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## AN ANALYSIS OF THE COMBINATIONS OF CLIMATIC ACTIONS ON BUILDING STRUCTURES

### 1. INTRODUCTION

Coincidence of actions plays an important role in structural calculations so the combinations of actions have been introduced into codes. They are expressed using combination factors. The values of these factors, named as factors, have usually been determined based on theoretical considerations. However, as climatic actions depend on the climatic conditions in different geographical locations, it is necessary to analyse the historical meteorological data. Among them wind and snow are most frequent to be used, also ambient air temperature. The paper deals with a probabilistic analysis of combinations of pairs of actions: wind and snow as well as wind and air temperature. The objective of the presented analyses is the verification of the combination factors for variable climatic actions given in the codes, e.g. [1] and to identify the values determined on the base of local meteorological data. Calculations are based on the data measured at Polish meteorological stations of the Institute for Meteorology and Water Management - a State Research Institute. Method of analysis as well as results based on data from two stations are presented here as examples. Annual maxima of these actions (and also minima of the air temperature) have been analysed as main data assuming that the climatic year lasts from October 1 to September 30. Gumbel probability distribution [2] has been used as a border probability distribution for all actions. The interdependence of snow load and wind velocity pressure as well as between wind and air temperature actions can be received for different return periods. Combination factors for characteristic values of those actions with return period of 50 years have been proposed.

### 2. METHOD OF THE ANALYSIS

An analysis of the coincidence of the snow load on the ground and wind velocity pressure is presented as an example of the method used in the paper also for combinations of wind and air temperature actions.

If one assumes that the probability density functions of the snow loads on the ground  $f_1(S)$  and wind speed  $f_2(V)$  are known then the simultaneous probability of the wind speed  $V_1 < V \leq V_2$  and snow load  $S_1 < S \leq S_2$  is [3] [4]

$$P[V_1 < V \leq V_2; S_1 < S \leq S_2] = \int_{S_1}^{S_2} \int_{V_1}^{V_2} f_1(S) \cdot f_2(V) \cdot dS \cdot dV. \quad (1)$$

The aim of the analysis is to find such values of wind speed and snow load which would be simultaneously exceeded with the accepted probability in appropriate reference period. Usually it is assumed, that so called characteristic value of one variable of climatic action may be exceeded once in 50 years on average. In any one year as a reference period the probability of exceeding of this value is 0.02. Other assumptions about return periods are also possible.

The probability of the simultaneously exceeding of values  $S_T$  and  $V_T$  in the return period  $T$  is

$$P(S > S_T, V > V_T) = \frac{1}{T}, \quad (2)$$

which may be written in terms of probability distribution functions  $F_1(S)$  and  $F_2(V)$

$$P(S > S_T, V > V_T) = P_1(S) \cdot P_2(V) = (1 - F_1(S_T)) \cdot (1 - F_2(V_T)). \quad (3)$$

The task is to find the values  $S_T$  and  $V_T$  simultaneously exceeded in the assumed return period that is to find the boundary probability distributions of the wind speed and the snow load on the ground. For this purpose the equation (3) should be solved taking into account equation (2).

$$(1 - F_1(S_T)) \cdot (1 - F_2(V_T)) = \left(\frac{1}{T}\right). \quad (4)$$

Putting to the equation (4) the Gumbel probability distribution of the snow load

$$F_1(S) = \exp(-\exp(-\alpha_s \cdot (S - U_s))) \quad (5)$$

and the Gumbel probability distribution of the wind speed

$$F_2(V) = \exp(-\exp(-\alpha_v \cdot (V - U_v))) \quad (6)$$

one have the following equation

$$1 - [\exp(-\exp(-\alpha_v \cdot (V - U_v)))] = \left(\frac{1}{T}\right) : [1 - \exp(-\exp(-\alpha_s \cdot (S - U_s)))] \quad (7)$$

The parameters of the Gumbel probability distribution of the annual maximal wind speeds are denoted as  $\alpha_v$  and  $U_v$  while  $\alpha_s$  and  $U_s$  are the parameters of the Gumbel distribution of annual maximal values of the snow loads on the ground.

After modification one obtains the formula for wind speed as a function of the return period  $T$  and probability parameters for wind speed and snow load

$$V = U_v - \frac{1}{\alpha_v} \ln \left\{ -\ln \left[ 1 - \left(\frac{1}{T}\right) : (1 - \exp(-\exp(-\alpha_s \cdot (S - U_s)))) \right] \right\}. \quad (8)$$

In order to compare wind action with snow load this equation should be transformed from wind speed into the velocity pressure using simple calculations. Taking  $\rho = 1,25 \text{ kg/m}^3$  and putting the right side of equation (8) to the power of 2 and recalculating from pascals into  $\text{kN/m}^2$  one obtains the dependence of the velocity pressure on the ground snow loads in the same units.

### 3. COMBINATIONS OF SNOW LOAD AND WIND VELOCITY PRESSURE

Five combinations of snow and wind actions have been considered. The maximum annual values are of key importance: the snow load on the ground and the 10-minute mean wind speed, at the anemometer height, regardless of the wind direction and the type of the terrain.

**Combination 1:** the maximum annual snow load and wind speed values, measured in the same climatic year but not at the same time. This is an extreme case, which constitutes the upper limit, as an envelope of all possible combinations.

**Combination 2a:** the maximum annual snow load and the 10-minute mean, maximum daily wind speed value measured on the same day as the snow load.

**Combination 2b:** the maximum annual snow load and the 10-minute mean, maximum daily wind speed value measured in a 15-day period, with the day on which the maximum annual snow load was measured in the middle of the period. It was assumed, quite arbitrarily but based on earlier analyses [5], that the snow cover whose weight is not much lower than the maximum annual measured value may remain for two weeks, one week before and one week after the maximum value was measured.

**Combination 3:** the 10-minute mean, annual maximum wind speed and the snow load measured on the same day. In this combination the snow load was often missing, as there was no snow cover; in the analysis the snow load was assumed to be  $S = 0 \text{ kN/m}^2$ .

**Combination 4:** the 10-minute mean, annual maximum wind speed and the snow load measured on the same day provided that there was a snow cover.

Combination 4 needs special approach. There were many cases of zero snow loads when the maximum wind speeds were recorded. Conditional probability should be used in this case. Taking into account this circumstance equation (5) may be replaced by the equation

$$F_1(s) = p_0 + (1 - p_0) \cdot F(s) \quad (9)$$

where  $(1 - p_0)$  is the frequency of snowy days when maximal yearly wind speeds are recorded,  $F(s)$  is the distribution of the maximal annual values of the snow load on the ground  $S > 0$ .

This approach may be more appropriate but there is another disadvantage: small number of data. Among 44 years of observations less than a half concern snow data when maximal annual wind speeds were recorded.

The absence of the correlation between analysed values was verified [4] and the parameters of the Gumbel probability distribution have been estimated using the maximum likelihood method. An example of the probability plot of the snow load on the ground is presented in Figure 1. Having the parameters of the Gumbel probability distribution it is possible to plot the wind velocity pressure against the snow load on the ground. An example is presented in Figure 2.

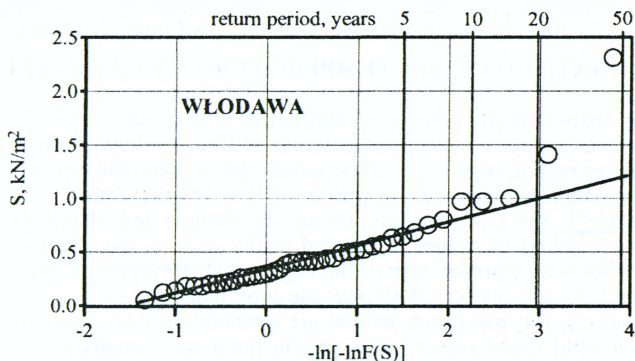


Figure 1 – Empirical distribution of the maximum annual (winter) values of the snow load on the ground on the Gumbel distribution probability plot.

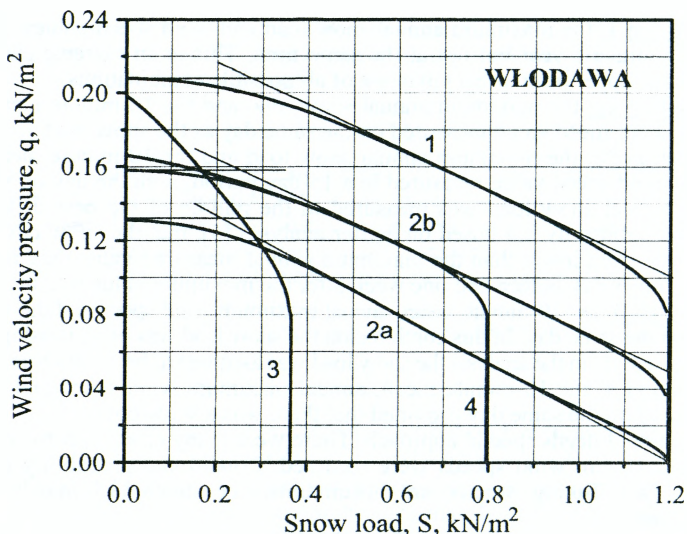


Figure 2 – Interdependence of the ground snow loads and 10. minute mean wind velocity pressure at the meteorological station Włodawa. Return period 50 years. The curves are shown according to approaches numbered above

It can be seen in Figure 2 that three curves may be simplified using straight lines according to Mathieu [6]. The curve 1, according to the first approach, may be considered as an envelope. The conclusive is rather the curve of combination number 2b. The wind velocity pressure may be taken as  $0.049 \text{ kN/m}^2$  when snow load is in the full characteristic value. If the velocity pressure would be in the full characteristic value then snow load may be  $0.25 \text{ kN/m}^2$  that is 21% of the full characteristic value equal to  $1.20 \text{ kN/m}^2$ . However, improved method of analysis on the base of conditional probability gives some value of snow load but never higher than given by the curve 1. The curve 3 shows that the combination of maximal wind speeds and corresponding snow loads may be neglected. On the base of results of the analysis of data from twelve meteorological stations it may be stated that the combination factor for snow and wind actions in Polish National Annex to the Eurocode [1] may be 0.3.

#### 4. COMBINATIONS OF AIR TEMPERATURE AND WIND VELOCITY PRESSURE

Six combinations of air temperature and wind actions have been considered. The maximum annual values are of key importance of the 10-minute mean wind speed, at the anemometer height, regardless of the wind direction and the type of the terrain as well as the annual minimum and maximum shadow air temperature.

**Combination 1:** the minimum annual air temperature and maximum annual 10-minute mean wind speed values, measured in the same climatic year but not at the same time. This is an extreme case of the negative air temperature, which constitutes the upper limit, as an envelope of all possible combinations.

**Combination 2:** the maximum annual air temperature and maximum annual 10-minute mean wind speed values, measured in the same climatic year but not at the same time. Similarly, this is an extreme case which constitutes the upper limit, here of the positive air temperature.



**Combination 3:** the minimum annual air temperature and maximum daily 10-minute mean wind speed value, measured on the same day.

**Combination 4:** the maximum annual air temperature and maximum daily 10-minute mean wind speed value, measured on the same day.

**Combination 5a:** the 10-minute mean annual maximum wind speed and the daily minimum air temperature measured on the same day.

**Combination 5b:** the 10-minute mean annual maximum wind speed and the daily maximum air temperature measured on the same day.

Data from three Polish meteorological stations were analysed up to day, namely from Suwałki, Warsaw and Zakopane. The absence of the correlation between analysed values was also verified and the parameters of the Gumbel probability distribution have been estimated, this time using the least squares method. Examples of the probability plots of the ambient air temperature and wind speed at Warszawa Okęcie meteorological station are presented in Figures 3, 4 and 5. Having the parameters of the Gumbel probability distribution it is possible to plot the wind velocity pressure against the ambient air temperature using the same equation (8) as for the combinations of snow load and wind velocity. Only the distribution parameters for snow loads should be replaced by the parameters of the distribution of extreme values of the ambient air temperature. An example is presented in Figure 6.

The curves 1 and 2 may be considered as an envelope never exceeded. The conclusive is rather the curve of combination number 5b. It shows that the full characteristic value of the velocity pressure occurs together with the air temperature slightly below zero of centigrade, that is about  $-4\text{ }^{\circ}\text{C}$  in Warsaw (at other two locations it is similar).

When the extreme values of air temperature occur then wind velocity is rather small. In combination 3 and 4 the highest value of the wind velocity pressure is  $0.08\text{ kN/m}^2$  that is 25.3% of the characteristic value  $0.316\text{ kN/m}^2$  (Fig. 6). At Warsaw meteorological station the value  $0.08\text{ kN/m}^2$  of the wind velocity pressure corresponds to the negative air temperature about  $-12\text{ }^{\circ}\text{C}$  and positive  $30\text{ }^{\circ}\text{C}$ . In both conditions, during winter and summer times, wind velocity pressure decreases to zero at the characteristic values of negative and positive air temperature (Fig. 6).

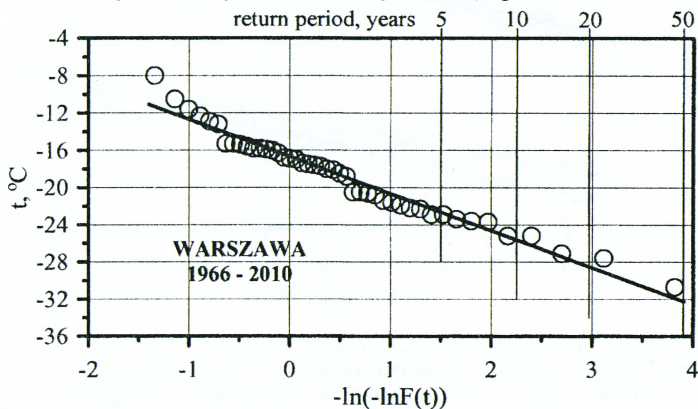


Figure 3 – Empirical distribution of the minimum annual values of the air temperature on the Gumbel distribution probability plot

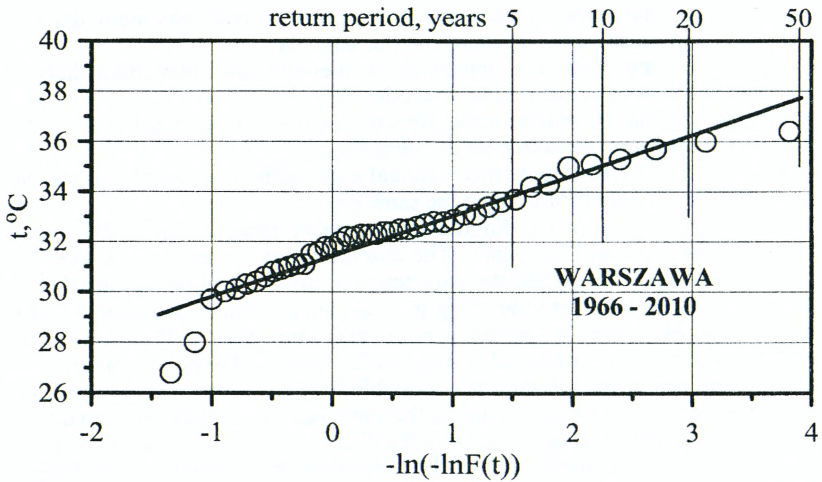


Figure 4 – Empirical distribution of the maximum annual values of the air temperature on the Gumbel distribution probability plot

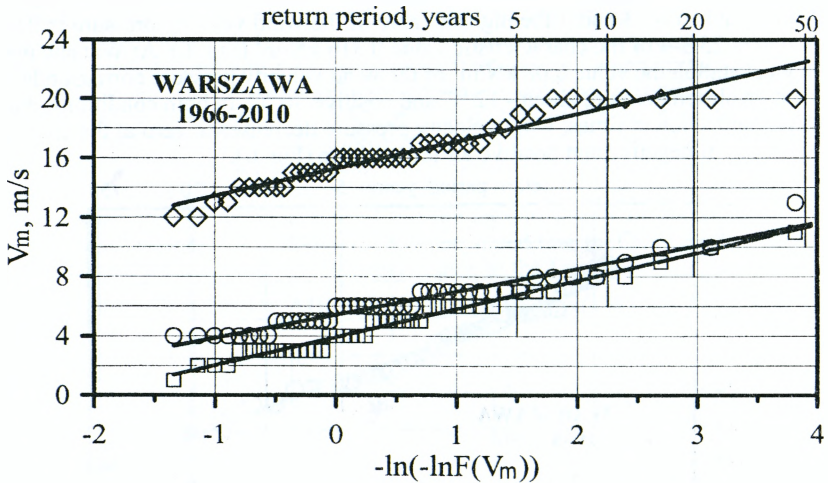


Figure 5 – Empirical distributions of the 10-minute mean wind velocity on the Gumbel distribution probability plot. Upper plot – annual maxima, lower plots – daily maxima on the days with annual minimum (squares) and annual maximum (circles) air temperature

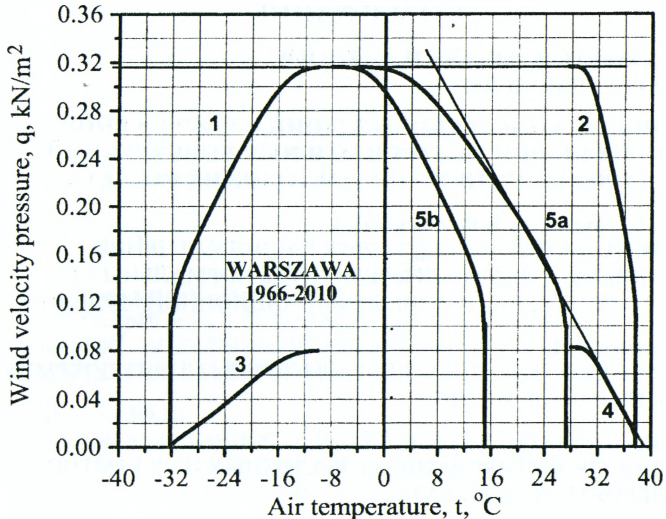


Figure 6 – Interdependence of the 10. minute mean wind velocity pressure and air temperature at the meteorological station Warszawa Okęcie. Return period 50 years. The curves are shown according to approaches numbered above

## 5. CONCLUSIONS

Presented analysis shows that it is possible to diminish the values of combination factor for snow, wind and ambient air temperature actions. As the results of the analysis done on the base of data from twelve Polish meteorological stations it was already proposed [4] to introduce the value  $\psi_0 = 0.3$  into the Polish National Annex to the Eurocode [1]. It is lower than the values given in the Eurocode, where they are: for snow load  $\psi_0 = 0.5$  and for wind actions  $\psi_0 = 0.6$ . It is also lower than in the Russian code [7]. Further analysis of the combinations of wind and air temperature actions is planned. It should also be mentioned that in both cases of combinations wind direction has not been taken into consideration, yet.

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