Some Aspects of Chaotic Time Series Analysis

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***Abstract:** We address two aspects in chaotic time series analysis, namely the definition of embedding parameters and the largest Lyapunov exponent. It is necessary for performing state space reconstruction and identification of chaotic behavior. For the first aspect, we examine the mutual information for determination of time delay and false nearest neighbors method for choosing appropriate embedding dimension. For the second aspect we suggest neural network approach, which is characterized by simplicity and accuracy.*

Keywords: - Chaos theory, embedding parameters, Lyapunov exponent, neural network.

1. INTRODUCTION

Chaotic behavior is characterized by highly sensitive to initial conditions and observed for many systems (stock market, EEG patterns of brainwave activity, central nervous system, etc.). The processing of chaotic time series may by divided into three stage, which are shown in Fig.1. The first stage is time series analysis. As a result of this stage we can identificate the chaotic behavior and estimate embedding parameters. Using the data from previous stage we can perform phase space reconstruction or build neural networks for optimal forecasting. The common test for chaos is calculation of the largest Lyapunov exponent, which should be positive [1]. Such a Lyapunov exponent is statistical measure of divergence between two orbits starting from slightly different initial conditions. Let d_o be initial divergence between two trajectories and d_n be divergence between such orbits after n steps. Then largest Lyapunov exponent is defined by

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \ln \frac{d_n}{d_0} \qquad (1)$$

Dealing with one dimensional time series we must first of all perform state space reconstruction. It is based on an embedding theorem [2], which guarantees that a full knowledge of the behavior a system is contained in time series of any a one measurement. As a result the full multivariate phase space can be constructed from the single time series.

To apply the embedding theorem it is necessary to define a suitable embedding dimension and time delay. The estimation such parameters provide a maximum predictability of chaotic time series and can be used for choosing of optimal windows size (number of input units) in forecasting neural network. The rest of the paper is organized as follows. In the section 2 and 3 we describe the approaches for choosing of embedding delay and embedding dimension. Section 4 and 5 are devoted to definition of the largest Lyapunov exponent.

2. CHOOSING OF EMBEDDING DELAY

For the choosing the optimal time delay τ can be used the following approaches: autocorrelation function, mutual information, etc. The optimal time delay is typically chosen in accordance with first zero of autocorrelation function or first minimum of mutual information.

The mutual information can be defined as follows [3]:

$$I(\tau) = -\sum_{i,j} P_{ij}(\tau) \cdot \ln \frac{P_{ij}(\tau)}{P_i \cdot P_j} \qquad (2)$$

where P_i is the probability to find a time series value

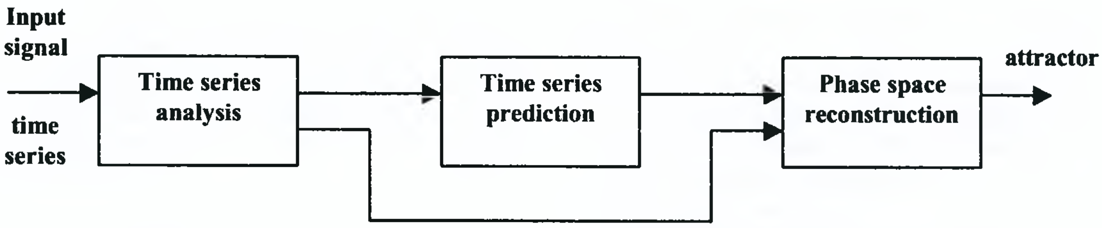

Figure 1. Functional diagram of the data processing

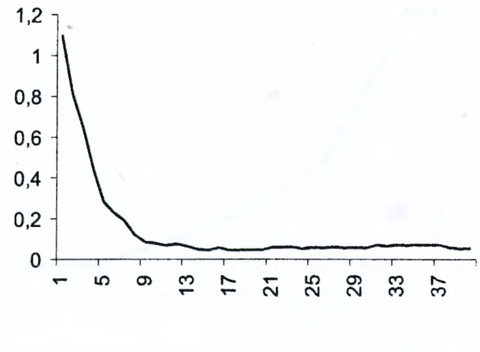

Figure 2. Graph of mutual information $I(\tau)$ versus τ for Henon attractor

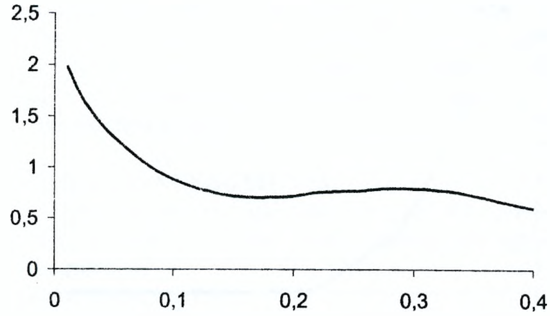

Figure 3. Graph of mutual information $I(\tau)$ versus τ for Lorenz attractor

in the i-th interval, and $P_{ij}(\tau)$ is the joint probability that an observation is located in the i-th interval and the observation time τ later is located in the j-th interval.

Estimating τ by such a way, we can get the coordinates as independent as it is possible.

In the Fig. 2 and 3 are represented the graphics of mutual information for Henon and Lorenz attractor respectively. As can be seen from the Fig.3 the first minimum of the mutual information is 0.16 for the Lorenz data. For the Henon data we can't define a minimum. In this case we can take $\tau = 1$ directly from the Henon equation.

3. EMBEDDING DIMENSION

As stated earlier embedding dimension is applied for state space reconstruction and definition of window size for predicting neural networks. Suppose we have a given one-variable time series represented by the N values as follows:

$$x(t) = F(x(t-1), x(t-2), \dots, x(t-k)) \quad (3)$$

where $t = k+1, N$.

Takens proved that dynamic reconstruction is possible, if

$$m \geq 2[d]+1 \quad (4)$$

where d is fractal dimension of original attractor, *[d]* denotes the integer part of d and $k = m-1 = 2 \cdot d$ characterizes the window size.In this case the reconstructed attractor embedded in the m-dimensional state space preserves important topological properties of the original attractor. There exist a lot of methods for estimating the embedding dimension m such as the false nearest neighbors, fractal dimension, principal component analysis and so on. Let's examine the false nearest neighbors approach [4]. It is based on idea, that geometric properties of the original and reconstructed attractor must be preserved.

The algorithm of the false nearest method is the following:

1. *Let m=1. Then we seek for each point* $\bar{x}(i)$ *in time series its nearest neighbor* $\bar{x}(j)$ *in m-dimensional space.*
2. *Calculate the distance* $|\bar{x}(i) - \bar{x}(j)|$. *After this we iterate both point and define*
3. $R_i = \dfrac{|\bar{x}(i+1) - \bar{x}(j+1)|}{|\bar{x}(i) - \bar{x}(j)|}$
4. *If* $R_i > R_t$, *where* R_t *is suitable threshold, then such a point is a false nearest neighbor. As a result we can get the number of the false nearest point* P.
5. *Calculate* P/N *and repeat algorithm for m=m+1.*
6. *The algorithm is continued until* P/N *is close to zero.*

Fig. 4 and 5 show graphics for determination of embedding dimension for Henon and Lorenz data respectively. From this figures we can define the embedding dimension equal 3 for Henon and equal 5 for Lorenz data.

4. ANALYTICAL APPROACH FOR DETERMENITION OF THE LARGEST LYAPUNOV EXPONENT

The standard approach for computing λ is calculated as follows.

1. *Starting from two points in the basin of attraction, separated by distance* d_0. *Usually* d_0 *is less than* 10^{-8}.
2. *Advance both orbits on one iteration ahead and calculate the new divergence between trajectories using Euclidian metric. As a result we evaluate* $\ln(d_1)$.

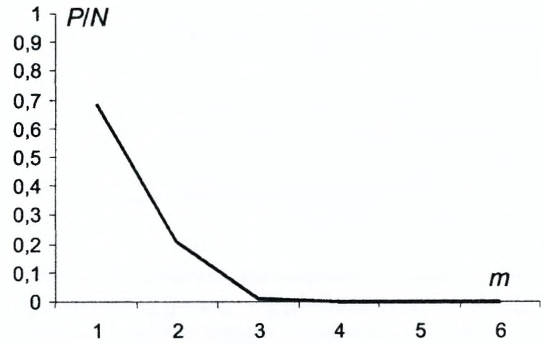

Figure 4. Determination of embedding dimention for Henon attractor by using the false nearest neighbors method

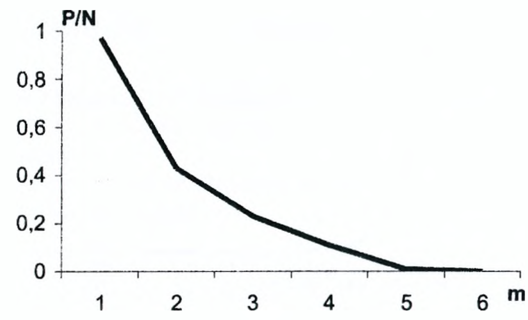

Figure 5. Determination of embedding dimention for Lorenz attractor by using the false nearest neighbors method

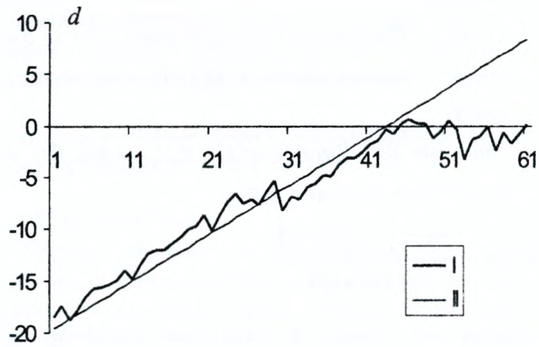

Figure 6. I - the evolution of distance between two nearby orbits for Henon attractor; II – regression line

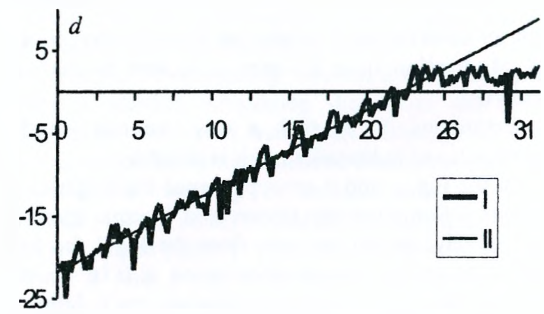

Figure 6. I - the evolution of distance between two nearby orbits for Lorenz attractor; II – regression line

3. *We repeat the last step for n points and calculate* $\ln(d_2)$, $\ln(d_3)$ ... $\ln(d_n)$.
4. *Plot the graph ln(d) versus n.*
5. *Using method of least square we construct straight line of regression, taking into account only points, for which* $\ln(d) < 0$. *The slope of the regression line estimates the largest Lyapunov exponent.*

Estimation λ by considered algorithm is a difficult task in general since initial divergence d_o must be less then 10^{-8}. This condition may be performed using a large length of an experimental data. However, it is very problematic to reach for real data. That's why the traditional approach has been limited in their applicability to many real world chaotic data. One way to avoid this problem is to use neural networks for computing largest Lyapunov exponent.

5. NEURAL NETWORK APPROACH

The key idea of proposed method [5] is to compute by help of neural network divergence between two orbits on n step ahead, using iterative approach. Such a procedure can be represented as follows:

1. *Train neural network using sliding window technique.*
2. *Select any point* $x(\tau)$ *from training set and form the following data point:* $\{x(\tau), x(\tau-1)...x(\tau-k)\}$ *where k is window size.*
3. *Compute* $\{x(\tau+1), x(\tau+2)...x(\tau+n)\}$, *using multistep prediction:* $x(\tau+i) = F(x(\tau+i-1), x(\tau+i-2)...x(\tau+i-k))$, *where* $i = \overline{1,n}$
4. *Compute* $x'(\tau) = x(\tau) + d_0$, *where* $d_0 \approx 10^{-8}$ *and repeat step 3 in order to get* $x'(\tau+i)$, $i = \overline{1,n}$.
5. *Define* $\ln d_i = \ln|x'(\tau+i) - x(\tau+i)|$, $i = \overline{1,n}$ *and mark point for which* $\ln d_i < 0$.
6. *Plot the graph ln(d) versus n.*
7. *Build line of regression for marked point and compute its slope, which equals to the largest Lyapunov exponent.*

Let's examine numerical experiments for estimating λ using feed forward neural network. In the experiments, a neural network with 7 input, 5 hidden units and 1 output nodes is trained to predict Henon and Lorenz time series. The hidden units are based on neurons of the sigmoid function and output unit on neuron of the linear function of activation respectively. The back propagation algorithm with adaptive step is used for training of neural network. The training set consists of 1500 patterns for Henon and 930 patterns for Lorenz time series respectively. The mean square error for Henon time series is $5.92 \cdot 10^{-5}$ after 1000 iteration. Fig. 6 shows the graph $\ln(d)$ versus t and regression line, which characterizes the largest Lyapunov exponent. The estimated value $\hat{\lambda} = 0.43$ is close to the desired value 0.419. The mean square error for Lorenz time series is $9.2 \cdot 10^{-4}$ after 700 iteration. The regression line and function $\ln(d)$ from t is shown on Fig. 7. The largest Lyapunov exponent is 0.98 (desired value is 0.906). We have seen, that neural network produced fairly accurate forecast.

As can be seen an obvious advantage of proposed approach in comparison with traditional is simplicity and accuracy.

6. CONCLUSION

We have shown in this paper both standard and novel approaches to time series analysis. It is necessary in order to perform state space reconstruction and identification of chaotic behavior. The new approach for estimation of the largest Lyapunov exponent is proposed. It is based on using of neural network and permits to decrease the computation complexity.

7. ACKNOWLEDGMENTS

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8. REFERENCES

[1] H. Kantz. A robust method to estimate the maximal Lyapunov exponent of a time series. *Phys. Lett. A* 185, 77 (1994).

[2] F. Takens. *Detecting Strange attractors in fluid turbulence*. Springer-Verlag, Berlin, 1981.

[3] A. Fraser, H. Swinneg. Independent coordinates for strange attractors from mutual information. *Phys. Rev. A* 33, 1134 (1986).

[4] M. Kennel, R. Brown, H. Abarbanel. Determining embedding dimension for phase-space reconstruction using a geometrical construction. *Phys. Rev. A* 45, 3403 (1992).

[5] Vladimir Golovko, Yury Savitsky, Nikolay Maniakov. Modeling Nonlinear Dynamic using Multilayer Neural Networks. *Proceedings of the Workshop Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications. (IDAACS'2001)*, Foros, Ukraine, July 1-4 2001, PP. 197-202