \text{mm}^4.$$

Calculate the curvature of the cracked section.

$$\left(1/r\right)_{cr}=\frac{M_{E,k}}{E_{c,eff}\cdot I_{cr}}=\frac{200\cdot 10^6}{8{,}15\cdot 10^3\cdot 7976\cdot 10^6}=3{,}08\cdot 10^{-6}\ \frac{1}{\text{mm}}.$$

Calculate M_{cr}.

From Table 3.1-3, the cracking strength of concrete, f_{ctm}, is given as $2{,}6\,\text{N}/\text{mm}^2$. Hence, form elastic bending theory and considering the uncracked concrete section, the moment that will just cause cracking of the section, M_{cr}, is given by:

$$M_{cr}=f_{ctm}\cdot\left(\frac{b\cdot h^2}{6}\right)=2{,}6\cdot\frac{300\cdot 700^2}{6}=63{,}7\ \text{kN}\cdot\text{m}.$$

Calculate ξ.

$$\xi=1-\beta\cdot\left(\sigma_{sr}/\sigma_s\right)^2=1-\beta\cdot\left(M_{cr}/M_{Ek}\right)=1-0{,}5\cdot\left(63{,}7/200\right)=0{,}95.$$

Calculate the average curvature $\left(1/r\right)$.

$$\left(1/r\right)=\xi\cdot\left(1/r\right)_{cr}+(1-\xi)\cdot\left(1/r\right)_{uc}=0{,}95\cdot 3{,}08\cdot 10^6+(1-0{,}95)\cdot 2{,}86\cdot 10^6=3{,}07\cdot 10^6\ \frac{1}{\text{mm}}.$$

For simply supported span subjected to a uniformly distributed load, the maximum mid-span deflection is given by the following expression:

$$a=0{,}104\cdot\left(1/r\right)\cdot L^2=0{,}104\cdot 3{,}07\cdot 10^{-6}\cdot 9500^2=3{,}03\ \text{mm}.$$

This value almost exactly matches the allowable value of $L/250=38$ mm and would be considered acceptable nothing the inherent uncertainty of some of the parameters used in the calculation.

5.3.2.3 Cases, where deflection calculations may be omitted. Basis of span-effective depth ratios

The calculation of deflection has been shown to be a tedious operation. However, for general use, rules based on limiting the span-effective depth ratio of a member are adequate to ensure that the deflections are not excessive.

The relationship between the deflection and the span-effective depth ratio of a member can be derived from equation:

$$a=K\left(1/r\right)_b\cdot L^2. \tag{5.3-15}$$

For small deflections it can be seen from Figure 5.3-4 that for unit length, *s*:

$$\varphi=\left(1/r\right)_b=\frac{\varepsilon_{cm}+\varepsilon_{rm}}{d}, \tag{5.3-16}$$

where: ε_{cm} is the maximum compressive strain in the concrete;

ε_{rm} is a tensile strain in the reinforcement;

K is a factor, which depends on the pattern of loading.

Therefore:

$$\frac{L}{d}=\frac{a}{L}\cdot\frac{1}{K}\cdot\left(\frac{1}{\varepsilon_{c,max}+\varepsilon_{rm}}\right). \tag{5.3-17}$$

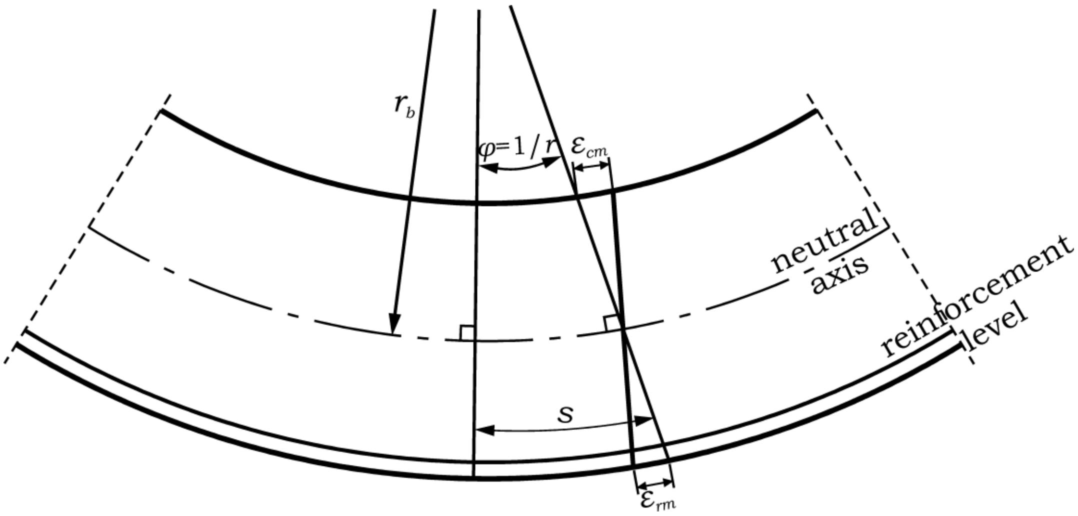


Figure 5.3-4 – To calculation of the member curvature

The strains in the concrete and tensile reinforcement depend on the areas of reinforcement provided and their stresses. Thus for a particular member section and a pattern of loading, it is possible to determine a span-effective depth ratio to satisfy a particular L/d limitation.

Provided that reinforced concrete beams or slabs in buildings are dimensioned so that they comply with the limits of span to depth ratio given in this clause, their deflections may be considered as not exceeding the limits set out in EN 1992 [N3] (clause p. 7.4.1(4),(5)). The limiting span-to-depth ratio may be estimated using Expression (5.3-18) and Equation (5.3-19) and multiplying this by correction factors to allow for the type of reinforcement used and other variables. No allowance has been made for any precamber in the derivation of these expressions:

$$\frac{l}{d}=K\cdot\left[11+1{,}5\cdot\sqrt{f_{ck}}\cdot\frac{\rho}{\rho_0}+3{,}2\cdot\sqrt{f_{ck}}\cdot\left(\frac{\rho}{\rho_0}-1\right)^{3/2}\right]\text{ if }\rho\le\rho_0; \tag{5.3-18}$$

$$\frac{l}{d}=K\cdot\left[11+1{,}5\cdot\sqrt{f_{ck}}\cdot\frac{\rho}{\rho-\rho'}+\frac{1}{12}\cdot\sqrt{f_{ck}}\cdot\sqrt{\frac{\rho'}{\rho_0}}\right]\text{ if }\rho\ge\rho_0. \tag{5.3-19}$$

where: l/d is the limit of the span to depth ratio;

K is the factor to take into account the different structural systems;

ρ_0 is the reference reinforcement ratio $= 10^{-3}\sqrt{f_{ck}}$;

ρ is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers);

ρ' is the required compression reinforcement ratio at mid-span to resist the moment due design loads (at support for cantilevers);

f_{ck} is in MPa unit of measurement.

Expression (5.3-18) and Expression (5.3-19) have been derived on the assumption that the steel stress, under the appropriate design load at SLS at a cracked section at the mid-span of a beam or slab or at the support of the cantilever, is 310 MPa, (corresponding roughly to $f_{yk} = 500$ MPa).

Where other stress levels are used, the values obtained using Expression (5.3-18) and Expression (5.3-19) should be multiplied by $310/\sigma_s$. It will normally be conservative to assume that:

$$310/\sigma_s = 500/\left(f_{yk} \cdot A_{s,red} \,/\, A_{s,prov}\right), \tag{5.3-20}$$

where: σ_s is the tensile steel stress at mid-span (at support for cantilevers) under the design load at SLS;

$A_{s,prov}$ is the area of steel provided at this section;

$A_{s,red}$ is the area of steel required at this section for ultimate limit state.

For flanged sections where the ratio of the flange breadth to the rib breadth exceeds 3,the values of l/d given by Expression (5.3-18) and Expression (5.3-19) should be multiplied by 0,8.

For beams and slabs, other than flat slabs, with spans exceeding 7 m, which support partitions liable to be damaged by excessive deflections, the values of l/d given by Expression (5.3-18) and Expression (5.3-19) should be multiplied by $7/l_{eff}$ (l_{eff} is in meters).

For flat slabs, where the greater span exceeds 8,5 m, and which support partitions liable to be damaged excessive deflections, the values of l/d given by Expression (5.3-18) and Expression (5.3-19) should be multiplied by $8{,}5/l_{eff}$ (l_{eff} in meters).

Recommended values of K are given in Table 5.3-2. Values obtained using Expression (5.3-18) and Expression (5.3-19) for common cases (C30/37, $\sigma_s = 310$ MPa, different structural systems and reinforcement ratios $\rho_l = 0{,}5$ % and $\rho_l = 1{,}5$ %) are also given.

The values given by Expression (5.3-18) and Expression (5.3-19) and Table 5.3-2 have been derived from result a parametric study made for a series of beams or slabs simply supported with rectangular cross section, using approach given in Section 5.3.2.2 . Different values of concrete strength class and a 500 MPa characteristic yield strength for reinforcement were considered. For a given are of tension reinforcement the ultimate moment was calculated and the qusi-permanent load combination was assumed as 50 % of corresponding total design load. The span/depth limit obtained satisfy the limiting deflection.

Table 5.3-2 – Basic ratios of span/effective depth for reinforced concrete members without axial compression

Structural System	K	Concrete highly stressed $\rho_l = 1{,}5$ %	Concrete lightly stressed $\rho_l = 0{,}5$ %
Simply supported beam, one- or two-way spanning simply supported slab	1,0	14	20
End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	1,3	18	26
Interior span of beam or one-way or two-way spanning slab	1,5	20	30
Slab supported on columns without beams (flat slab) (based on longer span)	1,2	17	24
Cantilever	0,4	6	8

Notes: *1. The values given have been chosen to be generally conservative and calculation may frequently show that thinner members are possible.*

2. For 2-way spanning slabs, the check should be carried out on the basis of the shorter span. For flat slabs the longer span should be taken.

3. The limits given for flat slabs correspond to a less severe limitation than a mid-span deflection of span/250 relative to the columns. Experience has shown this to be satisfactory.

CHAPTER 6

DETAILING

6.1 DISCUSSION OF THE GENERAL REQUIREMENTS

In this section the main features of the detailing requirements are arranged in a practical order and discussed.

6.1.1 COVER OF THE BAR REINFORCEMENT

According in EN 1992 [N3] in order to active the required design working life of the structure, adequate measures shall be taken to protect each structure element against the relevant environmental actions.

As it was defined in [8], the ***concrete cover*** is the distance between the surface of reinforcement closest to the nearest concrete surface (including links and stirrups and surface reinforcement where relevant) and the nearest concrete surface.

The nominal cover shell be specified on the drawings. It is defined as a minimum cover, c_{min}, plus an allowance in design for deviation, Δc_{dev}: $c_{nom} = c_{min} + \Delta c_{dev}$.

Minimum concrete cover, c_{min}, shall be provided in order ensure:

– the safe transmission of bond forces;

– the protection of the steel against corrosion (durability requirements);

– an adequate fire resistance.

In accordance with EN 1992 [N3], the greater value for c_{min}, satisfying the requirements for both bond and environmental condition shall be used:

$$c_{min} = \max\{c_{min,b};\, c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add};\, 10\ \text{mm}\},$$

where: $c_{min,b}$ is a minimum concrete cover due to bond requirement;

$c_{min,dur}$ is a minimum cover due to environmental conditions;

$\Delta c_{dur,\gamma}$ is an additive safety element;

$\Delta c_{dur,st}$ is a reduction of minimum concrete cover for use of stainless steel;

$\Delta c_{dur,add}$ is a reduction of minimum cover for use of additional protection.

In order to transmit bond forces and ensure adequate compaction of the concrete, the minimum cover should not be less than $c_{min,b}$ given in Table 6.1-1.

Table 6.1-1 – Minimum cover, $c_{min,b}$, requirements with regard bond (Table 4.2 from EN 1992 [N3])

Bond Requirement	
Arrangement of bars	Minimum cover $c_{min,b}$
Separated	Diameter of bar
Bundled	Equivalent diameter ($\varnothing_n$) EN 1992 [N3] (clause 8.9.1)
Note: *If the nominal maximum aggregate size is greater than 32 mm, $c_{min,b}$ should be increased by 5 mm.*	

The minimum cover values for reinforcement in normal weight concrete taking account of the exposure classes and structural classes is given by $c_{min,dur}$.

It should be pointed, that in accordance with EN 1992 [N3] structural classification and values of $c_{min,dur}$ for use in a country may be found in its National Annex. The recommended structural class (design working life of 50 years) is S4 and the recommended modifications to the structural class is given in Table 6.1-2. The recommended minimum structural class is S1. The recommended values of $c_{min,dur}$ are given in Table 6.1-3.

Table 6.1-2 – Recommended structural classification (Table 4.3N from EN 1992 [N3])

Structural Class							
Criterion	**Exposure Class**						
	XO	XC1	XC2 / XC3	XC4	XD1	XD2 / XS1	XD3 / XS2 / XS3
Design working life of 100 years	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2
Strength Class	$\geq$C30/37 reduce class by 1	$\geq$C30/37 reduce class by 1	$\geq$C35/45 reduce class by 1	$\geq$C40/50 reduce class by 1	$\geq$C40/50 reduce class by 1	$\geq$C40/50 reduce class by 1	$\geq$C45/55 reduce class by 1
Member with slab geometry (position of reinforcement not affected by construction process)	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1

Table 6.1-2 (end)

Special quality control of the concrete production ensured	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1
Notes: *1. The strength class and w/c ratio are considered to be related values. A special composition (type of cement, w/c value, fine fillers) with the intent to produce low permeability may be considered. 2. The limit may be reduced by one strength class if air entrapment of more than 4 % is applied.*							

Table 6.1-3 – Values of minimum cover, $c_{min,dur}$, requirements with regard to durability for reinforcement steel in accordance with EN 10080 [N8] (Table 4.4N from EN 1992 [N3])

Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	**Exposure Class**						
	X0	XC1	XC1/XC3	XC4	XD1/XS1	XD2/XS2	XD3/XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

6.1.2 SPACING OF BARS

The basic principle is that reinforcement bars in a member should be arranged in such a way that concrete can be placed and compacted satisfactorily so that adequate bond will develop between the bars and concrete.

Figure 6.1-1 defines the spacing s_{min} between bars in a layer and that between layers of reinforcement. The clear distance (horizontal or vertical) between individual parallel bars should be not less than:

– the maximum bar diameter ($\varnothing_{max}$); or

– (d_g + 5 mm) or 20mm, where d_g is the maximum size of aggregate.

Where bars a positioned in separate horizontal layers, the bars in each layer should be located vertically above each other. These should be sufficient space between the resulting columns of bars to allow access for vibrators and good compaction of the concrete.

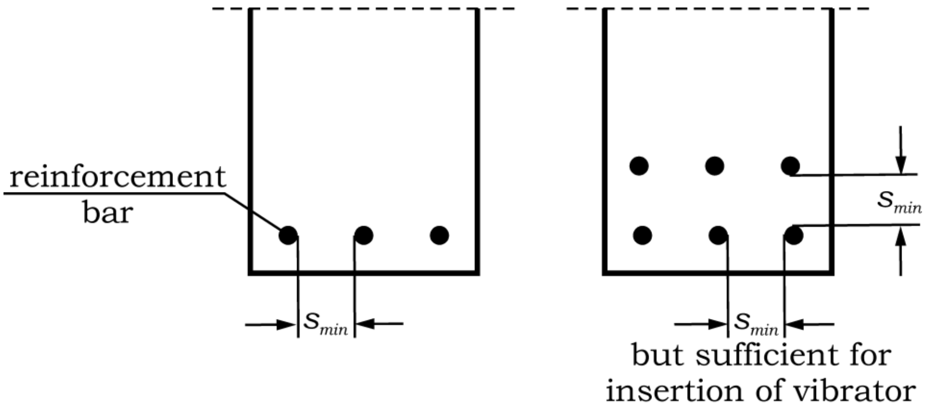


Figure 6.1-1 – Spacing of reinforcement bars

6.1.3 Mandrel diameters for bent bars

The minimum diameter to which a bar is bent shall be such as to avoid bending cracks in the bar, and to avoid failure of the concrete inside the bent of the bar.

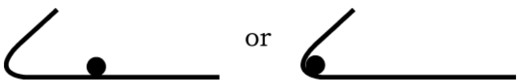
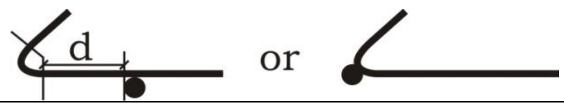
In order to avoid damage to the reinforcement the diameter to which the bar is bent (Mandrel diameter) should not be less than $\varnothing_{m,min}$ (see Table 6.1-4).

Table 6.1-4 – Minimum mandrel diameter to avoid damage to reinforcement (Table 8.1 from EN 1992 [N3])

a) for bars and wire

Bar diameter	Minimum mandrel diameter for bents, hooks and loops [see Figure 6.1-2]
$\varnothing \leq 16\,\text{mm}$	$4\varnothing$
$\varnothing > 16\,\text{mm}$	$7\varnothing$

b) for welded bent reinforcement and mesh bent after welding

Minimum mandrel diameter	
	
$5\varnothing$	$d \geq 3\varnothing$; $5\varnothing$; $d < 3\varnothing$, or welding within the curved zone: $20\varnothing$.
Note: *The mandrel size for welding within the curved zone may be reduced to* $5\varnothing$, *where the welding is carried out in accordance with EN ISO 17660.*	

The mandrel diameter need not be checked to avoid concrete failure if the following conditions exist:

– the anchorage of the bar does not require a length more than $5\varnothing$ past the end of the bend or he bar is not positioned at the edge (plane of bend close to concrete face) and there is a cross bar with a diameter $> \varnothing$ inside the bend.

– the mandrel diameter is at least equal to the recommended values given in Table 6.4N from EN 1992 [N3].

Otherwise, the mandrel diameter, $\varnothing_{m,min}$, should be increased in accordance with the following expression:

$$\varnothing_{m,min} \geq F_{bt} \cdot ((1 / a_b) + 1 / (2 \cdot \varnothing)) / f_{cd}, \qquad (6.1\text{-}1)$$

where: F_{bt} is the tensile force from ultimate loads in a bar or group of bars in contact at the start of a bend;

a_b for a given bar (or group of bars in contact) is half of the centre-to-centre distance between bars (or groups of bars) perpendicular to the plane of the bend. For a bar or group of bars adjacent to the face of the member, a_b should be taken as the cover plus $\varnothing / 2$.

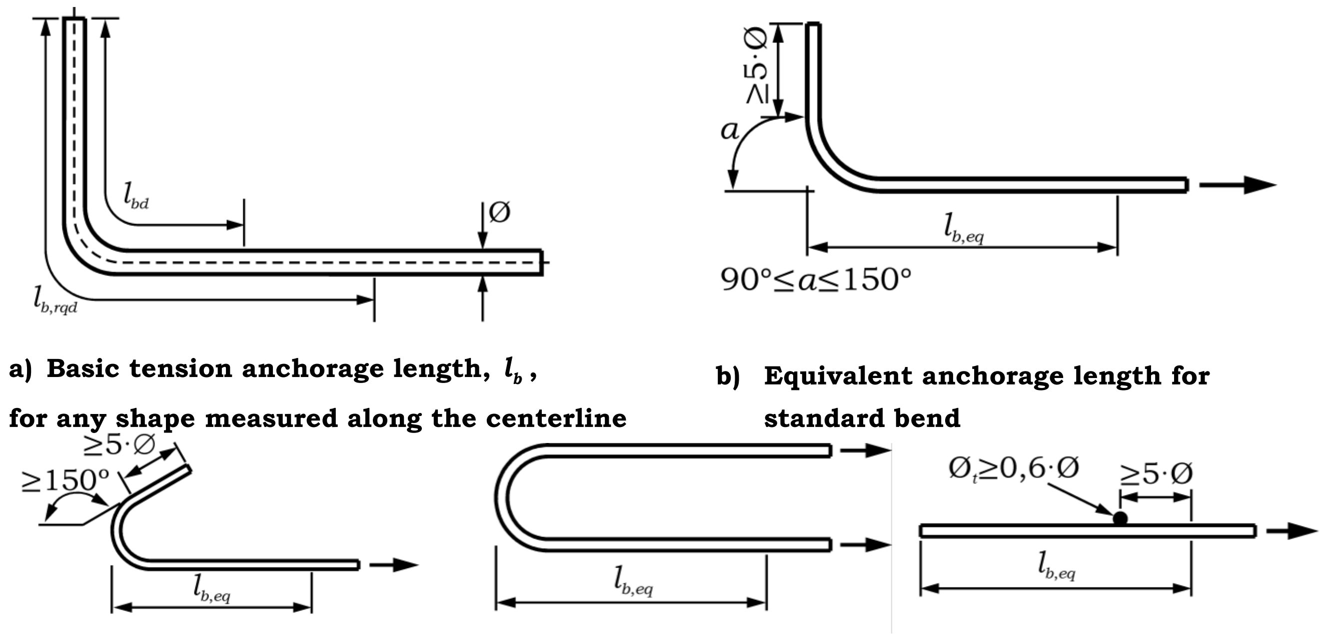
f_{cd} is the concrete design compressive strength, the value of which should not be taken greater than that for concrete class C55/67.

6.2 ANCHORAGE OF LONGITUDINAL REINFORCEMENT

6.2.1 GENERAL

When reinforcement is designed to carry stress, it needs to be anchored into adjacent parts such that 1) the required stress will be able to develop and 2) the force in the bar safely transmitted to the surrounding concrete without causing longitudinal cracks or spalling. Reinforcing bars, wires or welded mesh fabrics shall be so anchored that the bond forces are safely transmitted to the concrete avoiding longitudinal cracking or spalling. Transverse reinforcement shall be provided if necessary.

Methods of anchorage are shown in Figure 6.2-1.



a) Basic tension anchorage length, l_b, for any shape measured along the centerline

b) Equivalent anchorage length for standard bend

c) Equivalent anchorage length for standard hook

d) Equivalent anchorage length for standard loop

e) Equivalent anchorage length for welded transverse bar

Figure 6.2-1 – Methods of anchorage other than by a straight bar (see Figure 8.1 from EN 1992 [N3])

Bents and hooks do not contribute to compression anchorages.

6.2.2 ULTIMATE BOND STRESS

The ultimate bond strength shall be sufficient to prevent bond failure and basically depends upon the tensile strength of concrete and the location of the bars within the concrete, that is referred as «***bond conditions***». EN 1992 [N3] defines «*good*» and «*poor*» bond conditions (see Figure 6.2-1). There is also test evidence to show that the ultimate bond stress has some dependence on the size of the bar. EN 1992 [N3] de-rates the bond stress for earge-diameter bars (it is defined as bars larger than 40mm).

In accordance with EN1992 [N3] the design value of the ultimate bond stress, f_{bd}, for ribbed bars may be taken as follows:

$$f_{bd} = 2{,}25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd}, \tag{6.2-1}$$

where: f_{ctd} is the design value of concrete tensile strength. Due to the increasing brittleness of higher strength concrete, $f_{ctk,0.05}$ should be limited here to the value for C60/75, unless it can be verified that the average bond strength increases above this limit;

η_1 is a coefficient related to the quality of the bond condition and the position of the bar during concreting (see Figure 6.2-2). It is equal to 1,0, when «good» conditions are obtained, and it is equal to 0,7 for all the other cases and for bars in structural elements built with slip-forms, unless it can be shown that 'good' bond conditions exist;

η_2 is related to the bar diameter. It is equal to 1,0 for $\varnothing \le 32$ mm, and it is equal to $(132 - \varnothing)/100$ for $\varnothing > 32$ mm.

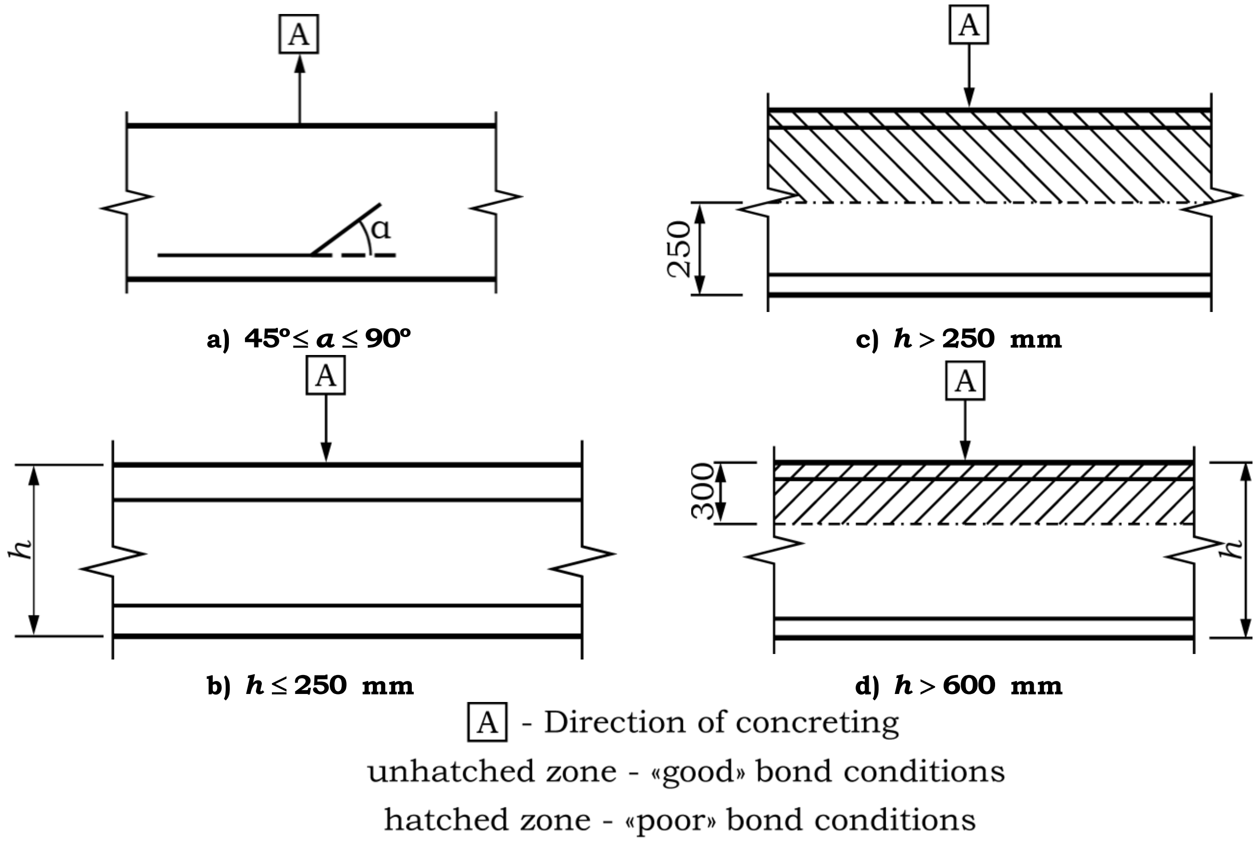


Figure 6.2-2 – Description of bond conditions (Figure 8.2 from EN 1992 [N3])

6.2.3 BASIC ANCHORAGE LENGTH

The calculation of the required anchorage length shall take into consideration the type of steel and bond properties of the bars.

The basic required anchorage length, $l_{b,rqd}$, for anchoring the force $A_s \cdot \sigma_{sd}$ in a straight bar assuming constant bond stress equal to f_{bd} follows from:

$$l_{b,rqd} = (\varnothing / 4) \cdot (\sigma_{sd} / f_{bd}). \qquad (6.2\text{-}2)$$

Where σ_{sd} is the design stress of the bar at the position from where the anchorage is measured from.

For bent bars the basic anchorage length, $l_{b,rqd}$, and the design length, l_{bd}, should be measured along the centre-line of the bar (see Figure 6.1-2 a).

Where pairs of wires/bars form welded fabrics the diameter, $\varnothing$, in Expression (6.2-2) should be replaced by the equivalent diameter $\varnothing_n = \varnothing\sqrt{2}$.

6.2.4 DESIGN ANCHORAGE LENGTH

The basic anchorage length can be modified to allow for such effects as the shape of the bars, the size of the concrete cover, confinement offered by transverse reinforcement or transverse pressure. In EN 1992 [N3], the design anchorage length is obtained by multiplying the basic anchorage length by a number of factors:

– $l_{bd} = a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdot l_{b,reqd} > max\{0,3 \cdot l_{b,reqd}; 10\varnothing; 100\ \text{mm}\}$ – for anchorage in tension;

– $l_{bd} = a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdot l_{b,reqd} > max\{0,6 \cdot l_{b,reqd}; 10\varnothing; 100\ \text{mm}\}$ – for anchorage in compression;

The product $a_2 \cdot a_3 \cdot a_5$ should be ≥0,7. Values of the different multipliers together with the conditions that should be met are given in the code and are not reproduced here. However, the applicability of these in practice are discussed below.

The following applies to bars in tension:

– a_1 allows for the shape of the bars. For straight bars, $a_1 = 1,0$. For curved bars, a_1 may be taken as 0,7, provided that the lesser of the side cover to the bars or half the clear spacing between bars is $> 3\varnothing$. It will be difficult to meet this condition in all but a few practical cases;

– a_2 allows for the effect of the size of concrete cover. For straight bars in tension, some reduction will be possible when the parameter c_d is between $\varnothing$ and $3\varnothing$, where c_d is the least of the side or bottom cover or half the clear spacing between the bars. In the case of curved bars this benefit does not accrue until c_d is larger than $3\varnothing$.

Again, advantage can be taken of this reduction only in a limited number of practical cases;

– a_3 allows for the effect of confinement offered by transverse reinforcement, which is not welded to main bars. The transverse reinforcement should be placed between the concrete surface and the bar that is being anchored. The reduction will be enhanced if the transverse reinforcement is in the form of links. Even so, the reduction that can be achieved in beams will only be 0,925 when $\sum A_{st} = 1{,}0 \cdot A_s$.

– $a_4 = 0{,}7$ can be used in all cases where the transverse reinforcement is welded to the main bars, provided the diameter of the transverse bar is at least $0{,}6\varnothing$, and it is located at least $5\varnothing$ inside $l_{b,reqd}$ from the free end of the bar.

– a_5 accounts for the effect of any pressure *p* (in MPa) transverse to the potential plane of splitting, and is taken as (1 – 0,04·*p*).

In summary, the conditions that need to be satisfied to take advantage of the reduction factors are such that they will only apply in a limited number of practical cases in building structures.

The values of coefficients a_1, a_2, a_3, a_4, a_5 are given in Table 6.2-1.

Table 6.2-1 – Values of a_1, a_2, a_3, a_4 and a_5 coefficients (Table 8.2 from EN 1992 [N3])

Influencing factor	**Type of anchorage**	**Reinforcement bar**	
		in tension	**in compression**
Shape of bars	Straight	$a_1 = 1{,}0$	$a_1 = 1{,}0$
	Other than straight (see Figure 6.2-1 b, c and d)	$a_1 = 0{,}7$ if $c_d > 3\varnothing$ otherwise $a_1 = 1{,}0$	$a_1 = 1{,}0$
Concrete cover	Straight	$a_2 = 1 - 0{,}15 \cdot (c_d - \varnothing)/\varnothing$ $\geq 0{,}7$ $\leq 1{,}0$	$a_2 = 1{,}0$
	Other than straight (see Figure 6.2-1 b, c and d)	$a_2 = 1 - 0{,}15 \cdot (c_d - 3\varnothing)/\varnothing$ $\geq 0{,}7$ $\leq 1{,}0$	$a_2 = 1{,}0$
Confinement by transverse reinforcement not welded to main reinforcement	All types	$a_3 = 1 - K \cdot \lambda$ $\geq 0{,}7$ $\leq 1{,}0$	$a_3 = 1{,}0$
Confinement by welded transverse reinforcement(2)	All types, position and size as specified in Figure 6.2-1 e	$a_4 = 0{,}7$	$a_4 = 0{,}7$
Confinement by transverse pressure	All types	$a_5 = 1 - 0{,}04 \cdot p$ $\geq 0{,}7$ $\leq 1{,}0$	–

Notes: *1.* $\lambda = \left(\sum A_{st} - \sum A_{st,min}\right)/A_s$, *where* $\sum A_{st}$ *is a cross-sectional area of the transverse reinforcement along the design anchorage length* l_{bd} *;* $\sum A_{st,min}$ *is a cross-sectional area of the minimum transverse reinforcement, that is equal to* $0{,}25 \cdot A_s$ *for beams and 0 for slabs;* A_s *is an area of a single anchored bar with maximum bar diameter; K is a coefficient, the values of which one are shown in Figure 6.2-3; p is a transverse pressure (in MPa) at ultimate limit stage along* l_{bd} *;*

2. For direct supports l_{bd} *may be taken less than* $l_{b,min}$ *provided that there is at least one transverse wire welded within the support. This should be at least 15 mm from the face of the support.*

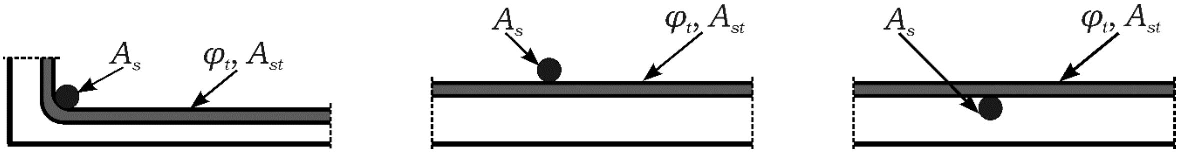


Figure 6.2-3 – Values of *K* for beams and slabs (Figure 8.4 from EN 1992 [N3])

6.2.5 ANCHORAGE OF LINKS AND SHEAR REINFORCEMENT

The anchorage of links and shear reinforcement should normally be effected by means of bends and hooks, or by welded transverse reinforcement. A bar should be provided inside a hook or bend.

The anchorage should comply with Figure 6.2-4. Welding should be carried out in accordance with EN ISO 17660 [N10] and have a welding capacity in accordance with EN 1992 [N3] (clause 8.6 (2)).

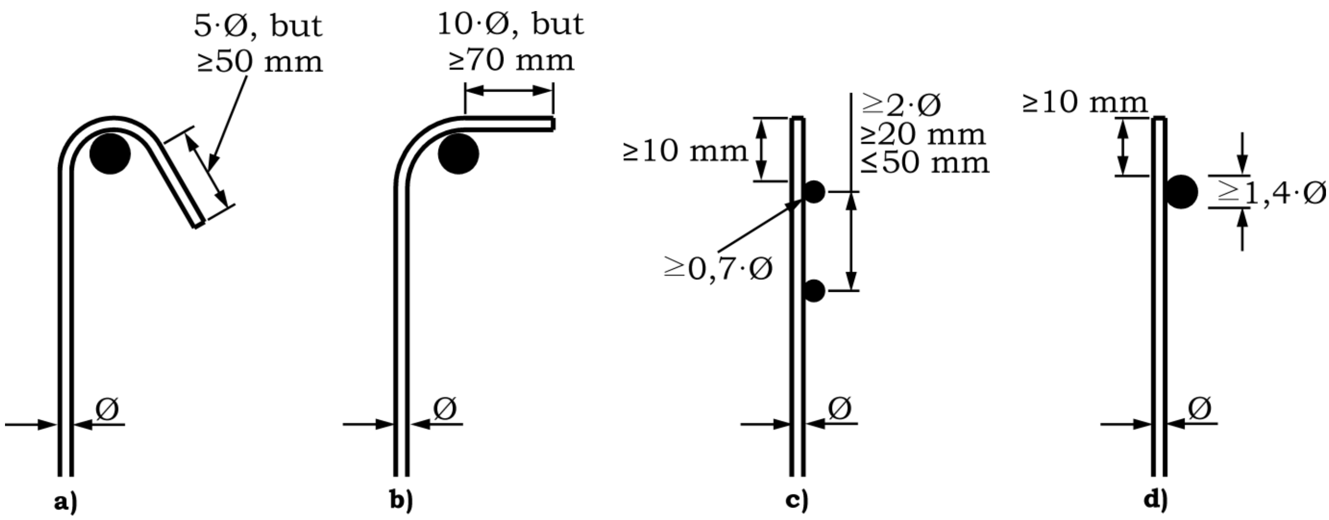


Figure 6.2-4 – Anchorage of links (Figure 8.5 from EN 1992 [N3])

6.3 DETAILING REQUIREMENTS FOR PARTICULAR MEMBER TYPES

6.3.1 BEAMS

6.3.1.1 Longitudinal reinforcement

Minimum area $A_{st,min}$:

– $A_{st,min} = 0{,}26 \cdot f_{ctm} \cdot b_t \cdot d / f_{yk}$, but not less than $0{,}0013 \cdot b_t \cdot d$, where f_{yk} is the characteristic yield stress of reinforcement and b_t denotes the mean width of the tension zone (for a T-beams with the flange in compression, only the width of the web is taken into account in calculating of the b_t value);

– at supports in monolithic construction where simple supports are assumed in the design, reinforcement $A_{st,sup}$ required to cope with partial fixity is at least $0{,}15 \cdot A_{st}$ span (see Figure 6.3-1).

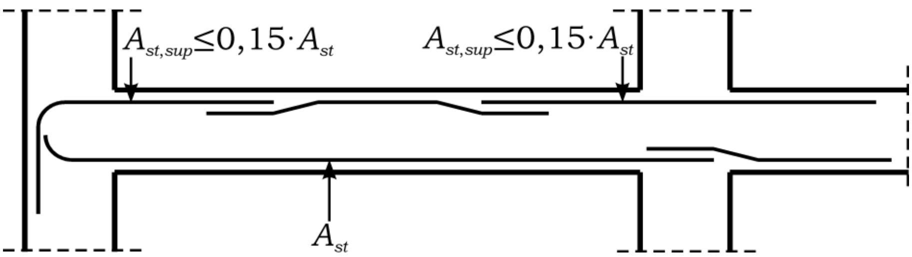


Figure 6.3-1 – Longitudinal reinforcement at supports in monolithic construction

Maximum area $A_{st,max}$ or $A_{sc,max}$ is equal to $0{,}04 \cdot A_c$, where A_c is the cross-section area of concrete.

At intermediate supports of continuous beams, the total area of tension reinforcement A_{st} of a flanged cross-section should be spread over the effective width of flange (see Chapter 2). Part of it may be concentrated over the web width (see Figure 6.3-2).

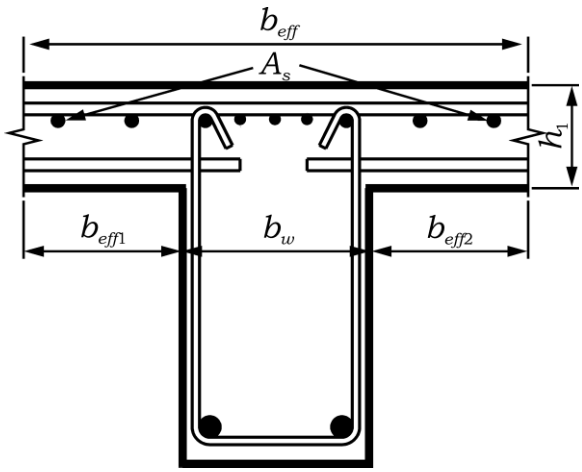


Figure 6.3-2 – Placing of tension reinforcement in flanged cross-section

Longitudinal compression reinforcement (diameter ∅) should be contained by link reinforcement, the maximum spacing of which should not exceed 15∅.

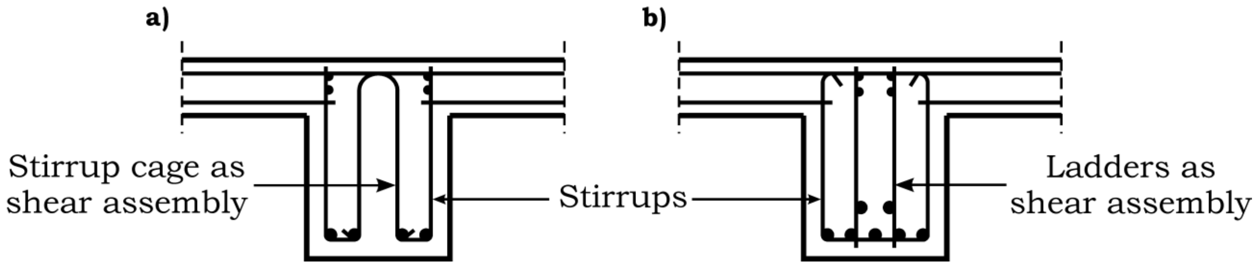
6.3.1.2 Shear reinforcement

General requirements:

– shear reinforcement should form an angle of 90°–45° with the mid-plane of the beam;

– shear reinforcement may consist of a combination of: links enclosing the longitudinal tensile reinforcement and the compression zone; bent-up bars; shear assemblies of cages, ladders, etc. which do not enclose the longitudinal reinforcement, but which are properly anchored in the compression and tension zones (see Figure 6.3-3);

– for combinations of links and shear assemblies, all shear reinforcement should be effectively anchored. Lap joints on the leg near the surface of the web are only permitted for high-bond bars. At least 50 % of the necessary shear reinforcement should be in the form of links.



a) – stirrup cage as a shear assembly; b) – ladders as a shear assembly

Figure 6.3-3 – Combination of links and shear assemblies

Minimum area A_{sw} should be calculated from the following expression: $\rho_{w,min} = (A_{sw} / s) \cdot b_w \cdot \sin\alpha = 0{,}08 \cdot f_{ck}^{0,5} / f_{yk}$, where $\rho_{w,min}$ is the minimum shear reinforcement ratio; A_{sw} is the area of shear reinforcement with a longitudinal spacing s; α is the angle between the shear reinforcement and the longitudinal steel.

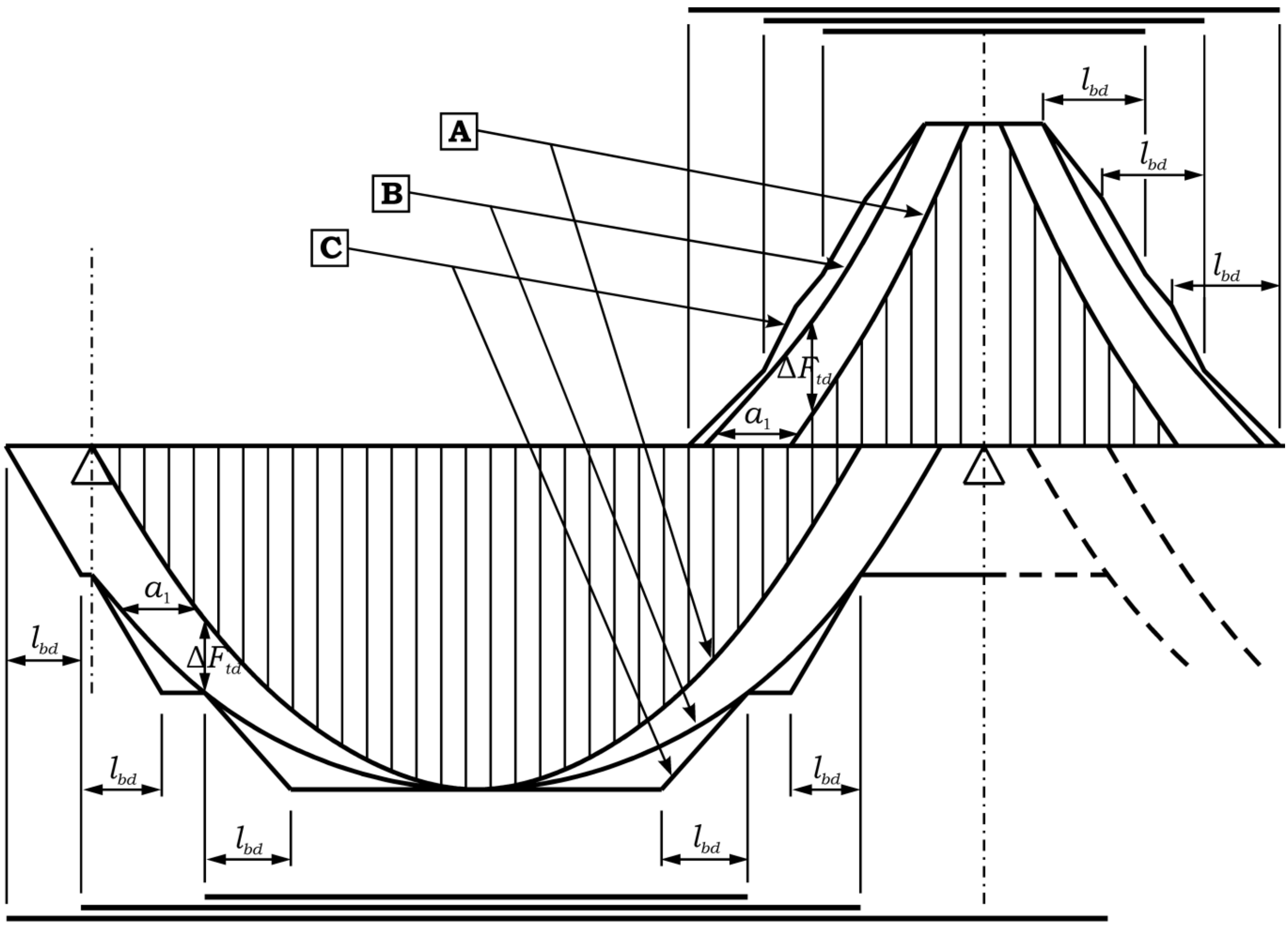
Maximum spacing of shear reinforcement s_{max}: maximum longitudinal spacing of links is equal to $0,75 \cdot d \cdot (1+\cot\alpha)$; maximum longitudinal spacing of bent-up bars is equal to $0,6 \cdot d \cdot (1+\cot\alpha)$.

The transverse spacing of a series of shear links should not exceed $0,75 \cdot d$ nor 600 mm.

6.3.1.3 Curtailment of longitudinal tension reinforcement

Any curtailed reinforcement should be provided with an anchorage length $l_{b,net}$, but not less than d from the point where it is no longer needed. This should be determined taking into account the tension caused by the bending moment and that implied in the truss analogy used for shear design. This can be done by shifting the point of the theoretical cut-off based on the bending moment by a_1 (see below for definition) in the direction of decreasing moment. This procedure is also referred to as the «***shift rule***»: $a_1 = z \cdot (\cot\theta - \cot\alpha)/2$, where θ is the angle of the concrete struts to the longitudinal axis and α is the angle of the shear reinforcement to the longitudinal axis; z normally can be taken as $0,75 \cdot d$.

For members with shear reinforcement, the additional force ΔF_{td} should be calculated in accordance with Section 4.2. The additional tensile force is illustrated in Figure 6.3-4.



$\boxed{A}$ - envelope of $M_{Ed}/z+N_{Ed}$ $\boxed{B}$ - acting tensile force F_s

$\boxed{C}$ - resisting tensile force F_{Rs}

Figure 6.3-4 – Illustration of the curtailment of longitudinal reinforcement, taking into account the effect of inclined cracks and the resistance of reinforcement within anchorage length

For reinforcement in the flange, placed outside the web, a_1 should be increased by the distance of the bar from the web.

6.3.1.4 Anchorage at supports

End support. When there is little or no fixity at an end support, at least one-quarter of the span reinforcement should be carried through to the support. The code recommends that the bottom reinforcement should be anchored to resist force of $\Delta F_{td} = V_{Ed} \cdot a_1/d + N_{Ed}$, where V_{Ed} is the shear force at the end, a_1 is as defined above for the shift rule, and N_{Ed} is the axial force, if any, in the member.

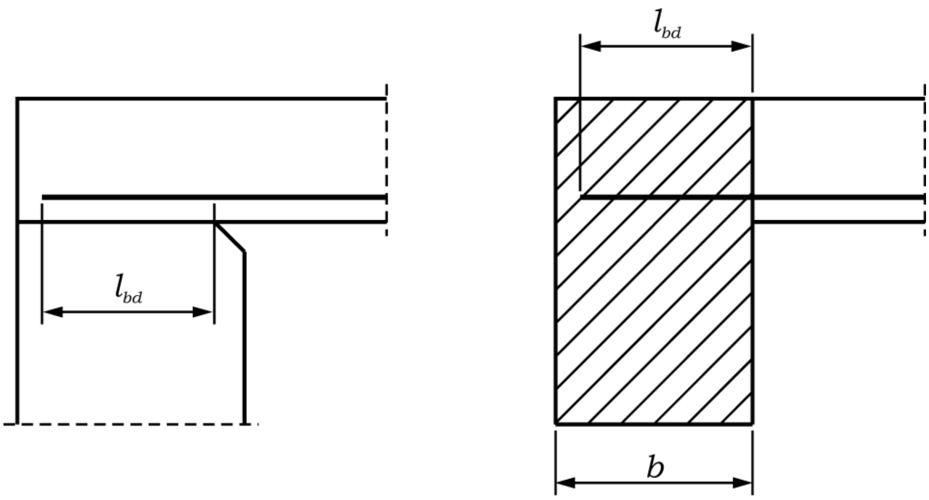
As simply supported ends, the bars should be anchored beyond the line of contact the member and its support (face of support) by:

– direct support for beams: 0,8 times of the value given in Table 6.3-1;
– all indirect supports: 1 time of value given in Table 6.3-1.

Table 6.3-1 – Typical values of anchorage and lap lengths for beams

	Bond conditions	Length in bar diameters		
		C25/30	C30/37	C35/40
Full tension and compression anchorage length $l_{b,rqd}$	good	36	32	31
	poor	48	43	41
Full tension and compression lap length l_0	good	42	37	35
	poor	56	49	47
Note: *The following is assumed: – bar size is not greater than 32 mm. If >32 mm, then the anchorage and lap lengths should be divided by a factor (132-bar size)/100; – normal cover exists; – no confinement by transverse pressure; – confinement by links-factor =0,9; – no more than 33 % of the bars are lapped at one place.*				

A «***direct***» support is one, where the reaction provides compression across the bar being anchored. All the other supports are considered as «***indirect***» (see Figure 6.3-5).



Direct support
Beam supported by wall or column

Indirect support
Beam intersecting another supporting beam

Figure 6.3-5 – Anchorage at end supports (Figure 5.34 from EN 1992 [N3])

Intermediate supports (general requirements):

– at intermediate supports, >25 % of the midspan bottom reinforcement should be carried to the support;

– the minimum anchorage of bottom reinforcement beyond the face of the support is 10∅ for straight bars; or the diameter of the mandrel for bars of 16 mm diameter or more and with hooks or bends; or twice the diameter of the mandrel in other cases;

– however, this does not mean that the support must be greater than 20∅ wide, as the bars from each side of the support can be lapped. It is, however, recommended that continuous reinforcement is provided to resist accidental forces (see Figure 6.3-6).

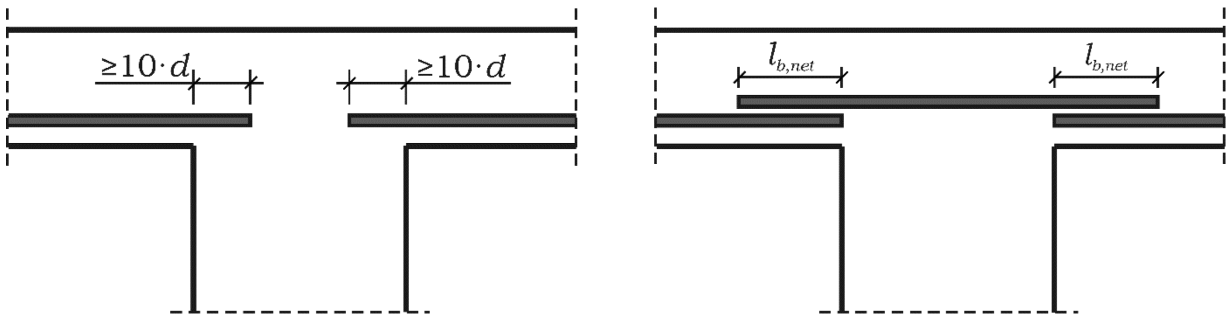


Figure 6.3-6 – Continuous reinforcement

The clear distance (spacing) between lapped bars should be in accordance with Figure 6.3-7.

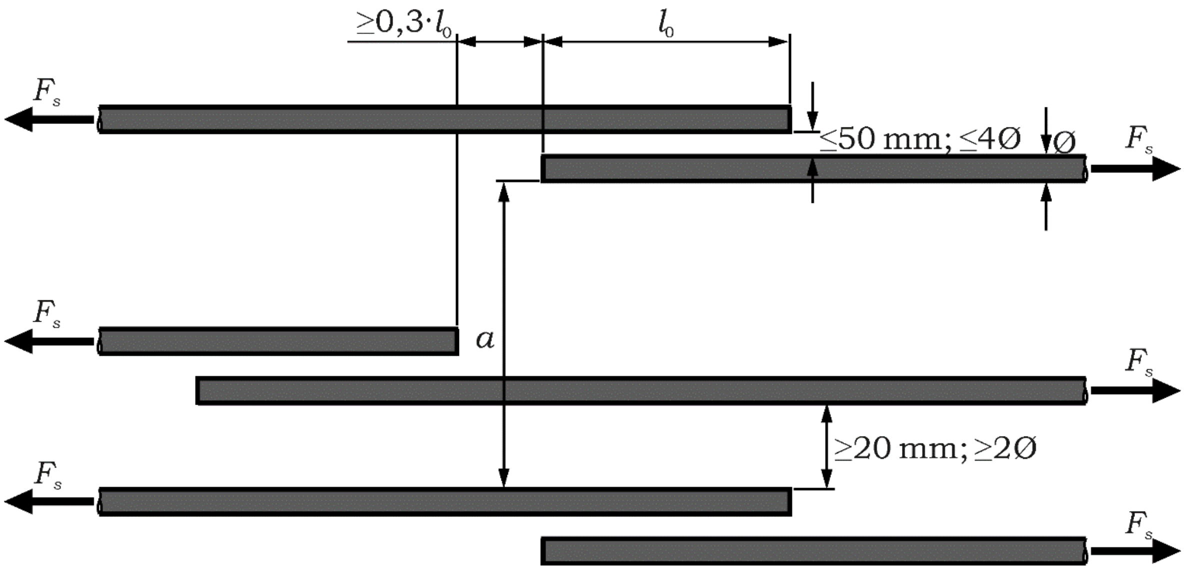


Figure 6.3-7 – Adjacent laps (Figure 5.29 from EN 1992 [N3])

It should be noted that, where the distance between lapped bars is greater than 50 mm or 4Ø, the lap length should be increased by the amount, by which the clear space exceeding 50 mm or 4Ø.

6.3.1.5 Skin reinforcement

Skin reinforcement to control cracking should normally be provided in beams over 1,0 m in depth where the reinforcement is concentrated in a small portion of the depth. This reinforcement should be evenly distributed between the level of the tension steel and the neutral axis and be located within the links.

6.3.1.6 Surface reinforcement

Surface reinforcement may be required to resist spalling of the cover, e.g. arising from fire or where bundled bars or bars greater than 32 mm are used. This reinforcement should consist of small-diameter, high-bond bars or wire mesh placed in the tension zone outside the links.

The area of surface reinforcement parallel to the beam tension reinforcement should not be less than $0{,}01 \cdot A_{ct,ext}$, where $A_{ct,ext}$ is the area of concrete in tension external to the links.

The longitudinal bars of the surface reinforcement may be taken into account as longitudinal bending reinforcement, and the transverse bars as shear reinforcement, provided that they meet the arrangement and anchorage requirements of these types of reinforcement (see Figure 6.3-8).

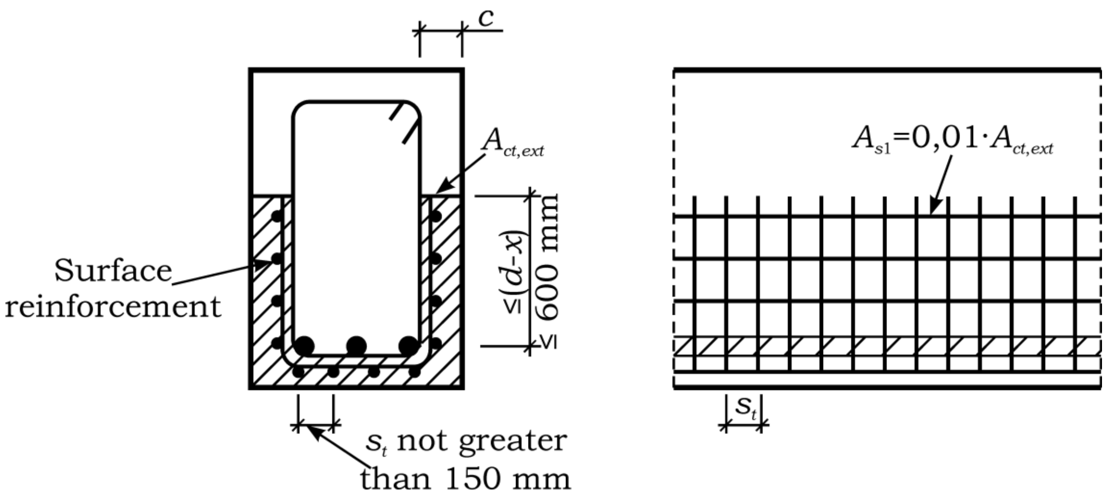


Figure 6.3-8 – Arrangement and anchorage requirements

6.3.2 SLABS

6.3.2.1 Longitudinal reinforcement

Minimum area $A_{st,min}$: is equal to $0{,}26 \cdot f_{ctm} \cdot b_t \cdot d / f_{yk}$, but have to be not less than $0{,}0013 \cdot b_t \cdot d$, where f_{yk} is the characteristic yield stress of reinforcement.

Maximum area $A_{st,max}$: is at least $0{,}04 \cdot A_c$, where A_c is the cross-section area of concrete.

Maximum bar spacing s_{max}:

– generally, $s_{max} = 3 \cdot h \le 400$ mm for main reinforcement and $3{,}5 \cdot h < 450$ mm for secondary reinforcement;

– local to concentrated loads, $s_{max} = 2 \cdot h < 250$ mm for main reinforcement and $3 \cdot h < 400$ mm for secondary reinforcement.

Reinforcement near supports:

– in simply supported slabs a minimum of 50 % of the reinforcement in the span should be anchored at supports (see Figure 6.3-9);

– where partial fixity is likely to exist despite the assumption of simple support in design, 25 % of the reinforcement required to resist the maximum span moment should be provided at the top of end supports;

– at the end supports, the reinforcement should extend from the face of the support, at least 0,2 times the adjacent span (see Figure 6.3-10).

– at intermediate supports, the reinforcement should be continuous across the support.

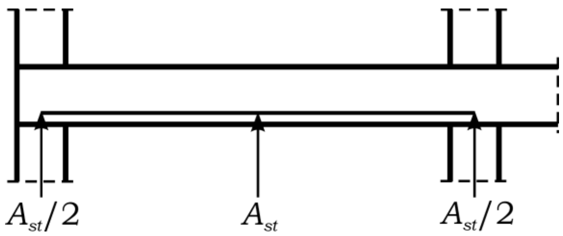


Figure 6.3-9 – Span reinforcement

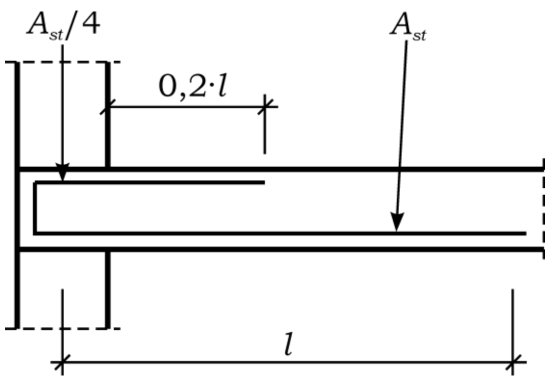


Figure 6.3-10 – End supports with partial fixity

6.3.2.2 Transverse reinforcement

The minimum area of the transverse reinforcement $A_{s,t}$ is 20 % of the longitudinal reinforcement.

6.3.2.3 Corner and edge reinforcement

Suitable reinforcement is required where slab corners are restrained against lifting. Normally, U bars extending $0,2 \cdot l$ into the span should be provided at all edges (see Figure 6.3-9).

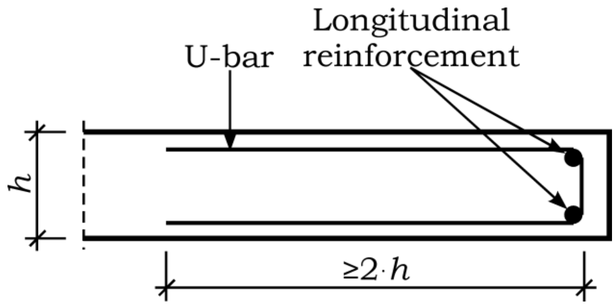


Figure 6.3-11 – Reinforcement at free edges

6.3.2.4 Shear reinforcement

Minimum slab depth $h_{min} = 200\text{ mm}$, where shear reinforcement is to be provided.

The requirements are given in clause 10.9.1 of the EN 1992 [N3] for beams apply generally to slabs, with the following modifications: form of shear reinforcement – shear reinforcement may consist entirely of bent-up bars or shear assemblies, where $V_{Ed} \leq 0{,}33 \cdot V_{Rd,max}$; maximum longitudinal spacing s_{max} is equal to $0{,}75 \cdot d \cdot (1 + \cot\alpha)$ for links and $1{,}0 \cdot d$ for bent up bars; maximum transverse spacing of shear reinforcement is equal to $1{,}5 \cdot d$.

6.3.2.5 Anchorage and lap length

Anchorage and lap lengths should be obtained from the Table 6.3-2 for bars and welded mesh fabric.

Table 6.3-2 – Typical values of anchorage and lap lengths for slabs

	Bond conditions	**Length in bar diameters**		
		C25/30	**C30/37**	**C35/40**
Full tension and compression anchorage length l_{bd}	good	40	36	34
	poor	58	51	49
Full tension and compression lap length l_0	good	46	42	39
	poor	66	59	56

Note: *1. The following is assumed: – bar size is not greater than 32 mm. If >32 mm, then the anchorage and lap lengths should be divided by a factor (132-bar size)/ 100;*
- *normal cover exists;*
- *no confinement by transverse pressure;*
- *no confinement by transverse reinforcement;*
- *no more than 33 % of the bars are lapped at one place.*

2. Lap lengths provided (for nominal bars, etc.) should not be less than 15 times the bar size or 200 mm, whichever is greater.

The clear spacing between two lapped bars should be in accordance with Figure 6.3-7.

6.3.3 Columns

6.3.3.1 Longitudinal reinforcement

Minimum diameter of the longitudinal reinforcement is 12 mm.

Minimum area $A_{s,min}$ is equal to $0{,}1 \cdot N_{Ed}/f_{yd}$, or $0{,}002 \cdot A_c$, whichever is greater, where N_{Ed} is the design axial force, f_{yd} is the yield strength of reinforcement and A_c is the area of concrete.

Maximum area $A_{s,max}$ is equal to $0{,}04A_c$ outside laps, and to $0{,}08A_c$ at laps.

Minimum number of bars is shown in Figure 6.3-12.

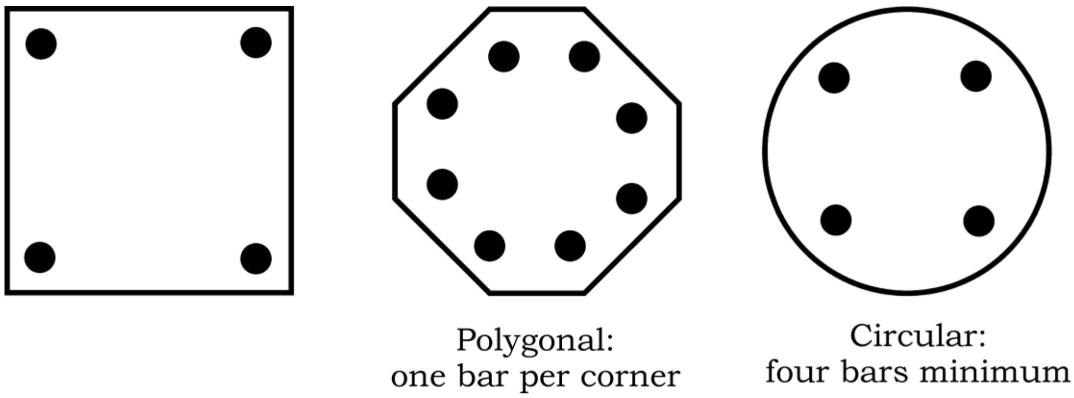


Figure 6.3-12 – Minimum number of bars

6.3.3.2 Transverse reinforcement

General requirements:

– all transverse reinforcement must be adequately anchored;

– every longitudinal bar (or group of bars) placed in a corner should be held by transverse reinforcement;

– no longitudinal bar in a compression zone should be further from a restrained bar than 150 mm.

Minimum diameter of the transverse bar should not be less than 6 mm or 0,25 times the diameter of the largest bar being restrained.

Spacing of the transverse reinforcement:

– generally, the maximum spacing s_{max} should be the least of the following: 20 times the diameter of the longitudinal bar; or the lesser dimension of the column; or 400 mm;

– for a distance equal to the larger dimension of the column, above and below slabs or beams the spacing noted above should be reduced by a factor of 0,6;

– the above reduced spacing is also required at laps of longitudinal bars of diameter greater than 14 mm. A minimum of three transverse bars should be evenly positioned over the lap length.

6.3.4 WALLS

6.3.4.1 Vertical reinforcement

Minimum area of the vertical reinforcement: $A_{sv,min} = 0{,}002 \cdot A_c$.

Maximum area of the vertical reinforcement: $A_{sv,max} = 0{,}04 \cdot A_c$.

The code EN 1992 [N3] permits this to be doubled if the designer can show that the integrity of concrete is not affected and the full strength can be achieved at the ultimate limit state.

Maximum bar spacing s_{max}: the distance between adjacent bars should not exceed three times the thickness of the wall or 400 mm, whichever is less.

When minimum reinforcement controls the design, 50 % of the minimum reinforcement should be placed on each face.

6.3.4.2 Horizontal reinforcement

Horizontal reinforcement should be placed between the vertical reinforcement and the face of the wall.

Minimum area of the horizontal reinforcement: $A_{sh,min} = 0{,}25 \cdot A_{sv}$, but not less than $0{,}001 \cdot A_c$.

Maximum spacing s_{max} is equal to 400 mm.

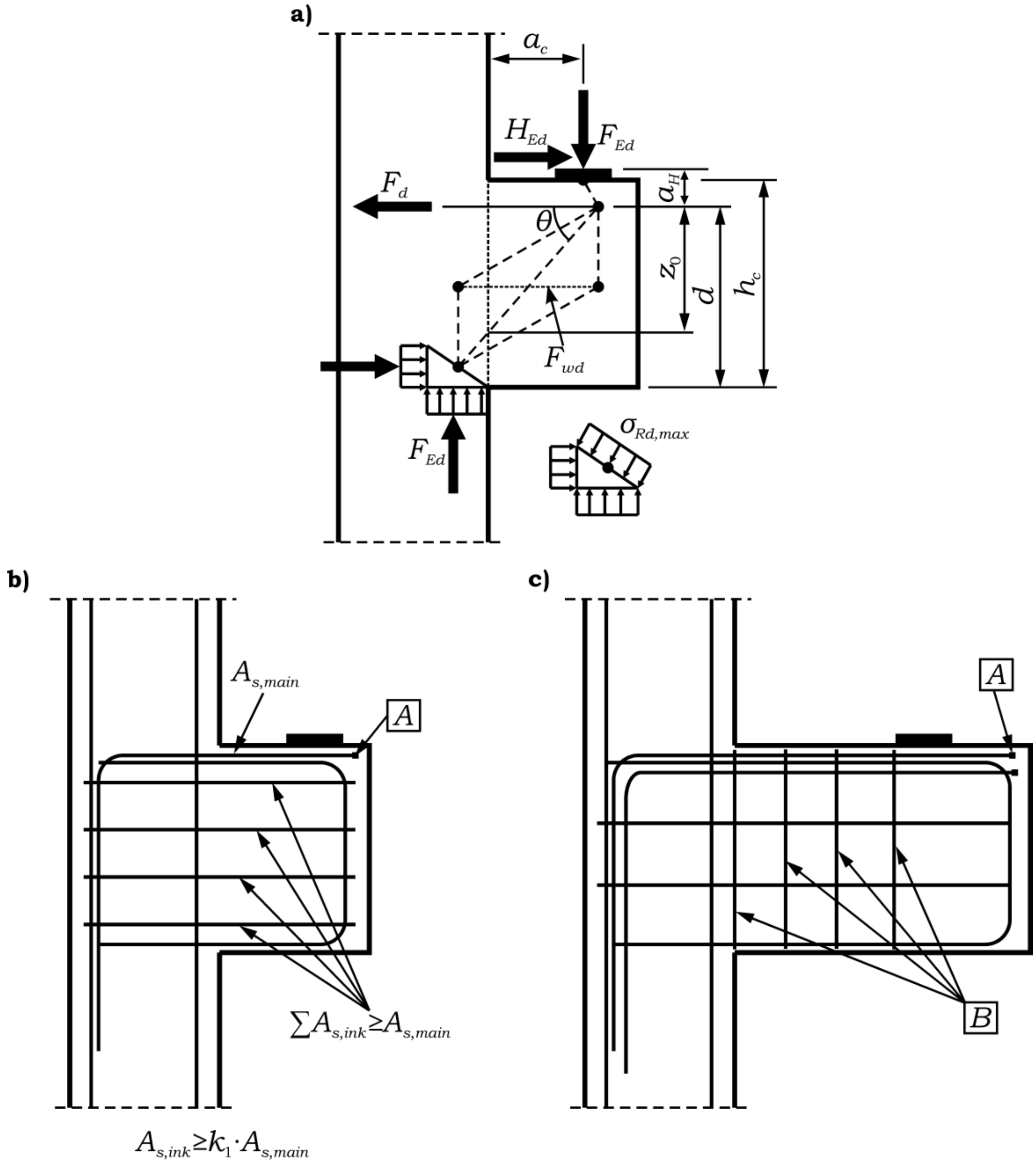
Minimum diameter: the code does not specify a value, but it will be prudent to use a minimum of 0,25 times the diameter of the vertical reinforcement.

6.3.4.3 Transverse reinforcement

Where the area of vertical reinforcement exceeds $0{,}02 \cdot A_c$ transverse reinforcement in the form of links should be provided in accordance with the requirements for columns.

6.3.5 CORBELS

General: where $a_c \le z_0$, a simple strut-and-tie model may be used (see Figure 6.3-13 a).



A Anchorage devices or loops B Links

a) – strut-and-tie model; b) – reinforcement for $a_c < 0{,}5 \cdot h_c$;

c) – reinforcement for $a_c \geq 0{,}5 \cdot h_c$

Figure 6.3-13 – Corbels

Anchorage of the primary tie reinforcement: unless a length l_{bnet} is available, the primary horizontal tie A_s should be anchored on both sides beyond the bearing area using U bars or a welded cross bar.

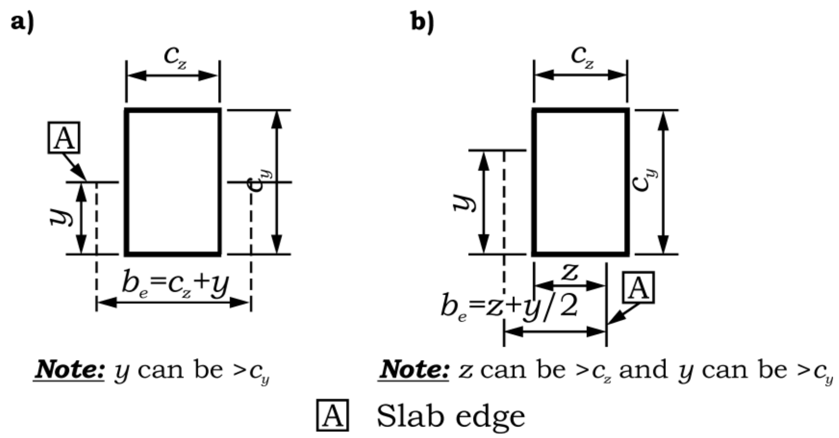
Provisions of links: when $a_c < 0{,}5 \cdot h_c$, closed horizontal or inclined links should be provided (see Figure 6.3-13 b). The area of the link should be at least 0,25 times the area of the primary tie reinforcement. When $a_c \geq 0{,}5 \cdot h_c$, and the load applied on the corbel exceeds $V_{Rd,c}$, vertical closed links should be provided in addition to the horizontal links (see Figure 6.3-13 c). The area of the vertical link should be at least $0{,}5 \cdot F_{Wd}/f_{yd}$, where F_{Wd} is the force in the main tie reinforcement.

6.3.6 REINFORCEMENT IN FLAT SLABS

Concentration of reinforcement over the columns will generally be required to meet the serviceability requirements. In the absence of rigorous calculations, top reinforcement with an area of $0{,}5 \cdot A_t$ should be placed in a width equal to the sum of 0,125 times the panel width on either side of the column. A_t represents the area of reinforcement required to resist the full negative moments in a panel.

At least two bars forming the bottom reinforcement in the slab should pass through the internal columns in each orthogonal direction.

Reinforcement required to transfer bending moments from slab to columns, at right angles to an edge, should be placed within an effective width as shown in Figure 6.3-14.

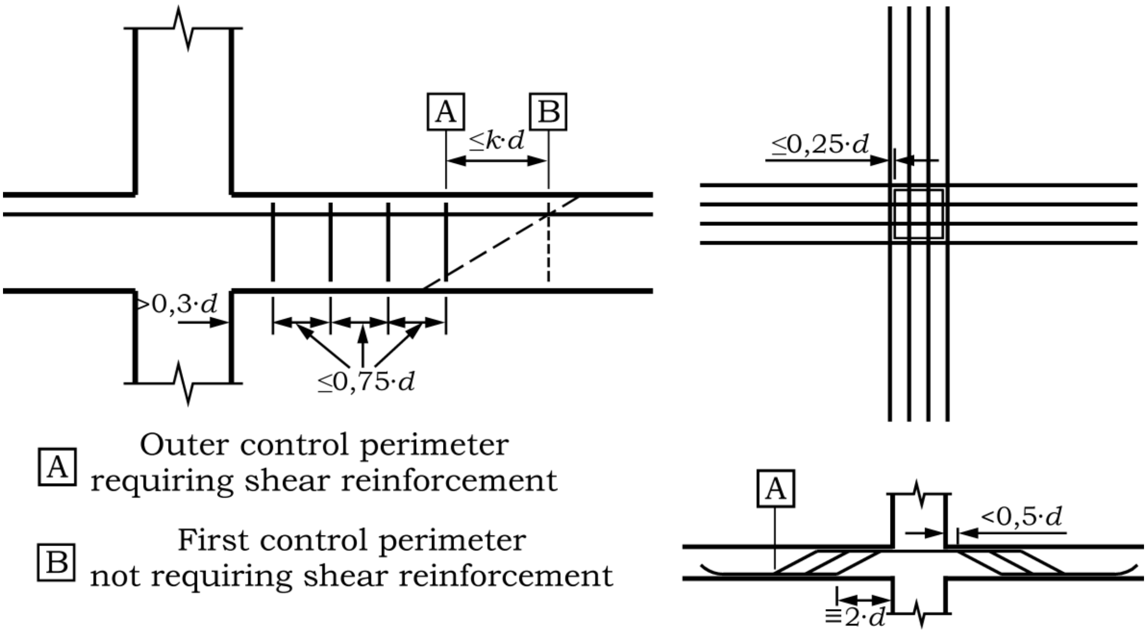


a) – edge column; b) – corner column

Figure 6.3-14 – Effective width, b_e, of a flat slab

EN 1992 [N3] recognizes the use of both proprietary shear reinforcement and conventional link reinforcement. In the case of the former, the design and detailing should comply with the relevant European technical approval.

Where punching shear reinforcement is required, it should be provided between the loaded area and $1{,}5 \cdot d$ within the control perimeter at which shear reinforcement is no longer required. Such reinforcement should be provided in at least two perimeters of spacing not exceeding $0{,}75 \cdot d$. The spacing of the perimeters in which links are provided should not exceed 0,75·*d*. The spacing of the legs of links around a perimeter should not exceed 1,5·*d* (see Figure 6.3-15).



a) – spacing of links; b) – spacing of bent-up bars

Figure 6.3-15 – Punching shear reinforcement

Where shear reinforcement is required, the area of one leg of link reinforcement should comply with $A_{sw,min} = (1,5 \cdot \sin a + \cos a)/s_r \cdot s_t \geq 0,08 \cdot f_{ck}^{0,5} / f_{yk}$, where a is the angle between the shear reinforcement and the longitudinal steel, and s_r and s_t are the spacings of the shear reinforcement in the radial and tangential directions, respectively.

Bent-up bars passing through the loaded area and within 0,25·*d*. on either side of it may be used as punching shear reinforcement.

CHAPTER 7

SLABS

7.1 GENERAL

In general case a slab is a flat two-dimensional planar structural element having thickness small compared to its other two dimensions. As it was stated in EN1992 [N3] a slab is a member for which the minimum panel dimension is not less than 5 times the overall slab thickness.

It provides a working flat surface or covering shelter in buildings. It primary transfer the load by bending in one or two directions. The floor system of a structure can take many forms such as in situ solid slabs, ribbed slabs (beam-and-girder floor with one-way and two-spanning slabs), flat plate and flat slabs; joist floor (waffle slab). In accordance with [17], the choice of type of slab for particular floor depends on many factors. Economy of construction is obviously an important consideration, but this is a qualitative argument until specific cases are discussed, and is a geographical variable. The design loads, required spans, serviceability requirements are all important.

7.2 CLASSIFICATION OF THE CONCRETE FLOOR SYSTEMS WITH ONE-WAY AND TWO-WAY SLABS

7.2.1 ONE-WAY AND TWO-WAY SLABS

In general case slabs are classified based on following main aspects:

Shape: square, rectangular , circular and polygonal;

Type of support: slab supported on stiff beams (***beam-supported floor***); slab supported on walls; slab supported on columns (beamless floor: Flat plate and Flat slabs);

Boundary (support) conditions: simply supported; cantilever slab; overhanding slab; fixed or continuous (continues) slabs;

Use: roof slabs; floor slabs;

Cross-section or sectional configuration: ribbed slab/beam-and-girder floor; filled slab; folded plate;

Spanning directions: one-way slab – spanning in one direction; two-way slab– spanning in two direction.

In general, rectangular one-way and two-way slabs are very common.

Additionally to proposed classification, concrete floor systems can be classified as: (a) beam-supported floor and, (b) beamless floors. They are further divided into several types, as shown in Figure 7.2-1.

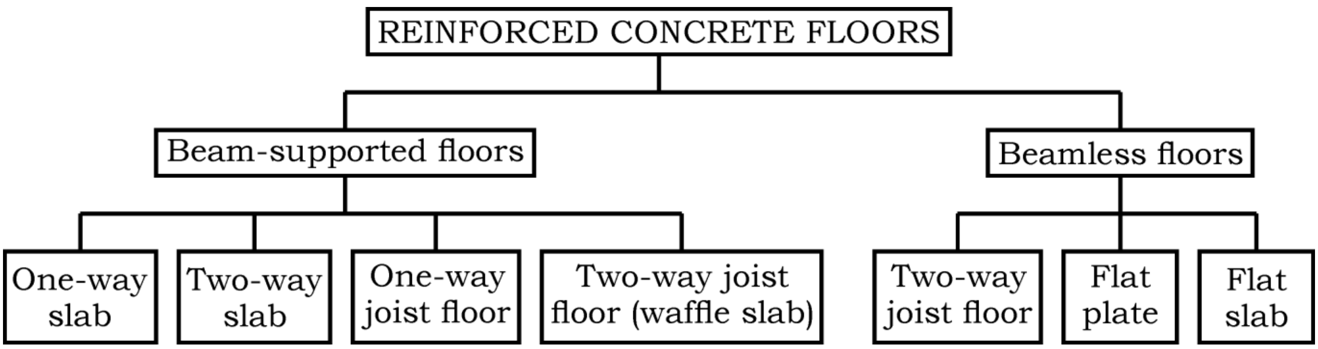
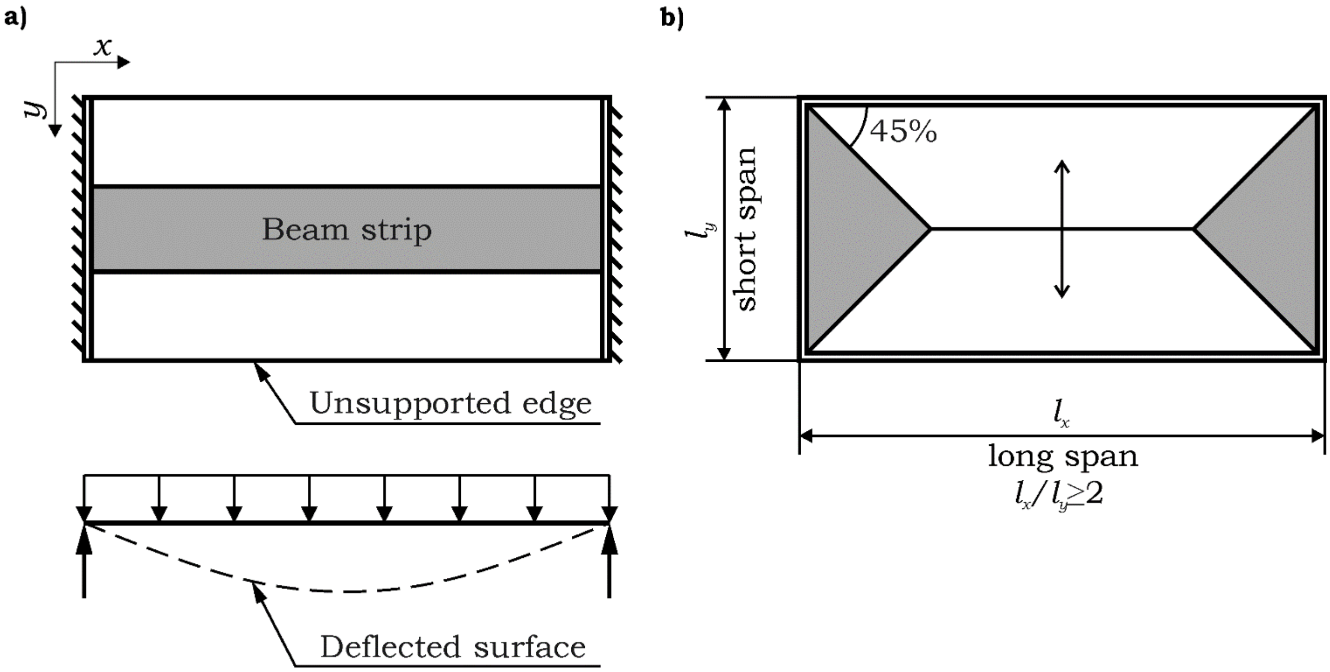


Figure 7.2-1 – Types of reinforced concrete slabs [18].

7.2.1.1 One-way solids slabs

When a slab is supported only on two parallel apposite edges, it spans only in direction perpendicular to two supporting edges (see Figure 7.2-2 a). Also, if the ratio of the long dimension (span) to the short dimension (span) of a four-side-supported slab panel is greater than or equal to 2,0, most of the load on the slab is transferred to the long pair of beams, that is, the load path is along the long dimension (span) of the slab is negligible (see Figure 7.2-2). Such a slab are also designed as one-way slab, because the load is effectively transferred along one direction. The reinforcement in a one-way slab is placed along the short direction, reffered as the «***primary reinforcement***» to distinguish it from the nominal reinforcement placed along the perpendicular direction, called the «***secondary reinforcement***». The main purpose of secondary reinforcement is to resist stressed caused by concrete shrinkage, thermal expansion and contraction of the slab.



a) – simply supported edge;

b) – most of the load from the slab is carried in short direction (this support)

Figure 7.2-2 – One-way solid slabs

7.2.1.2 Two-way solid slabs

If the ratio of the long to short span of a four-side-supported slab panel is less than 2,0, the slab is considered to behave as a two-way slab. However, real two-way slab behavior occurs when the ratio of the two spans is as close to 1,0 as possible (between 1,0 and 1,25). In a two-way slab, both directions participate in carrying the load. Reinforcement is, therefore, provided in both one-way and two-way slabs may occur in the same floor, Figure 7.2-3.

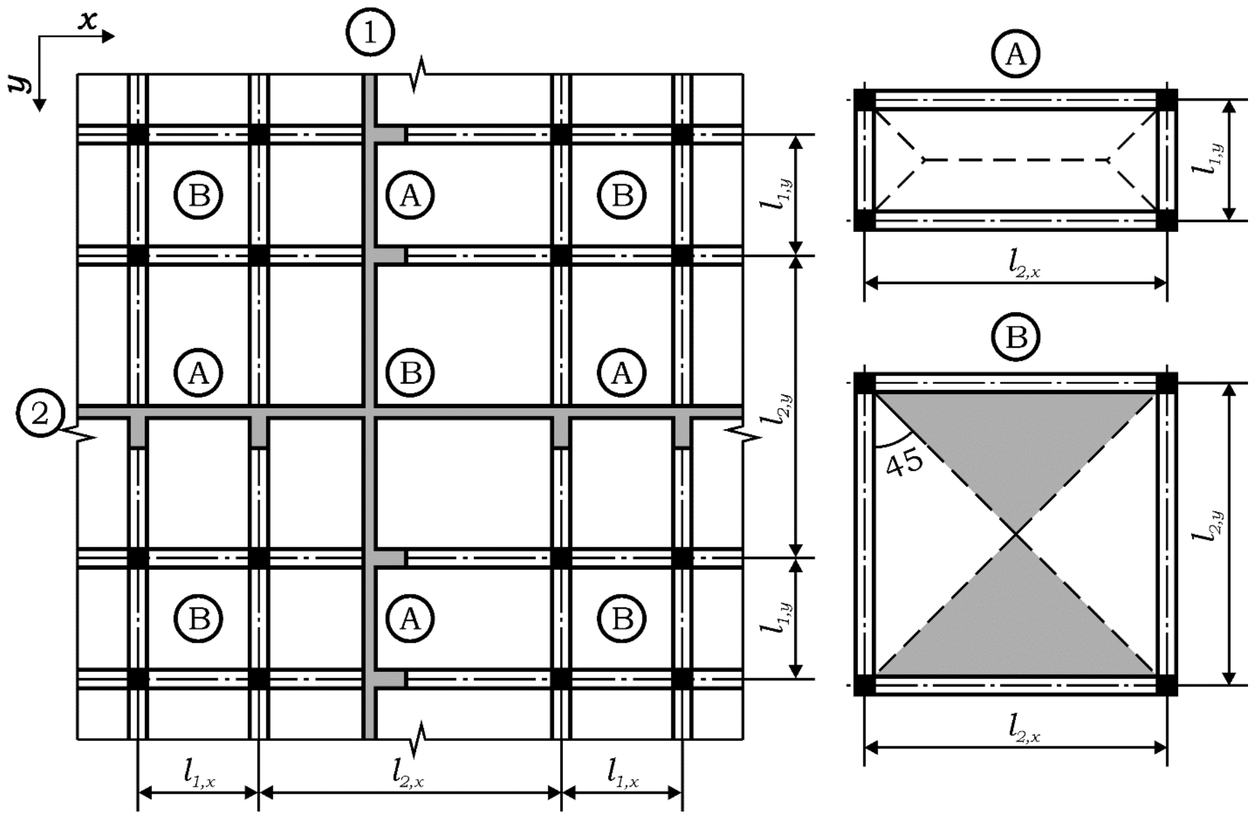


Figure 7.2-3 – Combination of the one-way (A) and two-way (B) solid slabs

Since, the slab rest freely on all sides, due to transverse load the corners tend to curl up and lift up. The slab looses the contact over some region. This is known as lifting of corner. These slab are called two-way simply supported slabs. At corner, the rotation occurs in both the direction and causes the corners to lift. If the corners of slab are restrained from lifting, downward reaction results at corner and the end strips gets restrained against rotation. However, when the end are restrained and rotation of central strip still occurs and causing rotation at corner (slab is acting as unit) the end strip is subjected to torsion.

7.2.2 BEAM-SUPPORTED FLOORS

7.2.2.1 Beam-and-girder floors

One-way and two-way solid slab become increasingly thick and hence uneconomical as their span increases. Generally, the use of a slab thicker than 200 mm is discouraged because it creates a large self-weight (dead load) on the floor. For one-way slab, an 200 mm slab thickness is reached with a span of approximately 5,0 m for a square two-way slab, a span of approximately 7,0 m requires 200 mm – thick slab. Because spans are relatively small for column spacing, one-way and two-way slabs are generally used in a beam-and-girder floor (Figure 7.2-4) or in a two-way beam-and-girder floor, Figure 7.2-5.

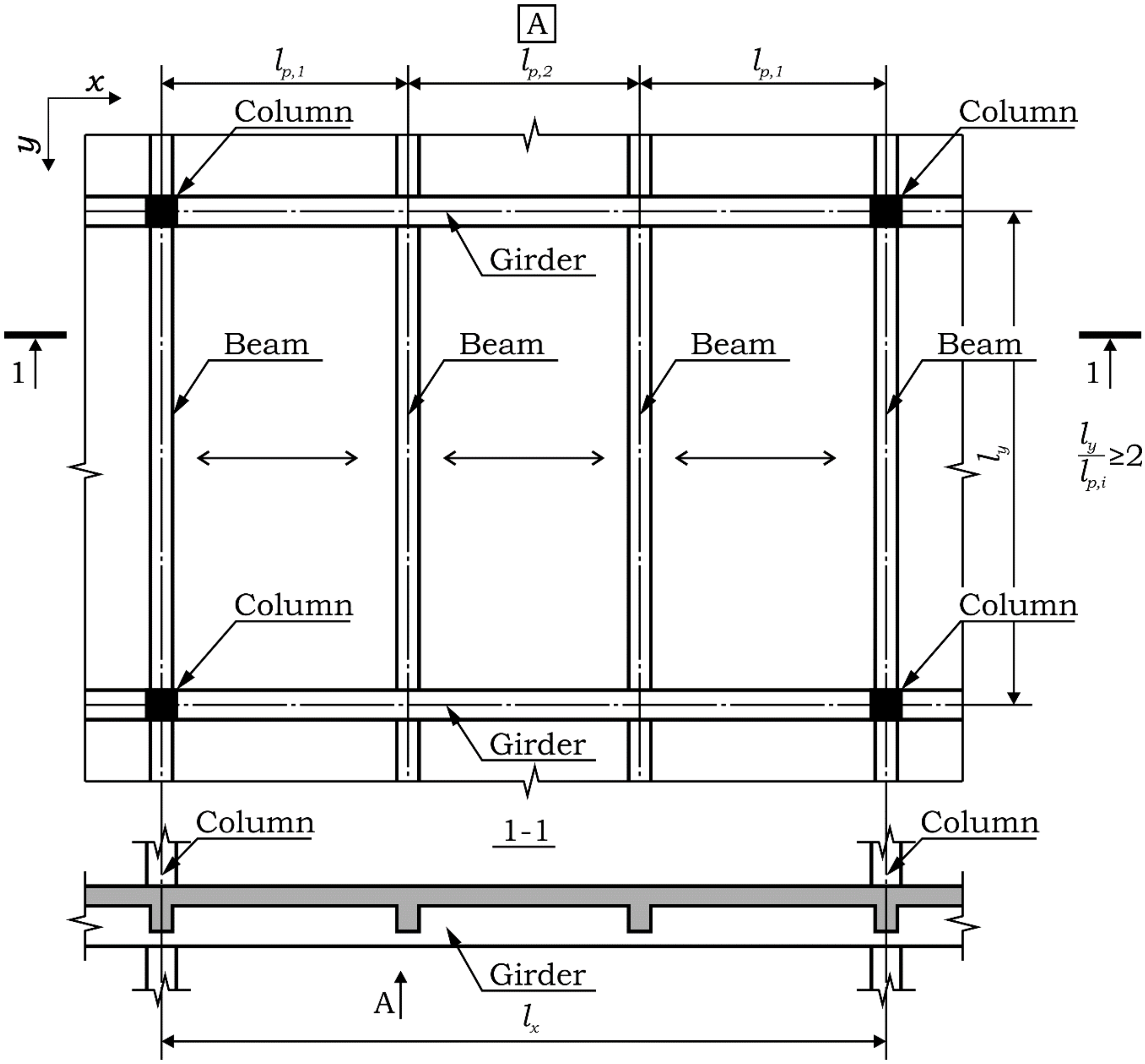


Figure 7.2-4 – Beam-and-girder floor with one-way slabs

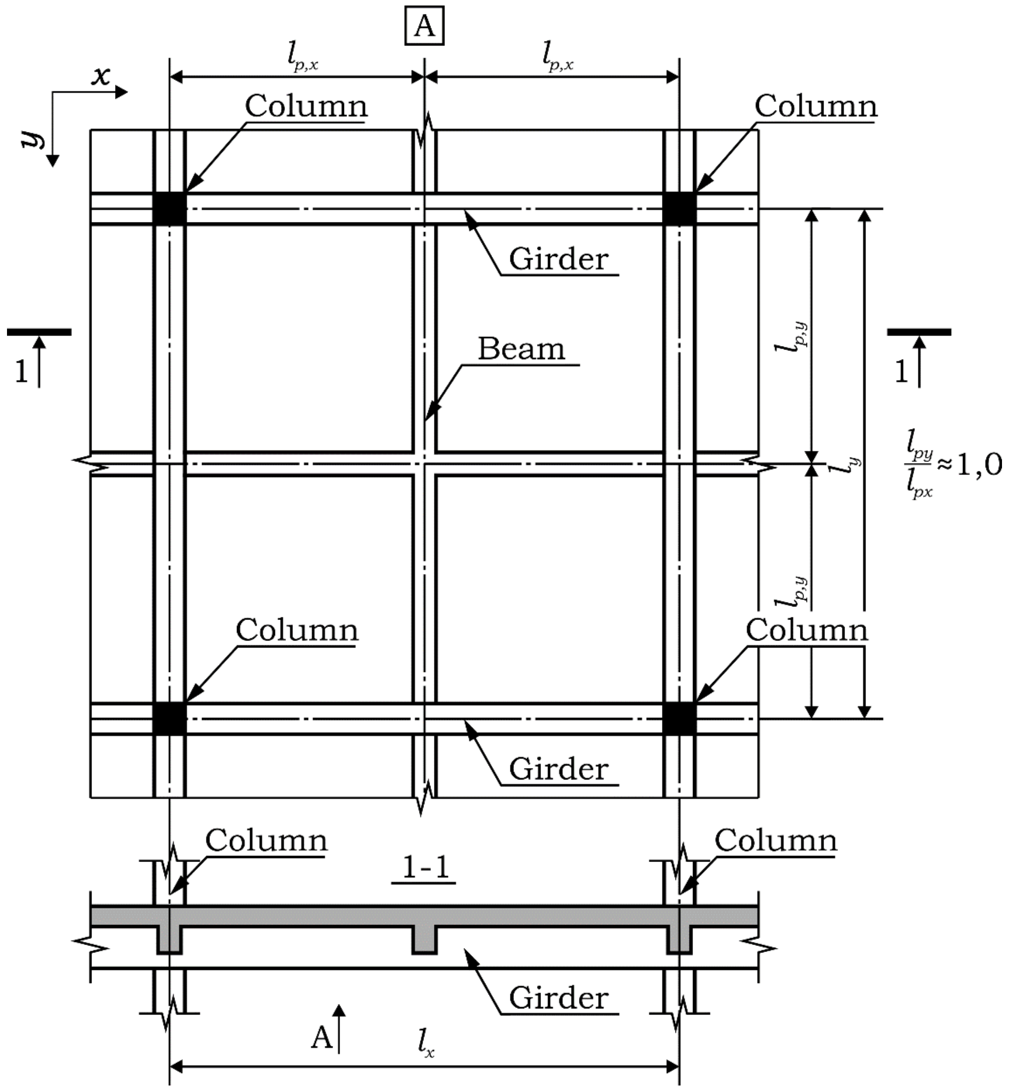


Figure 7.2-5 – Beam-and-girder floor with two-way spanning slabs

7.2.2.2 Band-beam floors

A reinforced concrete floor that cannot be constructed with a flat form deck becomes uneconomical. Therefore, the floor system shown in Figure 7.2-6 are relatively uncommon because of the complexity of the formwork resulting from deep beams around slab panels.

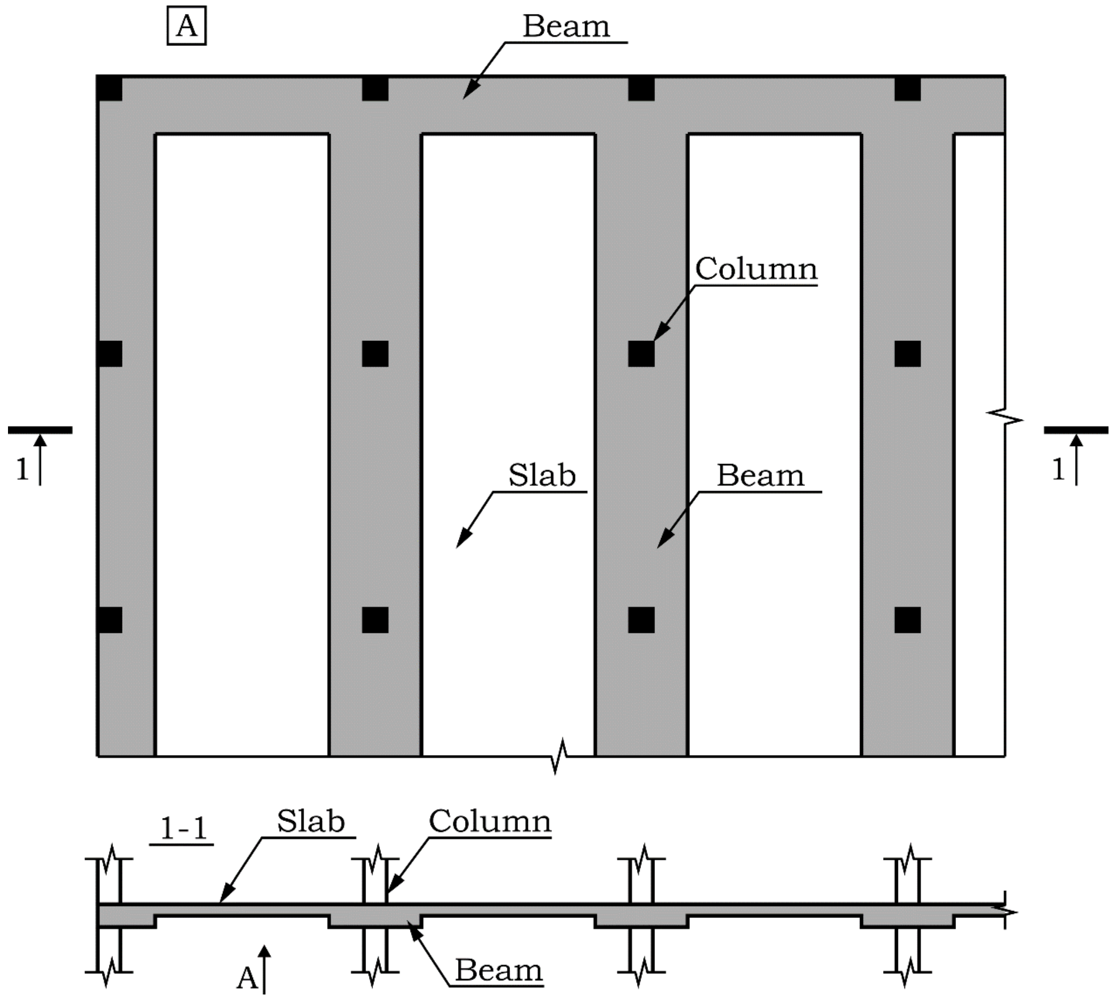


Figure 7.2-6 – Plan and section through a typical banded slab

A one-way slab floor with wide and shallow, continuous beams, referred to as band beams (in contrast with the conventional narrow beams), gives more economical formwork than the beam-and-girder systems in Figure 7.2-5. Because the beams are wide, the slab span is reduced, reducing the slab thickness. Additionally, because the beams are shallow, the floor-to-floor height is smaller, reducing the height of columns, interior partitions, and exterior cladding. A smaller floor-to-floor height also reduces the overall height of the building, which reduces the magnitude of lateral loads on the building.

7.2.2.3 One-way joist floors

A concrete floor that results from extremely economical formwork consists of closely spaced, narrow ribs in one direction supported on beams in the other direction, as it is shown in Figure 7.2-7. Because the ribs are narrow and closely spaced, the floor resembles a wood joist floor. It is, therefore, called a joist floor or a ribbed floor, but it is more commonly known as a one-way joist floor to distinguish it from the two-way joist floor described later.

A one-way joist floor is constructed with U-shaped pans as formwork placed over a flat-form deck. The gap between the pans represents the width of the joists, which can be adjusted by placing the pans closer together or farther apart, see Figure 7.2-8. The pans are generally made of steel or glass fiber-reinforced plastic (GFRP) and can be used repeatedly.

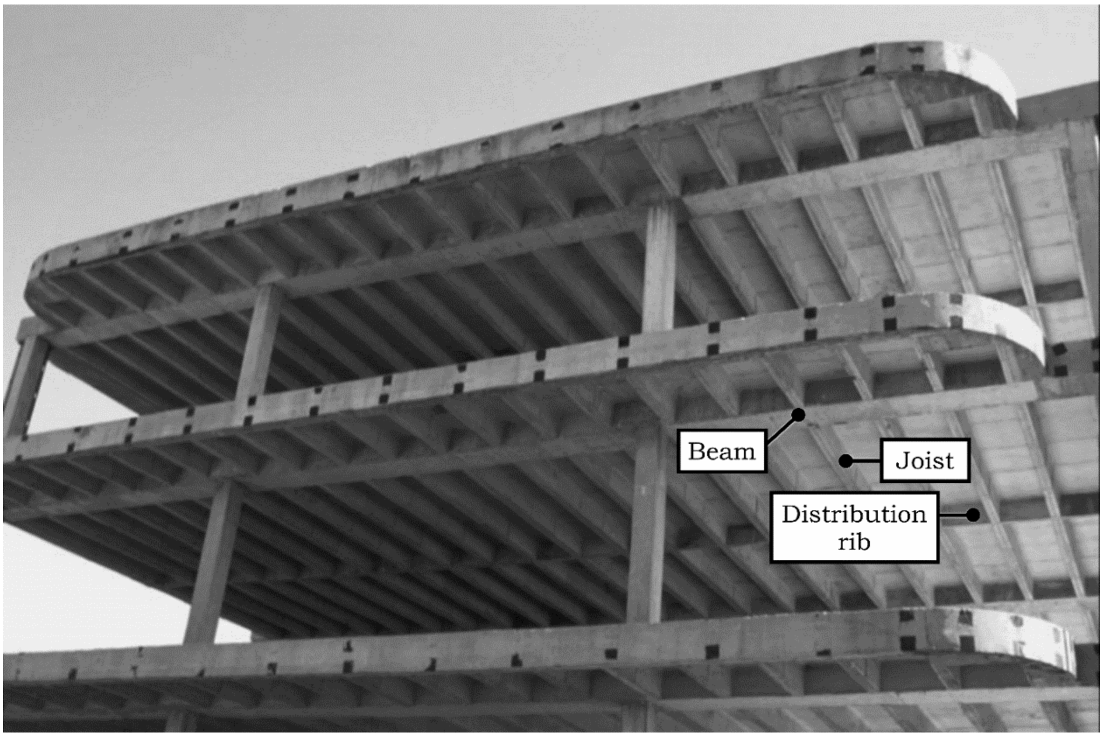


Figure 7.2-7 – A one-way joist floor

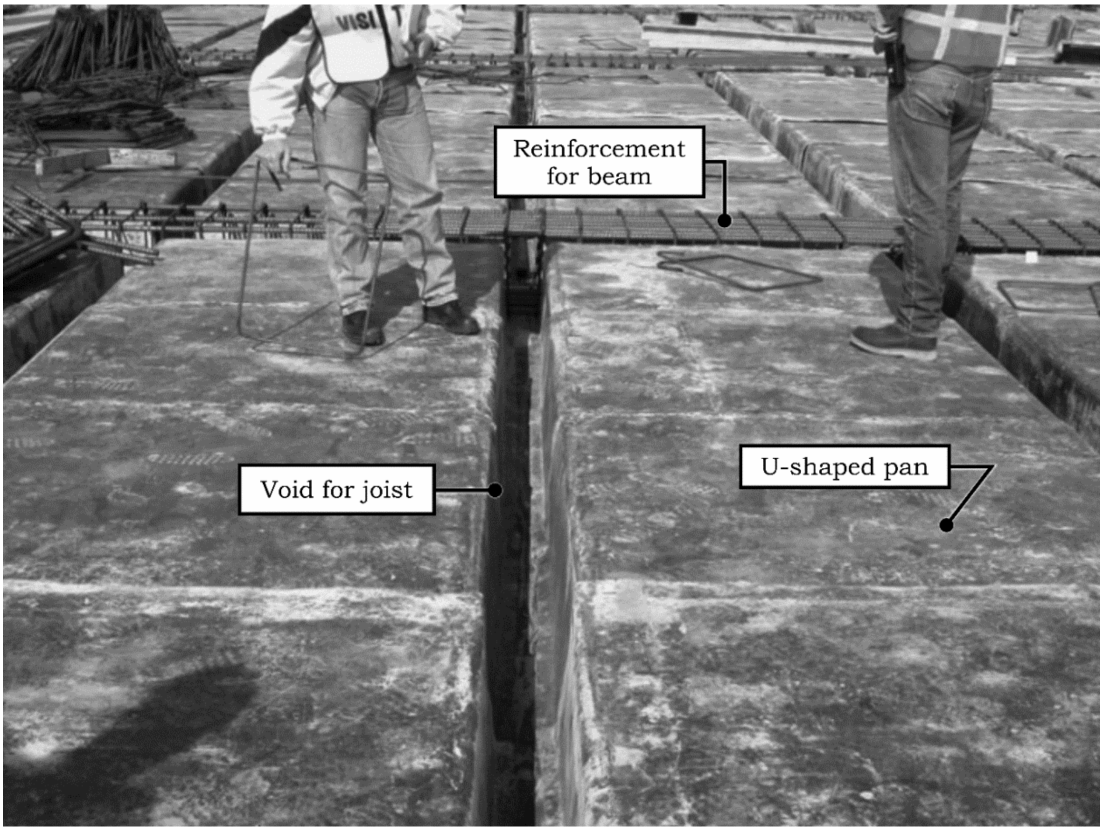


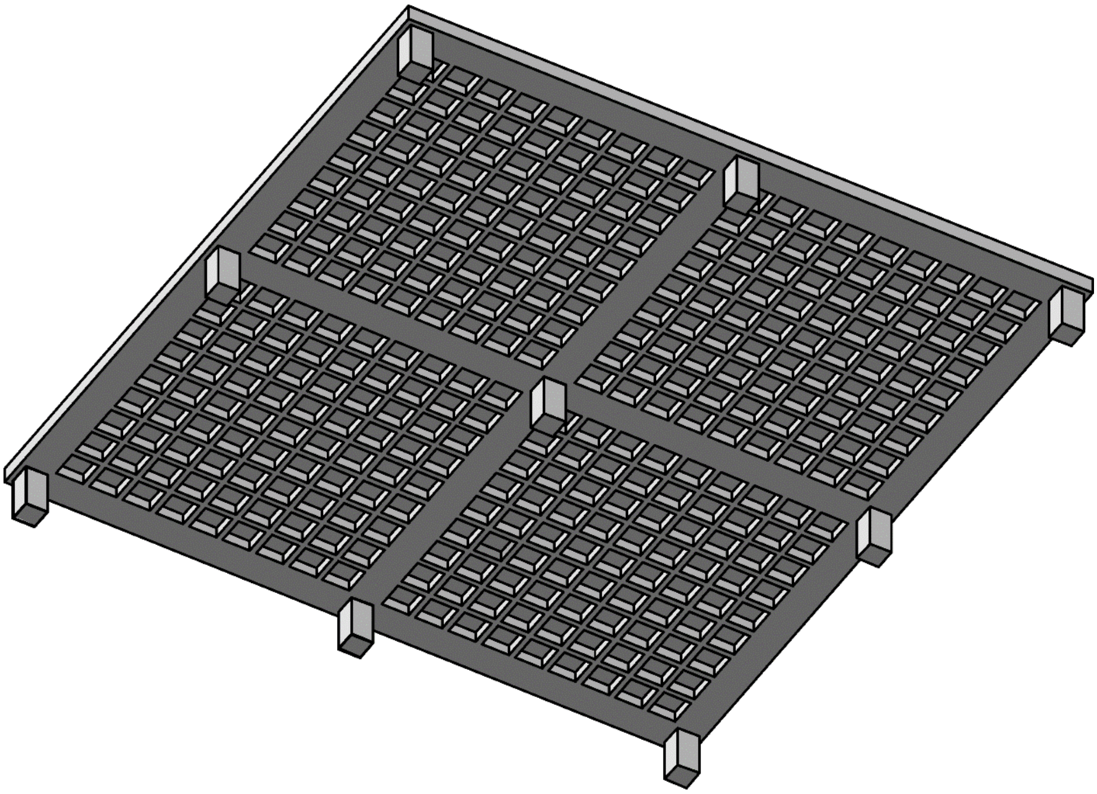
Figure 7.2-8 – Formwork for a one-way joist floor showing U-shaped pans

The vertical section through a pan tapers downward for easy stripping and has supporting lips at both ends. Pan widths and heights have been standardized to give two categories of one-way joist floors:

- standard-module one-way joist floor;
- wide-module one-way joist floor.

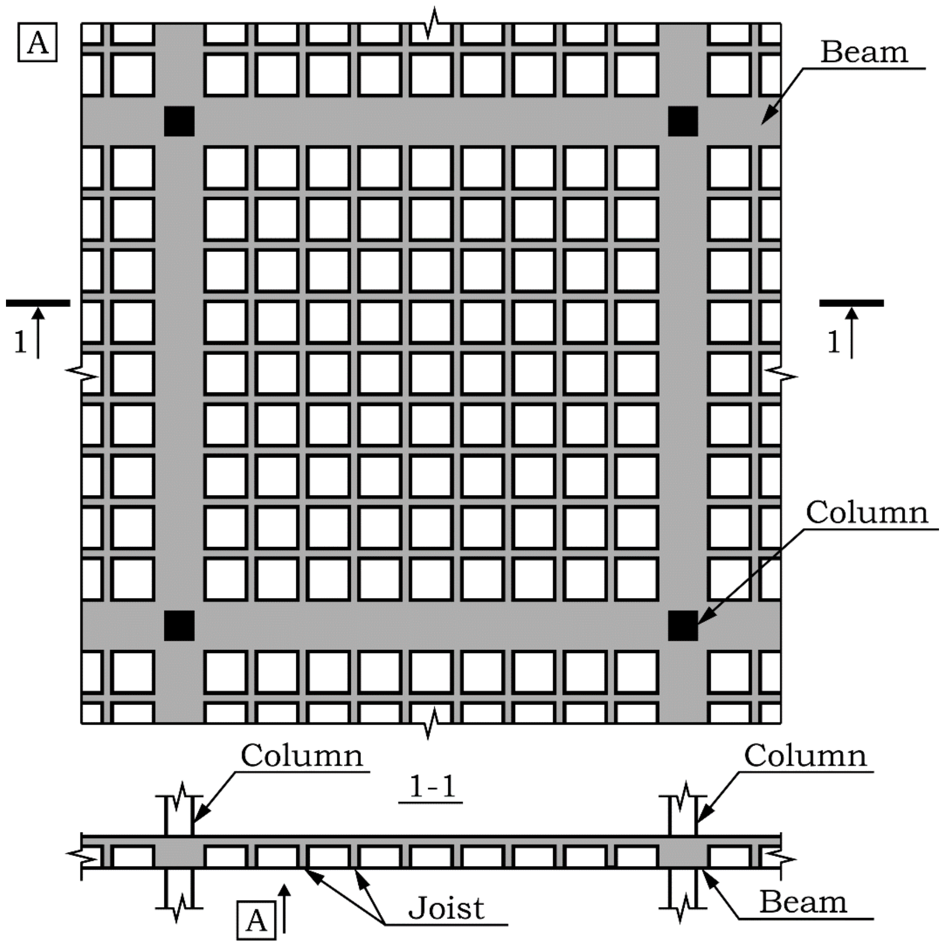
7.2.2.4 Two-way joist floors (waffle slabs)

A two-way joist floor, also called a waffle slab, consists of joists in both directions, Figure 7.2-9. For the same depth of joists, a waffle slab yields a stiffer floor than a one-way joist floor. It is, therefore, used where the column-to-column spacing lies between 10,5 and 15,0 m. A waffle slab is best suited for square or almost square column-to-column bays. When left exposed to the floor below, the waffle slab provides a highly articulated ceiling.



a)

Figure 7.2-9 – Two-way joist floor (waffle slab) supported on beams on all sides



b)

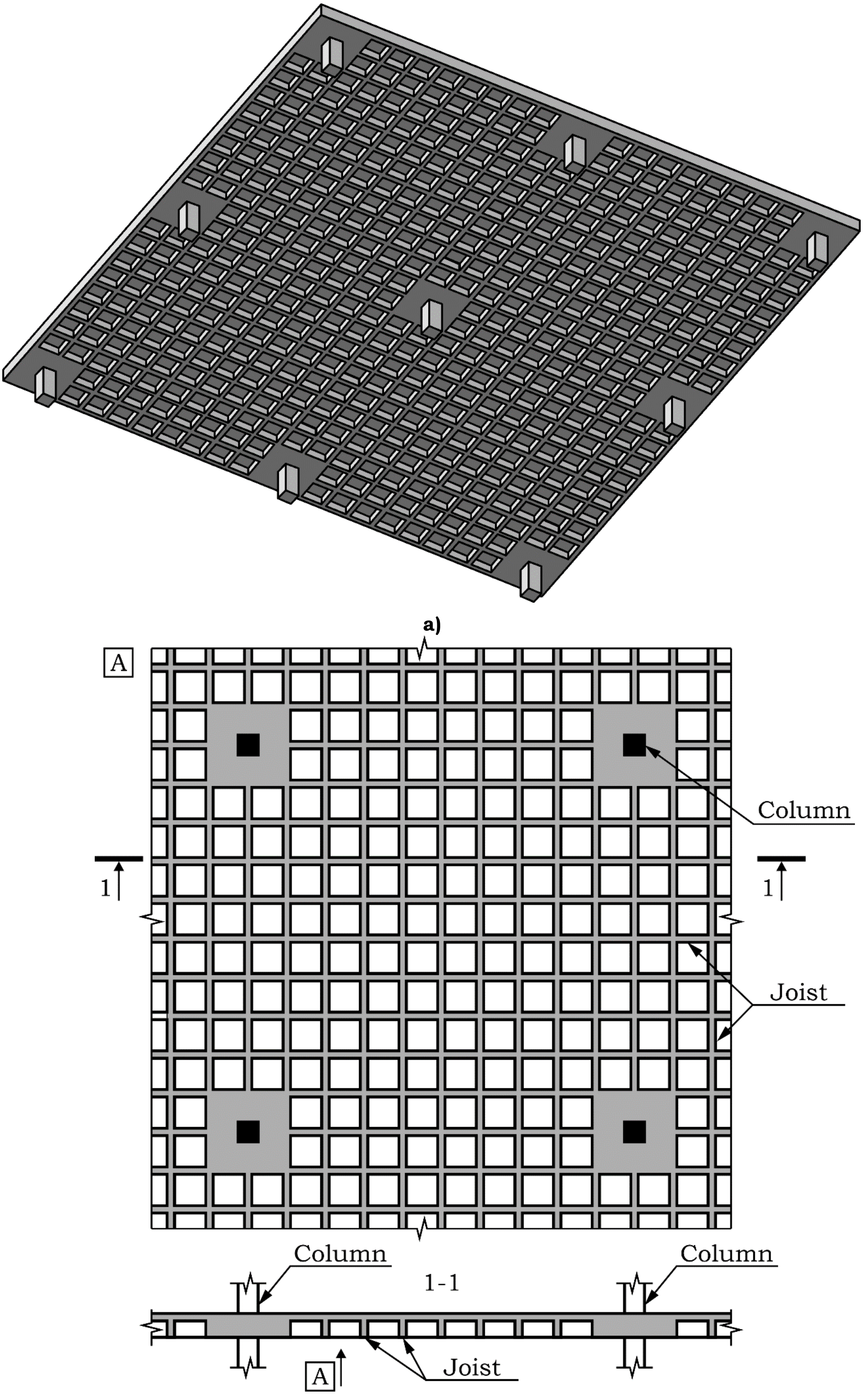
a) – isometric from below; b) – plan (looking-up) and section through the slab

Figure 7.2-9 (end) – Two-way joist floor (waffle slab) supported on beams on all sides

7.2.3 BEAMLESS CONCRETE FLOORS

7.2.3.1 Waffle slabs

A waffle slab is more commonly constructed as a beamless slab, as it is shown in Figure 7.2-10. In a beamless waffle slab, a few domes on all sides of a column are omitted so that the thickness of the slab at the columns is the same as the depth of the joists. The thickening of the slab at the columns provides shear resistance (against the slab punching through the columns).



a) – general view; b) – plan and section 1-1

Figure 7.2-10 – A beamless waffle slab

7.2.3.2 Flat plates

A flat plate consists of a solid slab supported directly on columns, as it is shown in Figure 7.2-11. A flat plate is similar to a two-way banded slab, except that the beam bands in both directions are concealed within the thickness of the slab. Therefore, the spans that can be achieved economically with a flat-plate floor are smaller than those obtained from one-way or two-way joist floors.

Flat-plate slabs are suitable for occupancies with relatively light variable imposed (live) loads, such as hotels, apartments, and hospitals, where small column-to-column spacing does not pose a major design constraint. Additionally, a drop ceiling is not required in these occupancies and HVAC ducts can be run within the corridors, where a lower ceiling height is acceptable.

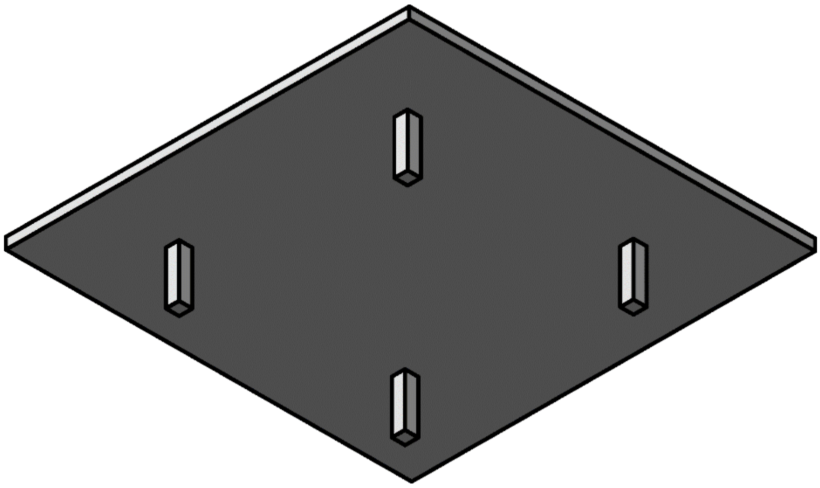
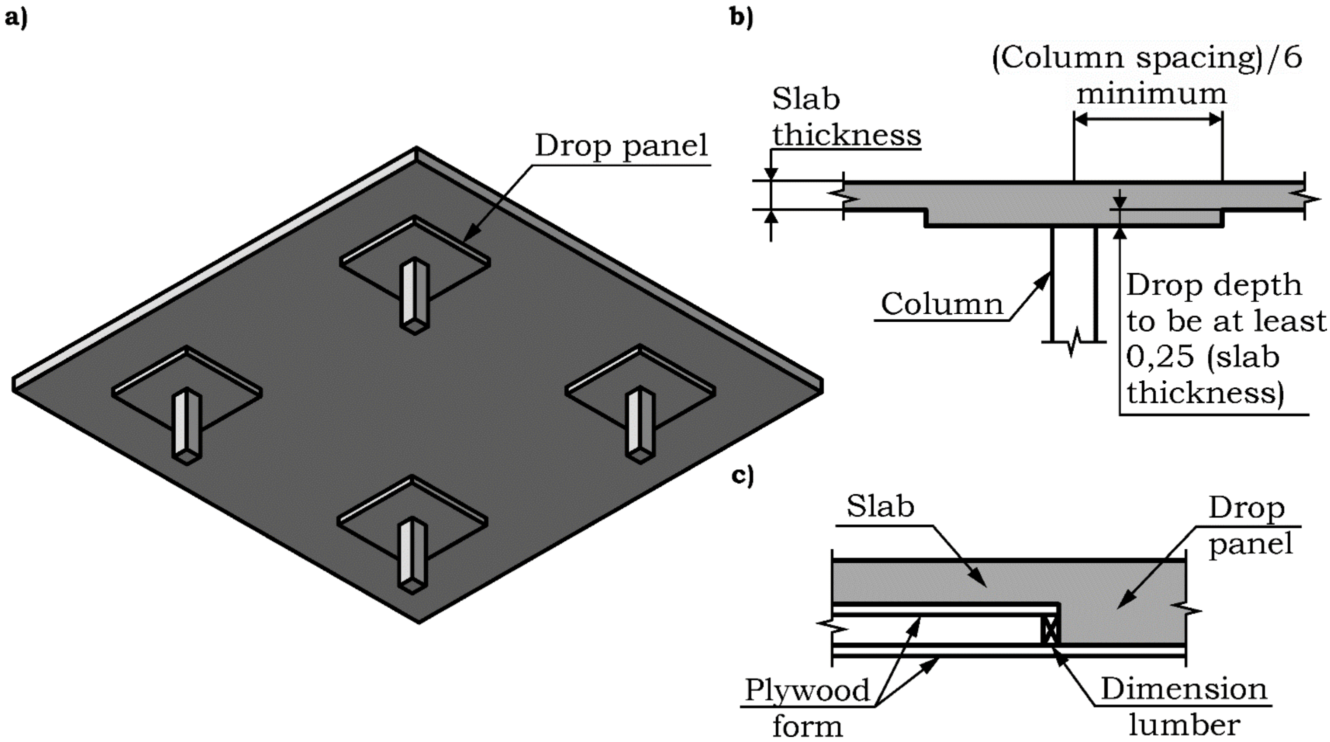


Figure 7.2-11 – A flat-plate slab in an office building (under construction)

A flat-plate slab results in a low floor-to-floor height, and its formwork is economical. Because the beams are concealed within the slab thickness, columns need not be arranged on a regular grid – a major architectural advantage. However, a flat plate is a two-way system; hence, the column spacing in both directions should be approximately the same. A slab thickness of approximately 150 mm is generally needed for 4,5×4,5 m column bays and approximately 200 mm for 6,0×6,0 m bays with residential loads.

7.2.3.3 Flat slabs

A flat slab is similar to a flat plate, but it has column heads, referred to as drop panels (see Figure 7.2-12 a). The primary purpose of drop panels is to provide greater shear resistance at the columns, where the shear maximizes.



a) – a typical flat slab; b) – minimum drop panel dimensions;
c) – drop depth of panel based on dimension lumber
Figure 7.2-12 – Typical details of a flat slab and minimum code requirements for drop panel dimensions

Structurally, the drop panel must extend a minimum of one-sixth of the slab span in each direction, and its drop below the slab must at least be 25% of the slab thickness (see Figure 7.2-12 b). For formwork economy, the drop depth is also based on lumber dimensions (see Figure 7.2-12 c). With round columns, however, manufacturers supply column forms that have built-in drop panels and column capitals.

A flat slab is generally used where the live loads are relatively high, such as in parking garages or storage or industrial facilities.

7.3 CHOICE OF TYPE OF SLAB FLOOR

As it was shown above, for beamless slabs, the choice between a flat slab and a flate plate is usually a matter of loading and span.

Flat plate resistance is often governed by punching shear resistance at the columns, and for service live (imposed) loads greater than perhaps (4,8 kN/m^2) and spans greater than about to 8 m the flat slab is often the better choice. If architectural and other requirements rule out capitals or drop panels, the shear can be improved by using metal shear heads or some other form of shear reinforcement, but the cost may be high.

Serviceability requirements must be considered, and deflections are sometimes difficult to control in reinforced concrete beamless slabs.

Large live (imposed) loads and small limits on permissible deflections may force the use of large column capitals. Negative-moment cracking around columns is sometimes a problem with flat plates, and again a column capital may be useful in its control.

Deflections and shear stresses may also be controlled by adding beams instead of column capitals. If severe deflection limits are imposed, the two-way slab will be most suitable, as the introduction of even moderately stiff beams will reduce deflections more than the largest reasonable column capital is able to. Beams are also easily reinforced for shear forces.

The choice between two-way and beamless slabs for more normal situation is complex.

In terms of economy of material, especially of steel, the two-way slab is often best because of the large effective depths of the beams. However, in terms of labor in building the floor, the flat plate is much cheaper because of the very simple formwork and less complex arrangement of steel. The flat slab is somewhat more expensive in labor than is the flat plate, but the forms for the column capitals are often available as prefabricated units, which can help limit costs.

The real cost parameter is the ratio of costs of labor relative to material. Few two-way slabs are built in areas of high labor costs unless there are definite structural reasons, and many are built where steel is the most costly item. Hollow-tile slabs are still built in some places, but only where the cost of both steel and cement is very high relative to labor.

7.4 DESIGN OF THE CONCRETE FLOOR SYSTEMS WITH ONE-WAY AND TWO-WAY SLABS

7.4.1 GENERAL

In accordance with [18, 19] the general design procedure for slabs to be adopted as follows:

1) Check that the cross section and cover comply with requirements for the fire resistance;
2) Check that cover and concrete grade (class) comply with requirements for durability;
3) Calculate bending moment and shear forces;
4) Calculate reinforcement;
5) Make final check on span/depth ratios;
6) For flat slabs check shear around columns (punching shear) and calculate shear reinforcement as necessary.

The effective span of simply supported slab should normally be taken as the clear distance between the faces of supports plus the slab thickness. However, where a bearing pad is provided between the slab and the support, the effective span should be taken as the distance between the centers of the bearing pads.

The effective span of a slab of a slab continuous over its supports should normally be taken as the distance between the centers of the supports (see Chapter 2).

The effective length of cantilever slab where this forms the end of a continuous slab is the length of cantilever from the center of the support.

7.4.2 FIRE RESISTANCE AND DURABILITY REQUIREMENTS

7.4.2.1 Fire resistance

The member size and reinforcement cover required to provide fire resistance are given in Table 7.4-1. (Table 5.1 [18]). The cover in the Table 7.4-1 may need to be increased for durability (see section 7.4.2.2).

Table 7.4-1 – Minimum dimensions and axis distances for reinforced concrete slabs (excluding flat slabs)

Standard fire resistance		Minimum dimensions (mm)						
		One-way[a,b] spanning slab	One-way[a,b] spanning slab $l_y / l_x \le 1{,}5$ [f]	One-way[a,b] spanning slab $1{,}5 < l_y / l_x \le 2$ [f]	Ribs in a two-way spanning ribbed slab[e]			
REI 60	$h_s =$	80	80	80	$b_{\min} =$	100	120	≥200
	$a =$	20	10[g]	15[g]	$a =$	25	15[g]	10[g]
REI 90	$h_s =$	100	100	100	$b_{\min} =$	120	160	≥250
	$a =$	30	15[g]	20	$a =$	35	25	15[g]
REI 120	$h_s =$	120	120	120	$b_{\min} =$	160	190	≥300
	$a =$	40	20	25	$a =$	45	40	30
REI 240	$h_s =$	175	175	175	$b_{\min} =$	450	700	-
	$a =$	65	40	50	$a =$	70	60	-

Notes:

(1) This table is taken from EN 1992-1-2, Tables from 5.8 to 5.11. For flat slabs refer to Chapter 7.

(2) The table is valid only if the detailing requirements (see note 3) are observed and in normal temperature design redistribution of bending moments does not exceed 15%.

(3) For fire resistance of R90 and above, for a distance of $0{,}3 \cdot l_{eff}$ from the centre line of each intermediate support, the area of top reinforcement should not be less than the following:

$$A_{s,req}(x) = A_{s,req}(0)(1 - 2{,}5(x / l_{eff}))$$

where:
x is the distance of the section begin considered from the centre line of the support;
$A_{s,req}(0)$ is the area of reinforcement required for normal temperature design;
$A_{s,req}(x)$ is the minimum area of reinforcement required at the section being considered but not less than that required for normal temperature design;
l_{eff} is the greater of the effective lengths of the two adjacent spans.

(4) There are three standard fire exposure conditions that need to be satisfied:
R Mechanical resistance for load bearing
E Integrity of separation
I Insulation

(5) The ribs in a one-way spanning ribbed slab can be treated as beams and reference can be made to Chapter 4. The topping can be treated as a two-way slab, where $1{,}5 < l_y / l_x \le 2$.

Key:

a The slab thickness h_s is the sum of the slab thickness and the thickness of any non-combustible flooring.

b For continuous solid slabs a minimum negative reinforcement $A_s \ge 0{,}005 \cdot A_c$ should be provided over intermediate supports if
1) cold worked reinforcement is used; or
2) there is no fixity over the end supports in a two span slab; or
3) where transverse redistribution of load effects cannot be achieved.

c In two way slabs the axis refers to the lower layer of reinforcement.

d The term two way slabs relates to slabs supported at all four edges. If this is not the case, they should be treated as one-way spanning slabs.

e For two-way ribbed slabs the following notes apply:
- The axis distance measured to the lateral surface of the rib should be at least (a+10).
- The values apply where there is predominantly uniformly distributed loading.
- There should be at least one restrained edge.
- The top reinforcement should be placed in the upper half of the flange.

f l_x and l_y are the spans of a two-way slab (two directions at right angles) where l_y is the longer span.

g Normally the requirements of EN 1992-1-1 will determine the cover.

7.4.2.2 Durability

The requirements for achieving durability given environment are the following:
- an upper limit to the water-to-cement ratio (in accordance EN206 [N4]);
- a lower limit to the cement content (in accordance EN206 [N4]);
- a lower limit to the nominal cover to the reinforcement (in accordance EN1992 [N3]);
- good compaction;
- adequate curing;
- good detailing (in accordance with EN1992 [N3]).

For a given value of nominal cover (expressed as minimum cover + an allowance for deviation, Δc_{dur}) Table B.2 (50 years) and Table B.2 (100 years) of Appendix B [N3] give values of concrete class, an upper limit to the water cement ratio and cement content which, in combination, will be adequate to ensure durability for various environments (see Chapter 3).

Where it is specified that only a contractor with a recognised quality system shall do the work $\Delta c_{dur} = 5$ mm, otherwise $\Delta c_{dur} = 10$ mm.

7.4.3 STRUCTURE ACTIONS

7.4.3.1 Distributed loads

Slabs should be designed to withstand the most unfavorable arrangements of design loads. For continuous slabs, subjected to predominantly uniformly distributed loads if will be sufficient to consider only the following arrangements of loads for ultimate state verification:

- alternate spans carrying the design permanent (dead) and imposed load (i.e. $1,35 \cdot G_k + 1,5 \cdot Q_k$), other spans carrying the design permanent (dead) load (i.e. $1,35 \cdot G_k$);

- all spans carrying the design permanent (dead) and imposed load (i.e. $1,35 \cdot G_k + 1,5 \cdot Q_k$). The moments obtained from elastic analysis may be redistributed up to a maximum of 30% except for plain or intended fabric for which the limit is 15%.

If should be noted that [18]:

- the resulting distribution of moments should remain in equilibrium with the applied load;

- the design redistributed moment at any section should not be less than 70% of the elastic design moment;

- there are limitation in the depth of the neutral axis of the section depending on the percentage of redistribution (see clause 4.5.2.4.1 from EN1992 [N3] and Chapter 2).

7.4.3.2 Concentrated loads

The bending moment arising from concentrated load may be distributed over a width of slab equal to the width of the load plus the lesser of the actual width or $1,2 \cdot (1 - x / l) \cdot x$ on each side of the load (see Figure 7.4-1), where x is the distance to the nearer support from the section under consideration, and l is the span.

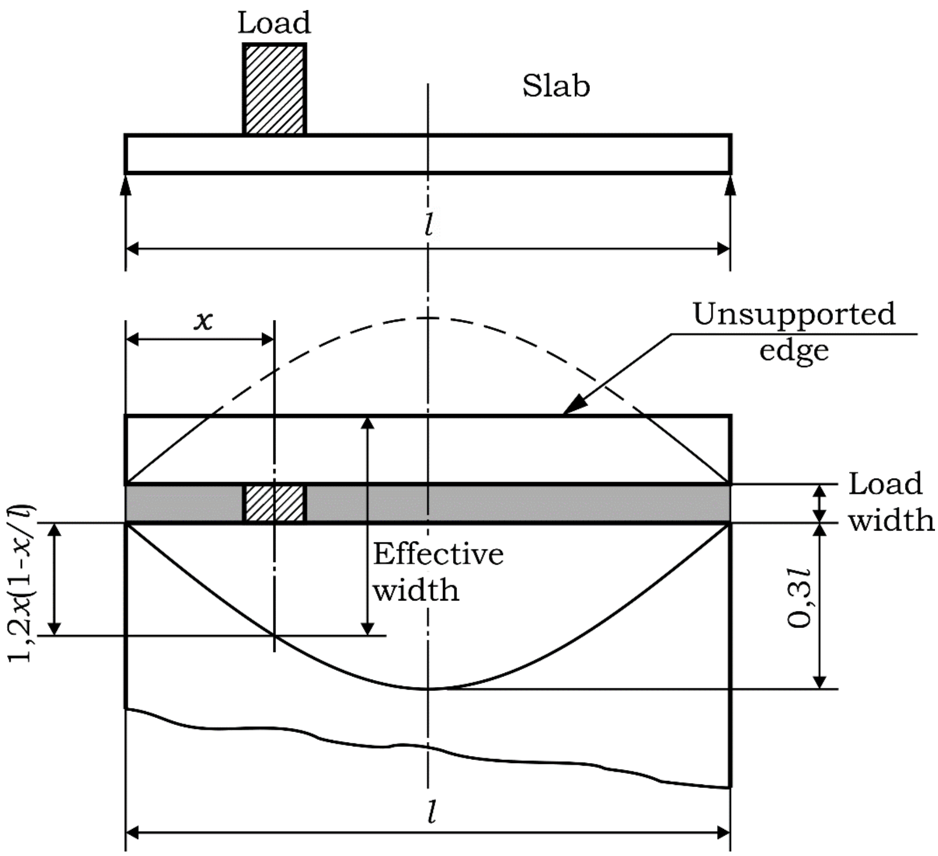


Figure 7.4-1 – Effective width of solid slab carrying a concentrated load near unsupported edge (Figure 5.1 from [18])

7.4.4 METHODS OF ANALYSIS

The analysis of slabs is extremely complicated because of the influence of number of factors stated above. Thus the exact (close form) solutions are not easily available. The main basic methods are:

(a) Classical methods – Levy and Naviers solution (Plate analysis);

(b) Yield line analysis – used for Ultimate limit state analysis;

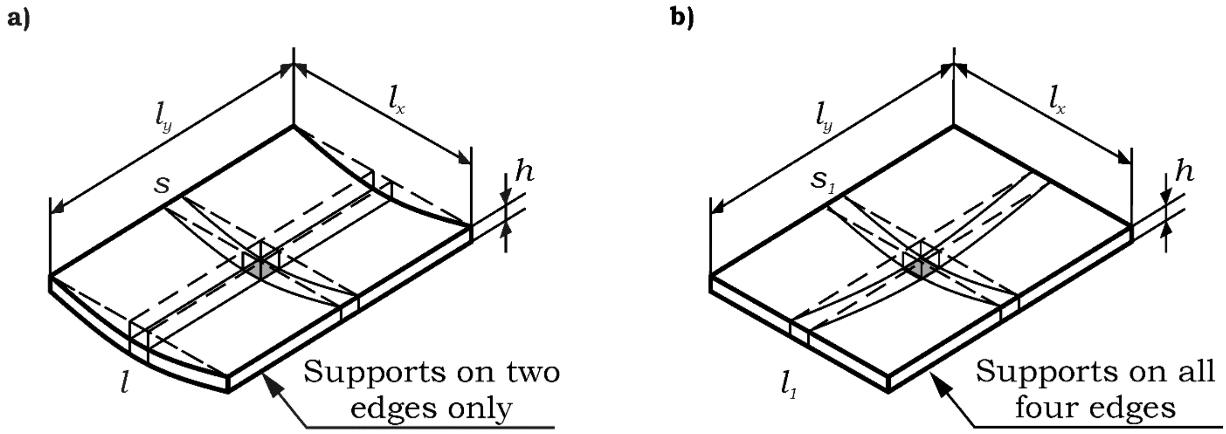
(c) Semi-empirical method – prescribed by codes for practical design which uses Coefficient methods;

(d) Numerical techniques – Finite element ant Finite difference methods.

7.4.5 ANALYSIS OF THE ONE-WAY AND TWO-WAY SLABS BY SEMI-EMPIRICAL COEFFICIENT METHODS

7.4.5.1 General

One-way slabs transfer the imposed loads in one direction only. They may be supported on two opposite sides only (see Figure 7.4-2 a), in which the structural action is essentially one-way, the loads being carried in direction perpendicular to the supporting beams or walls.



a) – one-way slab; b) – two-way slab

Figure 7.4-2 – Load transfer in slabs

But rectangular slabs often have such proportions and supports (e.g. relatively deep, stiff monolithic concrete beams) that result in two-way action (see Figure 7.4-2 b).

At any points, such slabs are curved in both directions resulting in biaxial moments. It is convenient to think of such slabs as consisting of two sets of parallel strips, in each direction and intersecting each other. So part of the load is carried by one set and remainder by the other.

Figure 7.4-2 b shows two center strips of a rectangular plate with spans l_x and l_y. For uniformly distributed loads of q (kN/m^2, kPa), each strips act approximately like a simple beam uniformly loaded by its share of q, i.e. q_x and q_y. Since they are part of the same slab, their midspan deflections must be the same (elastic stage):

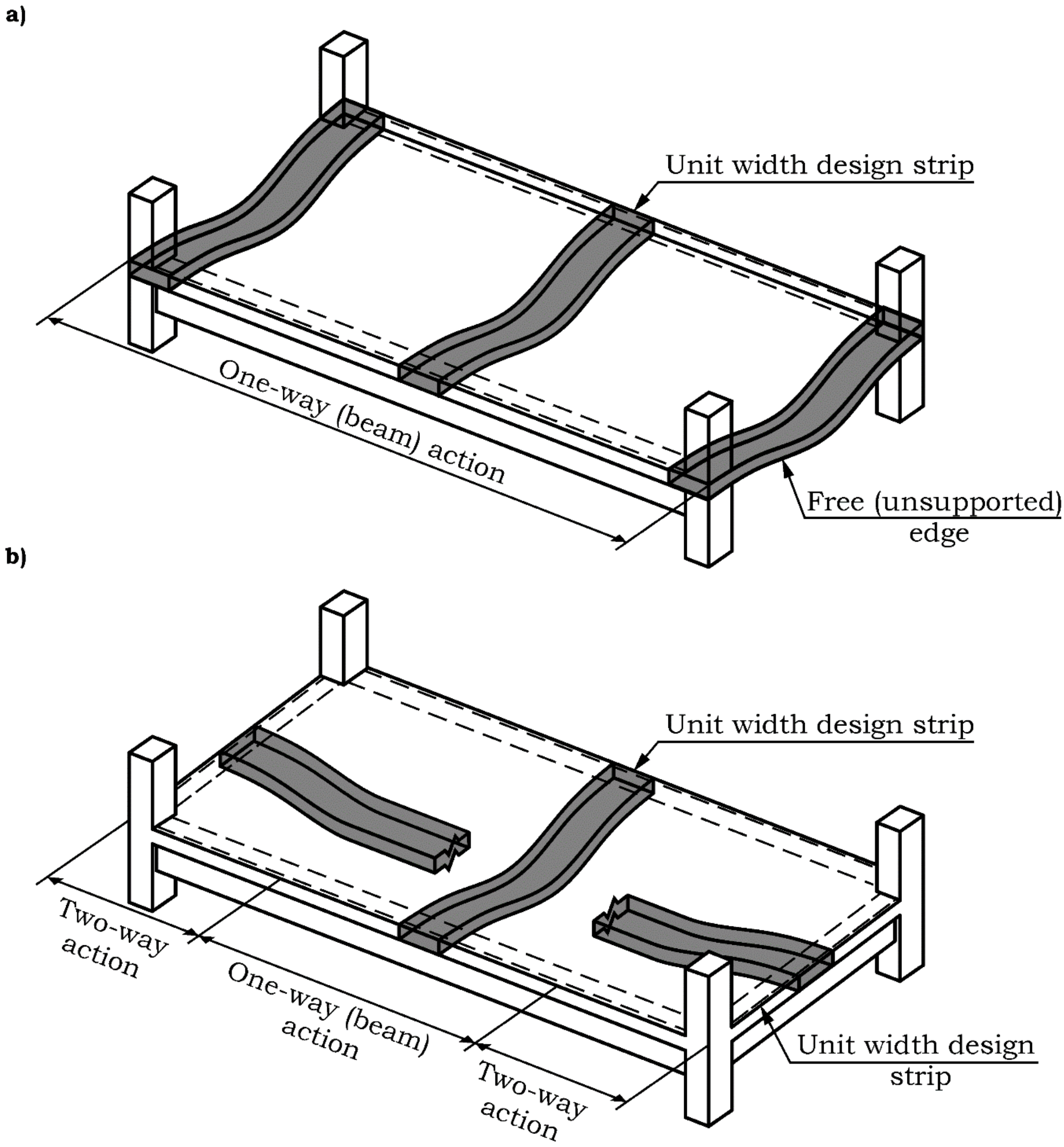
$$\frac{5q_x \cdot l_x^4}{384 \cdot E_{cm} I} = \frac{5q_y \cdot l_y^4}{384 \cdot E_{cm} I} \Rightarrow \frac{q_x}{q_y} = \left(\frac{l_y}{l_x}\right)^4 . \tag{7.4-1}$$

Therefore, large share of the load is carried in the shorter direction, the ratio of the two portion of the load being inversely proportional to the fourth power of the ration of spans. For example, if $l_y / l_x = 2$, $q_y / q_x = 16$, i.e., about 94 % of the load is carried in the shorter direction and only near 6 % in the longer direction.

7.4.5.2 One-way slabs

As it was shown above a slab subjected to dominantly uniformly distributed loads may be considered to be one-way spanning if either:

- it possesses two free (unsupported) and sensibly parallel edges (see Figure 7.4-3 a) or
- it is the central part of a sensibly rectangular slab supported on four edges with a ratio of the longer to shorter span greater than 2 (see Figure 7.4-3 b).



a) – supported on two sides; b) – supported on beams on all four sides

Figure 7.4-3 – One-way slab

However, this proportions also depends on the support conditions in each direction, because the maximum midspan deflection is $((q \cdot l^4)/(192 \cdot E_{cm} \cdot I))$ for hinger-fixed ends (simply supported) and $((q \cdot l^4)/(384 \cdot E_{cm} \cdot I))$ for fixed-fixed ends (F).

Therefore, if $l_y / l_x = 2$ and span l_x is simply supported, about 14 % of q is carried by hinger-fixed span l_x and 24 % by fixed-fixed span l_y. On the other hand, if l_y is simply supported, if carries only 2,4 % of q if l_x is hinged-fixed (simply supported) and 1,2 % of q if l_x is fixed-fixed.

For simply supported square slab,

$$l_y / l_x = 1,\ q_x = q_y = q/2; \tag{7.4-2}$$

So if only bending was present, the maximum bending moment in each slab would be equal:

$$M_{\max} = \frac{(q/2) \cdot l^2}{8} = 0{,}0625 \cdot q \cdot l^2 \tag{7.4-3}$$

However, the actual behavior of a slab is more complex than that of two intersecting strip. As shown in Figure 7.4-4, slab can be modeled as a grid, some strip of which (particularly the outer strips) are not only bent but also twisted. Consequently, the total load on the slab is carried out only by bending moments in two directions, but also twisting moments. For this reason, bending moments in elastic slabs would be smaller than that would be computed for sets in unconnected strips loaded by q_x and q_y.

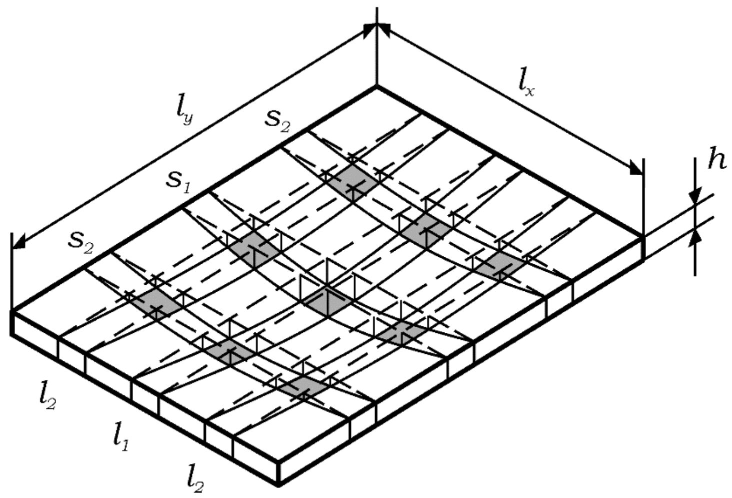


Figure 7.4-4 – Grid model of two-way slab

This subsection gives the requirement to fire resistance and durability, and bending and shear forces coefficient for one-way and two-way spanning slabs on linier supports, and flat slabs using solid, ribbed and coffered waffled constructions. The coefficients apply to slabs complying with certain limitations which are stated for each type.

For those cases where no coefficient are provided the bending moments and shear forces for one-way spanning slabs may be obtained by elastic analysis [20]. As it was pointed in [20], these moments may then be redistributed, maintaining equilibrium with applied loads, up to a maximum 30 %, although normally 15 % is considered as reasonable limit.

The treatment of shear around columns for flat slabs (punching shear) and the check of deflections for all types of slab are given, together with some notes on the use of precast slabs.

In heavily-loaded slabs, the thickness is often governed by shear of flexure, white in lightly-loaded slabs, the thickness is generally chosen based on deflection limitations.

Both lightly and heavily loaded slabs are typically dimensioned so that no shear reinforcement is required, as placing stirrups in slabs is perceived to be difficult and costly.

One-way spanning slabs are designed for flexure and shear on per meter width basis, assuming that they act as a series of independent strips. Thus one-way shear is slabs is often referred to as beam shear, and design for flexure and shear is carried out using a beam analogy. It is convenient to think of it as consisting of two sets of parallel strips, in each of the two directions, intersecting each other (mostly perpendicular to each other).

Evidently, part of the load is carried by one set and transmitted to one pair of edge support, and the remainder by the other.

In the example shown in Figure 7.4-3 b in which the slab is supported of stiff beams on all four sides, and where the aspect ratio in plan is not much greater than 2:1, some redistribution may be possible due to two-way action. However, in one-way slabs with an aspect ratio considerably greater than 2:1, redistribution due to two-way action may be negligible. Furthermore, in one-way slabs a supported on stiff supports along only two sides (Figure 7.4-3 a) no redistribution will be possible, and the full width of the slab may be called upon to resist the full shear.

To obtain internal forces in each way is necessary to compute load for corresponding direction. Because the imaginary strips actually are part of the same monolithic slab, their deflections at the intersection point must be the same. The line load can be obtain by means of split factor c_x based on assumptions of the identical center deflection of the short and long strip ($q_x = q_y$).

For continuous slabs with a) substantially uniform loading b) permanent (dead) load greater than or equal to imposed load and c) at least three spans that do not differ by more than 15 %, the bending moments and shear forces may be calculated using the coefficient given in Table 7.4-2.

Table 7.4-2 – Bending moments and shear forces for one-way slabs (Table 5.2 from [8])

	Simple		**Continuous**		**Penultimate support**	**Interior spans**	**Interior supports**
	End support	End span	End support	End span			
Moment	0	0,86·$F·l$	-0,04·$F·l$	0,075·$F·l$	-0,086·$F·l$	0,063·$F·l$	-0,063·$F·l$
Shear	0,4·F	-	-0,046·F	-	0,6·F	-	0,5·F

Notes: *F is the total design ultimate load ($1{,}35 \cdot G_k + 1{,}5 \cdot Q_k$) for each span;*
l is the span.

Allowance has been made in the coefficients in Table 7.4-2 for 20 % redistribution of moments.

7.4.5.3 Two-way spanning slabs on linear supports. Analysis with coefficient method

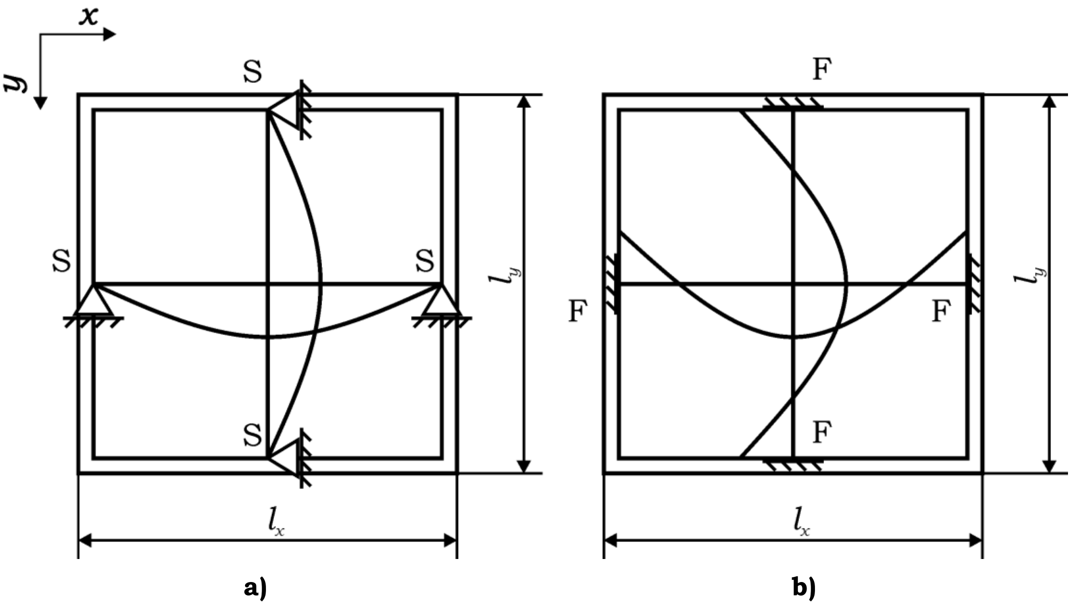
Two-way slabs on linear supports are surface members aching in a both directions. They are supported on all four sides. For this type of slabs, the ratio of length to width of the one slab should less than 2 otherwise one-way action is obtained, even though supports are provided on all side. In many cases the slabs are of such proportions and are supported in such way that two-way actions results.
In accordance with [18], the slab preliminary thickness design depends on boundary conditions (support) (see Figure 7.4-5):
– for simple supported at all sides (S), appropriate:

$$h_s = 1{,}1\left(\frac{l_x + l_y}{75}\right) \tag{7.4-4}$$

– for fixed support at all sides (F), appropriate:

$$h_s = 1{,}2\left(\frac{l_x + l_y}{105}\right) \tag{7.4-5}$$

For other boundary conditions is reasonable to keep thickness in range between all fixed (F) and supported (S) edges, but minimum depth is 80 mm.
It should be noted, that the thickness of two-way slabs and plates is typically governed by deflection limitations.



a) – simply supported edges; b) – fixed support at all sides

Figure 7.4-5 – Two-way spanning slabs (boundary conditions)

The determination of exact moments in two-way slabs with various support conditions is mathematically formidable and not suited to design practice. Various simplified methods are therefore adopted for determining moments, shear and reactions in such slabs. Quite popular and widely used among these methods is one using «***moment coefficient method***», for the special of two-way slabs supported on four sides by relatively stiff beams. The method used Tables of moment coefficients for a variety of support conditions. These coefficients are based on elastic analysis but also accounts for inelastic redistribution.

This method provides the values of $M_{x,\max}$ and $M_{y,\max}$ along the central strip of the slab, as demonstrated in Figure 7.4-7 for a slab simply supported on all sides. As shown in Figure 7.4-6, the maximum moments are less elsewhere. Therefore, other design values can be reduced according to the variation show. These variations in maximum moment across the width and length of a rectangular slab are accounted for approximately by designing the outer quarters of the slab span in each direction for a reduced moment.

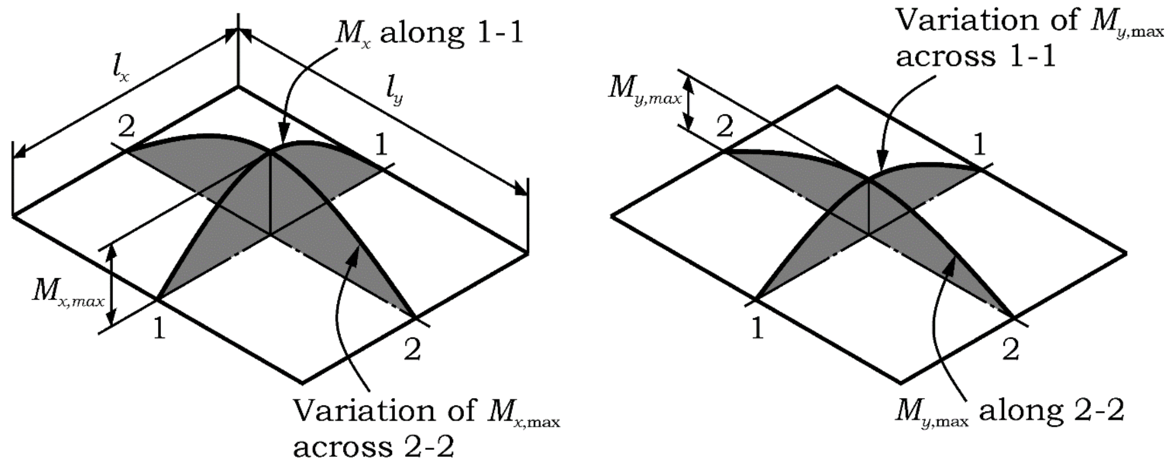


Figure 7.4-6 – Variation of moments in a uniformly loaded slab simply supported on all sides

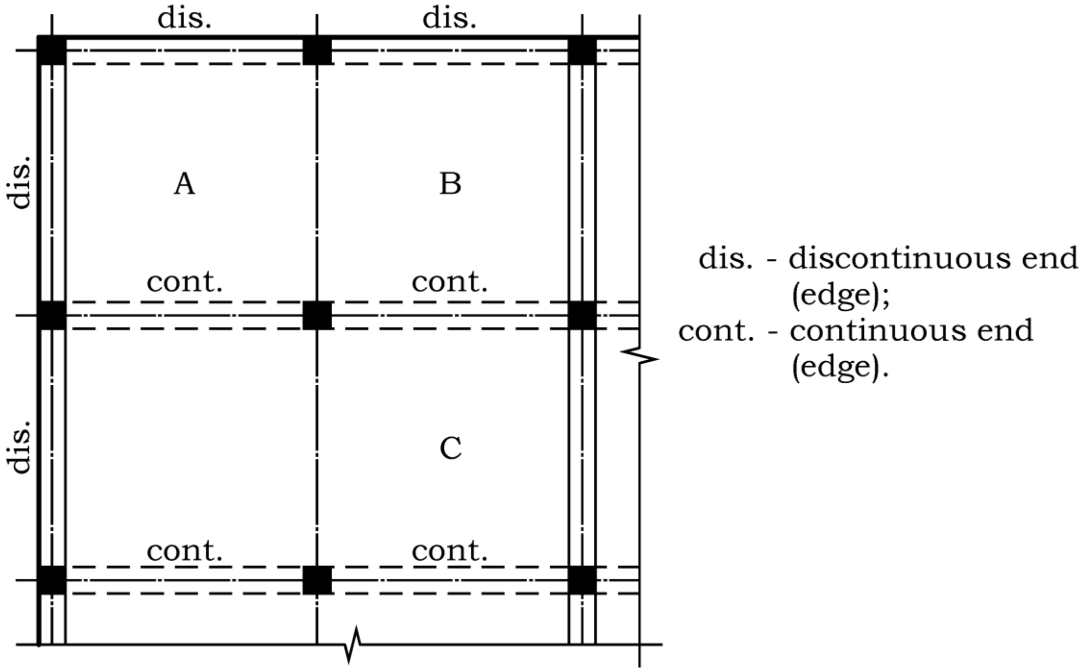


Figure 7.4-7 – Plan of typical slab

Bending moment in two-way slabs may be calculated by any valid method provided the ratio between support and span moments are similar to those obtained by the use of elastic theory with appropriate redistribution. In slabs where the corners are prevented from lifting, the coefficients in Table 7.4-3, may be used to obtain bending moments per unit with (m_{Ex} and m_{Ey}) in the two directions for various edge conditions, i. e.:

$$m_{Ex} = \beta_{Ex} \cdot q_d \cdot l_x^2; \tag{7.4-6}$$

$$m_{Ey} = \beta_{Ey} \cdot q_d \cdot l_x^2, \tag{7.4-7}$$

where: β_{Ex} and β_{Ey} are the coefficient from Table 7.4-3;

q_d is the total design ultimate load per unit area $(1{,}35 \cdot G_k + 1{,}5 \cdot Q_k)$;

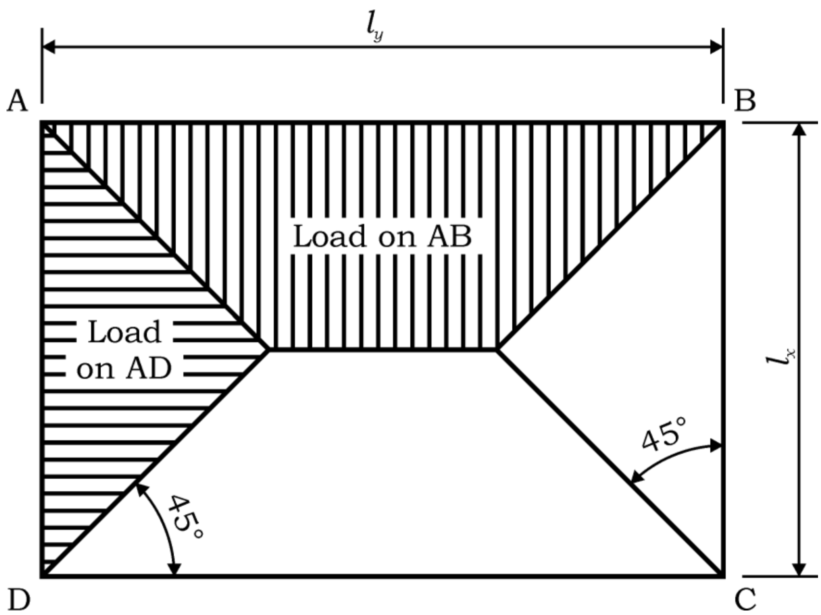
l_x is the shorten span.

The distribution of the reactions of two-way slabs on to their supports can be derived from Figure 7.2-20.

Class A reinforcement in accordance with EN 1992 [N3] is assumed to have sufficient ductility for use with this simplified design method or yield line analysis of two way slabs.

Table 7.4-3 Bending moment coefficients for two-way spanning rectangular slabs

Type of panel and moments considered	Short-span coefficients β_{Ex} values of l_y / l_x					Long-span coefficients β_{Ey} for all values of l_y / l_x
	1,00	1,25	1,50	1,75	2,00	
1. Interior panels:						
- negative moment at continuous edge	0,031	0,044	0,053	0,059	0,063	0,032
- positive moment at midspan	0,024	0,034	0,040	0,044	0,048	0,024
2. One short edge discontinuous:						
- negative moment at continuous edge	0,039	0,050	0,058	0,063	0,067	0,037
- positive moment at midspan	0,029	0,038	0,043	0,047	0,050	0,028
3. One long edge discontinuous:						
- negative moment at continuous edge	0,039	0,059	0,073	0,082	0,089	0,037
- positive moment at midspan	0,030	0,045	0,055	0,052	0,067	0,028
4. Two adjacent edges discontinuous:						
- negative moment at continuous edge	0,047	0,066	0,078	0,087	0,093	0,045
- positive moment at midspan	0,036	0,049	0,059	0,065	0,070	0,034



Notes: *1. The reactions shown apply when all edges are continuous (or discontinuous).*

2. When one edges is discontinuous, the reactions on all continuous edges should be increased by 10 % and the reaction on the discontinuous edge may be reduced by 20 %.

3. When adjacent edges are discontinuous, the reaction should be adjusted for elastic shear considering each span separately.

Figure 7.4-8 – Distribution of reactions from two-way slabs onto supports

(see Figure 5.2 from [18])

Compared to idealized «simply supported» slab, Figure 7.4-8 shows a more «realistic» scenario where a system of beams supports a two-way slab. For this slab, panel A has two discontinuous exterior edges and two continuous interior edges, panel B has one discontinuous and three continuous edges, while the interior panel C has all edges continuous. The design bending moments are zero at discontinuous ends, negative at continuous ends and positive at midspans.

7.4.5.4 Flat slabs

Concrete slabs are often carried directly by columns without the use of beams or girders.

Such slabs are described as Flat Plates (see Figure 7.2-11) and are commonly used where spans are not large and not particularly heavy.

A very similar construction Flat Slab (see Figure 7.2-12) is also beamless but incorporates a thickened slab region in the vicinity of columns (called Drop Panels) and often employs flared up column tops (Column Capitals). Both are devices to reduce stresses due to shear and negative bending around the columns.

In accordance with EN1992 [N3] (Annex I), flat slab should be analysed using a proven method of analysis, such as grillage (in which the plate is idealized as a set of interconnected discrete members), finite element, yield line or equivalent frame. Appropriate geometric and material properties should be employed.

In flat slab analysis, the full load is assumed to be carried by the slab in each direction. This is in apparent contrast to the analysis of two-way beam-supported slabs, in which the load is divided. In two-way slabs, as in flat slabs, equilibrium conditions require that the entire load is carried in each of two principal directions.

Through the structural analysis of flat slab can be carried out using computer based structural modeling, the two widely used methods for this purpose are the semi-empirical Direct Design Method and equivalent Frame Method.

7.4.5.4.1 Direct design method

If a flat slab has at least three spans or bays in each direction and the ratio of the longest span to the shortest does not exceed 1,2, the maximum values of the bending moments and shear forces in each direction may be obtained from Table 7.4-4. This assumes 20 % redistribution of bending moments.

Table 7.4-4 – Bending moment and shear force coefficients for flat slab panels of three or more equal span (Table 5.4 from [8])

	Outer support[(2)]	**Near middle of the end span**[(3)]	**At first interior support**	**At middle of interior span**	**At internal support**
Moment	-0,040·$F{\cdot}l$	0,086·$F{\cdot}l$	-0,086·$F{\cdot}l$	0,063·$F{\cdot}l$	-0,063·$F{\cdot}l$
Shear	0,460·F[(1)]	-	0,600F	-	0,500·F
Total column moments[(4)]	0,040·$F{\cdot}l$	-	0,022·$F{\cdot}l$	-	0,022·$F{\cdot}l$
Notes: *(1) –* F *is the total design ultimate load* ($1{,}35 \cdot G_k + 1{,}5 \cdot Q_k$)*; (2) – these moments may have to be reduced to be consistent with the capacity to transfer moments to the columns; (3) – the midspan moments must then be increased correspondingly; the total column moment should be distributed equally between the columns above and below; (4) – moment at supports may be reduced by* $0{,}15 \cdot F \cdot h_c$*, where* h_c *is the effective diameter of the column or column head.*					

7.4.5.4.2 Equivalent frame analysis

Where the conditions above do not apply, bending moment in flat slab should be obtained by frame analysis.

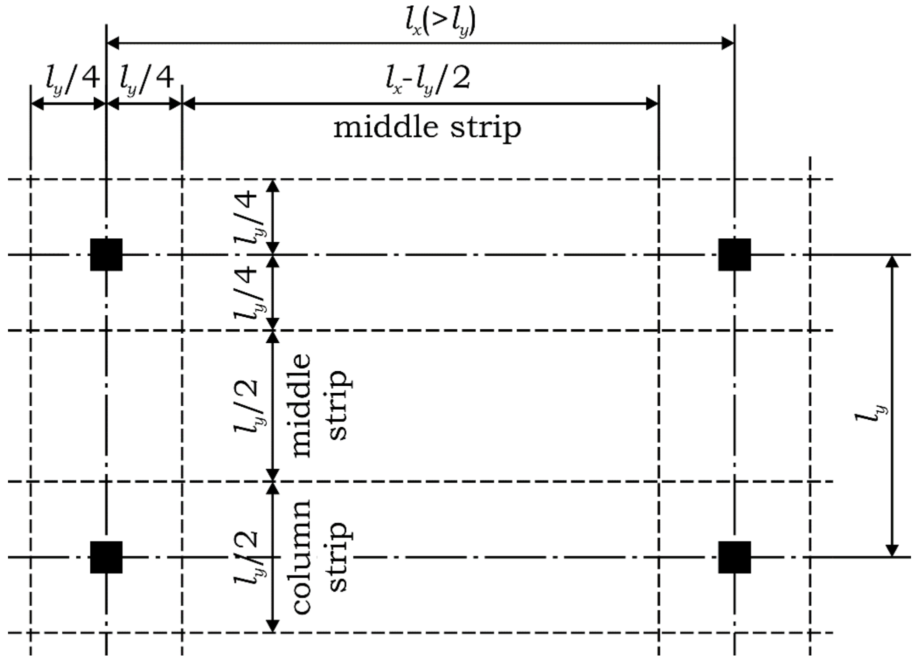
The structure should be divided longitudinally and transversely into frames consisting of columns and sections of slabs contained between the centre lines of adjacent panels (area bounded by four adjacent supports). The stiffness of members may be calculated from their gross cross-sections. For vertical loading 40 % of this value should be used to reflect the increased flexibility of the column/slab joints in flat slab structures compared to that of column/beam joints. Total load on the panel should be used for the analysis in each direction.

The total bending moments obtained from analysis should be distributed across the width of the slab. In elastic analysis negative moments tend to concentrate towards the center lines of the columns.

The panels should be assumed to be divided into column and middle strips (see Figure 7.4-9) and the bending moments should be apportioned as given in Table 7.4-5.

Table 7.4-5 – Simplified apportionment of bending moment for a flat slab in accordance with EN 1992 [N3]

	Negative moment	Positive moment
Column strip	(60–80) %	(50–70) %
Middle strip	(40–20) %	(50–30) %
Note: *Total negative and positive moments to be resisted by the column and middle strips together should always add up to 100 %.*		



Note: *When drops of width higher than ($l_y/3$) are used the column strips may be taken to be the width of drops. The width of middle strips should then be adjusted accordingly.*

Figure 7.4-9 – Division of panels in flat slabs (Figure I.1 from EN 1992 [N3])

In the assessment of the width of the column and middle strips, drops should be ignored if their smaller dimension is less than one-third of the smaller dimension of the panel.

Where the width of the column strip is different from $0{,}5 \cdot l_x$ as shown in Figure 7.4-9 and made equal to width of drop the width of middle strip should be adjusted accordingly.

The design moments obtained from analysis of the frames or from Table 7.4-5 should be divided between the column and middle strips in the proportions given in Tables 7.4-6.

Table 7.4-6 Recommended distribution of design moments of flat slab (Table 5.5 from EN 1992 [N3])

Design moment	Column strip, %	Middle strip, %
Negative	75	25
Positive	55	45
Note: *For the case where the width of column strip is taken as equal to that of the drop and the middle strip is there by increased in width, the design moments to be resisted by the middle strip should be increased in proportion to its increased width. The design moments to be resisted by the column strip may be decreased by an amount such that the total positive and the total negative design moments resisted by the column strip and middle strip together are unchanged.*		

In general, moments will be able to be transferred only between a slab and edge or corner column by a column strip considerably harrower than that appropriate for an internal panel. The breadth of this strip, b_e, for various typical cases is shown in Figure 7.4-10, and be should not be taken as greater than the column strip width appropriate for an interior panel.

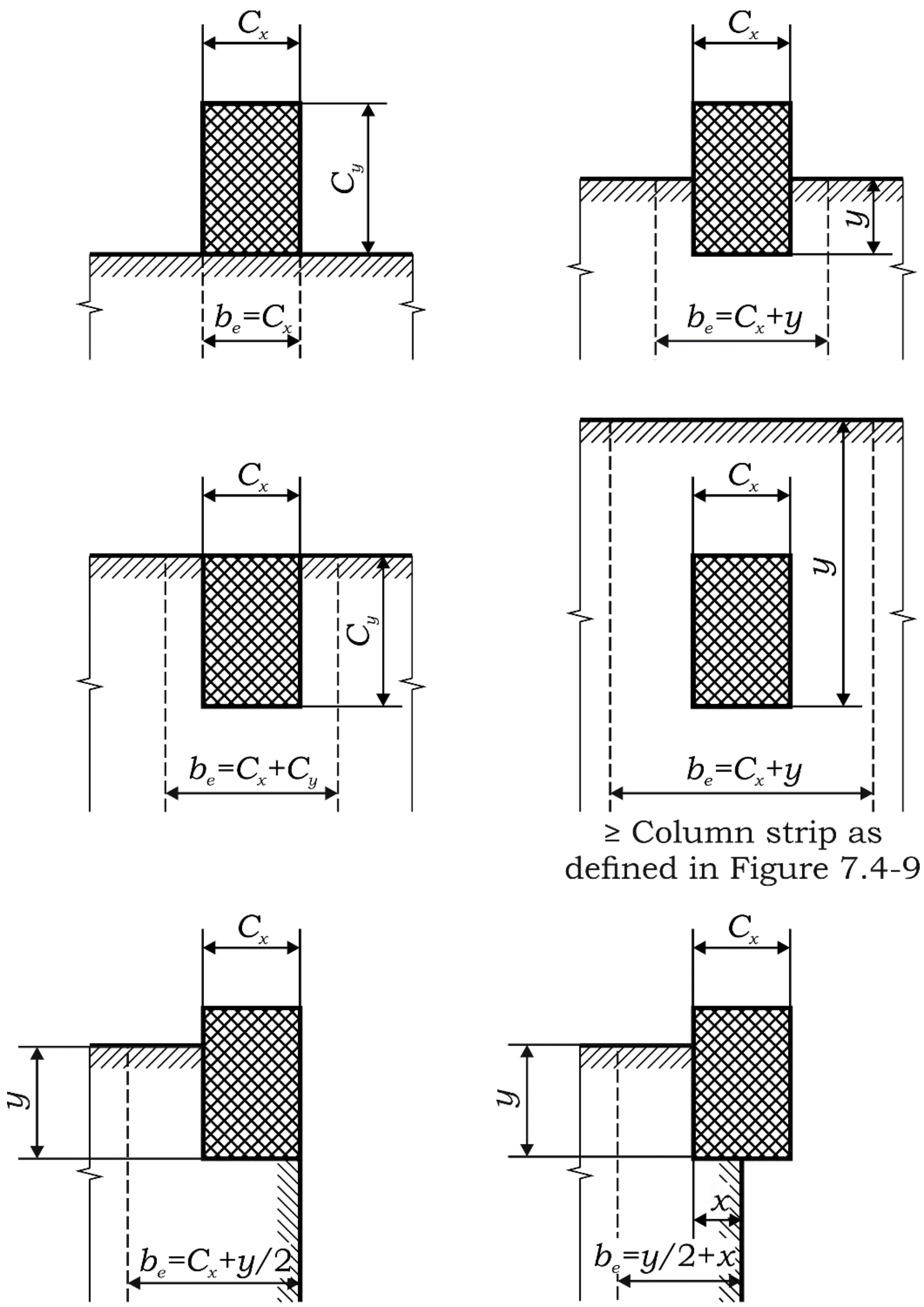


Figure 7.4-10 – Definition of width of effective moment transfer strip, *bₑ*, on plan

The maximum design moment that can be transferred to a column by this strip is given by the following expression:

$$M_{Ed}^{\max} = 0{,}17 \cdot f_{ck} \cdot b_e \cdot d^2 , \qquad (7.4\text{-}8)$$

where: d is effective depth for the top reinforcement in the column strip, and $f_{ck} \leq 35$ MPa.

Where the transfer moment at an edge column obtained from Table 7.4-5, is greater than $M_{Ed}^{\max}$ a further moment redistribution ≤ 10 % may be carried out.

Where the elastic transfer moment at an edge column obtained from frame analysis is greater than $M_{Ed}^{\max}$ moment redistribution ≤ 50 % may be carried out.

Where the slab is supported by the wall, or an edge beam with a depth greater than 1,5 times the thickness of the slab then:

- the total design load to be carried by the beam or wall should include those loads directly on the wall or beam plus a uniformly distributed load equal to one-quarter of the total design load on the panel;
- the design moments of half-column strip adjacent to the beam or wall should be one-quarter of the design moment obtained from analysis.

7.4.5.4.3 Effective shear forces in flat slabs

Generally the critical consideration for shear in flat slab structures in that of punching shear around the columns. This should be checked in accordance with Section except that the shear forces should be increased to allow for the effects of moment transfer as indicated below.

The design effective shear forces should be increased to allow for the column should be taken in accordance with EN1992 [N3] as follows: $V_{Eff} = 1,15 \cdot V_{Ed}$ – for internal column with approximately equal spans; $V_{Eff} = 1,4 \cdot V_{Ed}$ – for edge columns; $V_{Eff} = 1,5 \cdot V_{Ed}$ for corner columns (where V_{Ed} is the design shear transferred to the column and is calculated on the assumption that the maximum design load is applied to all panels adjacent column considered).

Where the adjacent spans differ by more than 25 % or the lateral stability depends on frame action V_{Eff} should be calculated in accordance with EN1992 (clause 6.4.3) [N3].

7.4.5.4.4 Irregular column layout

Where, due to the irregular layout of columns, a flat slab can not be sensibly analyzed using the equivalent frame method, a grillage or other elastic method may be used. In such a case the following simplified approach will normally be sufficient:

(1) analyze the slab with the full load, $\gamma_G \cdot G_k + \gamma_Q \cdot Q_k$, on all bays;

(2) the midspan and column moments should then be increased to allow for the effects of pattern loads. This may be achieved by loading a critical bay (or bays) with $\gamma_G \cdot G_k + \gamma_Q \cdot Q_k$ and the rest of the slab with $\gamma_G \cdot G_k$. Where there may be significant variation in the permanent load between bays, γ_G should be taken as 1,0 for the unloaded bays;

(3) the effects of this particular loading may then be applied to other critical bays and supports in a similar way.

7.4.6 YIELD LINE DESIGN

7.4.6.1 General

Yield Line Design is a well-founded method of designing reinforced concrete slabs, and similar types of elements. It uses Yield Line Design Theory to investigate failure mechanisms at ultimate limit state [19].

As it was shown above the most concrete slabs are designed for moments found by the methods, that are based on essentially elastic theory. On the other hand, reinforcement for slabs is calculated by non-liner resistance models for RC-elements that account for the actual inelastic behavior of members under load. As was stated in [19] limit analysis not only eliminates the inconsistency of combining elastic analysis with inelastic (non-liner) design but also accounts for the reserve strength characteristic of the most reinforced concrete structures and permits, within limits, an arbitrary readjustment of moments found by elastic analysis to arrive at design moments that permit more practical reinforcing arrangements.

The plastic hinge is defined as a location along a member in a continuous beam or frame at which, upon overloading, there would be large inelastic rotation at essentially a constant resisting moment. For slabs, the corresponding mechanism is the yield line.

In accordance with [19] a ***yield line*** is a crack in a reinforced concrete slab across which the reinforcing bars have yielded and along which plastic rotation occurs.

As it was Yield Line Theory is an ultimate load analysis. It establishes either the moments in an element (e.g. loaded slab) at the point of failure or the load at which an elements will fall. It may be applied to many types of slab, both with and without beams.

For the overloaded slab, the resisting moment per unit length measured along a yield line is constant as inelastic rotation occurs; the yield line serves as an axis of rotation for the slab segment.

7.4.6.2 Upper and lower bound theorems

Plastic analysis methods such as the Yield Line Theory derive from the general theory of structural plasticity, which states that the collapse load of a structure lies between two limits, an ***upper bound*** and a ***lower bound*** of the true collapse load. These limits can be found by well-establishes methods. A full solution by the theory of plasticity would attempt to make the lower and upper bounds converge to a single correct solution.

At was stated in [19] the lower bound theorem and the upper bound theorem, when applied to slabs, can be stated as follows:

Lower bound theorem: if, for a given external load, it is possible to find a distribution of moments that satisfies equilibrium requirements, with the moment not exceeding the yield moment at any location, and if the boundary conditions are satisfied, then the given load is a *lower bound of the true carrying capacity*.

Upper bound theorem: if, for a small increment of displacement, the internal work done by the slab, assuming that the moment at every plastic hinge is equal to the yield moment and that boundary conditions are satisfied, is equal to the external work done by the given load for that same small increment of displacement, than that load is an *upper bound of the true carrying capacity*.

If the lower bound conditions are satisfied, the slab can certainly carry the given load, although a higher load may be carried if internal redistributions of moment occur. If the upper bound conditions are satisfied, a load greater than the given load will certainly cause failure, although a lower load may produce collapse if the selected failure mechanism is incorrect in any sense.

The yield line method of analysis for slabs is an upper bound method, and consequently, the failure load calculated for a slab with known flexural resistances may be higher than the true value.

7.4.6.3 Rules for yield line

When a slab is on the verge of collapse because of the existence of a sufficient number of real or plastic hinges to form a mechanism, axes of rotation will be located along the lines of support or over point supports such as columns. The slab segments can be considered to rotate as rigid bodies in space about these axes of rotation. The yield line between any two adjacent slab segments is a straight line, being the intersection of two essentially plane surfaces. Because the yield line (as a line of intersection of two planes) contains all points common to these two planes, it must contain the point of intersection (if any) of the two axes of rotation, which is also common to the two planes. That is, the yield line (or yield line extended) must pass through the point of intersection of the axes of rotation of the two adjacent slab segments. The terms ***positive yield line*** and ***negative yield line*** are used to distinguish between those associated with tension at the bottom and tension at the top of the slab, respectively.

Guidelines for establishing axes of rotation and yield lines are summarized as follows:

1. Yield lines are straight lines because they represent the intersection of two planes.
2. Yield lines represent axes of rotation.
3. The supported edges of the slab will also establish axes of rotation. If the edge is fixed, a negative yield line may form, providing constant resistance to rotation. If the edge is simply supported, the axis of rotation provides zero restraint.
4. An axis of rotation will pass over any column support. Its orientation depends on other considerations.

5. Yield lines form under concentrated loads, radiating outward from the point of application.

6. A yield line between two slab segments must pass through the point of intersection of the axes of rotation of the adjacent slab segments.

Illustrations are given in Figure 7.4-11 of the application of the guidelines to the establishment of yield line locations and failure mechanisms for a number of slabs with various support conditions.

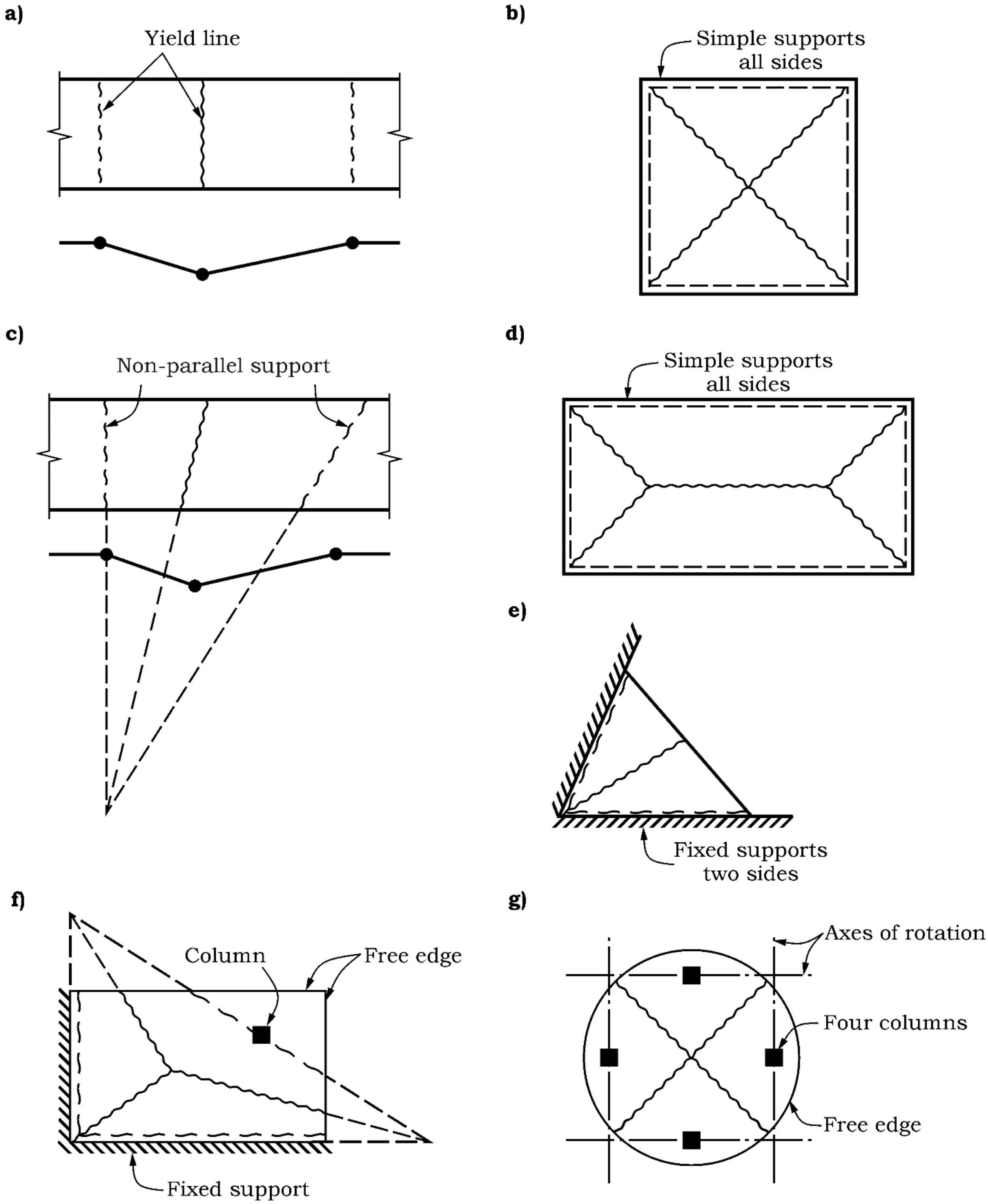


Figure 7.4-11 Typical yield line pattern [19]

Once yield line pattern has been postulated it is only necessary to specify the deflection at one point (usually the point maximum deflection) from which all other rotations can be found. These rules are illustrated in Figure 7.4-12.

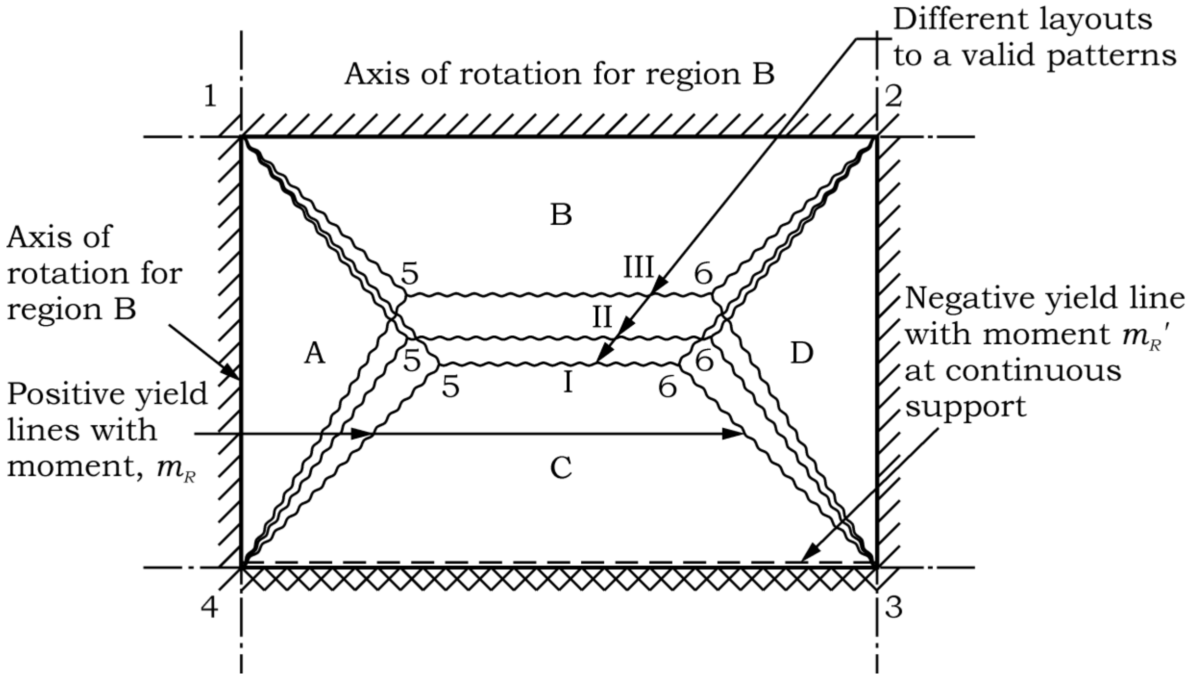


Figure 7.4-12 – Valid patterns for a two-way slab [19]

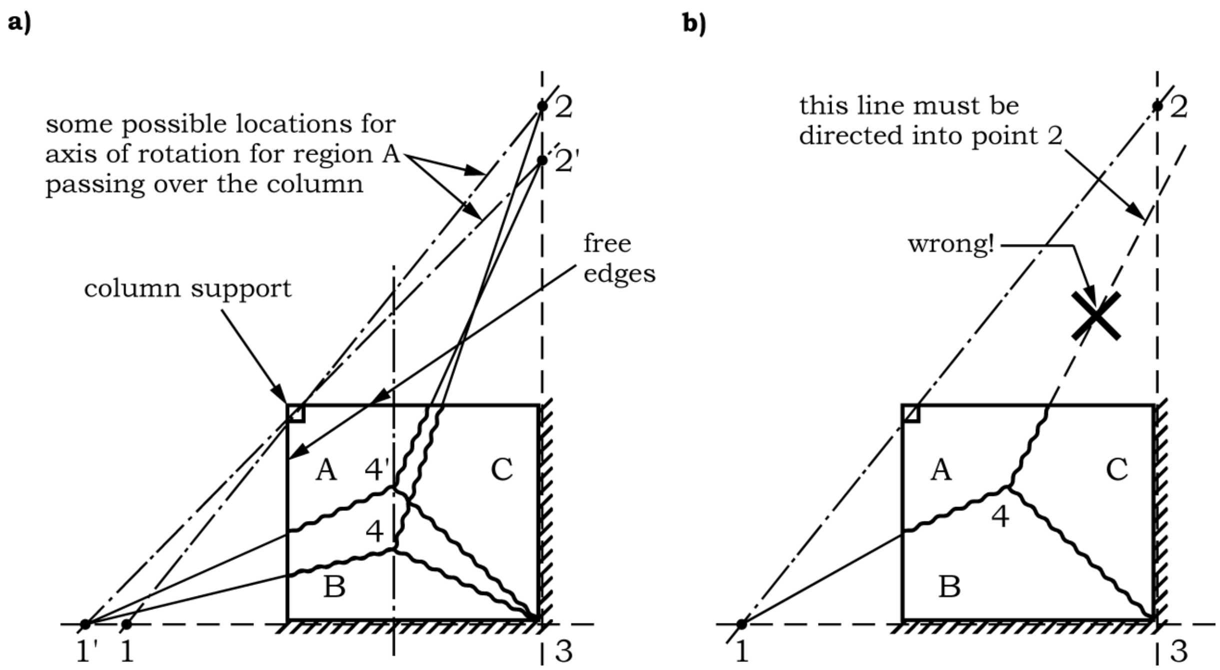
Figure 7.4-12 from [19] shows a slab with one continuous edge (along 3-4) and simply supported on the other three sides. The figure shows *three variations* of a valid yield line pattern. Successive applications of the virtual work method would establish which of the three would produce the *most unfavorable result.*

In this pattern, line 5-6 would be given unit deflection and this would then define the rotation of all the regions.

On the basis that a continuous support repels and a simple support attracts yield lines, layout III is most likely to be closest to the correct solution. As region C has a continuous support (whereas region B has not), line 5-6, must be closer to support 1-2 than support to 3-4.

It is always important to ensure that Rule 4 and Rule 6 (*Yield lines between adjacent rigid regions must pass through the point of intersection of the axes of rotation of those regions*) are observed in establishing a valid pattern. For the case under consideration, line 1-5, for instance, passes through the intersection of the axes of rotation of the adjacent regions A and B. Similarly line 2-6 passes through the intersection of the axes of rotation of adjacent regions B and D. Likewise line 5-6 in Figure 7.2-25 this line intersects the axes of rotation of adjoining regions B and C at infinity, i.e. line 5–6 is parallel to the axes of rotation.

Figures 7.4-13 shows the correct and incorrect application of Rule 4 and Rule 6 to a slab supported on two adjacent edges and a column.



a) – valid patterns for slab; b) – an invalid pattern

Figure 7.4-13. Slab supported on two adjacent edges and column [19]

7.4.6.4 Design methods

Once the general pattern of yielding and rotation has been established by applying the guidelines just stated, the specific location and orientation of the axes of rotation and the failure load for the slab can be established by either of two methods.

The first will be referred to as the method of segment equilibrium and will be presented as follows. It requires consideration of the equilibrium of the individual slab segments forming the collapse mechanism and leads to a set of simultaneous equations permitting solution for the unknown geometric parameters and for the relation between load capacity and resisting moments.

The second, ***the method of virtual work*** is based on equating the internal work done at the plastic hinges with the external work done by the loads as the predefined failure mechanism is given a small virtual displacement.

It should be emphasized that either method of yield line analysis is an upper bound approach in the sense that the true collapse load will never be higher, but may be lower, than the load predicted. For either method, the solution has two essential parts:

(1) establishing the correct failure pattern, and

(2) finding the geometric parameters that define the exact location and orientation of the yield lines and solving for the relation between applied load to the correct solution for the mechanism chosen for study, but the true failure load will be found only if the correct mechanism has been selected.

It is necessary to investigate all possible mechanisms for any slab to confirm that the correct solution, giving ***the lowest failure load***, has been found.

7.4.6.4.1 Analysis by virtual work method

Since the moments and loads are in equilibrium when the yield line pattern has formed, an infinitesimal increase in load will cause the structure to deflect further. The external work done by the loads to cause a small arbitrary virtual deflection must equal the internal work done as the slab rotates at the yield lines to accommodate this deflection. The slab is therefore given a virtual displacement, and the corresponding rotations at the various yield lines can be calculated. By equating internal and external work, the relation between the applied loads and the resisting moments of the slab is obtained. Elastic rotations and deflections are not considered when writing the work equations, as they are very small compared with the plastic deformations.

7.4.6.4.2 External work done by loads

An external load acting on a slab segment, as a small virtual displacement is imposed, does work equal to the product of its constant magnitude and the distance through which the point of application of the load moves, if the load is distributed over a length or an area, rather than concentrated, the work can be calculated as the product of the total load and the displacement of the point of application of its resultant.

In other words, the external energy expanded (W_e) is calculated by taking, in turn, the resultant of each load type (i.e. uniformly distributed load, line load or point load) acting on a region and multiplying it by its vertical displacement measured as a proportion of the maximum deflection implicit in the proposed yield line pattern.

For simplicity, the maximum deflection is taken as unity, and the vertical displacement of each load is usually expressed as a fraction of unity. The total energy expended for the whole slab is the sum of the expended energies for all the regions:

$$W_e = \sum (N \cdot \delta)_{\text{for all regions}}, \tag{7.4-9}$$

where N is the load(s) acting within a particular region (in kN);

δ is the vertical displacement of the load(s) N on each region expressed as a fraction of unity (in m).

7.4.6.4.3 Internal work done by resisting moments

The internal work done during the assigned virtual displacement is found by summing the products of yield moment m per unit length of hinge times the plastic rotation θ at the respective yield lines, consistent with the virtual displacement. If the resisting moment m_R is constant along a yield line of length l, and if a rotation θ is experienced, the internal work is:

$$W_i = m_R \cdot l \cdot \theta. \tag{7.4-10}$$

If the resisting moment varies, as would be the case if bar size or spacing were not constant along the yield line, the yield line is divided into n segments, within each one of which the moment is constant. The internal work W_i is then:

$$W_i = \left(m_{R,1} \cdot l_1 + m_{R,2} \cdot l_2 + \ldots + m_{R,n} \cdot l_n\right) \cdot \theta, \tag{7.4-11}$$

or:

$$W_i = \theta \cdot \sum_i m_{R,i} \cdot l_i \tag{7.4-12}$$

where: m_R is the moment or moment of resistance of the slab per meter run represented by the reinforcement crossing the yield line (in kN·m/m);

θ is the rotation of the about its axis of rotation (in m/m);

l is the length of the yield line.

In the other words, the internal energy dissipated (W_i), is calculated by taking the projected length of each yield line around a region onto the axis of rotation of that region, multiplying it by the moment acting on it and by the angle of rotation attributable to that region. The total energy dissipated for the whole slab is the sum of the dissipated energies of all the regions. (see Equation (7.4-10)).

Diagonal yield lines are assumed to be made up of small steps with sides parallel to the axes of rotation of the two regions it divides. The «length» of a diagonal or inclined yield line is taken as the summation of the projected lengths of these individual steps onto the relevant axes of rotation.

The angle of rotation of a region is assumed to be small and is expressed as being δ_{max}/length. The length is measured perpendicular to the axis of rotation to the point of maximum deflection of that region.

For the entire system, the total internal work done is the sum of the contributions from all yield lines. In all cases, the internal work contributed is positive, regardless of the sign of m_R, because the rotation is in the same direction as the moment. External work, on the other hand, may be either *positive* or *negative*, depending on the direction of the displacement of the point of application of the force resultant.

A fundamental principle of physics is that energy cannot be created or destroyed. So in the yield line mechanism, $W_e = W_i$. By equating these two energies the value of the unknown i.e. either the moment, m_R, or the load, Q, can then be established.

If deemed necessary, several iterations may be required to find the maximum value of moment m (or the minimum value of load capacity) for each chosen failure pattern.

7.4.6.4.4 The principles

For illustrate the principles, two straightforward examples from [19] are presented.

Consider a ***one-way slab*** simply supported on two opposite sides (span L and width b), supporting a uniformly distributed load of q_d in kN/m^2 (see Figure 7.4.-14).

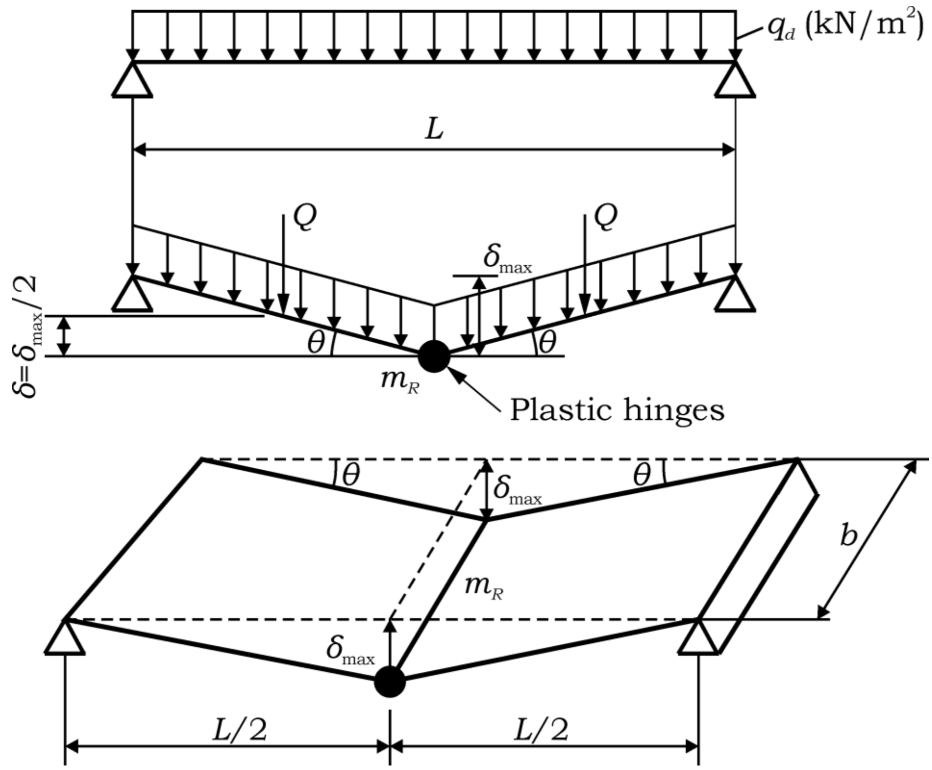


Figure 7.4-14 – A simply supported one-way slab

$$W_e = W_i, \text{ or}$$

$$\sum(N \cdot \delta) = \sum(m_R \cdot l \cdot \theta).$$

For simply supported one-way slab (see Figure 7.4-14):

$$2 \cdot q_d \cdot \frac{L}{2} \cdot b \cdot \frac{\delta_{max}}{2} = 2 \cdot m_R \cdot b \cdot \theta. \quad (7.4\text{-}13)$$

Taking into account, that $\theta = \delta_{max} / (L/2)$, the following equation is obtained:

$$\frac{2 \cdot q_d \cdot L \cdot b}{2} \cdot \frac{\delta_{max}}{2} = 2 \cdot m_R \cdot b \cdot \frac{2 \cdot \delta_{max}}{L}. \quad (7.4\text{-}13\text{ a})$$

Canceling gives:

$$\frac{2 \cdot q_d \cdot L}{4} = \frac{4 \cdot m_R}{L}. \tag{7.4-14}$$

Rearranging gives:

$$m_R = \frac{2 \cdot q_d \cdot L^2}{16} = \frac{q_d \cdot L^2}{8}. \tag{7.4-15}$$

The same principles apply to ***two-way spanning slabs***. Consider a square slab simply supported on four sides. Increasing load will firstly induce hairline cracking on the soffit, then large cracks will form culminating in the yield lines shown in Figure 7.4-15.

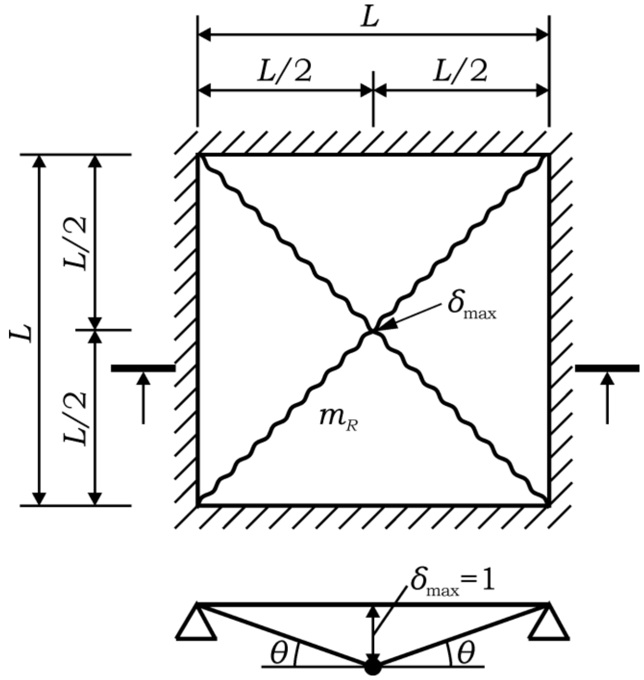


Figure 7.4-15 – Simply supported slab yield line pattern

Diagonal cracks are treated as stepped cracks, with the yield lines projected onto parallel axes of rotations. Assuming the slab measures $L \times L$ and carries a load of q kN/m²:

$$4 \cdot L \cdot \frac{L}{2} \cdot \frac{1}{2} \cdot q_d \cdot \frac{\delta_{max}}{3} = 4 \cdot m_R \cdot L \cdot \frac{\delta_{max}}{L/2}. \tag{7.4-16}$$

In this case the length of the projected yield line, l, for each region measured parallel to the axis of rotation is equal L.

$$\frac{4 \cdot L^2 \cdot q_d}{12} = 8 \cdot m_R, \tag{7.4-17}$$

or

$$\frac{q_d \cdot L^2}{24} = m_R. \tag{7.4-18}$$

7.4.6.5 Orthotropic reinforcement and skewed yield lines

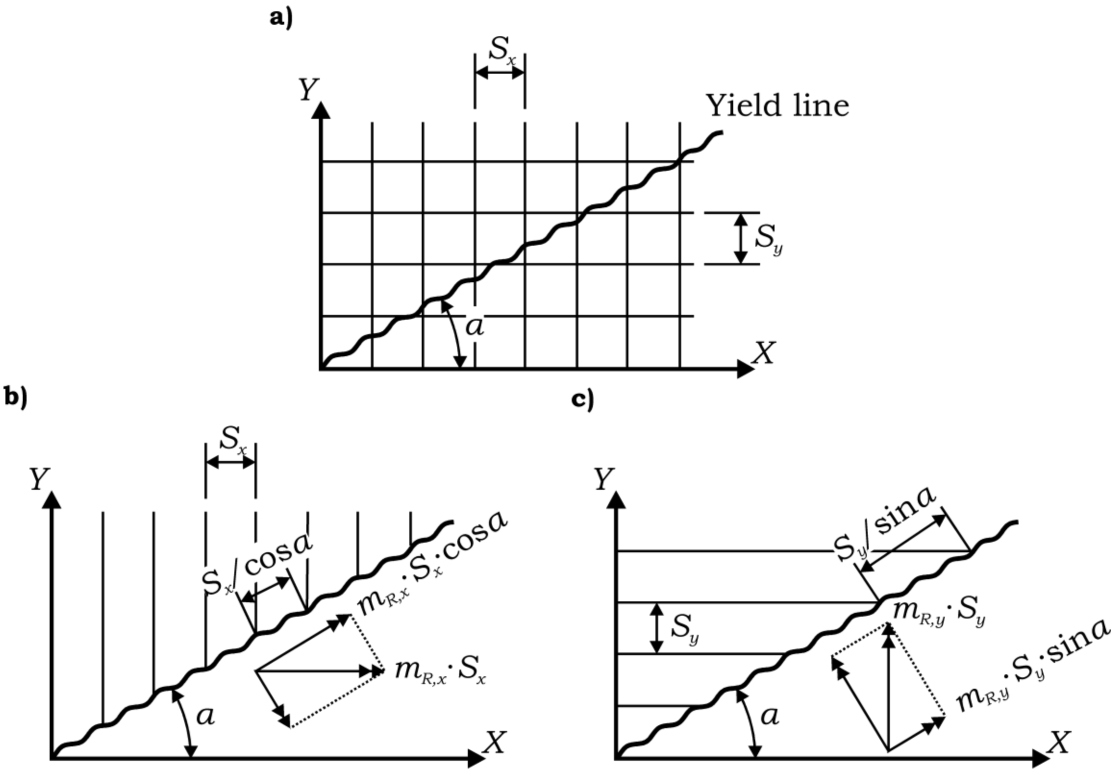
Generally, slab reinforcement is placed orthogonally, that is, in two perpendicular directions. The same reinforcement is often provided in each direction, but the effective depths will be different. In many practical cases, economical designs are obtained using reinforcement having different bar areas or different spacings in each direction. In such cases, the slab will have different moment capacities in the two orthogonal directions and is said to be orthogonally anisotropic, or simply orthotropic.

Often yield lines will form at an angle with the directions established by the reinforcement. For yield line analysis, it is necessary to calculate the resisting moment, per unit length, along such skewed yield lines. This requires calculation of the contribution to resistance from each of the two sets of bars.

Figure 7.4-16 shows an orthogonal grid of reinforcement, with angle α between the yield line and the X direction bars. Bars in the X direction are at spacing S_y and have moment resistance $m_{R,y}$ per unit length about the Y axis, while bars in the Y direction are at spacing u and have moment resistance $m_{R,x}$ per unit length about the X axis. The resisting moment per unit length for the bars in the Y and X directions will be determined separately, with reference to Figure 7.4-16 b and c, respectively.

For the Y direction bars, the resisting moment per unit length along the α axis provided by the Y direction bars is therefore.

$$m_{R,ay} = \frac{m_{R,x} \cdot S_x \cdot \cos a}{S_x / \cos a} = m_{R,x} \cdot \cos^2 a \tag{7.4-19}$$



a) – orthogonal grid and yield line; b) – *Y* direction bars; c) – *X* direction bars

Figure 7.4-16 – Yield line skewed with orthotropic reinforcement

For the bars in the X direction, the resisting moment per bar about the Y axis is $m_{R,y} \cdot S_y$, and the component of that resistance about the α axis is $m_{R,y} \cdot S_y \cdot \sin\alpha$. Thus, the resisting moment per unit length along the α axis provided by the X direction bars is

$$m_{R,\alpha y} = \frac{m_{R,y} \cdot S_y \cdot \sin a}{S_y / \sin a} = m_{R,y} \cdot \sin^2 a. \tag{7.4-20}$$

Thus, for the combined sets of bars, the resisting moment per unit length measured along the a axis is given by the sum of the resistances from Equation (7.4-17) and Equation (7.4-18):

$$m_{R,\alpha} = m_{R,x} \cdot \cos^2 a + m_{R,y} \cdot \sin^2 a. \tag{7.4-21}$$

For the special case where $m_{R,x} = m_{R,y} = m_R$, with the same reinforcement provided in each direction:

$$m_{R,a} = m \cdot (\cos^2 a + \sin^2 a) = m_R. \tag{7.4-22}$$

The slab is said to be isotropically reinforced, with the same resistance per unit length regardless of the orientation of the yield line.

The analysis just presented neglects any consideration of strain compatibility along the yield line and assumes that the displacements at the level of the steel during yielding, which are essentially perpendicular to the yield line, are sufficient to produce yielding in both sets of bars.

7.4.6.6 Fan patterns at concentrated loads

If a concentrated load acts on a reinforced concrete slab at an interior location, away from any edge or corner, a negative yield line will form in a more or less circular pat- tern, as in Figure 7.4-17, with positive yield lines radiating outward from the load point. If the positive resisting moment per unit length is m_R and the negative resisting moment m_R', the moments per unit length acting along the edges of a single element of the fan, having a central angle β, the arc along the negative yield line can be represented as a straight line of length $\beta \cdot r$.

Figure 7.4-17 shows the moment resultant obtained by vector addition of the positive moments m_R acting along the radial edges of the fan segment. The vector sum is equal to $m_R \cdot \beta$, acting along the length $\beta \cdot r$, and the resultant positive moment, per unit length, is therefore m. This acts in the same direction as the negative moment m_R', as shown in Figure 7.4-17 d. Figure 7.4-17 d also shows the fractional part of the total load P that acts on the fan segment.

Taking moments about the axis a − a gives:

$$\left(m_R + m_R'\right) \cdot r \cdot \beta - \frac{\beta \cdot P \cdot r}{2 \cdot \pi} = 0, \tag{7.4-23}$$

from which:

$$P = 2 \cdot \pi \cdot \left(m_R + m_R'\right). \tag{7.4-24}$$

The collapse load P is seen to be independent of the fan radius r. Thus, with only a concentrated load acting, a complete fan of any radius could form with no change in collapse load.

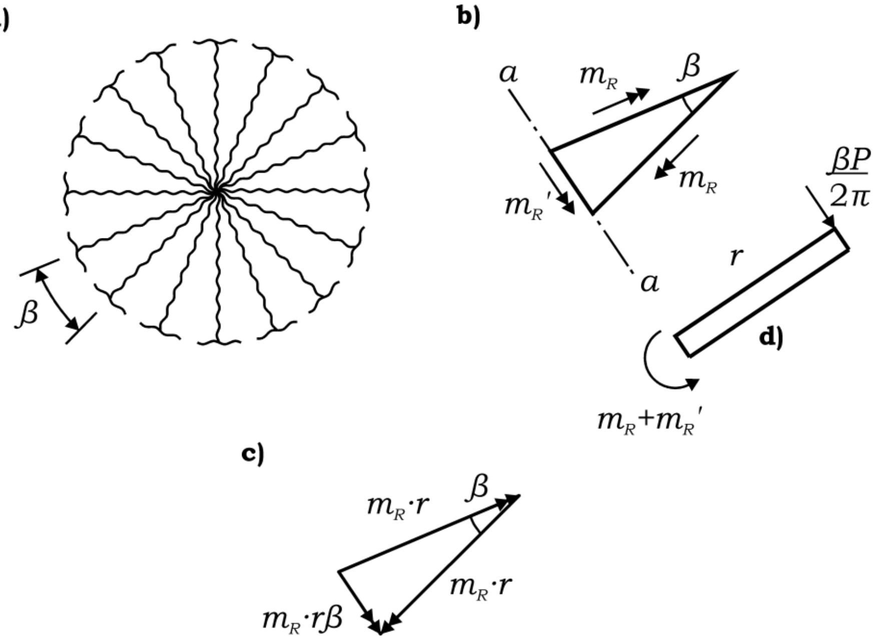


Figure 7.4-17 – Yield fan geometry at concentrated load [19]

7.4.6.7 Standard formulae for slabs design

Standard formulae that may be used for yield line solutions for common types of slab are presented in [19]. It may be regarded as a quick reference for common solutions.

The failure patterns produced by the yield lines in slabs depend on the nature of both the loading and support conditions.

7.4.7 ANALYSIS OF SLABS BY FINITE ELEMENT (FE) METHOD

7.4.7.1 General

As it was stated in [17, 20] before any analysis is carry out using computer software it is always good practice to carry out some simple hand calculations that can be used to verify that the results are reasonable.

It is particularly important to do this when using FE analysis, and not treat the computer as «a black box». Simple calculations can be carried out to determine the «free bending moment», i.e. calculate $q \cdot l^2 / 8$ for a span and then check that the FE results give the same value between the peak hogging and sagging moments. A discrepancy of 20 % is acceptable; outside of this limit further investigation should be carried out to determine the reason. Calculate the total load on the slab and compare these against the sum of the reactions from the model.

A recommended in accordance with [20] process of design using FE-analysis is given in Figure 7.4-18.

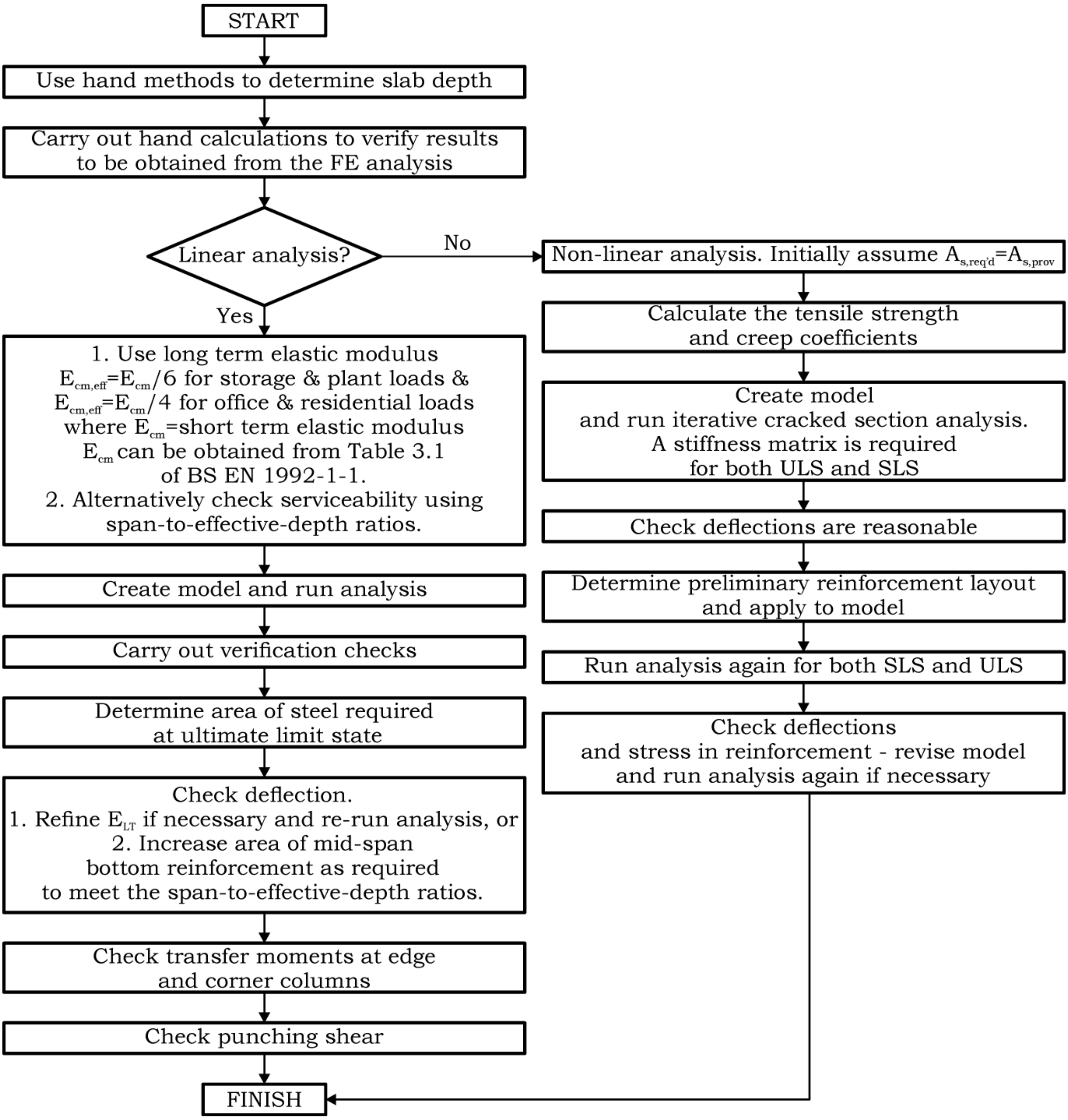


Figure 7.4-18 Design process using FE analysis (Figure 1 from [20])

The finite element method is commonly used to design the reinforcement in concrete slabs.

In order to simplify the analysis and to be able to use the superposition principle for evaluating the effect of load combinations, linear analysis is generally adopted even though concrete slabs normally have a pronounced non-linear response in ultimate limit states this can be justified since concrete slabs normally have good plastic deformability.

Theoretically, the design is based on the lower bound theorem of plasticity. Consequently, since the design is based on a moment (and force) distribution that fulfils equilibrium, the load carrying capacity will be sufficient provided that the structure has sufficient plastic deformation capacity. In Serviceability Limit States, the use of linear analysis is based on the assumption that the redistribution of moments (and forces) due to concrete cracking is limited.

For the slabs a linear analysis is normally performed in order to determine the load effects that will further be used for the detailed design of the structure. However in order to obtain a relevant basis for design a proper modelling and subsequently a proper interpretation and use of the FEM analysis results is required.

At support for instance, the sectional forces and moments are needed in the sections where a failure mechanism may occur.

For concrete slabs monolithically cast together with a supporting wall, the relevant moments in the slab are those at the face of the supporting wall. This consideration will influence the way in which the actual support is represented in the FE-model, the mesh density around the support point and the choice of relevant remits points.

Punching shear and deflection control are usually the governing criteria for flat slabs. Punching shear should be checked using code rules (see Chapter 4).

At is was shown in Chapter 5, deflection concrete structures is a complex phenomenon, which is dependent on the final tensile and compression strength, elastic modulus, shrinkage, time and duration of loading, and cracking of the member. Deflection prediction is based on assumptions and is therefore an estimate – even when using the most sophisticated computer software.

It should be noted, that deflection in a reinforced concrete slab is dependent on the age at first loading and the duration of the load because it will influence the time point at which the slab has checked (if at all) and is used to calculate the creep factor. A typical loading sequence in according with [20] is shown in Figure 7.4-19, which shows that in the early stages relatively high loads are imposed immediately after casting the slab above. Once a slab has cracked, it will remain cracked and the stiffness is permanently reduced.

In addition, in linear models, unrealistic concentrations of moments and shear forces will occur due to necessary simplifications in the model. In order to obtain an economical design these concentrations need to be distributed over a certain width, denoted as distribution or strip width [20]. Thus, three aspects of particular importance should be analyzed (reviewed) taking into consideration:

- modelling of support conditions;
- choice of result sections;
- choice of distribution width.

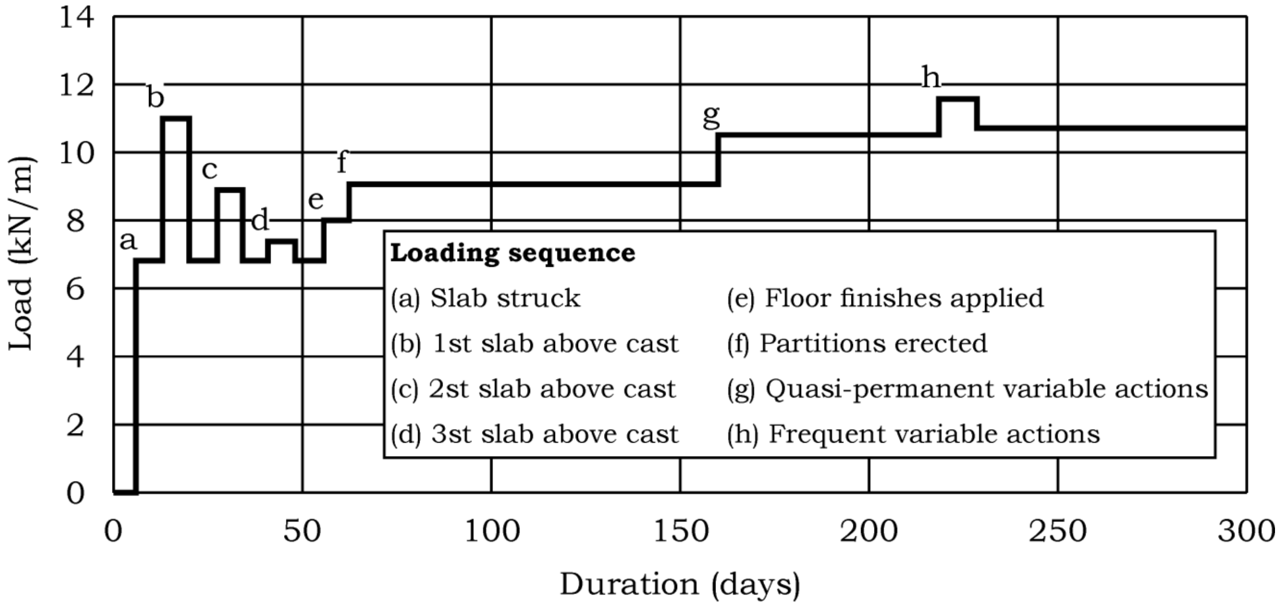


Figure 7.4-19 – Loading history for a slab (Figure 2 from [20])

7.4.7.2 Element types

When carrying out FE analysis, the selection of a particular type of element is no longer necessary as most commercially available software packages for slab design do not offer an option. For reference it is usual to use a «plate» element; this will provide results for flexure, shear and displacement. In the future it is likely that membrane action will be modelled and considered in the design, in which case a «shell» elements would be used.

Plate and shell elements are generally triangular or quadrilateral with node at each corner (see Figure 7.4-20). However, elements have been developed that include an additional node on each side, this gives triangle elements with six nodes and quadrilateral elements with eight nodes. By introducing more nodes into an element the accuracy of results is increased; alternatively, the number of elements can be reduced for the same number of nodes, so reducing computational time.

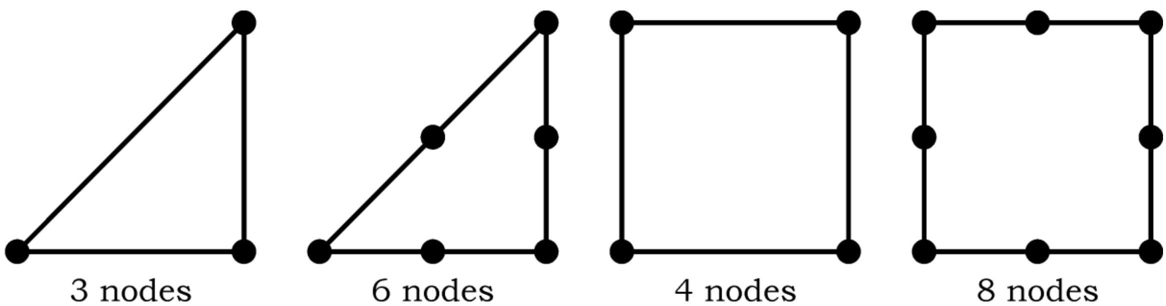


Figure 7.4-20 – Types of finite elements (FE)

The term «mesh» is used to describe sub-division of surface members into elements, with a finer mesh giving more accurate results.

As it was stated in [17] where a very coarse mesh was used (up to 5000 mm) it took just 30 second to analyze; although it is analytically correct it does not give sufficient detail. Conversely, when a much finer mesh was used (up to 500 mm) it took 15 minutes to analyze and gives the shape of bending moment diagram that would be expected. However, a mesh up to 1000 mm took just four minutes to analyze it gave very similar results and is considered to be sufficiently accurate for the purpose of structural design. From other hand, the 500 mm mesh has produced a higher peak moment; this is due to «singularities» or infinite stresses and internal forces that occur at the location of high point loads. This is due to assumptions that have been made in the model. Definite advice cannot be given as to the ideal size mesh size, but a good starting point is for elements to be not greater than span/10 or 1000 mm, whichever is the smallest. Elements should be «well conditioned», i.e. the ratio of maximum to minimum length of the sides should not exceed 2 to 1.

7.4.7.3 Modelling of support conditions

7.4.7. 3.1 General aspects

The support conditions in a finite element model of structure often have a decisive influence on the analysis results. Consequently, the modelling of the supports needs to be paid special attention.

In reality, the support from foundation or from other structural parts provides stiffness with respect to both translation and rotation. In the structural model, this is often simplified to free or fixed translations or rotations at the supports. In many situations these simplification can be motivated. However, in other cases such simplifications may have a critical influence on the analysis results. In such cases the supporting structure, or its supporting stiffness through translation or rotation springs, should be included in the model.

It also important to ensure that support conditions are introduced in the model at their correct locations and in correct directions. Note that constrained degrees of freedom in the model will control the deformation and rotation distribution in the analysis. Consequently, a small shift in the direction or position of a constraint may shift the deformed shape, and hence the internal distribution and magnitude of internal stresses, moments and forces.

For slabs supported by bearings or columns, the support conditions are often modelled as concentrated at single nodes. The effect of this is that a singularity is introduced in solution, with the sectional forces and moments tending to infinity upon mesh refinement. There are two principally different ways to deal with this problem: either the modelling of the support is improved so that the singularity is avoided or the results are evaluated in the failure-critical sections adjacent to the supports.

In most cases it is sufficient to model supports or connections to other structural parts in single points or lines. From the point of view of reinforcement design, the peak values that occur right at the connection are not of interest. Instead, design rules are needed in critical sections adjacent to the supports. For instance, if a slab is monolithically connected to a column, the effects of the singularity in the slab may be disregarded.

7.4.7.3.2 Modelling of supports at single points or lines

Generally, for analyses used as basis for detailed design, it is recommended to model supports through prescribed boundary conditions in single points or along single lines. In this way unintended rotation restraints in the numerical model different ways to model a wall support for continuous one-way slab (Figure 7.4-21) and compared the results with simple beam calculations.

The wall provides vertical support but the connection cannot transfer any moments, i.e. it is a «hinged» support.

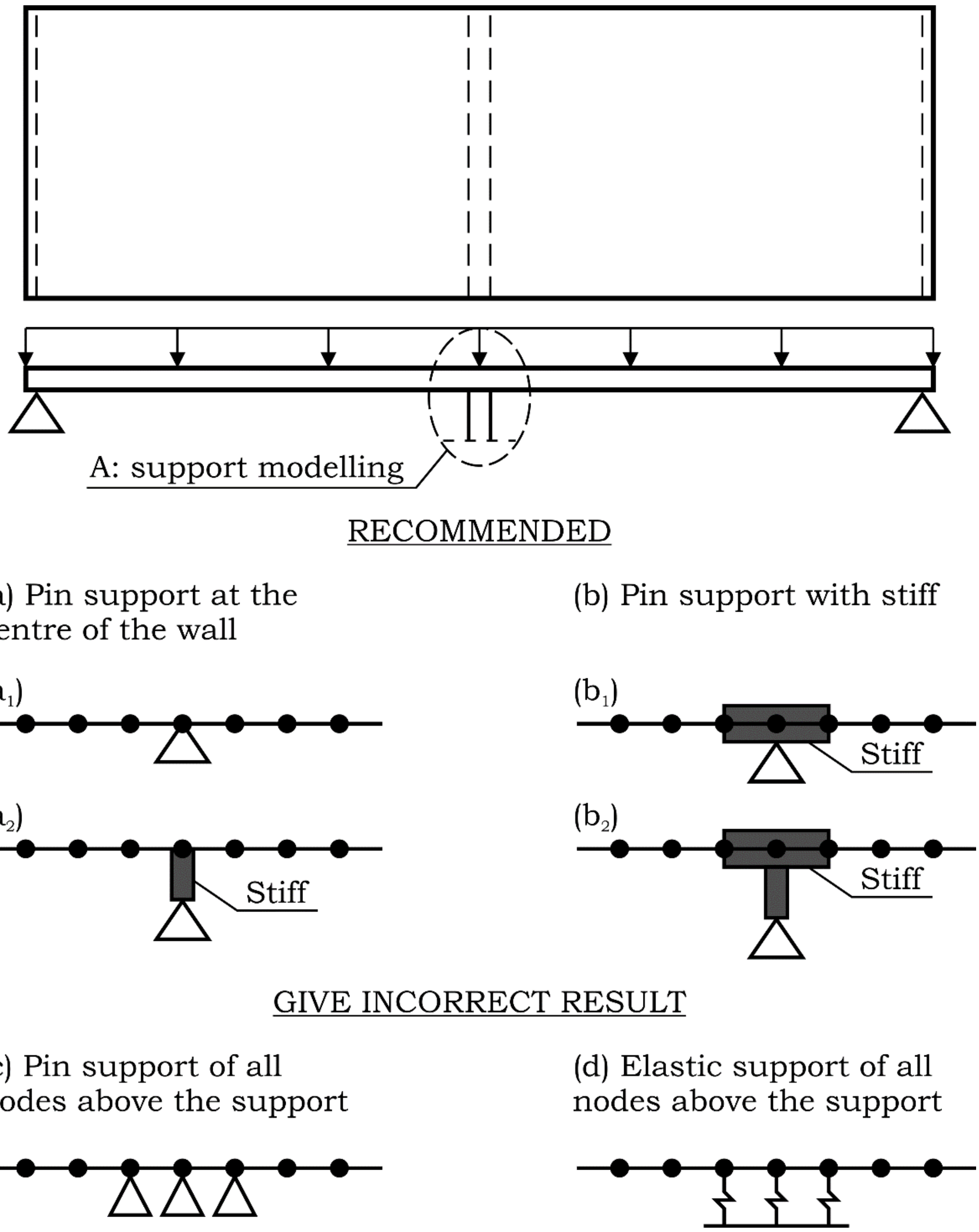


Figure 7.4-21 – Different ways to model a «hinged» line support for a slab modelled with linear shell elements, adapted from [17]

The pin support (a) is recommended. The pin support with stiff couplings (b) also gives good results while the other alternatives give incorrect results [17].

For the case of monolithic connection between the wall column and the slab, two modelling alternatives are shown in Figure 7.4-22.

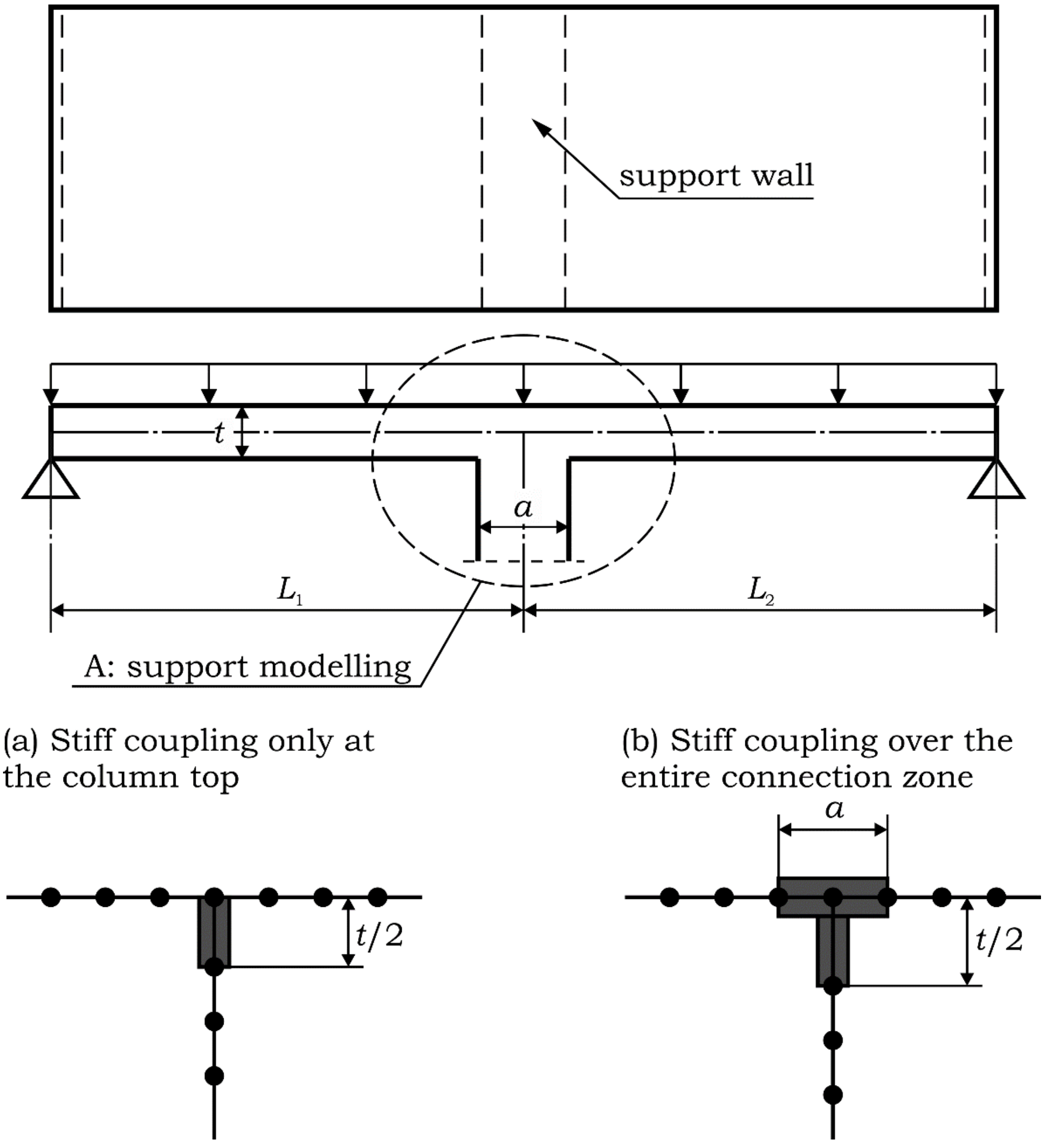


Figure 7.4-22 – Different ways to model a monolithic connection between the slab and the supporting wall/column [17]

Modelling alternative (a) (see Figure 7.2-35) involves a stiff coupling (rigid link) applied at the column top over a length equal to half slab width. This modelling alternative gives results in good agreement with continuum (solid) model if the slab thickness t and the wall (column) width a fulfil the following conditions:

$$\begin{cases} a < \min(l_{0,1}; l_{0,2}); \\ \dfrac{a}{t} < 2, \end{cases} \tag{7.4-25}$$

where: l_{01} and l_{02} represent the distances from the column center to joint of zero moment on either side of the column. These distance can be evaluated for a load case involving the permanent only. In case, the slab thickness is much smaller than the span, i.e. $a << \min(l_{01}; l_{02})$, the stiff coupling can for simplicity be left out and the wall or column be extend up the center of the slab.

Modelling alternative (b) (see Figure 7.4-23) gives higher moments at the supports and by consequence lower moments within the span. In addition, one should be careful not to introduce over constraint out of the plane that can cause too high moments and membrane forces for temperature loading.

Apart from the alternatives presented in Figure 7.4-21 there are several other ways to model the monolithic connection. One possibility would be to use alternative (b) but not extent the rigid coupling over the whole width a of the wall (or column) while another possibility would be to increase the thickness of the slab over the connections zone thus accounting for the increase in stiffness within this region.

If the width a of the wall (or column) and the thickness t of the slab do not respect the support conditions in more detail so that the stress model the support conditions in more detail described in a more realistic way (a modelling alternative avoiding singularities at the supports).

An alternative to model the support given by a column or bearing in a single point, and evaluate the results in adjacent critical sections, is to the solution is to replace the point reactions by surface loading as shown in Figure 7.4-23.

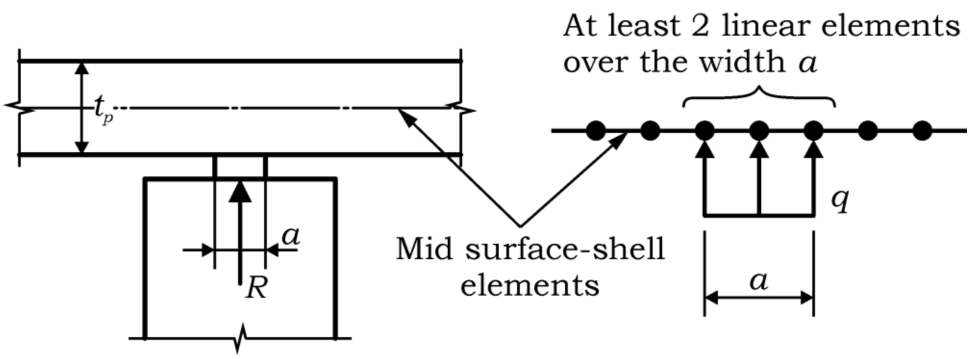


Figure 7.4-23 – Point reactions replaced by surface loading (Figure 2.4 from [17])

For elastomeric bearing, the surf load q can be approximated according to: $q = R/S$ where R is the support reaction force and S is the equivalent surface of the bearing. For more rigid supports and monolithic connections the support pressure will be concentrated towards the edges of the support surface.

This can be taken into account after evaluation of the support pressure distribution. Alternatively, equally distributed pressure can used as a conservative approximation. At least two firot order elements should be used over the width a. For supports that are wide compared to the slab thickness, more elements are needed.

In accordance with recommendation [17], the general analysis procedure is as follows:

- to computations are first performed with a point support at the centre of the bearing in order to determine the reaction force;

- the surface pressure is then computed and applied upwards at the support;

- the analysis is then performed once again, now adding the computed surface pressure at the support, in order to determine the actual reaction force at the support point should become (approximated) zero.

The solution presented above is not the only possible solution. An alternative approach is to model the bearing or support by springs according to the principle shown in Figure 7.4-24.

Note that the modelling alternative presented in Figure 7.4-24 does not introduce any spurious rotational restraints in the model. The stiffness properties of the springs can be determined from the stiffness properties of support, e.g. a bearing. Note also that the spring stiff ness must be different in the middle, on the side and at the corner of the support plate if discrete spring elements connecting the nodes are used to describe e.g. a constant surface stiffness.

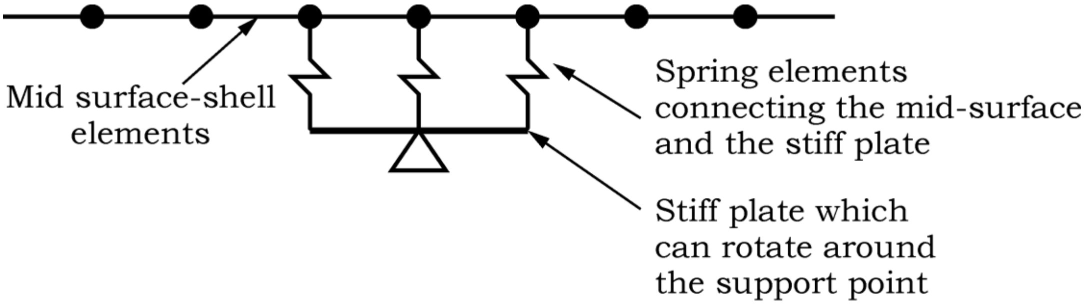
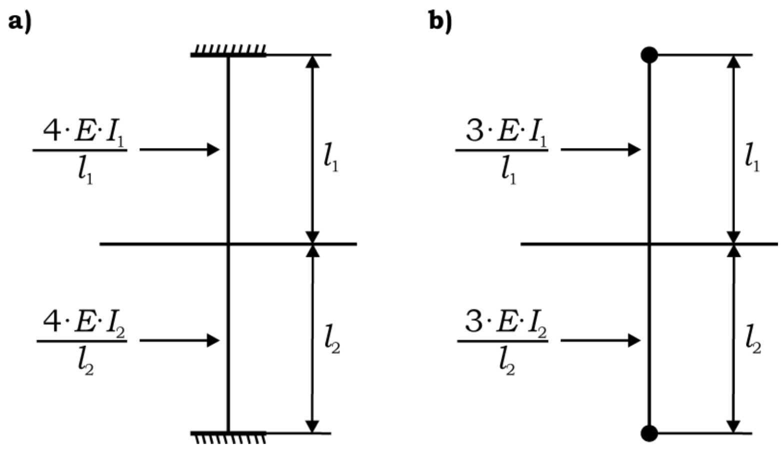


Figure 7.4-24 – Bearing support modelled by spring elements (Figure 2.5 from [17])

The modelling alternative described in Figure 7.4-24 should also be adopted if the support has a large minimum width compared to the span length and/or slab thickness. Typically, this situation occurs if wide columns are monolithically connected to the slab. In this case the column should be included in the model and the stiff plate in Figure 7.4-24 should be rigidly connected to the column top.

The stiffness of the columns should be modelled by using rotational spring stiffness. For a pin-ended column the stiffness can be taken as $k = 3 \cdot E \cdot L / l$ and for a fully fixed column $k = 4 \cdot E \cdot L / l$ (see Figure 7.4-25).



a) – far end fixed; b) – far end pinned

Figure 7.4-25 Modeling column stiffness (Figure 8 from [20])

However, for columns supporting the upper storeys, edges and corners the end condition will not be fully fixed and cracking can occur that will reduce their stiffness.

The rules for governing the maximum moment that can be transferred between the slab and column are given in EN1992 [N3] (Annex I: 1.2(5)). These rules are applicable even when using FE analysis. If the maximum transfer moments is exceeded the design sagging moment should be increased to reduce the hogging moment at the critical support.

7.4.7.4 Mesh density at support region

As it was shown in [17, 20] and stated above, when performing a linear finite element analysis of concrete slab, cross-sectional forces and moments become high at concentrated support, and will tend to infinity upon mesh refinement. However, when using the analysis as a basis for reinforcement design, the peak values of interest. Instead, the cross-sectional forces and moments in critical sections adjacent to the support are headed for design. When designing the slab reinforcement, averaged values of the bending moments over certain distribution widths perpendicular to the reinforcement direction will normally be used, see Chapter 4. The influence of the mesh density around a column on these averaged moment values will be much smaller than on the moment value at the critical section right at the edge of the column. Based on this observation, it is recommended *that the mesh density around the point support node in a slab (e.g. a column or an abutment), should be chosen such that there is at least one shell element regardless of order, between the support node and the critical cross-section.* Figure 7.4-26 illustrates an example of such a mesh refinement around column supports.

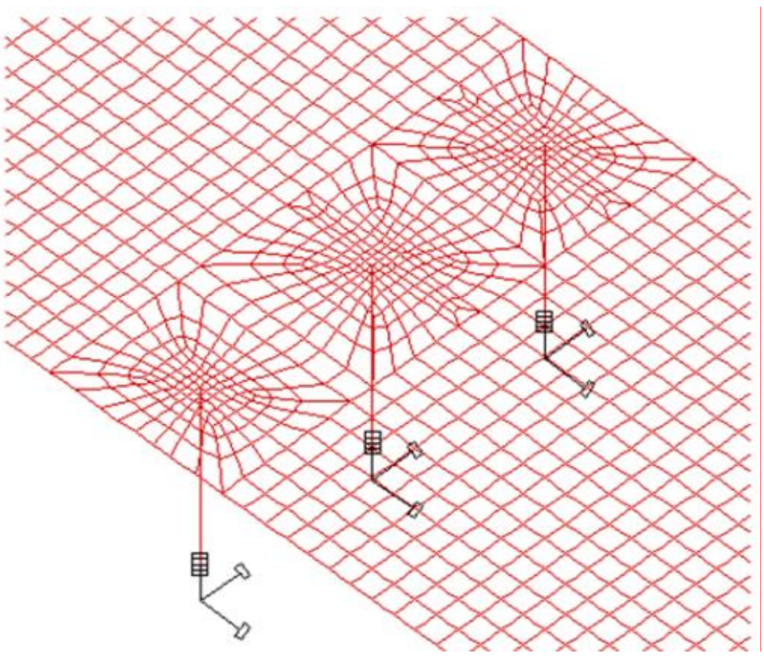


Figure 7.4-26 – Example of mesh refinements around column supports of a bridge slab (Figure 8 from[17])

For situations where a slab is supported by a line support, there is no problem with singularities. However, the element mesh needs to be fine enough to give accurate results in the adjacent critical sections. Also here it is recommended to provide at least one element length between the line support node and the critical cross-section, regardless of the shell element order.

Alternatively, the maximum moment and shear force at the line support can be used as a conservative approximation.

EN1992 [N3] gives some specific guidance in Annex I on how to deal with loading for unusual layout. When designing EN1992 [N3] the combination of the full factored dead load over the whole slab together with the factored live loading on alternate bays should be used (see Figure 7.4-27). These should be considered separately in each orthogonal direction. Note that a «chequer-board» pattern loading is an unlikely pattern and may not give the most unfavorable arrangements.

As it stated in [17, 20] all software will allow a number of load cases to be considered, and the engineer must assess how to treat pattern loading. It requires engineering judgement to determine the most unfavorable arrangement of design loads for a floor slab with an unusual geometry. However, EN 1992 [N3] gives some specific guidance in Annex I on how to deal with loading for unusual layout.

Where pattern loading is to be considered the maximum span moments for all flat slabs designed to EN 1992 [N3] can be obtained by using the combination of the full factored permanent (dead) load over the whole slab together with the factored variable (live) loading at alternate bays (see Figure 7.4-27). These should be considered separately in each orthogonal direction. Note that a «chequer-board» pattern loading is an unlikely pattern and may not give the most unfavorable arrangements [20].

For non-linear cracked section analysis, two stiffness matrices will be required, one each for the *ULS* and *SLS*. This is because the material properties will be different from those at the *ULS*. The load should be assigned to both cases with appropriate partial factors.

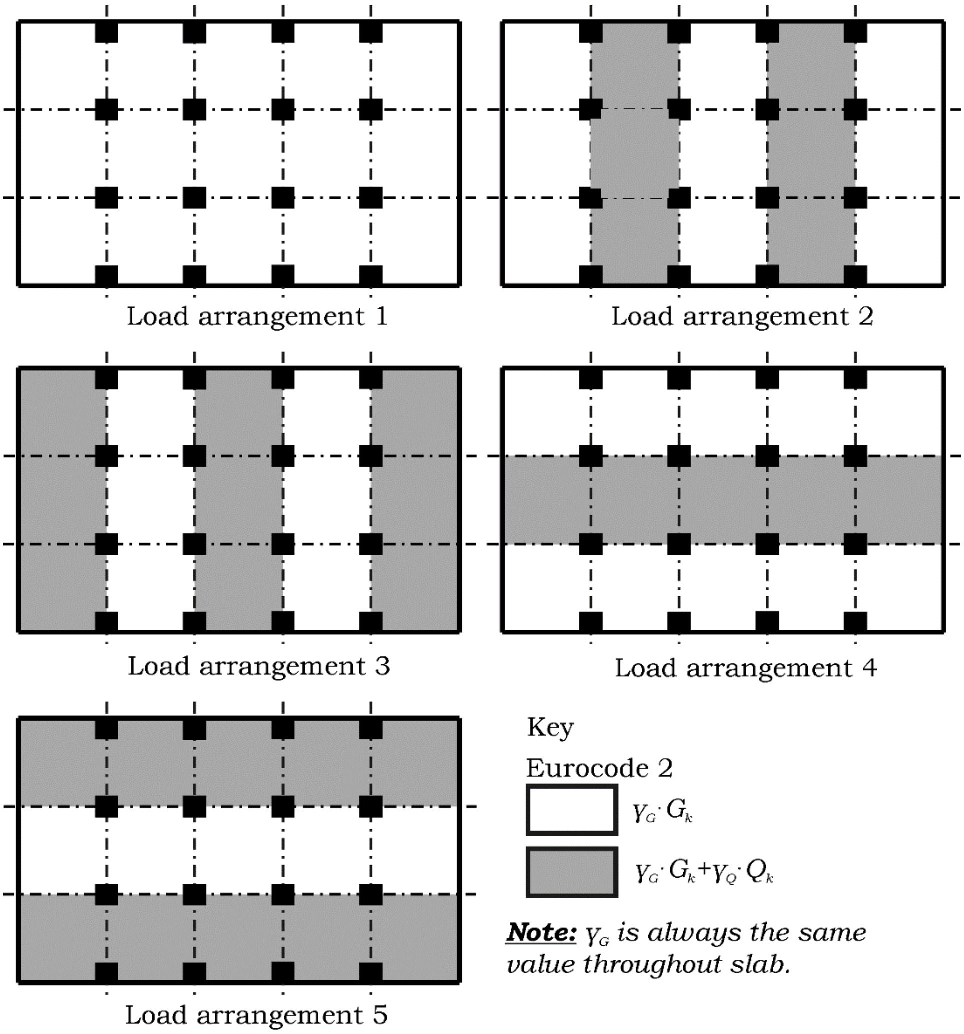


Figure 7.4-27 – Load arrangements for flat slabs [20]

7.4.7.5 Choice of result sections

7.4.7.5.1 Result sections for moments

(1) General

The exact loading on the slab and the maximum moment will depend on the loading on the distribution of the support pressure towards the bottom of the slab (see Figure 7.4-28). At the location where the maximum bending moment occurs a vertical bending crack will develop and eventually the reinforcement in this section will start to yield, possibly limiting the capacity with respect to bending. The same principle applies for a two-way slab; the critical sections for bending are situated at locations where the maximum moment occurs.

If the supports are modelled to describe the support pressure in a more realistic way, as it was shown in section above, the result sections for the moments are the sections where maximum moments are obtained in the FEM analysis. If, on the other hand, the supports are modelled in a simplified way, in single points or along discrete lines, the maximum bending moments obtained from the FEM analysis will over-estimate the real moments. At locations where the slab is supported at single points it will even tend to infinity upon mesh refinement. Here, the locations of the result section where FEM results should be evaluated depend on the design and the actual stiffness of the slab-support connection.

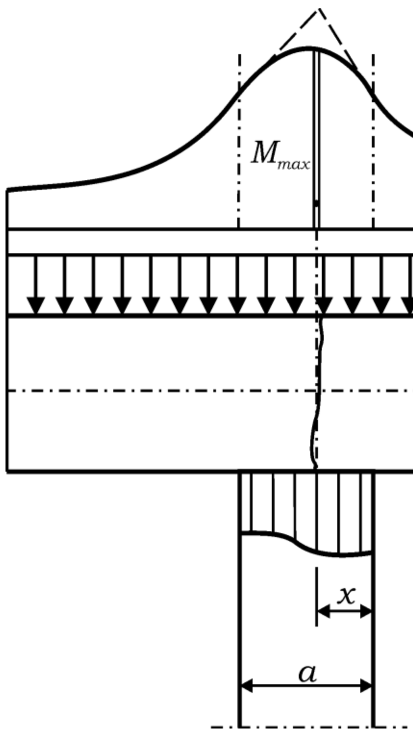


Figure 7.4-28 Bending moment variation and critical section for the bending moment in a slab with distributed support pressure [17]

(2) Monolithic connections modelled in single points or lines

If the slab is monolithically connected with its supports, columns or walls, it can be shown that the maximum stresses do not occur inside the connection region but instead appear at the border of the connection.

It should be noted, that the cross-sectional moments and forces in the slab are defined as integrals of the stresses over the cross-section and do not have a clear interpretation inside a connection region [17].

A critical bending crack will form no closer to the theoretical support point than along the surface of the column or the wall. This is also where the tensile reinforcement will start to yield.

Consequently, the critical cross-section for bending failure in the slab is along the surface of the column or wall (see Figure 7.4-29).

In [17] it was recommended to model monolithic connections between a slab and its supporting walls and columns along discrete lines and in single points.

For a monolithic connection modelled in this way the result section for moments can be taken as the section along the surface of the column or wall, see Figure 7.4-29. This corresponds to the recommendations given in Eurocode 2

(EN 1992 [N3], clause 5.3.2.2), provided that the support width is smaller than the slab thickness.

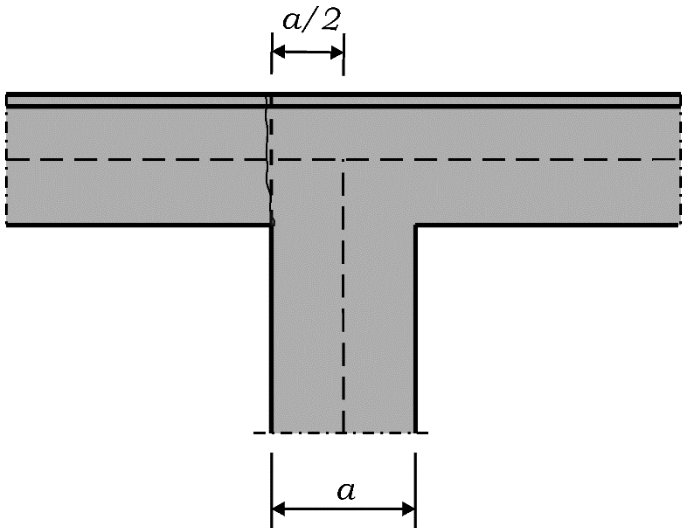


Figure 7.4-29 – Result section for bending moments, for a monolithic connection modelled as a connection in a single point or along a discrete line between structural finite elements (typically beam and shell elements (Figure 7.4-29 from [17]).

The width a of the column, in Figure 7.4-29, is the side length of a quadratic or rectangular column cross section. For a circular column, the real geometry can be approximated by an equivalent quadratic cross-section with:

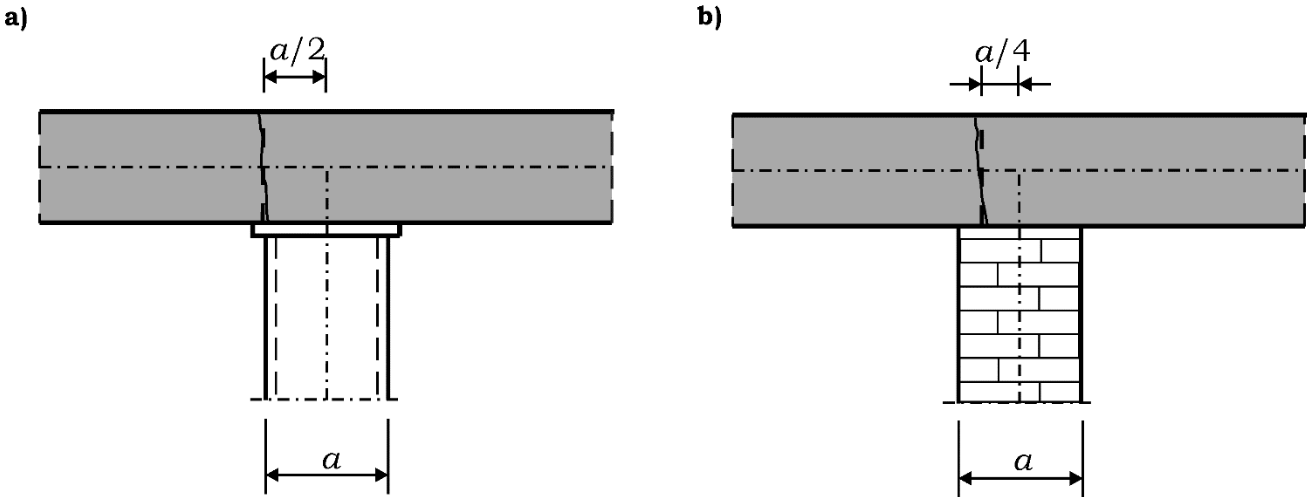
$$a = \frac{\sqrt{\pi} \cdot \varnothing}{2}, \tag{7.4-26}$$

where: Ø is the diameter of the column.

(3) Simple supports modelled in single points or lines

If the slab is simply supported on a wall or a column and the support transfers compression stresses only, the position of the critical section depends on the stiffnes of the support.

In case of a slab resting on a rigid support, the resultants of the support stresses for each support half will shift towards the edge of the support. As an approximation, the support resultants can be assumed to act at the face of the support. On the other hand, if the support is very weak, the support pressure will tend to approach a uniformly distributed support stress. These two cases can be seen as extremes regarding the support pressure distribution (see Figure 7.4-28). In case of column supports, the support pressure for rigid supports will tend to shift more towards the support edges than for wall. As a conservative approximation, case (b) can be assumed.



a) – rigid support; b) – weak support

Figure 7.4-30 Result section for bending moments for a simple support modelled in a single point or along a discrete line between structural finite elements (typically beam and shell elements).

According to clause 5.3.2.2 EN 1992 [N3], the design support moment, calculated with center-to-center distance between support points, can be reduced with:

$$\Delta M_{Ed} = \frac{R \cdot a}{8}, \tag{7.4-27}$$

where: R is the support reaction and a is the support width.

It is shown that the maximum support moment according EN 1992 [N3] assumption can be conservatively approximated from the theoretical moment distribution (with discrete supports at the support center) as the moment a distance $0{,}25 \cdot a$ from the support point. Similarly, it is shown that the assumption of support resultants at the edges of the support in the theoretical moment distribution (i.e. $0{,}5 \cdot a$ from the center support point). A weak support for a concrete slab could typically be a masonry wall or column. However, hot even for this support condition we would obtain an equally distributed support pressure.

For a concrete support of a one-way concrete slab that transvers both compressive and tensile strains, in [17] was shown that the support resultant for each support half be at 2/3 of the distance from the centre towards the edges, and for a steel support the edges, and for a steel support that transfers only compressive forces at 95 % of the distance towards the edges.

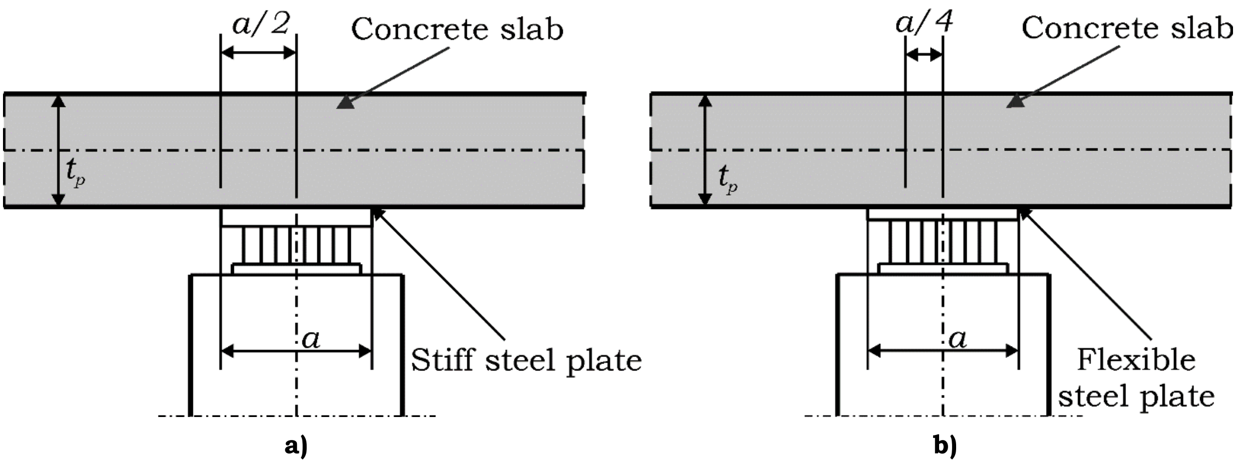
Without any detailed evaluation of the real support pressure, a distributed support pressure can be assumed as a conservative approximation. This mean that, for simple supports modelled in single points or lines, *the result section for moments can be taken as the section at half the distance between the center and edge of the column or wall* (see Figure 7.4-30).

This corresponds to the recommendation given in EN 1992 [N3] (see clause 5.3.2.2).

(4) Bearing supports modelled in single points or lines

A bearing consists principally of an elastomeric material between two steel plates. The steel plate that is in direct contact with the concrete slab is usually very stiff. For this case the result section can be assumed to be along the edge of the bearing top plate (see Figure 7.4-31 a).

On the other hand, if the steel plate can not be regarded as stiff, the support pressure will change towards a more distributed support pressure. For this case, and as a conservative assumption in general, the result section for moments can be taken as the section at half the distance between the center and the edge of the bearing top plate (see Figure 7.4-31 b).



a) – rigid bearing top plate; b) – flexible (or no) bearing top plate

Figure 7.4-31 – Result section for bending moments, for a bearing support modelled in a single points or long a discrete line (beam and shell models)

7.4.7.5.2 Result section for shear force

The location of result sections for shear forces at supports depend on where failure-critical shear cracks may occur.

The shear force in a slab section is caused by the part of the vertical load that is transferred towards the support across this section. Consequently, the shear force obtained from the slab analysis in the section at a distance $z \cdot \cot\theta$ (where z is the internal lever arm), is the shear force that needs to be transferred across the critical shear crack (see Figure 7.4-32). Any load that is acting on the slab top surface closer to the support than this will be transferred directly to the support. The self-weight of the slab can be treated as a load acting on the top of the slab.

It can be concluded that the critical result section in a slab with respect to shear forces are not located closer to the support edge than $z \cdot \cot\theta$ (see Figure 7.4-32). This is independent of the design and stiffness of the slab-support connection. For slabs without shear reinforcement, the critical shear crack can generally be assumed to have an inclination θ not steeper than 45°. In this case $z \cdot \cot\theta = z \approx d$.

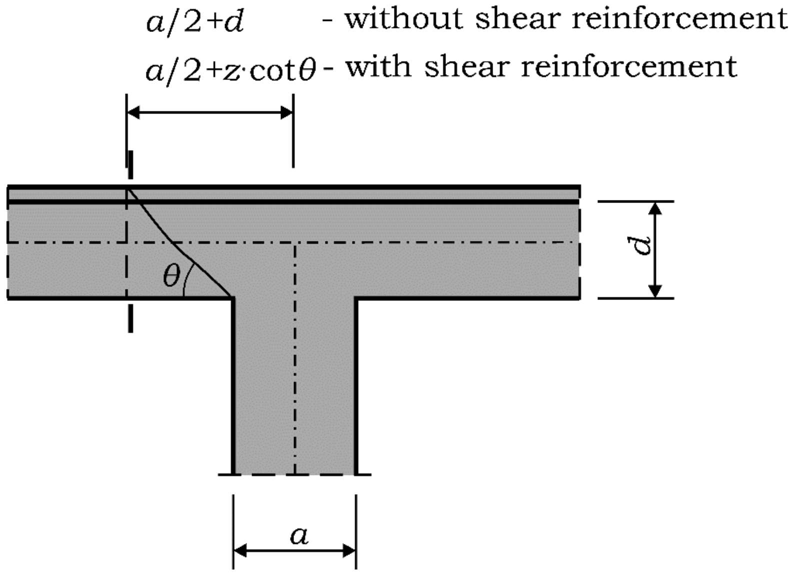


Figure 7.4-32 Critical section for shear force
(independent of the design and stiffness of the slab-support connections) [17]

In slabs with shear reinforcement, the risk for shear compression failure must also be checked in accordance with provisions given in Chapter 4.2. For this case, the entire shear force at the support edge must be accounted for.

7.4.7.6 Intepreting results. Redistribution of sectional forces and moments

The results from FE analysis will generally be in the form of contour plots of stresses and forces, althougth a «section» through the contour plots (either bending moment or areas of steel) can usually be obtained. These will show very large peaks in bending moment at the supports. The temptation to provide reinforcement to resist this peak moment should be avoided. This potential error stems from a lack of understanding of the assumptions made in the modelling. The reinforcement in the concrete will be distributed across a larger area; it is not therefore necessary to design to resist this peak moment. However, a method is required for distributing this peak moment across a larger area.

7.4.7.6.1 Design moments adjustment

As it was stated in [17] where high peak moments occur the concrete will crack and the reinforcement may yield if it’s the elastic limit is exceeded. The forces are then shed shed to the surrounding areas. Even if slab were designed to resist this moment it is unlikely that it would actually acluve this capacity for the following reasons:

- the construction process often leads to construction stage overload;
- the reinforcement in unlikely to be placed at exactly the point of peak moment.

It is therefore necessary to acknowledge that some of the peak moments to adjacent areas will occur due to the material properties of concrete, and not attempt to design against it.

In fact a recent paper by Scott and Whittle [17], concluded the redistribution occurs even at the *SLS* because of flu mismatch between the uniform flexural stuffness assumed and the variation in actual stuffness that occurs because of the variation in the reinforcement.

When using FE, especially for slabs with irregular geometry, it is not usually possible to carry out redistribution of the moment;

- It is not simple to determine where to distribute the hogging moment;
- If the software is carry out the design there is usually no method for changing the analysis output.

7.4.7.6.2 Effect of simplification in the modelling in accordance with [17]

As already pointed out, unrealistic concentrations of cross-sectional moments and shear forces will generally occur in linear FEM analyses due to simplifications in the modelling. Geometrical simplifications are typically simplified modelling of supports and connections between structural elements, or simplified modelling of concentrated loads e.g. wheel pressures on bridge slabs [17].The material simplifications are mainly related to the assumption that reinforced concrete behaves like a linear elastic and isotropic material. In reality however, reinforced concrete has a highly non-linear behaviour involving both cracking and crushing of concrete and yielding of reinforcement.

Simplifications made in the geometrical modelling often lead to very high concentrations of moments and shear forces at least locally. This occurs, for example, when a slab modelled with shell elements is supported by columns modelled with beam elements or by bearings modelled with boundary conditions applied at a single node.

The singularities that may occur are local disturbances of the moment and force fields, and do not influence the cross-sectional moments and shear forces a short distance from the support point where the singularity appears. In accordance with [17], *as long as the results in the critical sections are used and the finite element mesh is dense enough, modelling of support conditions in discrete points or lines does not influence the designing cross-sectional moments and shear forces.*

However, not even the high stress obtained in the critical sections do normally exist in reality. The concrete will crack already for service loads, leading to redistribution of moment and forces. In the ultimate limit state, the reinforcement will start to yield, leading to even large redistributions.

The material simplification introduced the assumption of linear elastic response will lead to higher cross-sectional moments that in reality, e.g. around a column or concentrated support, since cracking a subsequent yielding in the reinforcement is not included in the model.

7.4.7.6.3 Sectional forces and moments for reinforcement design

As it was shown in [17] in an ultimate limit state, the forces in each main reinforcement layer times its inner lever arm will result in a bending reinforcement moment resistance. These reinforcement moment resistance must balance the complete linear moment field, including torsional moment. The reinforcement moments m_{rx} and m_{ry} for design of reinforcement in two perpendicular directions x and y can be defined according to Equation (7.4-28) and Equation (7.4-29):

$$m_{ry,pos(neg)} = m_y \pm \mu \cdot |m_{xy}|; \qquad (7.4\text{-}28)$$

$$m_{ry,pos(neg)} = m_y \pm \mu \cdot |m_{xy}|, \qquad (7.4\text{-}29)$$

where: m_x and m_y are the linear bending moments in the x and y direction, i.e. moments generated by the normal stress in sections acting in x and y directions, respectively (and leading to reinforcement in the x and y directions, respectively).

Furthermore, m_{xy} is the torsional moment and μ is a factor that can be choses with respect to practical consideration, usually close to 1. In the above equations, the indices «pos» and «neg» refer to the top and bottom of the slab, respectively with the positive z direction pointing to the top.

In addition to the reinforcement moments, associated membrane forces can be evaluated as follows:

$$n_{rx,pos(ned)} = n_x \pm \mu \cdot |n_{xy}|; \qquad (7.4\text{-}30)$$

$$n_{ry,pos(ned)} = n_y \pm \frac{1}{\mu} \cdot |n_{xy}|. \qquad (7.4\text{-}31)$$

And included in the computation of the reinforcement areas. In Equation (7.4-30) and Equation (7.4-31), n_x ,n_y and n_{xy} are the membrane forces at the mid-surface of the slab. The indices «pos» and «neg» refer in this case to tension and compression, respectively.

If the reinforcement directions x and y are not orthogonal, Equation (7.4-26) and Equation (7.4-27) are replaced by Equation (7.4-28) and Equation (7.4-29):

$$m_{rx,pos(neg)} = \frac{1}{\sin^2\psi} \cdot \begin{bmatrix} m_1 \cdot \sin^2(\psi - \delta) + m_2 \cdot \cos^2(\psi - \delta) \pm \\ \pm |m_1 \cdot \sin\delta \cdot \sin(\psi - \delta) - m_2 \cdot \cos\delta \cdot \cos(\psi - \delta)| \end{bmatrix}; \qquad (7.4\text{-}32)$$

$$m_{ry,pos(neg)} = \frac{1}{\sin^2\psi} \cdot \begin{bmatrix} m_1 \cdot \sin^2\delta + m_2 \cdot \cos^2\delta \pm \\ \pm |m_1 \cdot \sin\delta \cdot \sin(\psi - \delta) - m_2 \cdot \cos\delta \cdot \cos(\psi - \delta)| \end{bmatrix}. \qquad (7.4\text{-}33)$$

In Equation (7.4-30) and Equation (7.4-31) m_1 and m_2 denote the principal moments at the considered location and the angles δ and ψ are defined in Figure 7.4-33.

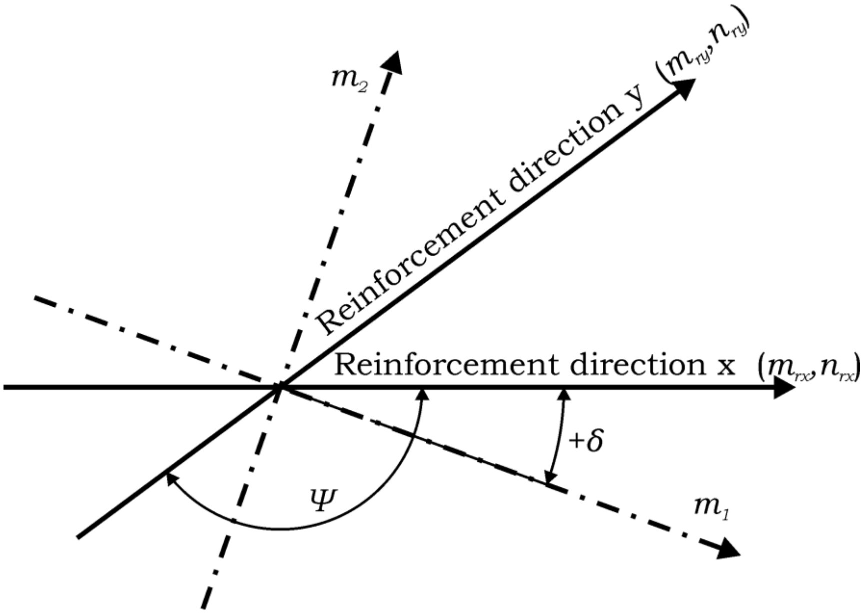


Figure 7.4-33 – Direction definition for skew reinforcement

Equation (7.4-28) and Equation (7.4-29) are modified in the same manner giving:

$$n_{rx,pos(neg)} = \frac{1}{\sin^2\psi} \cdot \begin{bmatrix} n_1 \cdot \sin^2(\psi - \delta) + n_2 \cdot \cos^2(\psi - \delta) \pm \\ \pm \left| n_1 \cdot \sin\delta \cdot \sin(\psi - \delta) - n_2 \cdot \cos\delta \cdot \cos(\psi - \delta) \right| \end{bmatrix}; \tag{7.4-34}$$

$$n_{ry,pos(neg)} = \frac{1}{\sin^2\psi} \begin{bmatrix} n_1 \cdot \sin^2\delta + n_2 \cdot \cos^2\delta \pm \\ \pm \left| n_1 \cdot \sin\delta \cdot \sin(\psi - \delta) - n_2 \cdot \cos\delta \cdot \cos(\psi - \delta) \right| \end{bmatrix}. \tag{7.4-35}$$

In Equation (7.4-32) and Equation (7.4-33) n_1 and n_2 denote the principal membrane forces at the considered location and the angles δ and ψ have the same significance as defined in Figure 7.4-33 (i.e. δ is the angle between x and the direction of n_1 and ψ is the angle between x and y).

In addition to the moments, the FE analysis of the slab will also provide shear forces in two directions. Any necessary shear reinforcement area should be computed for the resultant shear force defined as:

$$\upsilon_0 = \sqrt{\upsilon_x^2 + \upsilon_y^2}. \tag{7.4-36}$$

7.4.7.7 Redistribution of reinforcement moments

Owing to the capacity of plastic redistributions in concrete structures, the reinforcement moments (as well as the shear forces) can be redistributed over a certain width, here denoted w. The average value of the moment m_{av} can then be used to compute the necessary reinforcement which is normally placed within the distribution width.

The procedure can be illustrated for the simple example depicted in Figure 7.4-34. Considering a slab supported by four columns monolithically connected to the slab. The diagram at the bottom left shows (see Figure 4.2 b) the variation of the reinforcement moment m_{rx} along line L_1 in a direction parallel to the moment's direction (in this case the x direction).

The diagram in Figure 7.4-34 shows the distribution of m_{rx} along L_2 (length w) in direction orthogonal to the moment direction (in this case the y direction). The distribution of the moment m_{rx} along line L_2 is replaced by a constant distribution with the average value $m_{rx,av}$ computed according to the equation in Figure 7.4-34 c. In this equation the integral is nothing else than the total moment over a strip of width w. The averaging procedure aims then to design the reinforcement in the slab strip of width w for the total moment within the strip and distributing the reinforcement uniformly over the width of the strip. Note that the averaging procedure described above always takes place in a direction normal to the direction of the moment.

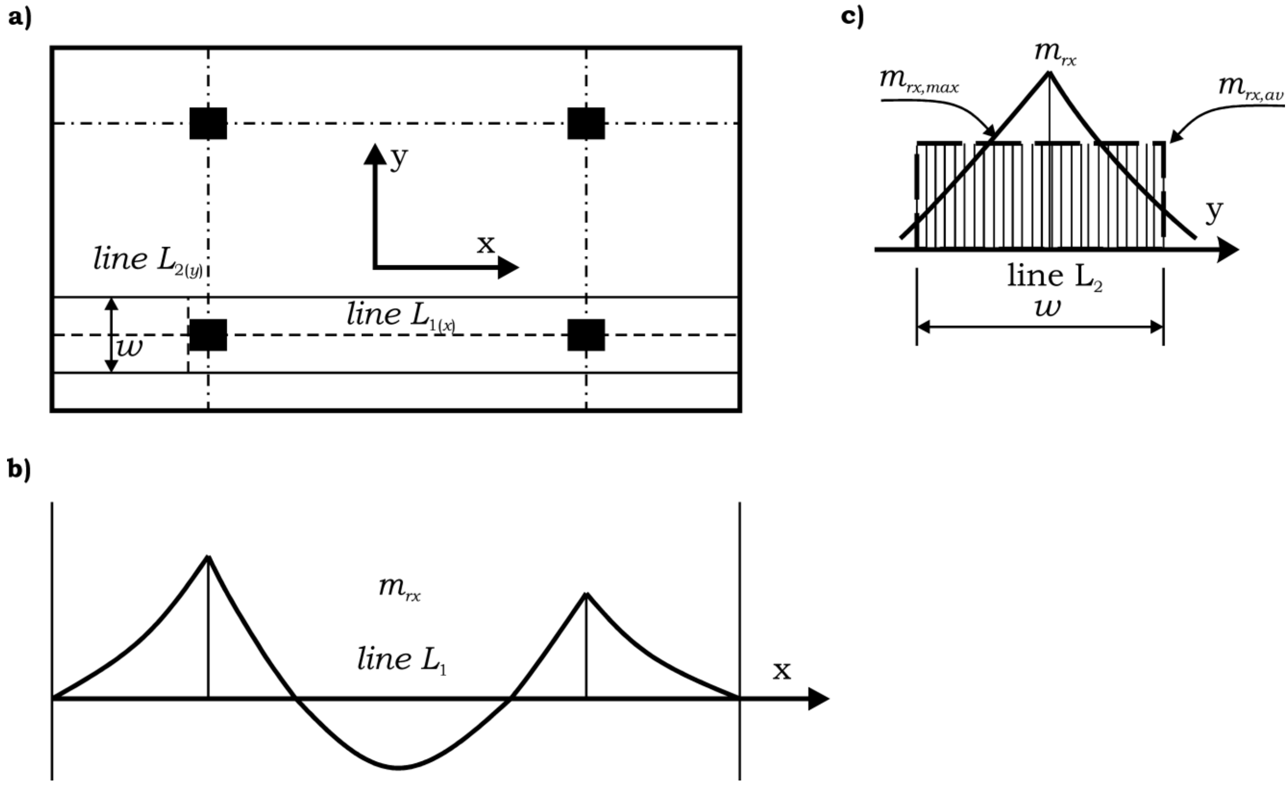


Figure 7.4-34 – Redistribution of the reinforcement moment m_{xt} over a width w

As an alternative, the averaging over the strip width can instead be made after calculating the corresponding required reinforcement (as continuous fields over the slab), thus giving:

$$A_{sx,av} = \frac{1}{w} \cdot \int_0^w A_{sx} dy. \tag{7.4-37}$$

Which approach that is preferred is a question of what is most convenient, for example depending on which approach that is implemented in the software used for the structural analysis.

As a general remark it should be noted that the distribution width (strip width) used for reinforcement design is, at least for ultimate limit states, *limited to the width over which yielding of the reinforcement can distribute without exceeding the rotational capacity in the point with the largest rotation. Consequently, what limits the distribution width is the rotational capacity of the slab.*

The recommendation given in [17] apply reinforcement moments and associated membrane effects and are based on the provisions given in EN 1992 [N3] but, since no specific guidelines are given for redistribution of moment and forces from linear FE analysis, and more detailed advices are based on what has been found in special literature, software manuals and on practical considerations from engineering practice [17].

The recommendation found in literature are generally based on the assumption that reinforced concrete slabs have good capabilities for plastic redistributions in ultimate limit state, but that the reinforcement need to be concentrated to regions with concentrated supports with response in service state.

For flat slabs, the reinforcement is typically arranged in support strips over the columns with a middle strip in between, in the two main directions.

The recommendation given in [17] are belived to be conservative. This implicates that there is a potential to improve them and to find more liberal provision based on improved knowledge on the response of concrete slabs.

7.4.7.7.1 Ultimate limit states

The distribution widths at a support (column or bearing) can be chosen according to the recommendation [17] as below:

$$w = \min(3 \cdot h; \frac{L_c}{10}), \tag{7.4-38}$$

for $\frac{x_u}{d} = 0,45$ (0,35 for concrete strength classes $\geq$C50/60);

$$w = \min(5 \cdot h; \frac{L_c}{5}), \tag{7.4-39}$$

for $\frac{x_u}{d} = 0,30$ (0,23 for concrete strength classes $\geq$ C50/60);

$$w = \frac{L_c}{4}, \tag{7.4-40}$$

for $\frac{x_u}{d} = 0,25$ (0,15 for concrete strength classes $\geq$ C50/60);

$$w = \frac{L_c}{2}, \tag{7.4-41}$$

for $\frac{x_u}{d} = 0,15$ (0,10 for concrete strength classes $\geq$ C50/60);

$$w = \min(5 \cdot h; \frac{L_c}{5}), \tag{7.4-42}$$

for $\frac{x_u}{d} = 0,0$.

In the above equations, h is the height of the section, x_u is the depth of the neutral axis at the ultimate limit state after redistribution and d is the effective depth of the section. L_c is the characteristic span width, determined differently for different categories of slabs in the following sections. For values of $\frac{x_u}{d}$ in between the limits above w can be determined by linear interpolation.

Regardless of the ductility requirements or any other of the limitations defined in the reminder of this report the value of the distribution width should never be taken less than $(2 \cdot h + a)$, i.e. $w \geq w_{\min} = (2 \cdot h + a)$, where a is the dimension of the support in the considered direction.

The following limitations apply for the distribution width determined using Equations (7.4-36)–(7.4-40):

1. The ratio of the averaged and maximum reinforcement moments (see also Figure 7.4-34) should be restricted to $\delta = \frac{m_{rx,av}}{m_{rx,\max}} \geq 0,6$.

2. If the column has a capital (or a drop panel) the distributions width should be chosen as shown in Figure 7.4-35. In addition, before redistribution, the reinforcement moments and associated membrane forces must be transformed so that they are defined with respect to the same reference line.

3. If the capital (drop panel) extends continuously over a line of columns or bearings it can be dimensioned as a beam. The beam forces (normal forces, bending moments and shear forces) can be obtained by integration from the shell (slab) results.

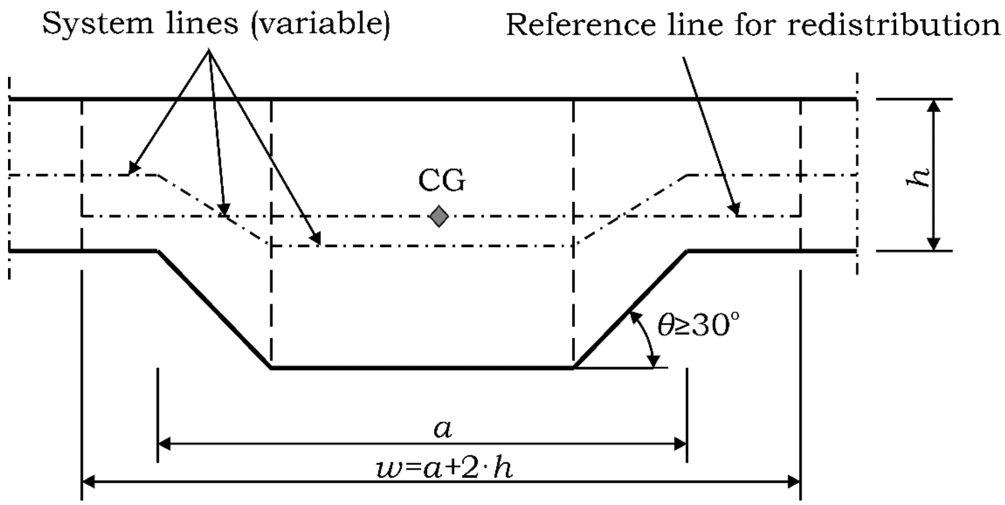


Figure 7.4-35 – Distribution widths for capitals (or drop panels) (Figure 4.4 from [17])

4. If the distribution width exceeds the distance between points of zero moment w_0 in the direction normal to the direction of the considered moment (i.e. the direction of redistribution) then the average value should be computed according to the principle illustrated in Figure 7.4-36. This principle is illustrated for the reinforcement moment in the x direction, i.e. m_{rx} but the same applies for m_{ry}.

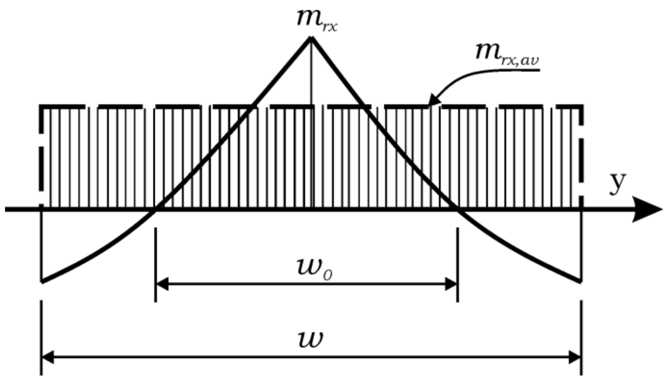


Figure 7.4-36 – Definition of the average value for cases where the distribution width exceeds the distance between points of zero moment (Figure 4.5 from [17])

5. For supports placed near the edge of the slab, the distribution width should be evaluated according to the principle illustrated in Figure 7.4-37. This amounts to choosing a w according to Equations (7.4-36)–(7.4-40) and then evaluating an effective width w_{eff} as indicated in the figure. This effective value should further be used in evaluating the averaged moment values.

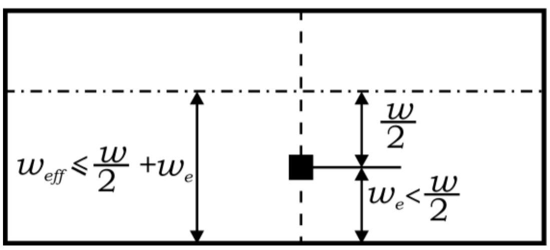


Figure 7.4-37 – Support near the edge of the slab [17]

7.4.7.7.2 Simplified approach in accordance with EN 1992 [N3]

EN 1992 [N3] deal with the peak in bending moment for flat slabs by averaging it over the column strip and middle strip (Annex I), with the column strip slab-divided into inner and outer areas. This method can be used for designing reinforcement using the results of an FE analysis. A section is taken across the bending moment diagram (i.e. in the y – direction for moments in the x – direction).

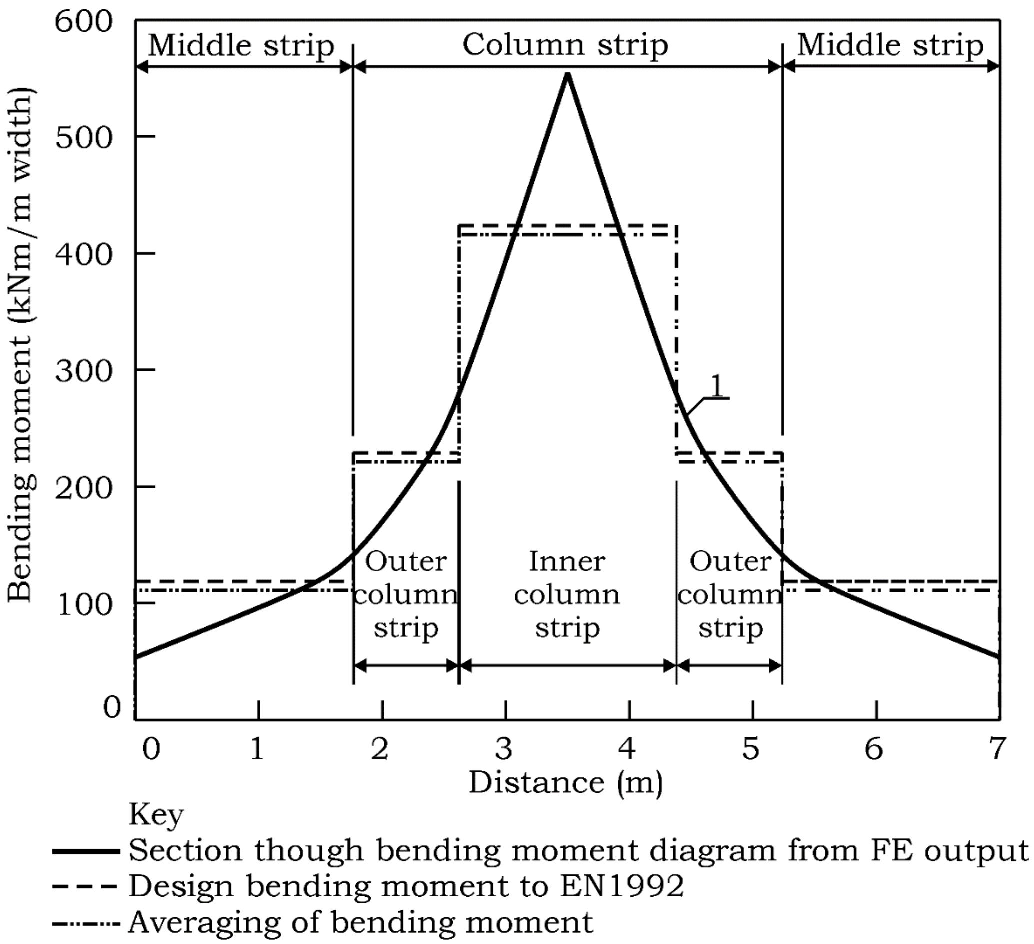


Figure 7.4-38 – Design bending moments compared with FE output (Figure 13 from [20])

At the face of the column (see Figure 4.7-38).The total bending moment is the area under the blue line (i.e. the integral), which can be apportioned according to rules given EN 1992 [N3].

The rules in EN 1992 [N3] (Annex I Table I.1) allows more flexibility in apportioning the total moment for the bay width to the column and middle strips. However EN 1992 [N3] is more rigid in terms of how much reinforcement should be applied to the inner column strip. In accordance with EN1992 [N3] (clause 9.4.1(2)) requires that the half the total reinforcement area for the bay width is placed in a strip that extends to a quarter of the bay width and is centered over the support.

In accordance with EN 1992 [N3], allow the design moment to be taken at the face of the support, indeed EN 1992 [N3] indicates this should be done. However, it may be prudent for the design moment at edge columns to be taken at the center of the support. This is because of uncertainties in the modelling and because it is critical that the moment is transferred from the slab to the column in these locations, if this has been assumed in the design.

An alternative method is to simply average the bending moment over a width of slab. However, the requirements of EN 1992 [N3] (clause 9.4.1(2)) should be adopted. The widths of these strips can be determined by the designer (an example is shown in Figure 7.4-38).

This method has the advantage that it can be used for a slab width is not required. It can also be used with area of steel results, removing the need to calculate the reinforcement areas by hand. It will be seen that both methods give a similar distribution of reinforcement when applied to the same strip widths.

An alternative way to determining design bay width is to use the method set out in [20]. This method has been developed for post-teusioned concrete design, assuming the analysis is at the serviceability limit state and for a homogeneous elastic plate. However, the principle that the bay width is taken as being the distance between the lines of «zero shear» may still be applied (see Figure 7.4-39).

This principle is particularly useful for an unusual geometries where using the lines of zero shear give a good basis on which to determine the bay width.

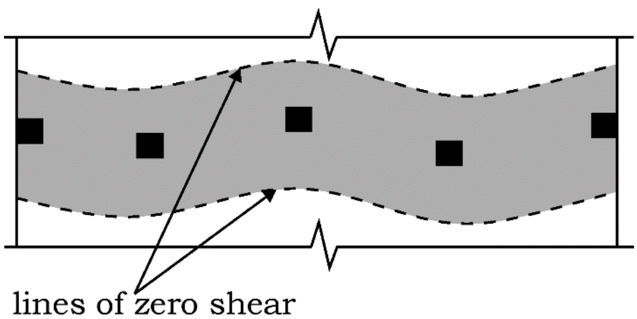


Figure 7.4-39 – Extract of shear diagram indicating lines of zero shear (Figure 14 from [20])

Whichever method is chose, engineering judgment should be applied for unusual design situation, making sure that there is sufficient reinforcement to resist the applied moment, without being overly- conservative.

A useful rule of thumb for verifying the results is that top reinforcement in the column strip will be in the order of twice the area of the bottom reinforcement (i.e. not the same as, or 4 times much as, the bottom reinforcement).

7.4.7.7.3 Serviceability limit states

(1) Distribution width for serviceability limit states design

This method has been developed for post-tensioned concrete design. However, the principles that the bay with is taken as being the distance between the lines of «zero shear» may still be applied. This principle is particularly use full for unusual geometries where using the lines of zero shear give a good-basis on which to determine the bay widths.

A use full rule of thumb for very tying the results is that top reinforcement in the column strip will be in the order of twice the area of the bottom reinforcement (i.e. not the same as, or 4 times as much as, the bottom reinforcement).

The choice of an appropriate distribution width for serviceability limit states is by far more intricate than for ultimate limit states and there are very few (if any) recommendations in the literature. This is mainly due to the fact that for serviceability limit states it is very difficult to determine the degree to which moment redistribution will take place. When a slab starts to crack, moments will redistribute from cracked areas to un-cracked areas (from support to field sections or vice versa). When the whole slab is cracked, the stiffer parts of the slab will attract larger moments. This means that the parts with larger maximum moments will contain a higher amount of reinforcement and hence become relatively stiffer after cracking. Consequently these parts will attract larger parts of the total moment after cracking than before.

In EN1992 [N3], it is pointed out that the reinforced distribution should reflect the behavior of the slab under working conditions, with a concentration of moments over the column. Unless rigorous checks are made for serviceability, half of the total top reinforcement should be concentrated into a column strip with the width:

$$w = \frac{l_1}{8} + \frac{l_2}{8}, \tag{7.4-42}$$

where: l_1 and l_2 are the distances from the column of the strip to the adjacent columns, in the direction perpendicular to the reinforcement. This leads generally to a larger concentration of reinforcement to the column strip than what is given by a linear analysis.

Given the above reasons the distribution width for serviceability limit states should be chosen more conservative than for ultimate limit states. Thus, for serviceability limit states the distribution width should be chosen between the limits given by:

$$\min(3 \cdot h; \frac{L_c}{10}) \le w \le \min(5 \cdot h; \frac{L_c}{5}). \tag{7.4-43}$$

(2) Approach to deflection calculation

Deflection is influenced by many factors, including the tensile and compressive strength of the concrete, the elastic modulus, shrinkage, creep, ambient conditions, restraint, loading, time, duration of loading, cracking.

Of the influences listed above, the three most critical factors are the values of tensile strength, elastic modulus, and creep, their effects have been discussed previously.

There are several situations where deflection are critical:

- deflection of the slab perimeter supporting cladding brackets/fixing on the slab perimeter prior to installation of the cladding;
- deflection of the slab after erection of the partions;
- where it affects the appearance.

The accuracy of the deflection calculation can be refined where the age of loading can be confidently predicted and the type of aggregates to be used is known.

This is more likely to be the case where the designer is working for a contractor. The time of striking and the time when additional formwork loads from the slab above are applied will have a major influence on the deflection.

This is because the slab is most likely to crack under these conditions and this will greatly influence the subsequent stiffness of the slab. The elastic modulus can be more accurately predicted when the type of aggregate in concrete is known, and this is more likely to be the case when the source of concrete has been determined.

When the loading sequence is known, the critical loading stage at which cracking first occurs can be established by calculation k for each stage, where:

$$k = \frac{f_{ctm}}{(q \cdot \sqrt{\beta})}, \tag{7.4-44}$$

where: f_{ctm} – tensile strength of the concrete;
q – loads applied at the stage;
β – 0,5 for long-term loads.

The critical load stage is where k is at its minimum and is usually when the slab above is cast (i.e. construction stage overload), and the tensile strength should be calculated for this stage.

The following methods can be used to carry out serviceability limit state design. They are listed in order of increasing.

Sophistication:
- span-to-effective depth ratios – compliance with code;
- linear finite element analysis with adjustment of elastic modulus;
- non-linear finite element analysis.

(3) Linear FE deflection analysis

In accordance with [17]. The linear FE-method should be used only to confirm that deflection is not critical and not a tool to estimate deflection. This method involves calculating the elastic modulus and slab stiffness by hand and adjusting the parameters used in the analysis. A cracked section analysis is carried out to determine the stiffness of the slab. The cracked section properties vary with the reinforcement size and layout, so this is an iterative process and should ideally be carried out for each element of slab. However, for initial sizing it is not unreasonable to assume that the cracked section stiffness is half the gross section stiffness [20], or to use a cracked section stiffness for a critical area of the slab and apply it globally, provided that it is not used to estimate deflection.

Changing the slab stiffness in an FE-model can not usually be carried out directly because most finite element packages calculate section properties from the thickness of the elements. The overall depth of the concrete should be used, as this gives the correct torsional constant.

However, to allow for reduction in slab stiffness, the elastic modulus can be adjusted by multiplying by the ratio of the cracked and uncracked slab stiffness, R, to model the correct slab thickness. So an appropriate long-term elastic modulus is determined as follows:

$$E_{eff} = \frac{R \cdot E_{cm}}{(1+\varphi)}. \qquad (7.4\text{-}45)$$

In general, the long-term elastic modulus is usually between a third (for storage loads) and a half (for residential loads) of the short-term value [20].

Therefore, allowing for the need to adjust for cracked stiffness, the long-term elastic modulus should be in the range one sixth to a quarter of the short-term elastic modulus.

(4) Non-linear FE deflection analysis

When using non-linear software, several analysis will often be required to obtain a final result. The software will carry out an iterative analysis to determine an initial deflection; this will be based on initial, assumed areas of reinforcement.

As discussed previously an important aspect to achieving a realistic estimate of deflection is to consider the loading history for the slab; once the slab has cracked (and hence has reduced in stiffness) this will affect the deflection throughout the life of the slab. This should be considered in the model.

The crack may not be cracked every where; rather it may be fully cracked in the zones of maximum moment, and in other planes it may be only partially cracked or not cracked at all. An accurate assessment of deflection can only be made where the appropriate section properties are calculated for each element in the slab. Software giving the most accurate deflection calculations will consider the shrinkage effects.

7.4.7.8 Punching shear

Although an FE model will produce shear stresses, where the columns are modelled as pins they have no effective shear perimeter and the shear force is infinite. In this case the simplest way to check punching shear is to take the reaction from the model and carry out the checks in the normal way using provisions in the Chapter 4. This can be automated by using a spreadsheet for the design of reinforced concrte.

If the area of the column has been modelled, then realistic shear stresses can be required in using them because there will be peaks which may exceed the design limits in the codes.

Some software can undertake the punching shear checks and design of the reinforcement, and the user should ensure that openings within the shear perimeter are considered in the softwear.

7.4.7.9 Validation

As with any analysis it is necessary to validate the results in order to avoid errors in the modelling and input of data. There is a risk of engineers assuming that because the computer can accurately and rapidly carry out complex calculations it must be right.

There are number of simple check of analysis that can be carried out and the results of these checks should always be included when the calculations are presented [17, 20]:

1) Are the supports correctly modeled?

2) Is the element size appropriate – particularly at locations with high stress concentrations?

3) Is there static equilibrium? Calculate by hand the total applied loads and compare these with the reactions from the model results;

4) Carry out simplified calculations, by making approximations if necessary (This could be investigated);

5) Do the contour plots look right? Are the peak deflections and moments where they would be expected? Sketch out by hand the expected results before carring out the analysis;

6) Is the span-to-effective depth ratio in line with normal practice.

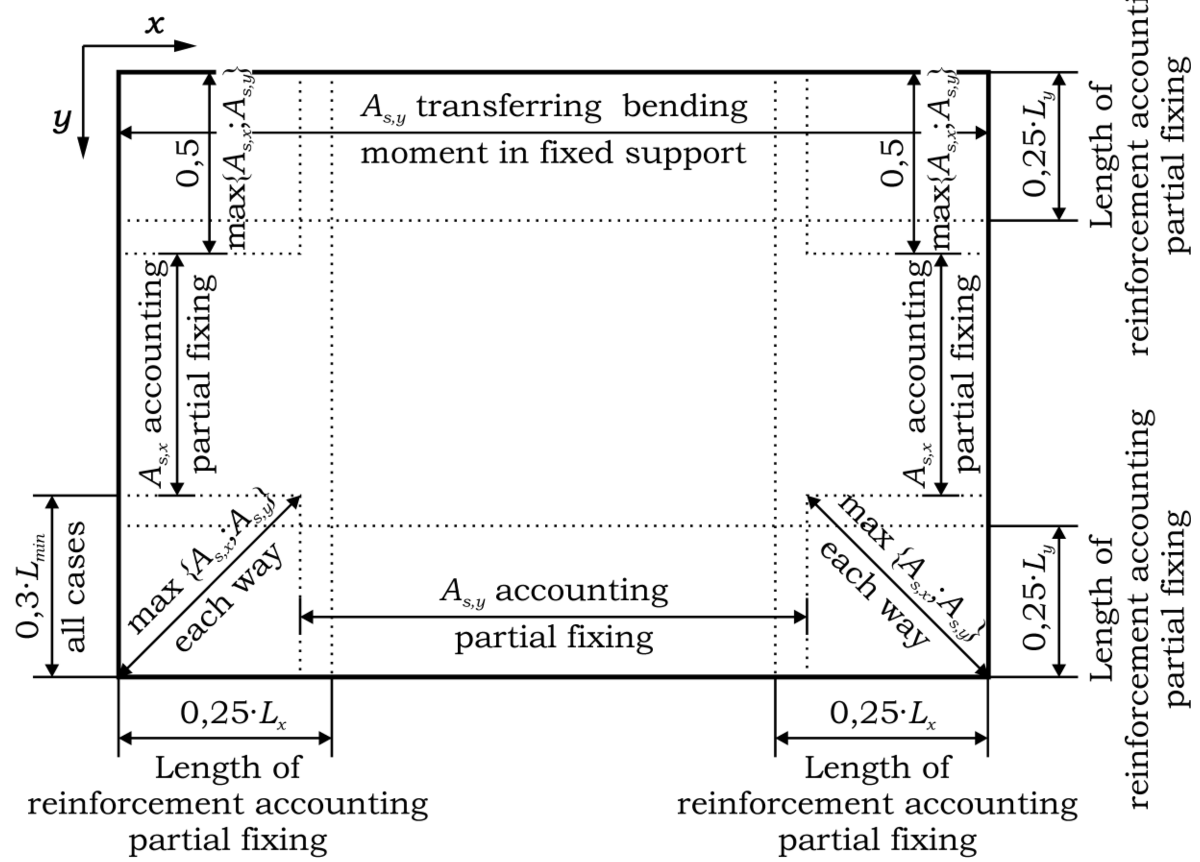
7.5 DETAILING

7.5.1 TWO-WAY SLABS ON LINEAR SUPPORTS

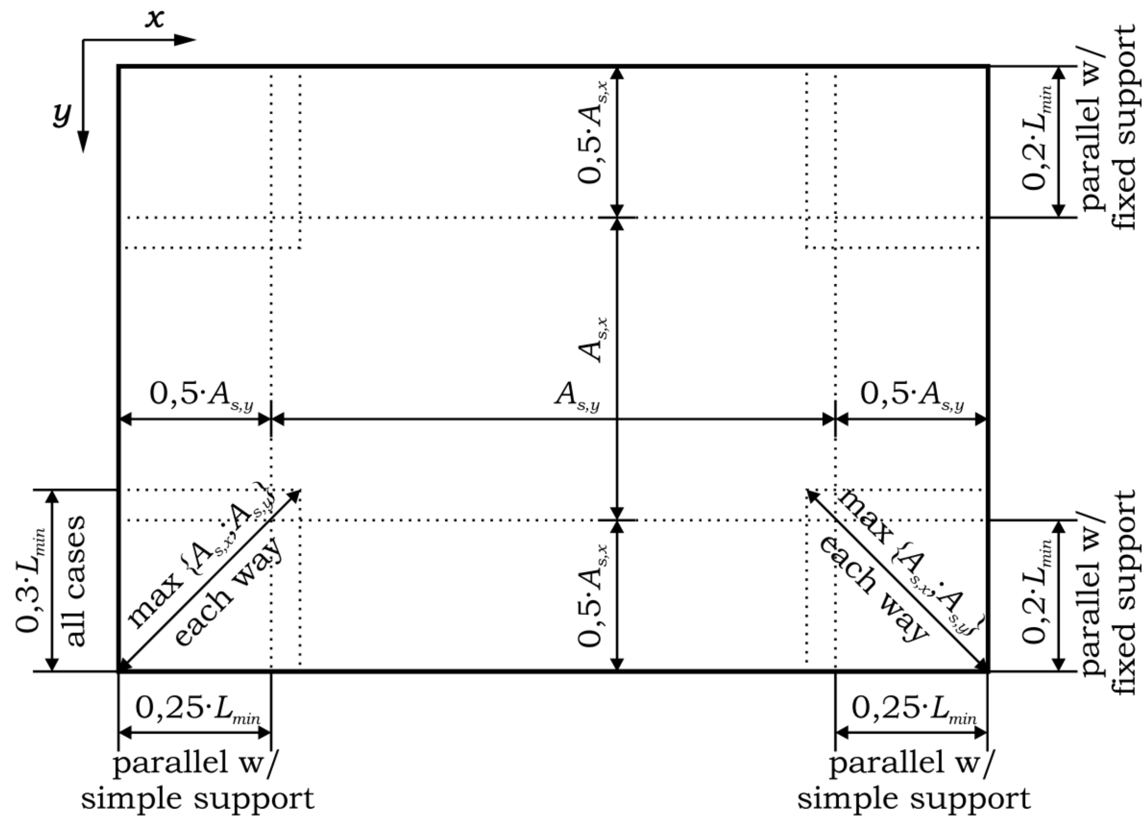
Longitudinal reinforcement must resist to dimensioning bending moments. In the performed analysis were neglected torsion moments in the corners of concrete slab. This simplification must be accounted in arrangement of longitudinal reinforcement according to detailing rules (see Figure 7.5-1):

- longitudinal reinforcement can be reduced in border zones to zones to design area in the middle of span;
- in the corners where simple supports meet to each other must be reinforcement transferring torsion moments. The minimum area of this reinforcement (in each way) is a maximum from values $A_{s,x}$ and $A_{s,y}$ in mid-span;
- top reinforcement is not reduced anywhere. Slab has to be reinforced by top surface in fixed or partial fixed supports (length of reinforcement is depicte above);
- in the corner where *simple supports meet* to each other must be reinforcement transferring torsion moments. The minimum area of this reinforcement (in each way) is a *maximum* from values $A_{s,x}$ and $A_{s,y}$ in mid-span;
- in corner where *simple support meets fixed support* is necessary one half of $\max(A_{s,x}; A_{s,y})$ placed parallel with fixed support;
- in corners where *fixed support meets fixed support* is not needed reinforcement for torsion moments (this case is not in scheme above);
- all reinforcement must be accompanied by distribution reinforcement;
- no surfaces can be without any reinforcement due to volume changes of concrete (shrinkage). Thereby the surfaces without reinforcement should be accompanied by welded wire mesh.

a)



b)



a) – top reinforcement; b) – bottom reinforcement

Figure 7.5-1 – Reinforcement arrangement (detailing rules for two-way spanning slab)

7.5.2 FLAT SLABS

Column and middle strip should be reinforced to withstand the design moment obtained from Chapter 7.4.

In general two-thirds of the amount of reinforcement required to resist the negative design moment in the column strip should be placed in a width to half that of the column strip symmetrically positioned about the centerline of the column.

The area of reinforcement in each directions should not be less than $0{,}00014 \cdot f_{ck}^{2/3} \cdot b \cdot h$ or $0{,}0015 \cdot b \cdot h$ (where: h is the overall depth of the slab (taken as $d/0{,}87$); b is the width for which the reinforcement is calculated).

If control of shrinkage and temperature cracking is critical, the area of reinforcement should not be less than 0,0065·b·h or 20 % of the area of main reinforcement.

The area of tension or compression reinforcement in either direction should not exceed 4 % of the area of concrete.

Main bars should not be less than 10 mm in diameter. To control flexural cracking the maximum bar spacing or maximum bar diameter of high-bond bars should not exceed the values given in Table 7.5-1, corresponding to the stress in the bar. In any case bar spacing should not exceed the lesser of 3·h or 500 mm.

Table 7.5-1 – Alternative requirements to control crack width to 0,3 mm for members reinforced with high bond bars (Table 5.6 from [N3])

Maximum bar diameter (mm)	Stress range (MPa)		Maximum bar spacing (mm)	Stress range (MPa)
40	150-165		300	≤160
32	165-190		275	160-180
25	190-210		250	180-200
20	210-230		225	200-220
16	230-260	or	200	220-240
12	260-290		175	240-260
10	290-320		150	260-280
8	320-360		125	280-300
			100	300-320
			75	320-340
			50	340-360

Notes: *The stress in the reinforcement may be estimated from the relationship:*

$$\sigma_s = \frac{f_{yk}}{\gamma_s} \cdot \left\{ \frac{\psi_2 \cdot Q_k + G_k}{1{,}5 \cdot Q_k + 1{,}35 \cdot G_k} \right\} \cdot \left(\frac{A_{s,req}}{A_{s,prov}} \right) \cdot \left(\frac{1}{\delta} \right),$$

where: ψ_2 *should be obtained from EN1990 [N1] for the particular type of loading considered;*

$\frac{f_{yk}}{\gamma_s}$ *may be taken as 435 MPa for 500 MPa reinforcement;*

$A_{s,req}$ *is the area of tension reinforcement required at the section considered for the ultimate limit state;*

$A_{s,prov}$ *is the area of the elastic ultimate moment at the section considered* ($\delta \le 1$)*;*

δ *is the ration of the elastic ultimate moment at the section considered* ($\delta \le 1$)*.*

ANNEX I

TERMS AND DEFINITIONS

COMMON TERMS USED IN EN 1990 TO EN 1999

Construction works: everything that is constructed or results from construction operations.

Note: *The term covers both building and civil works. It refers to the complete construction works comprising structural, non-structural and geotechnical elements.*

Type of building or civil engineering works: type of construction works designating its intended purpose, *e.g.* dwelling house, retaining wall, industrial building, road bridge.

Type of construction: indication of the principal structural n1aterial, *e.g.* reinforced concrete construction, steel construction, timber construction, masonry construction, steel and concrete composite construction.

Construction material: material used in construction work, *e.g.* concrete, steel, timber, masonry.

Structure: organised combination of connected parts designed to carry loads and provide adequate rigidity.

Structural member: physically distinguishable part of a structure, *e.g.* a column, a beam, a slab, a foundation pile.

Form of structure: arrangement of structural members.

Note: *Forms of structure are, for example, frames, suspension bridges.*

Structural system: load-bearing members of a building or civil engineering works and the way in which these members function together.

Structural model: idealisation of the structural system used for the purposes of analysis, design and verification.

SPECIAL TERMS RELATING TO DESIGN IN GENERAL

Design criteria: quantitative formulations that describe for each limit state the conditions to be fulfilled.

Design situations: sets of physical conditions representing the real conditions occurring during a certain time interval for which the design will demonstrate that relevant limit states are not exceeded.

Transient design situation: design situation that is relevant during a period much shorter than the design working life of the structure and which has a high probability of occurrence.

Note: *A transient design situation refers to temporary conditions of the structure, of use, or exposure, e.g. during construction or repair.*

Persistent design situation: design situation that is relevant during a period of the same order as the design working life of the structure.

Note: *Generally it refers to conditions of normal use.*

Accidental design situation: design situation involving exceptional conditions of the structure or its exposure, including fire, explosion, impact or local failure.

Design working life: assumed period for which a structure or part of it is to be used for its intended purpose with anticipated maintenance but without major repair being necessary.

Load arrangement: identification of the position, magnitude and direction of a free action.

Load case: compatible load anangements, sets of deformations and imperfections considered simultaneously with fixed variable actions and permanent actions for a particular verification.

Limit states: states beyond which the structure no longer fulfils the relevant design criteria.

Ultimate limit states: states associated with collapse or with other similar forms of sttuctural failure.

Note: *They generally correspond to the maximum load-carrying resistance of a structure or structural member.*

Serviceability limit states: states that correspond to conditions beyond which specified service requirements for a structure or structural member are no longer met.

Irreversible serviceability limit states: serviceability limit states where some consequences of actions exceeding the specified service requirements will remain when the actions are removed.

Reversible serviceability limit states: serviceability limit states where no consequences of actions exceeding the specified service requirements will remain when the actions are removed.

Serviceability criterion: design criterion for a serviceability linlit state.

Resistance: capacity of a member or component, or a cross-section of a member or component of a structure, to withstand actions without mechanical failure *e.g.* bending resistance, buckling resistance, tension resistance.

Strength: mechanical property of a material indicating its ability to resist actions, usually given in units of stress.

Reliability: ability of a structure or a structural nlember to fulfil the specified requirements, includ-ing the design working life, for which it has been designed. Reliability is usually ex-pressed in probabilistic terms.

Note: *Reliability covers safety, serviceability and durability of a structure.*

Reliability differentiation: measures intended for the socio-economic optimisation of the resources to be used to build construction works, taking into account all the expected consequences of failures and the cost of the construction works.

Basic variable: part of a specified set of variables representing physical quantities which characterise actions and environmental influences, geometrical quantities, and nlaterial properties including soil properties.

Maintenance: set of activities performed during the working life of the structure in order to enable it to fulfil the requirements for reliability.

Note: *Activities to restore the structure after an accidental or seismic event are normally outside the scope of maintenance.*

Nominal value: value fixed on non-statistical bases, for instance on acquired experience or on physical conditions.

TERMS RELATING TO ACTIONS

Action (F): a) set of forces (loads) applied to the structure (direct action);

b) set of imposed deformations or accelerations caused, for example, by temperature changes, moisture variation, uneven settlement or earthquakes (indirect action).

Note 1: *An accidental action can be expected in many cases to cause severe consequences unless appropri-ate measures are taken.*

Note 2: *Impact, snow, wind and seismic actions may be variable or accidental actions, depending on the available information of statistical distributions.*

Effect of action (E): effect of actions (or action effect) on structural members, *(e.g.* internal force, moment, stress, strain) or on the whole structure *(e.g.* deflection, rotation).

Permanent action (G): action that is likely to act throughout a given reference period and for which the variation in magnitude with time is negligible, or for which the variation is always in the same direction (monotonic) until the action attains a certain linlit value.

Variable action (Q): action for which the variation in magnitude with time is neither negligible nor monotonic.

Accidental action (A): action, usually of short duration but of significant magnitude, that is unlikely to occur on a given structure during the design working life.

Fixed action: action that has a fixed distribution and position over the structure or structural member such that the magnitude and direction of the action are determined unambiguously for the whole structure or structurallnember if this magnitude and direction are determined at one point on the structure or structural member.

Free action: action that may have various spatial distributions over the structure.

Single action: action that can be assumed to be statistically independent in time and space of any other action acting on the structure.

Static action: action that does not cause significant acceleration of the structure or structurallnelnbers.

Dynamic action: action that causes significant acceleration of the structure or structural members.

Quasi-static action: dynamic action represented by an equivalent static action in a static model.

Characteristic value of an action (F_k): principal representative value of an action.

Note: *In so far as a characteristic value can be fixed on statistical bases, it is chosen so as to conespond to a prescribed probability of not being exceeded on the unfavourable side during a «reference period» taking into account the design working life of the stucture and the duration of the design situation.*

Combination value of a variable action ($\psi_0 \cdot Q_k$): value chosen – in so far as it can be fixed on statistical bases – so that the probability that the effects caused by the combination will be exceeded is approximately the same as by the characteristic value of an individual action. It may be expressed as a determined part of the characteristic value by using a factor $\psi_0 \leq 1$.

Frequent value of a variable action ($\psi_1 \cdot Q_k$): value determined – in so far as it can be fixed on statistical bases – so that either the total tilne, within the reference period, during which it is exceeded is only a small given part of the reference period, or the frequency of it being exceeded is limited to a given value. It may be expressed as a determined part of the characteristic value by using a factor $\psi_1 \leq 1$.

Note: *For the frequent value of multi-component traffic actions see load groups in EN 1991-2.*

Quasi-permanent value of a variable action ($\psi_2 \cdot Q_k$): value determined so that the total period of time for which it will be exceeded is a large fraction of the reference period. It may be expressed as a detemined part of the characteristic va1ue by using a factor $\psi_2 \leq 1$.

Accompanying value of a variable action ($\psi \cdot Q_k$): value of a variable action that accompanies the leading action in a combination.

Note: *The accompanying value of a variable action may be the combination value, the frequent value or the quasi-permanent value.*

Representative value of an action (F_{rep}): value used for the verification of a limit state. A representative value may be the characteristic value (F_k) or an accompanying value ($\psi \cdot F_k$).

Design value of an action (F_d): value obtained by multiplying the representative value by the partial factor γ_i.

Note: *The product of the representative value multiplied by the partial factor $\gamma_F = \gamma_{Sd} \times \gamma_f$ may also be designated as the design value of the action.*

Combination of actions: set of design values used for the verification of the structural reliability for a limit state under the simultaneous influence of different actions.

TERMS RELATING TO MATERIAL AND PRODUCT PROPERTY

Characteristic value (X_k or R_k): value of a material or product property having a prescribed probability of not being attained in a hypothetical unlimited test series. This value generally corresponds to a specified fractile of the assumed statistical distribution of the particular property of the material or product. A nominal value is used as the characteristic value in some circumstances.

Design value of a material or product property (X_d or R_d): value obtained by dividing the characteristic value by a partial factor γ_m or γ_M, or, in special circumstances, by direct determination.

Nominal value of a material or product property (X_{nom} or R_{nom}): value normally used as a characteristic value and established from an appropriate document such as a European Standard or Prestandard.

TERMS RELATING TO GEOMETRICAL DATA

Characteristic value of a geometrical property (a_k): value usually corresponding to the dimensions specified in the design. Where relevant, values of geometrical quantities may correspond to some prescribed fractiles of the statistical distribution.

Design value of a geometrical property (a_d): generally a nominal value. Where relevant, values of geometrical quantities may correspond to some prescribed fractile of the statistical distribution.

Note: *The design value of a geometrical property is generally equal to the characteristic value. However, it may be treated differently in cases where the limit state under consideration is very sensitive to the value of the geometrical property, for example when considering the effect of geometrical imperfections on buckling. In such cases, the design value will normally be established as a value specified directly, for example in an appropriate European Standard or Prestandard. Alternatively, it can be established from a statistical basis, with a value corresponding to a more appropriate fractile (e.g. a rarer value) than applies to the characteristic value.*

TERMS RELATING TO STRUCTURAL ANALYSIS

Note: *The definitions contained in the clause may not necessarily relate to terms used in EN 1990, but are included here to ensure a harmonisation of tenns relating to structural analysis for EN 1991 to EN 1999.*

Structural analysis: procedure or algorithm for determination of action effects in every point of a structure.

Note: *A structural analysis may have to be perfooned at three levels using different models: global analysis, member analysis, local analysis.*

Global analysis: determination, in a structure, of a consistent set of either internal forces and moments, or stresses, that are in equilibrium with a particular defined set of actions on the structure, and depend on geometrical, structural and material properties.

First order linear-elastic analysis without redistribution: elastic structural analysis based on linear stress/strain or moment/curvature laws and performed on the initial geometry.

First order linear-elastic analysis with redistribution: linear elastic analysis in which the internal moments and forces are modified for structural design, consistently with the given external actions and without more explicit calculation of the rotation capacity.

Second order linear-elastic analysis: elastic structural analysis, using linear stress/strain laws, applied to the geometry of the deformed structure.

First order non-linear analysis: structural analysis, performed on the initial geometry, that takes account of the non-linear deformation properties of materials.

Note: *First order non-linear analysis is either elastic with appropriate assumptions, or elastic-perfectly plastic, or elasto-plastic or rigid-plastic.*

Second order non-linear analysis: structural analysis, perfornled on the geometry of the deformed structure, that takes account of the non-linear deformation properties of materials.

Note: *Second order non-linear analysis is either elastic-perfectly plastic or elasto-plastic.*

First order elastic-perfectly plastic analysis: structural analysis based on moment/curvature relationships consisting of a linear elastic part followed by a plastic part without hardening, performed on the initial geometry of the structure.

Second order elastic-perfectly plastic analysis: structural analysis based on moment/curvature relationships consisting of a linear elastic part followed by a plastic part without hardening, performed on the geometry of the displaced (or deformed) structure.

Elasto-plastic analysis: structural analysis that uses stress-strajn or moment/curvature relationships consisting of a linear elastic part followed by a plastic part with or without hardening.

Note: *In general, it is perfonned on the initial structural geometry, but it may also be applied to the geometly of the displaced (or deformed) structure.*

Rigid plastic analysis: analysis, performed on the initial geometry of the structure, that uses limit analysis theorems for direct assessment of the ultimate loading.

Note: *The moment/ curvature law is assumed without elastic defonnation and without hardening.*

ADDITIONAL TERMS AND DEFINITIONS USED IN EN 1992

Precast structures: precast structures are characterized by structural elements manufactured elsewhere than in the final position in the structure. In the structure, elements are connected to ensurethe required structural integrity.

Plain or lightly reinforced concrete members: structural concrete members having no reinforcement (plain concrete) or less reinforcement than the minimum amounts.

Unbonded and external tendons: unbonded tendons for post-tensioned members having ducts which are permanently ungrouted, and tendons external to the concrete cross-section (which may be encased in concrete after stressing, or have a protective membrane).

Prestress: the process of prestressing consists in applying forces to the concrete structure by stressing tendons relative to the concrete member. «Prestress» is used globally to name all the permanent effects of the prestressing process, which comprise internal forces in the sections and deformations of the structure. Other means of prestressing are not considered in this standard.

ANNEX II

BAR AREAS AND PERIMETERS

Table A.II-1 – Sectional areas of groups of bars (mm²)

Bar size (mm)	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28,3	56,6	84,9	113	142	170	198	226	255	283
8	50,3	101	151	201	252	302	352	402	453	503
10	78,5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	943	1260	1570	1890	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600

Table A.II-2 – Perimeters and weights of bars

Bar size (mm)	6	8	10	12	16	20	25	32	40
Perimeter (mm)	18,85	25,10	31,40	37,70	50,20	62,80	78,50	100,50	125,60
Weight (kg/m)	0,222	0,395	0,616	0,888	1,579	2,466	3,854	6,313	9,864
Note: *Bar weights based on a density of 7850 kg/m³.*									

Table A.II-3 – Sectional areas per metre width for various bar spacings (mm²)

Bar size (mm)	Spacing of bars								
	50	75	100	125	150	175	200	250	300
6	566	377	283	226	189	162	142	113	94,3
8	1010	671	503	402	335	287	252	201	168
10	1570	1050	785	628	523	449	393	314	262
12	2260	1510	1130	905	754	646	566	452	377
16	4020	2680	2010	1610	1340	1150	1010	804	670
20	6280	4190	3140	2510	2090	1800	1570	1260	1050
25	9820	6550	4910	3930	3270	2810	2450	1960	1640
32	16100	10700	8040	6430	5360	4600	4020	3220	2680
40	25100	16800	12600	10100	8380	7180	6280	5030	4190

Table A.II-4 – Shear reinforcement. A_{sw}/s_w for varying stirrup diameter and spacing

Stirrup diameter (mm)	Stirrup spacing (mm)										
	85	90	100	125	150	175	200	225	250	275	300
8	1,183	1,118	1,006	0,805	0,671	0,575	0,503	0,447	0,402	0,366	0,335
10	1,847	1,744	1,570	1,256	1,047	0,897	0,785	0,698	0,628	0,571	0,523
12	2,659	2,511	2,260	1,808	1,507	1,291	1,130	1,004	0,904	0,822	0,753
16	4,729	4,467	4,020	3,216	2,680	2,297	2,010	1,787	1,608	1,462	1,340

BIBLIOGRAPHY

(a) Eurocodes and other Codes

[N1] EN 1990: Eurocode 0 – Basis of Structural Design.

[N2] EN 1991: Eurocode 1 – Actions on Structures.

[N3] EN 1992: Eurocode 2 – Design of Concrete Structures – Part 1-1: General rules and rules for buildings. European Committee for standartization, Brussels, 2004.

[N4] EN 206: 2013 Concrete – Specification, performance, production and conformity. CEN – CENELEC Management Centre: B-1000, Brussels – 93 pp.

[N5] fib Model Code for Concrete Structures 2010. – Ernst and Sohn, 2013, – p.p. 389.

[N6] ISO 2394: 2015 General principles on reliability for structures. 111 pp.

[N7] EN 12390: 2009 Testing hardened concrete.

[N8] EN 10080: 2005 Steel for the reinforcement of concrete – Weldable reinforcing steel – General, Brussels.

[N9] ISO 4356: 2015 Basis for the design of structures – Deformations of building at Serviceability Limit States. 18 pp.

[N10] ISO 17660: 2015 Welding of reinforcing steel. 37 pp.

(b) Textbooks and other publications

[1] Zilch K., Zehetmaier G., Bemessung in konstructiven Betonbau. Nach DIN 1045-1 und DIN EN 1992-1-1 – Springer – Verlag Berlin Heildberg, 2006 – 536 p.

[2] Mehta P.K., Monteiro P.J.M., Concrete. Microstructure, Properties, and Materials – Third edition – Mc Grawttill, New Tork – DOI: 10.1036/0071462899.

[3] Commentary Eurocode 2 – European Concrete Platform ASBL, June 2008, Brussels.

[4] Bangash M.Y.H., Structural detailing in concrete, 2nd edition – Thomas Telford, London, 2007 – 469 p.

[5] Biazioli F., Mancini G., Just M., etc., Eurocode 2: Background and Applications. Design of Concrete Buildings. Worked examples – Joint Research Centre, European Commission, 2014 – 186 p.

[6] Mosley B., Bungey G., Hulse R., Reinforced Concrete Design to Eurocode 2 – 9th Edition – Palgrave Macmillan, London, 2014 – 420 p.

[7] Arya Ch., Design of Structural Elements – 3rd Edition – Spon Press, 2009 – 523 p.

[8] Narayanan R.S., Beeby A., Designers Guide to EN 1992-1-1 and EN 1992-1-2 Eurocode 2: Design of Concrete Structures. General rules and rules for buildings and structural fire design. Thomas Telford, 2005 – p.p. 213.

[9] Hofer A.A., Mc Cabe S.L., Comparison of shear capacity of T-beams using strut and tie analysis / Structural Engineering and Engineering Materials SM Report № 48 – University of Kansas, 1998 – 191 p.

[10] Leonhardt F., Vorlesungen über den Massivbau, Teil 1: Grundlagen zur Bemessung in Stahlbeton bau, Springer – Verlag, 3. Auflage, 1984.

[11] Mörsh E., Seine Theoric und Anwendung (Reinforced Concrete Theory and Application), Der Eisenbetonbau, Konrad Witter Verlag, Stuttgart, 1912.

[12] Ritter W., Die Bauweise Hennebique. Schwizerische Bauzeitung, vol. 33, № 7, February 1899, Zürich – p.p. 59–61, DOI: 10.5169/seals – 21308.

[13] Structural Analysis / Civil Engineering Course Material from IIT Kharagpur – Version 2 CE, IIT Kharagpur, 2008.

[14] Kong F.K., Reinforced Concrete Deep Beams. Taylor and Francis Books. Inc., 2003 – 299 p.

[15] Wight J.K., MacGregor J.G., Reinforced Concrete Mechanics and Design – 6th Edition, Pearson, Boston, 2015 – p. 1177.

[16] Lapko A., Jensen B.C., Podstawy projektowania i algorytmy obliczeń konstrukcji żelbetowych – Arkady, 2005 – 429 p.

[17] Pacoste C., Plos M., Johansson M., Recommendation for finite element analysis for the design of reinforced concrete slabs – Stockholm, Sweden, 2012 – 54 p.

[18] Manual for the design of concrete building structures to Eurocode 2 – ISE, 2006 – 141 p.

[19] Kennedy G., Goodchild C., Practical Yield Line Design – Reinforced Concrete – Telford, 2003 – 171 p.

[20] How to design Concrete Structures using Eurocode 2 – Eurocode, The Concrete Centre, 2011 – 107 p.

(c) Website

Eurocode expert – http: // www.eurocodes.co.uk/

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