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ROBUSTNESS ASSESSMENT OF PRECAST REINFORCED CONCRETE STRUCTURAL SYSTEMS IN AN ACCIDENTAL DESIGN SITUATION

V. V. Tur[1], A. V. Tur[2], A. A. Lizahub[3]

[1] Doctor of Technical Sciences, Professor, Head the Department of Concrete Technology and Building Materials, Brest State Technical University, Brest, Belarus, e-mail: profturvic@gmail.com
[2] PhD in Engineering, Associate Professor, Department of Building Structures, Brest State Technical University, Brest, Belarus, e-mail: aturphd@gmail.com
[3] Master of Engineering Science, Postgraduate of Department of Building Structures, Brest State Technical University, Brest, Belarus, e-mail: p_332_14lizogub@mail.ru

Abstract

The article presents the simplified implementation of the method of alternative load paths based on the energy balance approach. This method should be used to checking the global resistance of a damaged structural system after the occurrence of an accidental event. It is necessary to provide (reserve) enough ties with the required continuity and ductility in the original prefabricated structural system to ensure the integrity of the damaged system. We consider the system of ties as the “second line of defense” of the structural system after the exhaustion of the flexural resistance of its elements. The continuity and ductility of the tie elements provide a resistance of the damaged structural system under an accidental combination of actions by mobilizing alternative load paths after the support or key element has been removed. Ductility is the ability of the tie to obtain significant plastic bond elongation before rupture. The ductility is important properties of the ties for redistributing internal forces and getting large deflections. It is necessary for the realization of the chain (membrane) effect, as well as a measure that provides energy absorption (damping) during the dynamic application of an accidental action after the vertical support losing.

The article presents analytical solutions and the working example for the design of horizontal ties in precast hollow-core slabs floor, which are obtained based on the energy approach.

Keywords: robustness, dynamic resistance, ties, energy balance method, membrane effect.

ОЦЕНКА ЖИВУЧЕСТИ СБОРНЫХ ЖЕЛЕЗОБЕТОННЫХ КОНСТРУКТИВНЫХ СИСТЕМ В ОСОБОЙ РАСЧЕТНОЙ СИТУАЦИИ

В. В. Тур, А. В. Тур, А. А. Лизогуб

Реферат

В статье представлена упрощенная реализация метода альтернативных траекторий нагрузок на основе энергетического подхода. Этот метод следует использовать для проверки глобального сопротивления поврежденной конструктивной системы после реализации особого события. Для обеспечения целостности поврежденной системы в исходной сборной конструктивной системе необходимо предусматривать (резервировать) достаточное количество связей, обладающих требуемой степенью неразрывности и пластической деформативности. Система связей рассматривается как «вторая линия защиты» конструктивной системы после исчерпания ее элементами сопротивления изгибу. Неразрывность и пластичность связевых элементов обеспечивают сопротивление поврежденной конструктивной системы при особой комбинации воздействий за счет мобилизации альтернативных путей нагрузки после удаления опоры или ключевого элемента. Пластичность подразумевает собой способность связи достигать значительного пластического удлинения перед разрывом и является важным свойством связей для перераспределения внутренних сил и развития больших прогибов. Что в свою очередь необходимо для реализации цепного (мембранного) эффекта, который обеспечивает поглощение (демпфирование) энергии при динамическом приложении особого воздействия после удаления вертикальной опоры.

В статье представлены аналитические решения и пример для расчета горизонтальных связей в перекрытии из сборных пустотных плит, полученные на основе энергетического подхода.

Ключевые слова: живучесть, динамическое сопротивление, связи, метод энергетического баланса, мембранный эффект.

1 Introduction

Resonant building disasters over the last century [1], [2] have shown that checking the robustness of damaged systems in accidental design situations should be considered as one of the most important stages of the design and detailing of structural elements of the building. Structural system should fulfil the requirements regarding robustness at the stage of conceptual design, considering the use of various strategies for protection against progressive collapse.

We should note that in the scientific and technical literature, definitions of the term “*robustness*” are widely presented [1]–[6]. For example, ***fib*** Bulletin 43 [7] guidelines define *structural robustness as the insensitivity of a structural system to local failure*. In this context, insensitivity is understood as the state of a modified structural system, when damage to individual elements (so-called key elements respect to the system as a whole) causes only insignificant changes in its structural behaviour (its response). The ability of the system to redistribute additional action effects that appear after damaging the structure under the accidental actions achieves this. In this case, we expect to observe a ductile (not brittle) behaviour for structural components without global collapse mode for the structural system.

In the current standards [8]–[13], the requirement for robustness checking is implicit where an accidental situation causes by events such as fires, explosions, impacts of vehicles in parts of the building, the consequences of human errors made at various stages of the structure's lifetime.

It should be noted that almost all known definitions of the term “robustness” [2] are based mainly on the phenomenon of disproportionate collapse, and only a few, for example, [5] consider robustness as an aspect of the safety of a structural system. According to [13] *“robustness is a specific aspect of structural safety that refers to the ability of system subject to accidental of exceptional loadings (such as fire, explosions, impact or consequences of human error) to sustain local damage to some structural components without experiencing a disproportionate degree of overall distress or collapse”*.

The draft new *fib* MC2020 develops provisions related to the assessment of the robustness of structural systems, which are based on the risk assessment format as presented in ISO 2394:2015 [14]. According to *fib* MC2020 structural robustness, checks should include the following basic steps: (1) identification of the intended hazard (*H*) or the list of hazards to which the structural system is likely to be exposed during a lifetime. At the same time, it should be taken into account that we may not identify some hazards at the designing stage (for example, terrorist and/or criminal attacks); (2) determination of the local resistance of an individual key element (*D*); (3) determination of subsequent indirect damage to the system (*S*) following direct local failure, also described as progressive collapse; (4) quantifying the values of direct C_{dir} and indirect C_{ind} consequences, including economic, social, environmental losses, as well as the cost of loss of human lives (human victims) in monetary terms according to ISO 2394:2015. Direct costs (damage) are usually localized because of damage to individual structural components, while indirect losses are associated with the loss of system functionality because of the implementation of direct losses. The total risk R_{tot} associated with a system failure in an accidental design situation is calculated according to ISO 2394:2015.

The main strategies for protecting structural systems from progressive collapse and requirements for assessing the robustness of reinforced concrete structural systems are detailed in [1], [10], [12], [15]–[18]. In this article, we will consider only the alternative load path strategy (ALP) in more detail.

The combination of horizontal (internal and perimetric) and vertical ties placed in floor elements, columns and walls ensures the integrity of the structural system. In an accidental design situation, the system of ties is considered as the "*second line of defence*" of the structural system after the exhaustion of the flexural resistance of its elements.

When the internal support is removed in the floor elements, the arched effect, bending (beam) and membrane (chain) effects can be realized in succession (depending on the vertical displacement development for the different boundary conditions). If the slab deflection exceeds the critical value and the ties collapse or lose anchorage in adjacent spans, this will indicate that the limit state has been exceeded.

Compared to monolithic reinforced concrete structural systems, precast RC- buildings are more sensitive to the effects of accidental actions. This is due to the presence of different types of butt joints that ensure the integrity of the structural system and the continuity of alternative load paths. At the same time, prefabricated systems distinguish between joints working in tension, compression, bending, torsion, and shear. When designing precast buildings, all requirements are taken into account, both strategies for protection against progressive collapse, and checks of the robustness of the structural system.

In traditional prefabricated reinforced concrete systems, friction forces on the contact of elements, restraining deformations on supports (arch effect) and welded joints of embedded parts slightly increase the resistance of the system under the action of vertical (gravity) loads. However, this is not enough to ensure sufficient resistance of the structural system in accidental design situations. In this way, in the original precast RC-structural system, it is necessary to reserve enough ties that have the required continuity and ductility to ensure the integrity of the damaged system. The continuity of the tie elements provides resistance to an accidental combination of actions by mobilizing alternative load paths after the support have been removed. Ductility is the ability to obtain significant plastic bond elongation before rupture. Such property is important for redistributing forces and obtaining large deflections necessary for the realization of the chain (membrane) effect, as well as a measure that provides energy absorption (damping) during the dynamic application of an accidental action after the vertical support losing.

In structural systems made of prefabricated reinforced concrete elements, all key elements, which failure can lead to the disproportionate collapse of the complete system, should be identified at the stage of conceptual design. Therefore, at the first stage, it is recommended to analyse the local resistance of key elements, as it is performed, for example, in case of the panel buildings designing.

In a two-stage design, it performed a structural system robustness check using non-linear static (NLS) or dynamic (NLD) models that consider the spatial work (3D) of the structural system. Adequate modelling of ties is important when using computer software and it should be based on fairly simple and reasonable relationships. ("*Make everything as simple as possible, but not simpler*" – Albert Einstein).

The article presents simplified analytical solutions for the design of horizontal ties in precast hollow-core slabs floor, which are obtained on the basis of the provisions of the energy balance approach [16]–[19]. Using the example of a real prefabricated floor, we compared the calculation results of the required parameters of horizontal ties designed according to the proposed method and calculation models included in the structural codes of various countries [8]–[13], [20]. It has been established that the ductility of ties is one of the basic parameters that should be controlled when calculating ties.

2 Analytical models for horizontal ties resistance

2.1 Membrane (chain) effects in a damaged structural system

As shown above, redundancy of alternative load paths is considered as the main strategy for protecting the structural system from progressive collapse. Alternative load paths in a damaged structural system are realized through "chain" (or "membrane") effects for floor slabs, cantilever and beam effects for precast walls, vertical suspension of walls and columns, diaphragm effect in the floor plane. When implemented chain (membrane) of the mechanism in the damaged structural system, all gravitational loading perceived due to reactions in the tensioned horizontal ties.

As follows from [2], until now there is no consensus on the magnitude of the vertical deflection, after exceeding which chain effects are accounted in the structural system resistance. It is generally accepted that this is a state when compressive axial forces become tensile, or a state in which the tie elements begin to actively perceive tensile forces.

In RC frames, the beam-end-moment effect is initially implemented. Flexural plastic hinges are formed in the near support sections. After the exhaustion of their bending resistance at large deflections, chain (membrane) resistance mechanisms come into operation.

In accordance with the requirements of the standards [8], [10]–[13], [20] calculation of chain (membrane) forces in a deformed structural system are performed, as a rule, separately, without taking into account its bending behaviour during the formation of plastic hinges.

Chain (membrane) effects should be considered as the "second line of defence" of the structural system against progressive collapse, if the damaged structural system is capable of mobilizing alternative loading paths.

2.2 Basic assumptions of simplified analytical models

For damaged structural systems, the resistance will depend on the dynamic effects during the transition to a deformed shape under an accidental action combination, as well as the nonlinear behaviour of the connections. In the design, we should consider these effects in the calculation model. Bulletin 43 [7] proposed a simplified approach for such an analysis. The basic provisions of a simplified model for calculating modified systems with alternative load paths based on the application of the energy approach were developed in [16]–[19]. We apply the considered model for simplified analysis of the damaged load-bearing structural systems for which the global resistance depends on the resistance of the horizontal ties loaded by tension. However, the basic principles adopted in described model are valid also to the analysis of another type of collapse mechanism where the plastic displacements are localized in connections. Considering the collapse mechanisms of the structural system, authors carried the development of analytical models of the resistance of horizontal tensile ties based on the following assumptions:

1) we assume the key element to be removed from the structural system suddenly after the accidental action applied;
2) we assume that gravity forces only load the damaged system with the removed element. The accidental combination includes the characteristic value of dead load and quasi-permanent value of the imposed load. Basic rules for accidental load combinations when checking damaged structural systems are discussed in detail in [21], [22];
3) prefabricated elements under displacement of the system are assumed to be perfectly rigid bodies connected by deformable ties;
4) the global resistance of the damaged structure depends only on the resistance of some critical ties. During development of the deflection of the damaged structural system, maximum forces arise in the ties;
5) at the stage when the support has suddenly been removed these connections providing alternative load paths are assumed to be unstrained.

The gravity forces on the system are modelled by the resultant $Q = mg$ applied at the centre of gravity of the prefabricated elements. The actual position of the damaged system is determined by the generalized displacement a_q at the centre of gravity and a rotation θ (here, in the general case, a_{qz} is the vertical component of displacement). It is possible to establish simple geometric relationships between the vertical deflection of the system a_{qz} and the linear elongation w_i of the ductile joints by assumptions (2) and (4). The load-displacement relationship "*N-w$_i$*" should describe the nonlinear behaviour for each tie connection i (see section 2.2).

2.3. Modelling the dynamic resistance of a structural system based on energy balance: design equations

According to assumption 5, immediately after support is removed, the vertical displacements of the structural system are practically not limited because the ductile joints are unloaded. The system when moving down is under acceleration. The resultant tie forces in the damaged system can be taken as the system resistance R, which balances the gravity force acting in its centre of gravity. We can define resistance as static (quasi-static) or dynamic. According to the energy approach [16]–[19], the static resistance varies depending on the value of displacement and can be expressed by the resistance function R_{stat} (a_{qz}) associated with the "*N-w*" relationship for ductile ties in the joints of precast elements [7].

In the general case, the energy balance equation for the vertical displacement a_q and rotation θ of the moving system can be written in the traditional form as:

$$\frac{m}{2}\cdot\left(\frac{da_q}{dt}\right)^2+\frac{I_m}{2}\cdot\left(\frac{d\theta}{dt}\right)=m\cdot g\cdot a_{qz}-\sum_{i=1}^{n}N_i\left(w_i\right)dw_i \qquad (1)$$

The first term on the right-hand side of equation (1) describes the potential energy, and the last term is the absorbed strain energy of the tie. The two terms on the left-hand side of this equation describes the kinetic energy because of displacement and rotation, respectively. To get a deformed state of equilibrium, the motion of the system must stop. At the downward position, the kinetic energy of the structural system has the value W_k = 0. In this case, we assume that the maximum vertical deflection $a_{qz,max}$ of the centre of gravity of the damaged part of the structural system and of the tie linear displacements w_i to be reached. The equilibrium equation for the deformed state of a system with single-degree-of-freedom (SDOF) in the first half-period of oscillations can be written in the traditional way (Eq. 2):

$$Q\cdot a_{qz,max}=\sum_{i=1}^{n}\int_{0}^{w_{i,max}}N_i\left(w_i\right)dw_i \qquad (2)$$

where $a_{qz,max}$ is the maximum vertical deflection in the point where the driving force Q applied, when the downward motion stop;

$w_{i,max}$ is horizontal displacement of the i-th connections.

According to [7], the strain energy capacity of the tie can be obtained from the relationship *"N-w"* as follows:

$$\xi\left(w\right)=\frac{W_{int}\left(w\right)}{N_u\cdot w}=\frac{\int_{0}^{w}N\left(w\right)dw}{N_u\cdot w} \qquad (3)$$

Therefore, Eq. 2 expressing the energy balance of the deformed system can be written:

$$Q\cdot a_{qz,max}=\sum_{i=1}^{n}\xi_i\left(w_{i,max}\right)\cdot N_{i,u}\cdot w_{i,max} \qquad (4)$$

At the stage when the motion stops at the downward position, the system is not necessarily in equilibrium. Therefore, besides Eq. 4, the following inequality should be met:

$$R_{stat}\left(a_{qz,max}\right)\geq m\cdot g \qquad (5)$$

If inequality (5) is not met, the accepted value of $a_{qz,max}$ is not correct, since before it is reached, the tie elements are broken. The process of successive destruction of the tie elements with increasing displacement of the modified system is called the zipper-type mode.

The conditions of the equilibrium of forces in the deformed state for the proposed collapse mechanism are checked using Eq. 4 and Eq. 5. The dynamic resistance of the damaged system gets based on the resistance of horizontal ties to the maximum driving force $Q = mg$ after a sudden removal of the column.

As follows from Eq. 4, the dynamic resistance R_{dyn} ($a_{qz,max}$) depends on the maximum vertical deflection $a_{qz,max}$, which is chosen to consider 1) the availability of free space for the downward movement of the system (for example, according to [7] and [11] it is the distance to the underlying floor) and 2) the ductility of the ties..

A quantitative assessment of the uncertainties of the proposed simplified method based on energy balance, in comparison with direct nonlinear dynamic analysis, is considered in [16]–[19], in particular in the most recent of them [23]. In [23], it was noted that instead of the cumbersome nonlinear dynamic analysis (NLD), which contains a number of uncertainties (for example, load history, damping coefficient, etc.), the method based on energy balance (EBM) is a promising approach for determining the maximum dynamic response of the structure. Despite some errors adopted in the estimation, authors [23] show that the method based on the energy balance is quite accurate and effective both 1) in implementing the bending mechanism (the formation of plastic hinges at small deformations) and 2) at the stage of implementation of the membrane (chain) effect in ties that perceive tension (the stage of large displacements). Studies [23] show that the model describing the uncertainty (modelling error) of the energy balance method (EBM) compared to nonlinear dynamic analysis (NLD) well describes by a lognormal distribution with the following statistical parameters LN (0.95; 0.20). (*It should be noted that there is a certain amount of slyness here: the finite element model should be tested based on classical laws, and not vice versa*).

3 Prefabricated building with hollow-core slabs

3.1 Static and dynamic resistance

As shown above (see Eqs. 2-4), the resistance of a damaged system with alternative loading paths almost directly depends on the ductility of the tie connections.

Let us consider a prefabricated floor with hollow-core slabs of equal spans. The internal support of the continuous girder is removed under accidental action. When the support is removed, a longitudinal strip of prefabricated floor together with the ties forms an alternative load-bearing bridging system. In accordance with the formulated assumptions (see Section 2.2), in the ultimate state, prefabricated floor elements are considered as rigid bodies connected by ductile ties.

After the sudden removal of the mid-column of the continuous girder, the prefabricated slabs rotate at the adjacent supports and move in the horizontal direction.

As follows from [2] and [7], the resistance model considers a longitudinal strip of prefabricated floor elements (for example, hollow-core slabs). In this simplified model, the resistance in the transverse direction, arch, and beam effects in the longitudinal direction are neglected and are not taken into account.

We assume that the horizontal ties of the system have the same mechanical characteristics; therefore, for any state of deflection, the three ties have the same tensile force and the same elongations, because the characteristic load-deflection *N-w* relationships for each tie are the same. For each precast floor element, the resultant Q, which is assumed to be placed in the centre of gravity of the element, represents the self-weight and other permanent loads. The deformed state is described by the deflection a_{qz} of the driving force [7] (see Fig. 1).

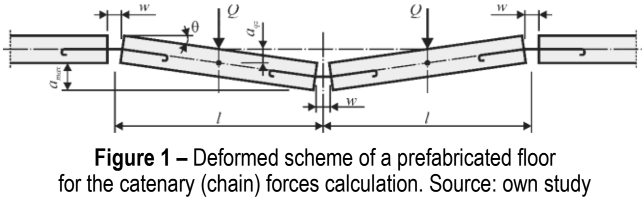


Figure 1 – Deformed scheme of a prefabricated floor for the catenary (chain) forces calculation. Source: own study

From the conditions of static equilibrium of the system in a deformed state, we can write:

$$Q \cdot \frac{l}{2} = N(w) \cdot 2 \cdot a_{qz} \quad (6)$$

For the certain displacements w of the horizontal ties, the vertical deflection a_{qz} can be calculated directly from the deformed geometric scheme (see Fig. 1) as:

$$a_{qz} = \frac{\sqrt{3 \cdot l \cdot w}}{2} \quad (7)$$

where l is the length of the prefabricated elements.

Considering Eq. 7, static resistance expresses as:

$$R_{max}\left(a_{qz,max}\right) = 2 \cdot N(w) \cdot \sqrt{\frac{3 \cdot w}{l}} \quad (8)$$

The maximum value of the static resistance is associated with the maximum displacement $a_{qz,max}$, at which the downward movement of the system must be stopped and is determined by the formula:

$$R_{\max}\left(a_{qz,\max}\right) = 2 \cdot N_u \cdot \sqrt{\frac{3 \cdot w_{\max}}{l}} \quad (9)$$

We can express the energy equilibrium condition for the doubled span system as:

$$2Q \cdot a_{qz,max} = 3\xi(w_{max}) \cdot N_u \cdot w_{max} \quad (10)$$

By introducing Eq. 7 and Eq. 9 into Eq. 10, we obtain dynamic resistance as:

$$R_{dyn}\left(a_{qz,max}\right) = \frac{1}{2}\xi(w_{max}) \cdot R_{max} \quad (11)$$

As stated in [2] even if the tie will have an ideally plastic response, the static response of the system based on the chain (catenary) actions increases almost linearly with increasing displacement, therefore Eq. 11 uses a factor of 1/2.

3.2 Modelling of the tie elements. "*N-w*" relationship for reinforcing bar

The "*N-w*" relationship relates the tensile force N in the tie connection and its end displacement w (local end-slip displacement). To obtain the "*N-w*" diagram, it is necessary to have an adequate local bond-slip relationship "τ-s" (Fig. 2). Based on extensive experimental research [24], including our own, we adopted the dependence in accordance with [8] and [13].

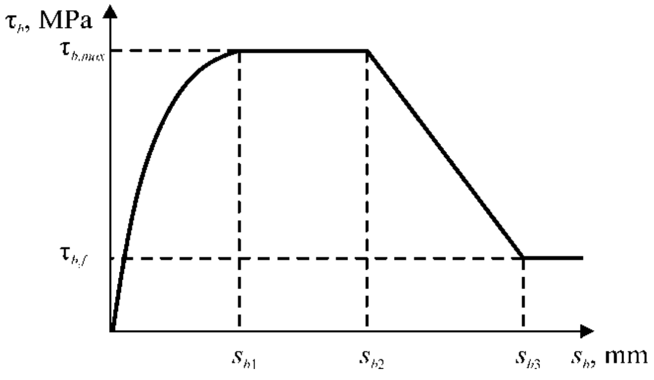


Figure 2 – Analytical bond stress-slip relationship [8], [13]

The advantage of the relationship (Fig. 2) is the applicability for both ribbed and plain bars, as well as an almost complete range of concrete classes, including high-strength ones [8], [13], [24]. The bond-slip behavior for reinforcement bars is modelled according to [13].

According to [24] for the "*N-w*" diagram developing, it is necessary (a) to determine the transmission length l_t and (b) the strain $\varepsilon_s(x)$ and $\varepsilon_{ct}(x)$ distribution along this length.

In the general case, the transmission zone length increases with increasing tensile stress. In this case, for a steel bar embedded in concrete, the following cases are possible: (1) the length of the transmission zone is shorter or equal to the anchorage length; (2) the transmission zone length is greater than the anchorage length. It should be noted that for continuous tie elements passing through the overlap, case (1) is usually valid.

According to [8] and [13] for monotonic loading the reference value of τ_b of the bond stresses between concrete and reinforcing bar can be calculated as follows:

$$\tau_b = \tau_{b,max} \cdot s_b^{0.4} \quad (12)$$

where $\tau_{b,max}$ is the maximum bond-shear stresses between concrete and reinforcing bar in accordance with [8];

s_b is the current value of the relative slip displacement of the reinforcing bar in concrete.

If the anchorage length is greater than the transmission length, to determine the slip displacement we assume to consider the concrete element rigid in relation to the reinforcing bar. Such an assumption gives a slight overestimation of the designed value of the end slip. According to works [7] and [24] the relation (13) is valid if the following requirements are satisfied: 1) the steel bar works in an elastic stage; 2) the net end-slip is less than 1.0 mm:

$$w_{end} = 0.288 \cdot \left(\frac{\varnothing \cdot \sigma_s^2}{\tau_{b,max} \cdot E_s}\right)^{0.714} + \frac{\sigma_s}{E_s} \cdot 2 \cdot \varnothing \quad (13)$$

where $\tau_{b,max} = 2.5\sqrt{f_{cd}}$ for "good" bond conditions; (14a)

$\tau_{b,max} = 1.25\sqrt{f_{cd}}$ for "all other" bond conditions; (14b)

$\varnothing$ is bar diameter, in [mm].

In Eq. 13 the first term on the right-hand side describes the end-slip displacement caused by bond stresses along that part of the transmission length where bond stresses appear, here defined as the "net end-slip" $s_{end,net}$:

$$s_{end,net} = 0.288 \cdot \left(\frac{\varnothing \cdot \sigma_s^2}{\tau_{b,max} \cdot E_s}\right)^{0.714} \quad (15)$$

The last term of Eq. 13 considers the effect of local concrete failure near the free end over a length of approximately $2\varnothing$.

The relationship between stress σ_s and a given end-slip displacement can be rewritten from Eq. 15 as follows:

$$\sigma_s = 2.39 \cdot \sqrt{\frac{\tau_{b,max} \cdot E_s}{\varnothing} \cdot s_{end,net}^{1.4}} \quad (16)$$

where $$s_{end,net} = w_{end} - \frac{\sigma_s}{E_s} \cdot 2 \cdot \varnothing \quad (17)$$

The transmission length according to works [7] and [24] is calculated as:

$$l_t = 0.583 \cdot \frac{\varnothing \cdot \sigma_s}{\tau_{b,max} \cdot s_{end,net}^{0.4}} + 2 \cdot \varnothing \quad (18)$$

When yielding in steel reinforcement starts, the end-slip $w_{end,y}$ and the transmission length $l_{t,y}$ can be obtained by inserting $\sigma_s = f_{yd}$ into Eq. 16 and Eq. 18. In the general case, the relationship "N-w_{end}" is nonlinear before steel yielding is reached. However, the pull-out stiffness of the joint $k_a(w_{end})$ is generally defined as a secant at point $N(w_{end})$:

$$k_a(w_{end}) = \frac{N(w_{end})}{w_{end}} \tag{19}$$

As the first approximation, we can get the value of the connection pull-out stiffness based on the stiffness it reached just before yielding:

$$k_a = \frac{N_y}{w_{end,y}} \tag{20}$$

where N_y is the force corresponding to the steel yielding in the tie connection;

$w_{end,y}$ is end-slip corresponding to $\sigma_s = f_{yd}$.

It should be noted that Eq. 19 underestimates the stiffness for loads less than N_y. A more accurate value of the axial stiffness calculates by Eq. 20 for a given loading range or the end-slip.

According to [7], the "plastic zone length" defines as the part of the transmission length where the reinforcement bar has reached yielding. Within the "plastic zone length", the bond-shear stress decreases due to steel yielding to [24].

If the anchorage length of the reinforcement bar in the concrete body is sufficient, the maximum value of the "plastic zone length" along the transmission length can be calculated accounting that the reinforcing steel reaches rupture tensile strength f_u. Along the "plastic zone" length, the tensile stress in reinforcement increases from the value of yield strength f_y to the value of the ultimate tensile strength f_{ud} at the loaded end of the bar.

The ultimate value of the plastic zone length can be calculated according to [24] as:

$$l_{t,pl} = \left(\frac{f_{ud} - f_{yd}}{\tau_{bm,pl}} \right) \cdot \frac{\varnothing}{4} \tag{21}$$

where $\tau_{bm,pl}$ is the average value of bond-shear stress, calculated by Eq. 22.

To calculate the average shear-bond stress for ribbed bars of ductile type (classes B and C according to [8] determined as the "*high ductility*") in [7], [24] it has been proposed the following formula:

$$\tau_{bm,pl} = 0.27\tau_{b,max} \tag{22}$$

where $\tau_{b,max}$ is determined by Eq. 14a or Eq. 14b depending on the bond conditions.

The ultimate end-slip of the tie bar can be calculated as follows:

$$w_{end,u} = l_{t,pl} \cdot \varepsilon_{sm,pl} + w_{end,y} \tag{23}$$

where $\varepsilon_{sm,pl}$ is the average strain of the reinforcing bar along the plastic zone length, according to [7] can be estimated as $\varepsilon_{sm,pl} = 0.5\varepsilon_{su}$.

As follows from Eq. 23, with an increase of the plastic zone length $l_{t,pl}$, the ultimate displacement of the tie increases. An idealized three-line the "*N-w*" relationship shown in Fig. 3 can be proposed based on the recommendations of [7].

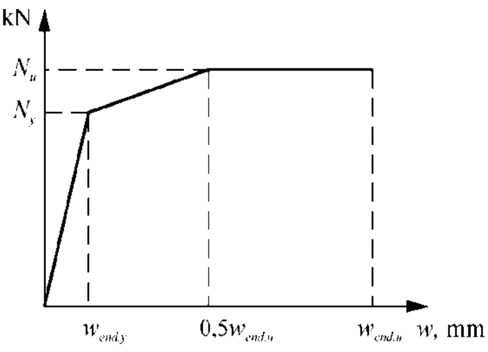


Figure 3 – Idealized "*N-w*" relationship [7], [24]

4 Example of checking the robustness of precast multi-story building

4.1 Structural system description, input data for analysis

As an example, we will consider the structural system of an 18-storey building, the plan of which is shown in Fig. 4. In the considered structural system, monolithic (cast-in-place) columns with a section of 300x300 mm (along axes 2, 6) and prefabricated panel walls (along axes 1, 4, 7) are used as supporting vertical elements. Floors are made of prefabricated hollow-core slabs (1.2x6.0x0.22 m) with modification in the support nodes. The slabs are supported on prefabricated girders with a height of 0.26 m (see Fig. 4). At the stage of preliminary analysis and design, it designed an integrated system of horizontal and vertical ties under the requirements of [8] and [10].

In accordance with the input data, the following characteristic values of actions were adopted for the design: (1) dead load of floor slabs g_{k1} = 3.05 kPa; (2) dead load of the floor finishing g_{k2} = 0.6 kPa; (3) imposed load q_k = 1.5 kPa. An accidental load combination is taken as:

$$p_A = g_{k1} + g_{k2} + \psi_2 \cdot q_k = 3.05 + 0.6 + 0.3 \cdot 1.5 = 4.1 \text{ kPa}.$$

According to the proposed analytical model, we check the robustness of the structural system, taking into account the chain (membrane) effects.

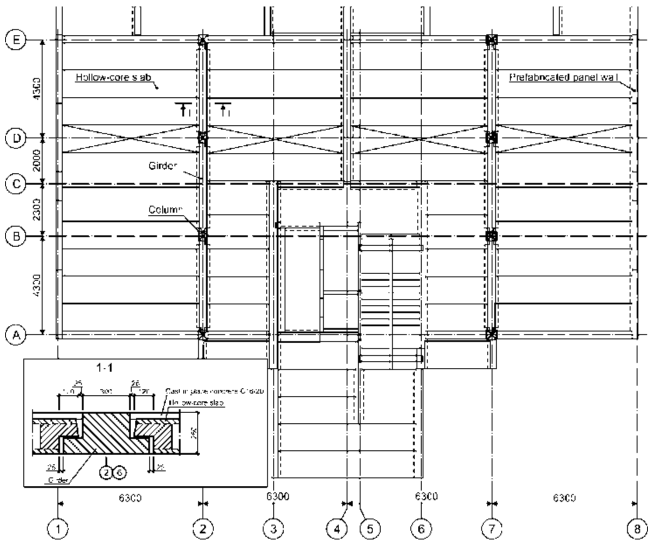


Figure 4 – The first floor of an analyzed 18-storey prefabricated framed building.

Within the framework of the approaches in the current codes and guidelines [8]–[12] the resistance of precast floor slabs in one direction and resistance of prefabricated girders in the other (transverse) direction are considered separately. To determine the required cross-sectional areas of horizontal ties in girders and slabs, the principle of compatibility of vertical displacements at the point of removal of the column is used. In accordance with this principle (the principle of compatibility), the force-deformation connections parameters of the ties in the beams and slabs are determined from the condition of equality of displacements: $a_{max,p} = a_{max,b}$ (here, $a_{max,p}$ and $a_{max,b}$ is the maximum deflection of slabs and girders at the remote element, respectively).

4.2 Modelling of the horizontal tie

We accept plain bars Ø28S240, which have significant plastic deformability, as tie connections. The following reinforcement steel properties are taken: f_{yk} = 240 MPa; $(f_u/f_y)_k$ = 1.3; E_s = 200 GPa; $\varepsilon_{suk} = 1.5 \cdot 10^{-2}$; A_s = 616 mm^2. It is assumed, that the tie bar is anchored in confined concrete of compressive strength class C20/25 (f_{ck} = 20 MPa; f_{cm} = 20 + 8 = 28 MPa), "good" bond conditions. In Tab. 1 presents the values of the main parametric points of the "*N-w*" relationship (Fig. 3) for tie connections, which are calculated based on the proposed end-slip approach.

Table 1 – The "*N-w*" relationship basic points for plain bars Ø28S240

№	Reinforcement	Parameters for "*N-w*" diagram			
		N_y, kN	N_u, kN	$w_{end,y}$, mm	$w_{end,u}$, mm
1	2Ø28S240	295.68	384.38	0.658	28.94
2	4Ø28S240	591.40	768.76	0.658	28.94
Note: general view of the "*N-w*" diagram see Fig. 3.					

4.3 Analytical solution for ties sections area

Let us consider a strip of girders along axis 2 (Fig. 4). We consider that the internal tie connections in the direction of axis 2 are concentrated in prefabricated girders. The girders are loaded with an accidental loads combination as follows: Q = 4.1x6.0x4.0 = 98.4 kN. The girders have a loop connection at the column (2Ø25S500) and have additionally reserved horizontal tie connections (4Ø28S240). The deflection of the middle joint (at the joint with the removed column) should not exceed 2.3 m (free floor space). In Tab. 2 presents the main design parameters and the results of robustness checking.

Table 2 – The robustness checking results for an analysed precast building (by EBM)

Calc. step	Design Parameter	Reference	Value	Units
Girders (l = 4.3 m)				
1	w_{max}	Tab. 1	0.02894	m
2	$a_{max,b} = 2\,a_{qz,max}$	Eq. 6	0.61	m
3	R_{max}	Eq. 9	218.47	kN
4	R_{dyn}	Eq. 11	218.47	kN
5	$R_{dyn} > Q$	Eq. 5	105.8>98.4	
Hollow-core slabs (l = 6.3 m)				
6	$a_{max,s} = a_{max,b}$	see note(1)	0.61	m
7	w_{max}	Eq. 6	0.0206	m
8	R_{max}	Eq. 9	77.94	kN
9	R_{dyn}	Eq. 11	37.80	kN
10	$R_{dyn} > Q$	Eq. 5	37.8>29.52	
Notes: 1) based on the compatibility hypothesis $a_{max,s} = a_{max,b}$.				

Based on the accepted concept of the deflection compatibility, we will show how to determine the required cross-sectional area of the tie connections for a given type of reinforcement (S240).

Based on the results from Table 2, the required ultimate force to break the bar can be calculated from Eq. 9 as:

$$N_u = \frac{R_{max}}{2\cdot\sqrt{\dfrac{3\cdot w_{max}}{6.0}}} = \frac{60.86}{2\cdot\sqrt{\dfrac{3\cdot 0.0206}{6.0}}} = 299.8\ kN;$$

Since 2Ø28S240 N_u = 384 kN > 299.8 kN, required reinforcing bar area is equal:

$$A_{s,req} = \frac{N_u}{1.3\cdot f_{yk}} = \frac{299.8\cdot 10^3}{1.3\cdot 240} = 960.9\ mm^2;$$

We accept 2Ø25S240 (A_s = 982 mm^2).

4.4 Comparison of the required tie sections area and dynamic resistance designed by the energy balance method (EBM) and by the current standards

At the first stage, we verified the proposed model based on the results of our own investigations [25] obtained by testing span-to-span hollow-core slab fragments under uniformly distributed load and sudden support removal (see Fig. 5). The authors found that the obtained experimental results have a good agreement with the calculation based on EBM.

Figure 5 – Experimental investigation of the fragment of the hollow-core slabs [25]

Let us compare now the calculation results obtained by the proposed model (EBM) and the models included in the codes and standards of various countries. In Tab. 3 gives the results of calculating the required cross-sectional area of horizontal tie connections according to the current standards.

Analysis of standards [8], [10]–[12] shows that all the design models for calculation of the chain (membrane) force presented in Tab. 3 are based on the equations, which are got from the of static equilibrium of the deflected system at maximum vertical displacement:

$$T_j = (1+\alpha)\cdot\frac{(g_k + \psi_i\cdot q_k)\cdot l_b^2}{2\cdot\delta_s} \tag{24}$$

where δ_s is the vertical displacement of the joint with the removed element.

Table 3 – Designing of the horizontal ties according to the current standards

№	Reference	Expression	Tie Force, T_j		a_{max}(2)	δ_s	Reinforcement (A_s, mm^2)
			kN	kN/m	m		
1	[10]	$0.8(g_k^{(1)} + q_k)sL$	75	62.5	1.3	$\le \dfrac{l_s}{4.8}$	1Ø20S240 (314)
2	[10] [11] [12]	$\dfrac{(g_k+q_k)}{7.5}\cdot\dfrac{l_r}{5}\cdot F_t$; $F_t = 20 + 4n_s$ or 60 kN/m	62.69	51.91	1.97	$\le \dfrac{l_s}{3.2}$	1Ø20S240 (314)
3	[12] [8]	$3(1.2\,g_k + 0.5q_k)l_b$; $3(g_k + 0.3q_k)l_b$	116.34	96.57	1.05	$\le \dfrac{l_s}{6}$	1Ø25S240 (491)
Notes: 1) input data l_b = 6.3 m; g_k = 3.65 kPa; q_k = 1.5 kPa; 2) the value of the maximum deflection $a_{max} = \delta_s = (g_k + q_k)\cdot l_b^2/(2\cdot T_j)$							

In work [2] it is shown that the design model of [10] and [11] is based on the following formula for determining the vertical displacement:

$$\delta_s = 18.75\cdot(1+\alpha)\cdot\frac{l_b}{T_j} \tag{25}$$

When determining α from 0 to 1.5 (according to [2]) and F_t from 24 to 60 kN/m (when the number of floors changes from 1 to 10 or more), we obtain the maximum displacement $a_{max} = \delta_s \leq l_b / 1.28$(!). The design model of [12] was obtained with $a_{max} = \delta_s \leq l_b / 6$.

Based on the analysis of the results got from testing of the full-scale slab-to-slab joint tests carried out by PCA [2], it is implied that the catenary action will stop at an ultimate deflection greater than $\delta s = lb / 6.67$ which agrees well with the [12] requirements and other research studies. Furthermore, experimental studies [2], [18], [23] shown that in bar fracture failure mode the system is collapsed at $\delta s \approx lb / 10$. The discrepancy in the value of an ultimate vertical displacement according to the different standards [8], [10], [11] and another research provision (e.g. energy balance method) is remarkable and standard [12] is more relaxed.

Comparison of calculation results obtained using energy balance method and standard methods [8]–[12] (see Tab. 3) shows that the required areas of reinforcing bars used as horizontal ties are significantly different. So, when calculating according to the codes [8]–[11] 1Ø20S240 is required, [12] 1Ø25S240 standards (see Tab. 3), while from the calculation according to the energy balance method using the dependences "*N-w*" is 2Ø25S240.

Assuming of fixed values of maximum vertical deflection (from $l_b / 1.28$ to $l_b / 10$) in the codes [8]–[12] leads to rather optimistic and relaxed results when the horizontal ties designed.

Tab. 4 compares the values of the parametric points of the "*N-w*" relationship for the tie connections which were calculated according to standards [8], [9] and Tab. 5 presents robustness criteria checking results of the precast floor using the resistances R_{max}, R_{dyn}, assessed based on the energy balance method.

The calculation result presented in Tab. 5 shows that checking criterion $R_{dyn} \geq Q$ for horizontal ties designed according to the standards [8], [10]–[12] is not satisfied. At the same time, the designed tie connection, despite the significant plastic deformability of the reinforcement S240 ($\varepsilon_{suk} = 1.5 \cdot 10^{-2}$), does not provide the *a priori* assumed vertical deflections without the bar rupture. A sudden failure mode of the structural system occurs. The vertical deflection of the damaged floor a_{max} = 0.73 m with horizontal steel ties 1Ø20S240, determined by the energy balance method considering the ultimate (rupture) steel force, turns out to be insufficient for resisting the accidental action effects. Tie forces determined by the standards [8], [10]–[12] correspond to a_{max} from 1.05 m to 1.968 m, which is not realistic for this type of reinforcement (Ø20S240).

Table 4 – The "*N-w*" relationship basic parameters for analysed horizontal ties according to [7] and [24]

№	Reinforcement	Parameters of the "*N-w*" diagram			
		N_y, kN	N_u, kN	$w_{end,y}$, mm	$w_{end,u}$, mm
1	1Ø25S240	117.84	153.19	0.492	38.49
2	1Ø20S240	75.4	97.97	0.612	30.8
Note: "*N-w*" diagram see Fig. 5.					

Table 5 – Design values of resistances R_{max}, R_{dyn} for the damaged system

№	Reference	Required reinforcement	Resistance of ties, kN		Maximum displacement a_{max}, m		Criterion $R_{dyn} \geq Q$
			R_{max}	R_{dyn}	according to the standards(1)	according to the energy balance(2)	
1	[10]	1Ø20S240	23.73	11.03	1.3	0.73	not done
2	[10]–[12]				1.968	0.73	not done
3	[8], [12]	1Ø25S240	41.63	19.44	1.05	0.85	not done
4	according to EBM	2Ø25S240	60.87	29.52	-	0.61	done
Note: 1) max deflection corresponding to the ultimate force in the tie calculated by the current standards; 2) max deflection by the energy balance method (EBM)							

As follows from Eq. 24, obtained from the equilibrium condition of the deflected system, at a constant value of the tie force $T_j = f_{yd} \cdot A_{st}$ (after yielding of steel), the global resistance of the structure linearly depends on the value of the vertical deflection δ_s. After rewriting equation (Eq. 24), considering that $(g_k + \psi_i \cdot q_k) = R_{max}$, we obtain:

$$R_{max} = \frac{2 \cdot f_{yd} \cdot A_{st}}{l_b^2} \cdot \delta_s \quad (26)$$

For the considered case of horizontal tie 1Ø25 S240 at l_b = 6300 mm, A_{st} = 491 mm²:

$$R_{\max} = 0.0059 \cdot \delta_{s,\max} \quad (27)$$

4.5 Reliability assessment of the load-bearing capacity models

The next stage of the comparison of the proposed energy balance method (EBM) and standard methods considered in actual codes was performed based on the reliability assessment of the damaged system with the horizontal ties designed according provisions (requirements) of the codes and EBM. To determine failure probability, the probabilistic model for the dynamic resistance is combined with the probabilistic model for accidental load combination acting on the typical floor. We calculated failure probabilities for the damaged system according to the following limit state function g(***X***):

$$g(\boldsymbol{X}) = \theta_R \cdot R_{dyn} - \theta_E \cdot (G + Q) \quad (28)$$

Probabilistic models for most important basic variables adopted in the probabilistic models for the dynamic resistance and effects of actions, which are used in limit state function (Eq. 28) are listed in Tab. 6.

Table 6 – Probabilistic models of basic variables for reliability analyses

Category of variables	Name of basic variables	Sym. X	Dimension	Distrib.	Mean μ_X	St. dev. σ_X
Actions	Permanent	G	kN	N	G_k = 27,59	$0{,}1\mu_G$ = 2,759
	Imposed	Q	kN	GU	$0{,}2Q_k$ = 2,268	$1{,}1\mu_Q$ = 2,495
Material strengths	Concrete (C20/25)	f_c	MPa	LN	28	4,8
	Reinforcement (S240)	f_y	MPa	LN	300	30
	Reinforcement (S500)	f_y	MPa	LN	560	30
Model uncertainties	Load effect factor	θ_E	–	N	1	0,10
	Resistance factor	θ_R	–	N	1	0,05
Notes: N – normal distribution; LN – lognormal distribution; GU – Gumbel distribution; l_s = 6,3 m; b_s = 1,2 m; g_k = 3,65 kPa; q_k = 1,5 kPa; $G_k = g_k \times l_s \times b_s$ = 27,59 kN; $Q_k = q_k \times l_s \times b_s$ = 11,34 kN						

The probability density distribution functions for the different analyzed design cases are shown in Fig. 6 and Fig. 7. The results of the failure probability calculations are presented in Tab. 7. Probabilistic modelling of the limit state function was performed with usage Monte Carlo simulation method (N=10⁸).

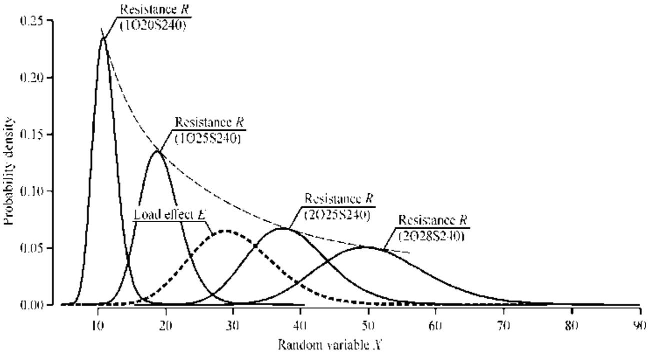


Fig. 6 – Load effect E and resistance R as random variables for ties from reinforcement class S240

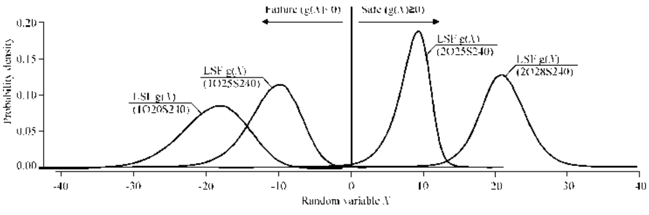


Figure 7 – Limit state function $g(\boldsymbol{X})$ as random variables for ties from reinforcement class S240

Table 7 – Results of probability simulation of performance function of damaged system with hollow-core slabs

№	Reference	Req. reinfor-cement	Resistance, R		Load Effect, E		Performance function, $g(\boldsymbol{X})$		p_f $(g(\boldsymbol{X})<0)$	β
			mean	sd	mean	sd	mean	sd		
Ties from reinforcement of class S240 (k = 1.3; ε_u = 15%)										
1	[10]-[12]	1Ø20 S240	11.03	1.74	30.14	6.28	-19.12	4.74	0.999999	< -4.8
2	[8], [12]	1Ø25 S240	19.26	3.03			-10.88	3.70	0.999971	-4.02
3	according to EBM	2Ø25 S240	38.53	6.07			8.39	2.53	0.007649	2.42
Force-equivalent ties from reinforcement of class S500 (k = 1.08; ε_u = 5%)										
1	[10]-[12]	1Ø20 S500	7.25	0.64	30.14	6.28	-22.89	5.70	0.999999	< -4.8
2	[8], [12]	1Ø25 S500	12.67	1.12			-17.47	5.28	0.999998	-4.6
3	according to EBM	2Ø25 S500	25.34	2.23			-4.80	4.33	0.877255	-1.16
Notes: sd – standard deviation; $k = f_u / f_y$; β is reliability index according to the Laplace function.										

As we can see from results presented in Tab.7, only energy balance method (EBM) in which ductility of the steel ties considered, allow to design reliable structural system in damaged state (failure probability p_f(g($\boldsymbol{X}$)<0) = 0.007649 in case 2Ø25S240). From analysis of the numerical results (see Tab. 7) one can conclude that the design under actual design according to codes [8], [10]–[12] is non-robust and would collapse in case of the notional column removal, even if the requirements by the codes are fulfilled. The very close conclusions were formulated in [26] based on own numerical investigations ("*It is concluded that in case of the removal of an inner column, the original design according to the Eurocodes is very likely to fail.*"). We relate the main reason of this problem of the non-robust designing with requirements of the actual codes in which neglecting the ductility and rotation capacity of the slab elements in the damaged system.

5 Brief algorithm for simplyfied calculation of the dynamic resistance

As shown earlier, one of the main issues in the tie force assessment is to determine the value of deflection at which the catenary effect is mobilized. For a statically indeterminate structural system, this point can be determined using a simplified approach. As the deflection at which the catenary effect starts, point a_0 of the "*F*-δ" relationship should be taken as shown in Fig. 8. We assumed it as the point where the nonlinear flexural response crosses with a straight-line response of the catenary effect (see Fig. 8).

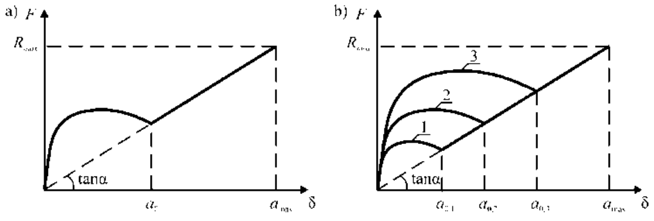
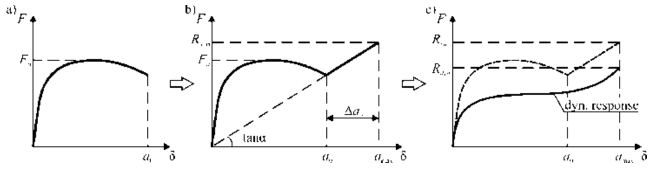


Figure 8 – For the position of point a_0 assessment

It should be borne in mind that the horizontal tie in the structure is either unloaded before being put into operation (when it is designed as an independent link), or has compressive strained (when it is part of the reinforcement). The amount of horizontal tie reinforcement should be designed in such a way that a chain (membrane) effect is provided for the perception of an accidental combination of actions and that a smooth transition from a disengaging flexural plastic hinge to an engaging tensile tie is ensured. Here, the amount and ductility properties of flexural reinforcement should provide a sufficient length of the plastic deformation branch of the "*F*-δ" response to achieve the deflection a_0 (see Fig. 9). We should base the structural design procedure for robustness checks on ensuring a smooth and consistent transition to the mobilization of alternative loading paths.



a) the response "*F*-δ" when a flexural plastic hinge realizing;
b) common response "*F*-δ" for the flexural joint and horizontal tie connection; c) the dynamic response of the damaged system

Figure 9 – The calculation steps to the determination of the parameters of a system of the horizontal ties

In this case, the maximum deflection a_{max} and the resistance R_{max} should be determined based on the energy balance equations, as shown earlier. To ensure compatibility (consistency) in the response of flexural hinges and horizontal tensile ties in statically indeterminate systems, the following procedure can be proposed:

1) a nonlinear calculation of the modified structural system is performed and the nonlinear reaction "*F*-δ" is determined taking into account only flexural plastic hinges behavior (Fig. 9a). A linear reaction "*F*-δ" will pass through this point (a_0), which describes, with an acceptable approximation, the operation of the horizontal tie. The slope tangent is the axial stiffness of the horizontal tie;
2) the parameters of the horizontal tie connections necessary to ensure the resistance of the accidental combination are calculated. For a given R_{max}, the deformation parameters of the ties are determined, which will ensure the achievement of the maximum displacement a_{max} (Fig. 9b);
3) to perform complex nonlinear analysis of the damaged structural system with flexural and tension plastic hinges using computer software;
4) to calculate parametric points of a dynamic diagram and determine the global resistance of the damaged structural system (Fig. 9c), accounting of the value of the global safety factor according to [11].

5 Conclusions

Based on the obtained results, we can make the following conclusions:

1. The proposed method for determining membrane (chain) forces based on the provisions of the energy balance method of the damaged structural system (EBM) is a promising method for calculating its maximum dynamic response. This method for determining the total dynamic response of a system can be successfully applied both in the case of simple analytical models and for complex nonlinear finite element models instead of cumbersome nonlinear dynamic analysis (NLD), which contains a number of uncertainties (for example, load history, damping coefficient, modeling error etc.).
2. Comparison of the calculation results according to the current standards [8]–[12] with proposed energy balance method, has shown that the calculation models of the codes can give an unsafe result, for example, underestimating the required cross-sectional area of horizontal ties. This is because it based all dependencies for calculating the tie force on constant values of the ultimate deflection (usually from 1/6 to 1/10 of the span) without checking the ultimate deformability of horizontal ties. As follows from the analysis performed, with the unchanged value of the accidental combination of actions, the calculated

tie force (for which its cross-section is selected) will change in inverse proportion to the deflection. The approach adopted in the standards [8]–[12] can lead to unrealistic results when the adopted reinforcement is horizontal tie cannot ensure the achievement of the *a priori* maximum deflection due to insufficient deformability. The model included in [12], in which the constant deflection of 1/10 span is used to derive the design equations, is most similar to the solutions based on the energy balance. Changes should be made to the current standards [8]–[10] in terms of the application of methods based on the energy balance of the system for the design of horizontal ties.

3. Taking into account a number of assumptions made in the formulation of the basic provisions of the method based on the energy approach, it is necessary to perform a statistical analysis of the uncertainty modeling based on the results of experimental studies, but not the results of dynamic calculations of the finite element model as done in [23].

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