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RESEARCH OF DYNAMIC CHARACTERISTICS OF POWER TRANSFERS BY FRICTIONAL TYPE FLEXIBLE LINK

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Abstract

Functioning of power transfer by flexible link of frictional type in conditions of a dynamic loading is theoretically described. Small motions of power transfer by flexible link of frictional type which allows to forecast unstable transfer operating modes are presented. Experimental research in the obtained dependences are given and recommendations as to the choice of rational operational transfer parameters which ensure the rise in stability of its movement are offered.

Keywords: power transfer, frictional flexible coupling, dynamic loading, unstable operation, operating parameters, movement stability

ИССЛЕДОВАНИЕ ДИНАМИЧЕСКИХ ХАРАКТЕРИСТИК ПЕРЕДАЧИ МОЩНОСТИ ГИБКИМ ЗВЕНОМ ФРИКЦИОННОГО ТИПА

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Реферат

Теоретически описано функционирование передачи мощности гибким звеном фрикционного типа в условиях динамической нагрузки. Представлены малые движения передачи мощности гибким звеном фрикционного типа, что позволяет прогнозировать нестабильные режимы работы передачи. Приведены экспериментальные исследования полученных зависимостей и предложены рекомендации по выбору рациональных эксплуатационных параметров передачи, обеспечивающих повышение устойчивости ее движения.

Ключевые слова: передача мощности, фрикционная гибкая муфта, динамическая нагрузка, нестабильная работа, рабочие параметры, устойчивость движения.

Introduction

Despite the quasi-constant nature of the transmitted torque, slip oscillations are inevitable in a frictional belt drive (FBD), which in some cases leads to slipping of the belt relative to the pulleys. and thus affects the stability of the gear movement. This phenomenon contributes to the occurrence of vibration in the drives on the whole, and the vibrations of the belt legs contribute to the intense production of noise. The durability of belts in suchlike conditions is also low, however, theoretically, such a phenomenon has not been described up to now.

When analyzing large displacements, it was shown [1] that in certain modes the FBD, as a coupling, is nonholonomic. In the work [2], a linear equation of nonholonomic coupling was obtained, which is the result of considering non-vibrational loading of the transmission. The authors of works [3, 4] made an attempt to experimentally substantiate slip zones, in which transmission can be represented as a holonomic system. The drawback of these works is that the authors do not take into account the change in the slip coefficient when the load fluctuates, and that is why they could not explain the significant discrepancy between empirical and analytical results in the resonant modes of operation. Thus, the issue of the criterion for transmission holonomy at small vibrations remains open.

The aim of this work is to describe small motions of a system with a FBD in a nonholonomic setting, which makes it possible to predict unstable transmission modes. It is known that a nonholonomic dynamical system will occur if the generalized coordinates of the system are related to the generalized velocities of the system and the kinematic coupling equation is not integrable.

Main part

To derive a FBD coupling equation, let's consider the processes of changing the speeds of the driving and driven pulleys over a period of time Δt when oscillating relative to stationary motion. Similar processes of changing the speeds of the transmission pulleys occur in non-stationary loading conditions and in the absence of torsional vibrations. For the simplicity of analytical conclusions, let's assume the calculated FBD gear ratio equal to one and $\Delta t_1 = \Delta t_2 = \Delta t$ in accordance with Fig. 1. Furthermore, we linearize the sections of speed change over the period Δt, corresponding to the drop and increase in the load. At t_0, the angular velocities of the driving and driven pulleys are respectively equal to $\dot{\varphi}_{10}$ and $\dot{\varphi}_{20}$.

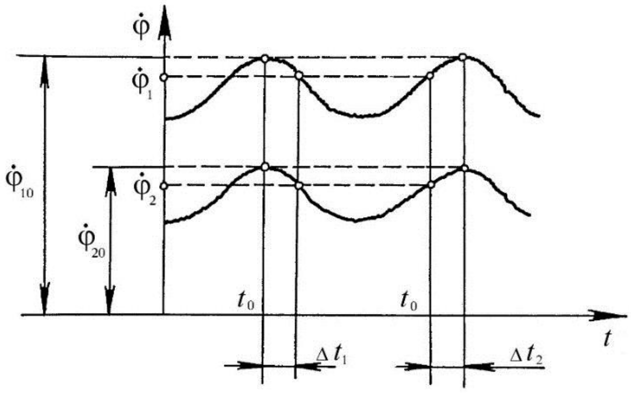

Figure 1 – Change of angular speeds of the leader and follower pulleys of power transmission by flexible link of frictional type at loading oscillations

During the time Δt, these speeds will change and become equal, respectively, $\dot{\varphi}_1$ and $\dot{\varphi}_2$. In this case, the angles of rotation of the pulleys during the time Δt will be equal: for the case Δt_1

$$\varphi_1 = \dot{\varphi}_{10}\Delta t_1 - \frac{\varepsilon_1 \Delta t_1^2}{2}, \quad (1)$$

$$\varphi_2 = \dot{\varphi}_{20}\Delta t_1 - \frac{\varepsilon_2 \Delta t_1^2}{2}, \quad (2)$$

where ε_1 and ε_2 — angular acceleration of the transmission pulleys.

For the case Δt_2

$$\varphi_1 = \dot{\varphi}_{10}\Delta t_2 + \frac{\varepsilon_1 \Delta t_2^2}{2}, \quad \varphi_2 = \dot{\varphi}_{20}\Delta t_2 + \frac{\varepsilon_2 \Delta t_2^2}{2}.$$

With uniform rotation, the angular accelerations of the pulleys are equal to:

for the case Δt_1

$$\varepsilon_1 = \frac{\dot{\varphi}_{10} - \dot{\varphi}_1}{\Delta t_1}, \tag{3}$$

$$\varepsilon_2 = \frac{\dot{\varphi}_{20} - \dot{\varphi}_2}{\Delta t_1}. \tag{4}$$

for the case Δt_2

$$\varepsilon_1 = \frac{\dot{\varphi}_1 - \dot{\varphi}_{10}}{\Delta t_2}, \quad \varepsilon_2 = \frac{\dot{\varphi}_2 - \dot{\varphi}_{20}}{\Delta t_2}.$$

We substitute expressions (3) and (4), respectively, in (1) and (2) and express the time from these dependencies. We perform a similar procedure with other formulas.

Taking into account that the time Δt is the same for the driving and driven pulleys, we obtain the FBD coupling equation, both for the case of Δt_1 and for the case of Δt_2 in the form

$$\frac{\varphi_1}{\dot{\varphi}_1 + \dot{\varphi}_{10}} = \frac{\varphi_2}{\dot{\varphi}_2 + \dot{\varphi}_{20}}, \tag{5}$$

or

$$\varphi_1\left(\dot{\varphi}_2 + \dot{\varphi}_{20}\right) = \varphi_2\left(\dot{\varphi}_1 + \dot{\varphi}_{10}\right). \tag{6}$$

Since the type of connections and their properties determine the form of the differential equations of motion of the system, it is necessary to find out the properties of the obtained kinematic coupling equation. Let's make an integrability analysis with the help of the elements of the Pfaffian forms theory [5].

We write the equation (6) in the following form

$$\varphi_1 \frac{d\varphi_2}{dt} - \varphi_2 \frac{d\varphi_1}{dt} + \left(\varphi_1\dot{\varphi}_{20} - \varphi_2\dot{\varphi}_{10}\right) = 0,$$

$$\varphi_1 d\varphi_2 - \varphi_2 d\varphi_1 + \left(\varphi_1\dot{\varphi}_{20} - \varphi_2\dot{\varphi}_{10}\right)dt = 0. \tag{7}$$

It follows (7)

$$Adx + Bdy + Cdz = 0, \tag{8}$$

where $A = \varphi_1;\ \ B = -\varphi_2;\ \ C = \left(\varphi_1\dot{\varphi}_{20} - \varphi_2\dot{\varphi}_{10}\right);$
$dx = d\varphi_2;\ \ dy = d\varphi_1;\ \ dz = dt.$

An integrating factor for the differential equation (8) exists if the condition is fulfilled

$$A\left(\frac{\delta B}{\delta z} - \frac{\delta C}{\delta y}\right) + B\left(\frac{\delta C}{\delta x} + \frac{\delta A}{\delta z}\right) + C\left(\frac{\delta A}{\delta y} - \frac{\delta B}{\delta x}\right) = 0. \tag{9}$$

We calculate particular derivatives entering the condition (9)

$$\frac{\delta B}{\delta z} = -\frac{\delta \varphi_2}{\delta t} = -\dot{\varphi}_2;\quad \frac{\delta A}{\delta z} = \frac{\delta \varphi_1}{\delta t} = \dot{\varphi}_1;$$

$$\frac{\delta C}{\delta y} = \frac{\delta\left(\varphi_1\dot{\varphi}_{20} - \varphi_2\dot{\varphi}_{10}\right)}{\delta \varphi_1} = \dot{\varphi}_{20};\quad \frac{\delta A}{\delta y} = \frac{\delta \varphi_1}{\delta \varphi_1} = 1;$$

$$\frac{\delta C}{\delta x} = \frac{\delta\left(\varphi_1\dot{\varphi}_{20} - \varphi_2\dot{\varphi}_{10}\right)}{\delta \varphi_2} = -\dot{\varphi}_{10};\quad \frac{\delta B}{\delta x} = -\frac{\delta \varphi_2}{\delta \varphi_2} = -1.$$

Substituting these values of the derivatives into the condition (5), after a number of transformations we find

$$\frac{\varphi_1}{\dot{\varphi}_{10} - \dot{\varphi}_1} = \frac{\varphi_2}{\dot{\varphi}_{20} - \dot{\varphi}_2};\ \text{или}\ \frac{\varphi_1}{\Delta\dot{\varphi}_1} = \frac{\varphi_2}{\Delta\dot{\varphi}_2}.$$

This condition means that the increment of the rotation angles per unit of speed change for the driving and driven pulleys of the transmission must remain equal during torque fluctuation.

However, in real conditions, because of the presence of an irreparable lag of the driven pulley due to elastic sliding, we have

$$\varphi_1 > \varphi_2;\ \Delta\dot{\varphi}_1 \neq \Delta\dot{\varphi}_2;\ \text{или}\ \frac{\varphi_1}{\dot{\varphi}_{10} - \dot{\varphi}_1} \geq \frac{\varphi_2}{\dot{\varphi}_{20} - \dot{\varphi}_2}.$$

Consequently, the equation of the kinematic connection of the FBD (6) is non-integrable, that is, a kinematic connection carried out by a belt in a friction-type transmission — flat-belt, V-belt is nonholonomic. Its equation in general form [5]

$$\sum_{j=1}^{n} A\nu_j \dot{\varphi}_j + a_\nu = 0,$$

where $\upsilon = 1, 2, 3, \ldots, \rho$; ρ – the number of linear nonholonomic connections, for our case $\rho = 1$; n – the number of the system generalized coordinates ($n = 2$);

$$A_{11} = -\varphi_2,\ \ A_{12} = \varphi_1,\ \ a_1 = \left(\varphi_1\dot{\varphi}_{20} - \varphi_2\dot{\varphi}_{10}\right). \tag{10}$$

In differentials an equation general form can be presented in the following way:

$$\sum_{j=1}^{n} A\nu_j d\varphi_j + a_\nu dt = 0. \tag{11}$$

As is known from analytical mechanics, a coupling type (11) meets basic Hertz-Hölder principle for the analytical mechanics of nonholonomic systems. It follows from this principle that the relations for coordinates $\delta\varphi_j$ variations in the presence of nonholonomic coupling are obtained from the equations of differential connections by discarding the term which, in the equation, contains value dt (54) and replacing $d\varphi_j$ by $\delta\varphi_j$. Thus, the coordinate variations $\delta\varphi_j$ are bound by the relation $\sum_{j=1}^{n} A\nu_j \delta\varphi_j = 0.$

For the case under consideration, this allows to obtain

$$\sum_{j=1}^{2} A\nu_j \delta\varphi_j = 0,\ \text{или}\ A_{12}\delta\varphi_2 + A_{11}\delta\varphi_1 = 0,$$

$$\varphi_1\delta\varphi_2 - \varphi_2\delta\varphi_1 = 0.$$

Considering small oscillations with respect to stationary motion and assuming that the possible displacements $d\varphi_j$ meet a number of coordinate variation values $\delta\varphi_j$, we write, for small oscillations, a coupling equation without a free term in differential form $\varphi_1 d\varphi_2 - \varphi_2 d\varphi_1 = 0.$

Dividing this equation by dt we obtain the constraint equation for small oscillations

$$\varphi_1\dot{\varphi}_2 - \varphi_2\dot{\varphi}_1 = 0,\ \text{или}\ \dot{\varphi}_2 = \frac{\varphi_2}{\varphi_1}\dot{\varphi}_1 = 0. \tag{12}$$

Thus, a free term a_1 does not have a significant effect on dynamic processes during vibrations of friction belt drives and actually determines only the initial state of the system. In the further analysis, we use equations with indefinite Lagrange multipliers and a coupling equation (12). Such an equation has the form

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$$\frac{d}{dt}\frac{\delta T}{\delta x_j}-\frac{\delta T}{\delta x_j}=-\frac{\delta \Pi}{\delta x_j}-\frac{\delta \Phi}{\delta x_j}+\lambda_1 A_1+\ldots+\lambda_l A_j e, \quad (13)$$

where j = 1, 2, ..., n – the number of generalized coordinates; l – the number of linear nonholonomic links; T – the kinetic energy of the system; Π – potential energy of the system; Φ – dissipation function.

In addition, a linear nonholonomic coupling is laid on FBD system in the form

$$A_{1R}\dot{x}_1+A_{2R}\dot{x}_2+\ldots+A_{SR}\dot{x}_R; \quad R=1, 2, ..., l. \quad (14)$$

To use these equations in the study of small oscillations with respect to stationary motion, it is necessary to pass from the original equations to the equations in variations, replacing the coordinates x_j with q_i j

$$\left.\begin{aligned} x_j &= x_j(t)+q_j \\ x_j &= x_j(t)+q_j \end{aligned}\right\}.$$

For the PBd, the generalized coordinates are φ_1 и φ_2 (n = 2). That's why

$$\left.\begin{aligned}\varphi_1 &= \varphi_1(t)+q_1\\ \dot{\varphi}_1 &= \dot{\varphi}_1(t)+q_1\end{aligned}\right\}, \quad \left.\begin{aligned}\varphi_2 &= \varphi_2(t)+q_2\\ \dot{\varphi}_2 &= \dot{\varphi}_2(t)+q_2\end{aligned}\right\}, \; \varphi_2\dot{\varphi}_1-\varphi_1\dot{\varphi}_2=0. \quad (15)$$

Thus, a dynamic FBD system has one degree of freedom of movement, since n = 1.

The transition to equations in variations is carried out as follows. We perform the calculations of the terms $\frac{d}{dt}\frac{\delta T}{\delta x_j}$, $\frac{\delta T}{\delta x_j}$, $\frac{\delta \Pi}{\delta x_j}$, and then, in the obtained equations, we expand all the terms in the degrees of variation and restrict ourselves to only terms of the lowest orders.

Generalized forces are calculated taking into account the selected new variables. These forces can be calculated as coefficients for possible displacements in the expression of virtual work. The transition from the coordinates x_j to their variations q_j must also be performed for the nonholonomic coupling equation (14). For the considered movement of the belt drive, we will have a nonholonomic coupling equation in the form

$$A_1\dot{\varphi}_1+A_2\dot{\varphi}_2=0. \quad (16)$$

Expanding $A_1 = A_1(\varphi_1;\ \varphi_2)$ и $A_2 = A_2(\varphi_1;\ \varphi_2)$ in the degrees of variation and restricting ourselves to terms of the lowest orders, we obtain

$$\begin{aligned} A_1 &= A_1 I_0+\frac{\delta A_1}{\delta\varphi_1}I_0 q_1+\frac{\delta A_1}{\delta\varphi_2}I_0 q_2+\ldots;\\ A_2 &= A_2 I_0+\frac{\delta A_2}{\delta\varphi_1}I_0 q_1+\frac{\delta A_2}{\delta\varphi_2}I_0 q_2+\ldots \end{aligned} \quad (17)$$

When decomposing, we use a usual method: we assume all coefficients to be constant. In the equations (13), the terms with the index "0" correspond to stationary motion.

The equations with indefinite factors for the FBD then take the form

$$\frac{d}{dt}\frac{\delta T}{\delta\dot{\varphi}_1}-\frac{\delta T}{\delta\varphi_1}=Q_1+A_1\lambda, \quad \frac{d}{dt}\frac{\delta T}{\delta\dot{\varphi}_2}-\frac{\delta T}{\delta\varphi_2}=Q_2+A_2\lambda,$$

where Q_1, и Q_2 – generalizing forces.

An indefinite factor λ is the function of coordinates and velocities. Expanding it into a Taylor series after the transformations, we obtain

$$\lambda=\lambda_0+\sum_{j=1}^{n}\frac{\delta\lambda}{\delta\varphi_j}I_0 q_j+\sum_{j=1}^{n}\frac{\delta\lambda}{\delta\varphi_j}I_0\dot{q}_j.$$

For simplicity, we denote all lower-order terms by λ'. Then

$$\lambda=\lambda_0+\lambda'.$$

Kinetic energy is expressed as a quadratic form of generalized velocities with coefficients a_{ij}, which are determined by the coordinates of the system

$$T=\frac{a_{11}\dot{\varphi}_1^2+2a_{12}\dot{\varphi}_1\dot{\varphi}_2+a_{22}\dot{\varphi}_2^2}{2}. \quad (18)$$

After the differentiation operations with respect to generalized coordinates, we have

$$\begin{aligned} &a_{11}\ddot{\varphi}_1+a_{12}\ddot{\varphi}_2+\frac{1}{2}\frac{\delta a_{11}}{\delta\varphi_1}\dot{\varphi}_1^2+\frac{\delta a_{11}}{\delta\varphi_2}\dot{\varphi}_1\dot{\varphi}_2-\frac{1}{2}\frac{\delta a_{22}}{\delta\varphi_1}\dot{\varphi}_2^2+\\ &+\frac{\delta a_{11}}{\delta\varphi_2}\dot{\varphi}_2^2=Q_1(\varphi_1;\varphi_2;\dot{\varphi}_1;\dot{\varphi}_2)+A_1\lambda, \end{aligned} \quad (19)$$

$$\begin{aligned} &a_{12}\ddot{\varphi}_1+a_{22}\ddot{\varphi}_2+\frac{1}{2}\frac{\delta a_{22}}{\delta\varphi_2}\dot{\varphi}_2^2+\frac{\delta a_{22}}{\delta\varphi_1}\dot{\varphi}_1\dot{\varphi}_2-\frac{1}{2}\frac{\delta a_{11}}{\delta\varphi_2}\dot{\varphi}_1^2+\\ &+\frac{\delta a_{12}}{\delta\varphi_1}\dot{\varphi}_1^2=Q_2(\varphi_1;\varphi_2;\dot{\varphi}_1;\dot{\varphi}_2)+A_2\lambda. \end{aligned} \quad (20)$$

We obtain the equation of disturbed motion. Using (15), limiting ourselves to terms of lower orders and omitting transformations, we have the following equations:

- for stationary movement

$$\begin{aligned} &\frac{1}{2}\frac{\delta a_{11}}{\delta\varphi_1}I_0\dot{\varphi}_1^2(t)+\frac{\delta a_{11}}{\delta\varphi_2}I_0\dot{\varphi}_1(t)\dot{\varphi}_2(t)+\\ &+\frac{\delta a_{12}}{\delta\varphi_2}I_0\dot{\varphi}_2^2(t)=Q_1 I_0+A_1 I_0\lambda_0, \end{aligned} \quad (21)$$

$$\begin{aligned} &\frac{1}{2}\frac{\delta a_{22}}{\delta\varphi_2}I_0\dot{\varphi}_2^2(t)+\frac{\delta a_{22}}{\delta\varphi_1}I_0\dot{\varphi}_1(t)\dot{\varphi}_2(t)+\\ &+\frac{\delta a_{12}}{\delta\varphi_1}I_0\dot{\varphi}_1^2(t)-\frac{1}{2}\frac{\delta a_{11}}{\delta\varphi_2}I_0\dot{\varphi}_1^2(t)=Q_2 I_0+A_2 I_0\lambda_0. \end{aligned} \quad (22)$$

- for oscillations (in variations)

$$\begin{aligned} &a_{11}I_0\ddot{q}_1+a_{12}I_0\ddot{q}_2+\frac{\delta a_{11}}{\delta\varphi_1}I_0\dot{\varphi}_1(t)I_0\dot{q}_1+\frac{\delta a_{11}}{\delta\varphi_2}I_0\dot{\varphi}_2(t)I_0\dot{q}_1+\\ &+\frac{\delta a_{11}}{\delta\varphi_2}I_0\dot{\varphi}_1(t)I_0\dot{q}_2+2\frac{\delta a_{12}}{\delta\varphi_2}I_0\dot{\varphi}_2(t)I_0\dot{q}_2-\\ &-\frac{\delta a_{12}}{\delta\varphi_1}I_0\dot{\varphi}_2(t)I_0\dot{q}_2=\frac{\delta Q_1}{\delta\varphi_1}I_0 q_1+\frac{\delta Q_1}{\delta\varphi_2}I_0 q_2+\frac{\delta Q_1}{\delta\varphi_1}I_0\dot{q}_1+\\ &+\frac{\delta Q_1}{\delta\varphi_2}I_0\dot{q}_2+\frac{\delta A_1}{\delta\varphi_1}I_0\lambda_0 q_1+\frac{\delta A_1}{\delta\varphi_2}I_0\lambda_0 q_2+A_1 I_0\lambda', \end{aligned}$$

$$\begin{aligned} &a_{12}I_0\ddot{q}_1+a_{22}I_0\ddot{q}_2+\frac{\delta a_{22}}{\delta\varphi_2}I_0\dot{\varphi}_2(t)I_0\dot{q}_2+\frac{\delta a_{22}}{\delta\varphi_2}I_0\dot{\varphi}_2(t)I_0\dot{q}_1+\\ &+\frac{\delta a_{22}}{\delta\varphi_1}I_0\dot{\varphi}_1(t)I_0\dot{q}_2+2\frac{\delta a_{12}}{\delta\varphi_1}I_0\dot{\varphi}_1(t)I_0\dot{q}_1+\\ &+\frac{\delta a_{11}}{\delta\varphi_2}I_0\dot{\varphi}_1(t)I_0\dot{q}_1=\frac{\delta Q_2}{\delta\varphi_1}I_0 q_1+\frac{\delta Q_2}{\delta\varphi_2}I_0 q_2+\frac{\delta Q_2}{\delta\varphi_1}I_0\dot{q}_1+\\ &+\frac{\delta A_2}{\delta\varphi_1}I_0\lambda_0 q_1+\frac{\delta A_2}{\delta\varphi_2}I_0\lambda_0 q_2+A_2 I_0\lambda'. \end{aligned}$$

We divide the generalized forces into two categories. To the first, we refer the forces that have potential (П), to the second – all dissipative ones (Φ)

$$\Pi = \frac{1}{2}\left(b_{11}\varphi_1^2 + 2b_{12}\varphi_1\varphi_2 + b_{22}\varphi_2^2\right),$$

$$\Phi = \frac{1}{2}\left(l_{11}\varphi_1^2 + 2l_{12}\varphi_1\varphi_2 + l_{22}\varphi_2^2\right).$$

Then

$$Q_1 l_0 = -\frac{\delta\Pi}{\delta\varphi_1}l_0 - \frac{\delta\Phi}{\delta\dot{\varphi}_1}l_0;\quad Q_2 l_0 = -\frac{\delta\Pi}{\delta\varphi_2}l_0 - \frac{\delta\Phi}{\delta\dot{\varphi}_2}l_0;$$

$$\frac{\delta Q_1}{\delta\varphi_1}l_0 = -\frac{\delta^2\Pi}{\delta\varphi_1^2}l_0;\quad \frac{\delta a_1}{\delta\varphi_2}l_0 = \frac{\delta Q_2}{\delta\varphi_1}l_0 = \frac{\delta^2\Pi}{\delta\varphi_1\delta\varphi_2};$$

$$\frac{\delta Q_2}{\delta\varphi_2}l_0 = -\frac{\delta^2\Pi}{\delta\varphi_2^2}l_0;\quad \frac{\delta Q_2}{\delta\dot{\varphi}_1}l_0 = -\frac{\delta^2\Phi}{\delta\dot{\varphi}_1^2}l_0;$$

$$\frac{\delta Q_1}{\delta\dot{\varphi}_2}l_0 = \frac{\delta Q_2}{\delta\dot{\varphi}_1}l_0 = -\frac{\delta^2\Phi}{\delta\dot{\varphi}_1\delta\dot{\varphi}_2}l_0;\quad \frac{\delta Q_2}{\delta\dot{\varphi}_2}l_0 = -\frac{\delta^2\Phi}{\delta\dot{\varphi}_2^2}l_0.$$

If we introduce a double notation, then the equations of small oscillations of the system under consideration with respect to stationary motion can be written in the form:

$$\begin{aligned} &a_{11}\ddot{q}_1 + a_{12}\ddot{q}_2 + \left(l_{11} + \tilde{l}_{11}\right)\dot{q}_1 + l_{12}\dot{q}_2 + \tilde{\gamma}_1 q_2 + \\ &+\left(b_{11} + b'_{11}\right)q_1 + \left(b_{12} + b'_{12}\right)q_2 = A_1 l_0\lambda', \\ &a_{12}\ddot{q}_1 + a_{22}\ddot{q}_2 + l_{12}\ddot{q}_1\left(l_{22} + \tilde{l}_{22}\right)\dot{q}_2 + \tilde{\gamma}_2 q_1 + \\ &+\left(b_{12} + b'_{21}\right)q_1 + \left(b_{22} + b'_{22}\right)q_2 = A_2 l_0\lambda', \end{aligned} \tag{23}$$

where

$$a_{11} = a_{11}l_0;\ a_{12} = a_{12}l_0;\ a_{22} = a_{22}l_0;\ l_{11} = -\frac{\delta Q_1}{\delta\dot{\varphi}_1}l_0;\ l_{22} = -\frac{\delta Q_2}{\delta\dot{\varphi}_2}l_0;$$

$$l_{12} = -\frac{\delta Q_1}{\delta\dot{\varphi}_2}l_0;\ l_{21} = -\frac{\delta Q_2}{\delta\dot{\varphi}_1}l_0;\ \tilde{l}_{11} = \frac{\delta a_{11}}{\delta\varphi_1}l_0\dot{\varphi}_1 + \frac{\delta a_{11}}{\delta\varphi_2}l_0\dot{\varphi}_2;$$

$$\tilde{l}_{22} = \frac{\delta a_{22}}{\delta\varphi_2}l_0\dot{\varphi}_2 + \frac{\delta a_{11}}{\delta\varphi_1}l_0\dot{\varphi}_1;\ \tilde{\gamma}_1 = \frac{\delta a_{11}}{\delta\varphi_2}l_0\dot{\varphi}_1 + 2\frac{\delta a_{12}}{\delta\varphi_2}l_0\dot{\varphi}_2 - \frac{\delta a_{22}}{\delta\varphi_1}l_0\dot{\varphi}_1;$$

$$\tilde{\gamma}_2 = \frac{\delta a_{11}}{\delta\varphi_2}l_0\dot{\varphi}_2 + 2\frac{\delta a_{22}}{\delta\varphi_1}l_0\dot{\varphi}_1 + \frac{\delta a_{22}}{\delta\varphi_1}l_0\dot{\varphi}_2;\ b_{12} = \frac{\delta Q_1}{\delta\varphi_2}l_0 = -\frac{\delta Q_2}{\delta\varphi_1}l_0;$$

$$b_{11} = -\frac{\delta Q_1}{\delta\varphi_1}l_0;\ b_{22} = -\frac{\delta Q_2}{\delta\varphi_2}l_0;\ b'_{11} = \frac{\delta A_1}{\delta\varphi_1}l_0\lambda_0;\ b'_{12} = \frac{\delta A_1}{\delta\varphi_2}l_0\lambda_0;$$

$$b'_{21} = \frac{\delta A_2}{\delta\varphi_1}l_0\lambda_0;\ b'_{22} = \frac{\delta A_2}{\delta\varphi_2}l_0\lambda_0.$$

In this case, the terms $l_{11}\dot{q}_1$, $l_{12}\dot{q}_2$, $l_{22}\dot{q}_2$ have the form of dissipative forces, although they are not connected with the dissipative function. The terms $\tilde{\gamma}_1\dot{q}_2$ и $\tilde{\gamma}_2\dot{q}_1$ are "gyroscopic" forces силы $\left|\tilde{\gamma}_1\right| = \left|\tilde{\gamma}_2\right| = \gamma$.

Then, after a series of transformations, the equations in variations can be written in the form

$$\begin{aligned} &a_{11}\ddot{q}_1 + a_{12}\ddot{q}_2 + \left(l_{11} + \tilde{l}_{11}\right)\dot{q}_1 + \left(l_{12} + \tilde{l}_{12}\right)\dot{q}_2 + \\ &+\gamma\dot{q}_2 + \left(b_{11} + b'_{11}\right)q_1 + \left(b_{12} + b'_{12}\right)q_2 = A_1 l_0\lambda', \end{aligned} \tag{24}$$

$$\begin{aligned} &a_{12}\ddot{q}_1 + a_{22}\ddot{q}_2 + \left(l_{12} + \tilde{l}_{22}\right)\dot{q}_1 + \left(l_{22} + \tilde{l}_{22}\right)\dot{q}_2 - \\ &-\gamma\dot{q}_1 + \left(b_{12} + b'_{21}\right)q_1 + \left(b_{22} + b'_{22}\right)q_2 = A_2 l_0\lambda'. \end{aligned} \tag{25}$$

We obtain the value λ_0 using equations (21) or (22) and assume it to be known. Since in the above two equations (24) and (25) there are three unknown values − q_1, q_2 и λ', it is necessary to add the nonholonomic coupling equation to them (16), replacing the variables in it with their variations

$$\begin{aligned} &\dot{\varphi}_1\frac{\delta A_1}{\delta\varphi_1}l_0 q_1 + \dot{\varphi}_1\frac{\delta A_1}{\delta\varphi_2}l_0 q_2 + \dot{\varphi}_2\frac{\delta A_2}{\delta\varphi_1}l_0 q_1 + \\ &+\dot{\varphi}_2\frac{\delta A_2}{\delta\varphi_2}l_0 q_2 + +A_1 l_0\dot{q}_1 + A_2 l_0\dot{q}_2 = 0. \end{aligned} \tag{26}$$

To determine the expansion terms of the lowest orders λ', which are equal to $\lambda' = \sum_{j=1}^{n}\frac{\delta\lambda}{\delta\varphi_j}l_0 q_0 + \sum_{j=1}^{n}\frac{\delta\lambda}{\delta\varphi_j}l_0 q_j$, , it is necessary to find an indefinite factor λ as the function of the coordinates and velocities of the system $\lambda = \lambda\left(\varphi_j;\ \dot{\varphi}_j\right)$. To do this, it is necessary to differentiate the nonholonomic coupling equation (6) $\frac{d}{dt}\left(\sum_{j=1}^{n}A_{\nu j}q_j + a_\nu\right) = 0.$

Transforming (19) and (20) to the form $\ddot{\varphi}_1 = \frac{Q_1 + A_1\lambda}{a_{11}}$, $\ddot{\varphi}_2 = \frac{Q_2 + A_2\lambda}{a_{22}}$, and substituting them in (10), we express $\lambda = \lambda\left(\varphi_j;\ \dot{\varphi}_j\right)$, which expands into a Taylor series with the definition of the expansion terms of the lowest orders.

To draw up the equations of motion of the belt contour as an autonomous drive system, it is necessary to determine the kinetic and potential energy of the system.

Kinetic energy:

− for the drive pulley

$$T_1 = \frac{1}{2}J_1\dot{\varphi}_1^2,$$

− for driven pulley

$$T_2 = \frac{1}{2}J_2\dot{\varphi}_2^2,$$

where J_1 и J_2 are respectively the reduced moments of inertia of the driving and driven pulleys.

On the other hand, in the accepted notation

$$T_1 = \frac{1}{2}a_{11}\dot{\varphi}_1^2;\ T_2 = \frac{1}{2}a_{22}\dot{\varphi}_2^2.$$

We define potential energy as

$$\Pi = \frac{C_\varphi\left(\varphi_1 - \varphi_2\right)^2}{2},$$

where C_φ is the torsional stiffness of the transmission.

Since the influence of elastic slip on the relative rotation of the pulleys is several times higher than the deformations of the belt legs, the value C_φ will be set as depending on the value of resilient slip ξ

$$C_\varphi = \frac{M_2 l_0}{2\pi\xi}.$$

The angle of rotation of the driven pulley can be determined from the coupling equation (12), setting the angle of rotation of the driving pulley

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$$\varphi_2 l_0 = \frac{\dot{\varphi}_2}{\dot{\varphi}_1} l_0.$$

Since J_1 и J_2 do not depend on the angles of rotation of the FBD shafts, the equations of stationary motion (21) and (22) will take the form

$$Q_1 l_0 + A_1 l_0 \lambda_0 = 0,$$
$$Q_2 l_0 + A_2 l_0 \lambda_0 = 0. \quad (27)$$

From the equation (27) we find λ_0

$$Q_1 l_0 = -\frac{\delta \Pi}{\delta \varphi_1} l_0 = C_\varphi \left(\varphi_1 - \varphi_2 + \frac{\dot{\varphi}_2^2 \varphi_1}{\dot{\varphi}_1^2} \right) A_1 l_0 = \varphi_2 l_0,$$

$$\lambda_0 = \frac{Q_1}{A_1} l_0 = C_\varphi \left(\frac{\varphi_1}{\varphi_2} - 1 + \frac{\varphi_2}{\varphi_1} \right) l_0.$$

The equations in variations (24) and (25) for the belt drive at $J_1 = \text{const}$ и $J_2 = \text{const}$ will assume the form

$$a_{11}\ddot{q}_1 + l_{11}\ddot{q}_1 + l_{12}\dot{q}_2 + \gamma q_2 + \left(b_{11} + b'_{11}\right) q_1 + \\ + \left(b_{12} + b'_{12}\right) q_2 = A_1 l_0 \lambda', \quad (28)$$

$$a_{22}\ddot{q}_2 + l_{12}\ddot{q}_1 + l_{22}\dot{q}_2 + \left(b_{12} + b'_{21}\right) q_1 + \\ + \left(b_{22} + b'_{22}\right) q_2 = A_2 l_0 \lambda'. \quad (29)$$

We reveal the values of the terms on the left-hand sides of the equations (28) and (29)

$$l_{11} = C_\varphi \left(\frac{\varphi_2}{\dot{\varphi}_2} + \frac{\varphi_2}{\varphi_1} - \frac{\dot{\varphi}_2^2 \varphi_1}{\dot{\varphi}_1^4} \right) l_0 = C_\varphi N_1 l_0,$$

$$l_{12} = C_\varphi \left(-\frac{\varphi_1}{\dot{\varphi}_2^2} - \frac{\varphi_1}{\dot{\varphi}_1} + \frac{2\varphi_2}{\dot{\varphi}_1} \right) l_0 = C_\varphi N_2 l_0,$$

$$b_{11} = C_\varphi \left(1 - \frac{\dot{\varphi}_2}{\dot{\varphi}_1} + \frac{\dot{\varphi}_2^2}{\dot{\varphi}_1^2} \right) l_0 = C_\varphi N_3 l_0.$$

$$b'_{11} = 0; \quad b_{12} = C_\varphi \left(\frac{\dot{\varphi}_1}{\dot{\varphi}_2} - 1 + \frac{\dot{\varphi}_2}{\dot{\varphi}_1} \right) l_0 = C_\varphi N_4 l_0,$$

$$b'_{12} = \lambda_0; \quad l_{22} = C_\varphi \left(-\frac{\varphi_1 \dot{\varphi}_1}{\dot{\varphi}_2^3} + \frac{\varphi_1}{\dot{\varphi}_2} + \frac{\varphi_1}{\dot{\varphi}_1} \right) l_0 = C_\varphi N_5 l_0, \quad (30)$$

$$b_{22} = C_\varphi \left(-\frac{\dot{\varphi}_1^2}{\dot{\varphi}_2^2} - \frac{\dot{\varphi}_1}{\dot{\varphi}_2} + 1 \right) l_0 = C_\varphi N_6 l_0,$$

$$b'_{21} = -\lambda_0, \quad b'_{22} = 0.$$

In what follows, for simplicity of notation, the index "0" will be omitted, i.e. $\lambda_0 = \lambda$.

Differentiating the nonholonomic coupling equation (17), we obtain

$$\ddot{\varphi}_2 = \frac{\varphi_2 - \ddot{\varphi}_1 - \dot{\varphi}_1 \dot{\varphi}_{20} + \dot{\varphi}_2 \dot{\varphi}_{10}}{\varphi_1}. \quad (31)$$

From the equations of stationary motion (21), we express φ_1 и φ_2, respectively

$$\ddot{\varphi}_1 = -\frac{C_\varphi}{J_1} \left(\varphi_1 - \varphi_2 + \frac{\varphi_2 \dot{\varphi}_2}{\dot{\varphi}_1} \right) + \frac{\varphi_2}{J_1} \lambda, \quad (32)$$

$$\ddot{\varphi}_2 = -\frac{C_\varphi}{J_2} \left(\frac{\varphi_1 \dot{\varphi}_1}{\dot{\varphi}_2} - \varphi_1 + \varphi_2 \right) - \frac{\varphi_1}{J_2} \lambda. \quad (33)$$

Substituting (32) and (33) into (31), we obtain

$$\lambda = k_1 q_1 + k_2 q_2 + k_3 \dot{q}_1 + k_4 \dot{q}_2.$$

Expanding (18) in a Taylor series and discarding higher-order terms, we obtain

$$\lambda = \frac{\frac{C_\varphi}{J_2}\left(\frac{\varphi_1 \dot{\varphi}_1}{\dot{\varphi}_2} - \varphi_1 + \varphi_2 \right) - \frac{C_\varphi \varphi_2}{J_1 \varphi_1}\left(\varphi_1 - \varphi_2 + \frac{\varphi_2 \dot{\varphi}_2}{\dot{\varphi}_1} \right) - \frac{\dot{\varphi}_1 \dot{\varphi}_{20}}{\varphi_1} + \frac{\dot{\varphi}_2 \dot{\varphi}_{10}}{\varphi_1}}{\frac{\varphi_2^2}{J_1 \varphi_1} + \frac{\varphi_1}{J_2}}, \quad (34)$$

where

$$k_1 = \frac{\frac{C_\varphi}{J_2}\left(\frac{\dot{\varphi}_1}{\dot{\varphi}_2} - 1 + \frac{\dot{\varphi}_2}{\varphi_1} \right) + \frac{C_\varphi \varphi_2}{J_1 \varphi_1}\left(1 - \frac{\dot{\varphi}_2}{\dot{\varphi}_1} - \frac{\varphi_2^2}{\varphi_1^2} \right) + \frac{\dot{\varphi}_1 \dot{\varphi}_{20}}{\varphi_1} - \frac{\varphi_2 \varphi_{10}}{\varphi_1^2}}{\frac{1}{J_2} - \frac{\varphi_2^2}{J_1 \varphi_1^2}},$$

$$k_2 = \frac{\frac{C_\varphi}{J_2}\left(-\frac{\varphi_1^2}{\varphi_2^2} + 1 + \frac{\dot{\varphi}_1}{\dot{\varphi}_2} \right) - \frac{C_\varphi}{J_1 \varphi_1}\left(\frac{\dot{\varphi}_1}{\dot{\varphi}_2} - 1 + \frac{\dot{\varphi}_2}{\varphi_1} \right) + \frac{\dot{\varphi}_2 \dot{\varphi}_{20}}{\varphi_2^2} + \frac{\dot{\varphi}_1 \dot{\varphi}_{10}}{\varphi_1^2}}{\frac{2\varphi_2}{J_1 \varphi_1} + \frac{\varphi_1}{J_2 \varphi_2}},$$

$$k_3 = \frac{\frac{C_\varphi}{J_2}\left(\frac{\varphi_1}{\dot{\varphi}_2} - \frac{\varphi_2}{\dot{\varphi}_2} - \frac{\dot{\varphi}_2 \dot{\varphi}_1}{\dot{\varphi}_1^2} \right) + \frac{C_\varphi \varphi_2}{J_1 \dot{\varphi}_1^2}\left(\frac{\varphi_2}{\dot{\varphi}_2} - \frac{\dot{\varphi}_2 \varphi_1}{\dot{\varphi}_1^2} - \frac{\varphi_2 \dot{\varphi}_2}{\dot{\varphi}_1^2} \right) - \frac{\dot{\varphi}_{20}}{\varphi_1} + \frac{\varphi_2 \dot{\varphi}_{10}}{\varphi_1^2}}{\frac{\varphi_2}{J_2 \dot{\varphi}_2} - \frac{\varphi_2 \dot{\varphi}_2}{J_1 \dot{\varphi}_1^2}},$$

$$k_4 = \frac{\frac{C_\varphi}{J_2}\left(\frac{\varphi_1 \dot{\varphi}_1}{\dot{\varphi}_2} + \frac{\varphi_2 \dot{\varphi}_1}{\dot{\varphi}_2} + \frac{\varphi_1}{\dot{\varphi}_1} \right) - \frac{C_\varphi}{J_1 \varphi_1}\left(-\frac{\varphi_2 \dot{\varphi}_1}{\dot{\varphi}_2^2} - \frac{\varphi_1}{\dot{\varphi}_1} + \frac{\varphi_2}{\dot{\varphi}_1} \right) - \frac{\dot{\varphi}_{20}}{\varphi_2} + \frac{\dot{\varphi}_{10}}{\varphi_1}}{\frac{2\varphi_1 \dot{\varphi}_2}{J_1 \dot{\varphi}_2^2} - \frac{\varphi_2 \varphi_1^2}{J_2 \dot{\varphi}_1^2}}.$$

After substituting the obtained coefficients (30) into the equations (28) and (29), taking into account (34), we will obtain

$$J_1 \ddot{q}_1 + C_\varphi N_1 \dot{q}_1 + C_\varphi N_2 q_2 + C_\varphi N_3 q_1 + \left(C_\varphi N_4 + \lambda_0 \right) q_2 = \\ = A_1 \left(k_1 q_1 + k_2 q_2 + k_3 \dot{q}_1 + k_4 \dot{q}_2 \right), \quad (35)$$

$$J_2 \ddot{q}_2 + C_\varphi N_3 \dot{q}_1 + C_\varphi N_5 q_2 + \left(C_\varphi N_4 - \lambda_0 \right) q_1 + C_\varphi N_6 q_2 = \\ = A_2 \left(k_1 q_1 + k_2 q_2 + k_3 \dot{q}_1 + k_4 \dot{q}_2 \right). \quad (36)$$

To the obtained equations of motion (35) and (36), it is necessary to add a nonholonomic coupling equation in variations, which, taking into account (26), will have the form

$$\dot{\varphi}_1 q_2 - \dot{\varphi}_2 q_1 + \varphi_2 \dot{q}_1 - \varphi_1 \dot{q}_2 = 0.$$

For the joint solution of (35) and (36), it is necessary to determine from the coupling equation in variations the variation of the coordinate q_1, equal to $q_1 = \frac{\dot{\varphi}_1 q_2 - \varphi_2 \dot{q}_1 - \varphi_1 \dot{q}_2}{\dot{\varphi}_2}$ and substitute it into the equation of motion of the driven pulley (36).

Similarly, the deduced coordinate variation must be substituted into the equation of motion of the driving pulley (35).

Omitting the transformations, having performed these operations and made the adduction of similar terms, we obtain an FBD analytical model for torsional vibrations, i.e. equation of small oscillations regarding stationary movement

$$\left.\begin{aligned} J_1\ddot{q}_1 + z_{11}\dot{q}_1 + z_{12}q_1 + z_{13}\dot{q}_2 = 0 \\ J_2\ddot{q}_2 + z_{21}\dot{q}_2 + z_{22}q_2 + z_{23}\dot{q}_1 = 0 \end{aligned}\right\}, \quad (37)$$

where

$$z_{11} = C_\varphi N_1 - \left(C_\varphi N_4 + \lambda_0\right)\frac{\varphi_2}{\dot{\varphi}_1} + \frac{\varphi_1\varphi_2 k_2}{\dot{\varphi}_1} - A_2 k_3,$$

$$z_{12} = C_\varphi N_3 + \left(C_\varphi N_4 + \lambda_0\right)\frac{\varphi_2}{\dot{\varphi}_1} + \frac{\varphi_1\varphi_2 k_2}{\dot{\varphi}_1} - A_2 k_1,$$

$$z_{13} = C_\varphi N_2 + \left(C_\varphi N_4 + \lambda_0\right)\frac{\varphi_1}{\dot{\varphi}_1} + \frac{\varphi_1^2 k_2}{\dot{\varphi}_1} - A_2 k_4,$$

$$z_{21} = C_\varphi N_5 - \left(C_\varphi N_4 - \lambda_0\right)\frac{\varphi_1}{\dot{\varphi}_2} + \frac{\varphi_1^2 k_1}{\dot{\varphi}_2} - A_2 k_4,$$

$$z_{22} = C_\varphi N_6 + \left(C_\varphi N_4 - \lambda_0\right)\frac{\dot{\varphi}_1}{\dot{\varphi}_2} + \frac{\varphi_1\dot{\varphi}_1 k_1}{\dot{\varphi}_2} - A_2 k_2,$$

$$z_{23} = C_\varphi N_2 - \left(C_\varphi N_4 - \lambda_0\right)\frac{\varphi_2}{\dot{\varphi}_2} + \frac{\varphi_1\varphi_2 k_1}{\dot{\varphi}_2} - A_2 k_3.$$

To study the stability of the FBD motion, it is required to solve a system of motion equations (37) or obtain a characteristic equation and solve its roots. We use the operator notation. The system (12) is transformed to the form

$$\left.\begin{aligned} \left(J_1 p^2 + z_{11}p + z_{12}\right)q_1 + z_{13}pq_2 = 0, \\ z_{23}pq_1 + \left(J_2 p^2 + z_{21}p + z_{22}\right)q_2 = 0 \end{aligned}\right\}.$$

As a result, a homogeneous system of linear algebraic equations was obtained. For its solution to be non-trivial, the determinant of this system must be equal to zero. Expanding the determinant, we obtain a characteristic equation of the system in operator form:

$$\begin{aligned} D_p = J_1 J_2 p^4 + \left(J_2 z_{11} + J_1 z_{21}\right)p^3 + \\ + \left(z_{11}z_{21} + J_1 z_{22} - z_{13}z_{23}\right)p^2 + \\ + \left(J_2 z_{12} + z_{12}z_{21} + z_{11}z_{22}\right)p + z_{12}z_{22} = 0. \end{aligned} \quad (38)$$

In view of a significant complexity of calculating the roots of the characteristic equation of high orders, we will use the method of approximate calculation with the help of stability criteria. We use Mikhailov stability criterion [6]. This criterion envisages the use of a frequency hodograph $D(j\upsilon)$, which can be obtained from the characteristic equation at $p = j\upsilon$.

We replace p by $j\upsilon$ in polynomial (23) and select substantial and imaginary parts

$$D(j\nu) = U(\nu) + jV(\nu). \quad (39)$$

Taking into account (116), the expression (117) is transformed to the form

$$U(\nu) = J_1 J_2 p^4 - \left(z_{11}z_{21} + J_1 z_{22} - z_{13}z_{23}\right)\nu^2 + z_{12}z_{22},$$

$$V(\nu) = -\left(J_2 z_{11} + J_1 z_{21}\right)\nu^3 - \left(J_2 z_{12} + z_{12}z_{21} + z_{11}z_{22}\right)\nu.$$

The curve described by the end of the vector $D(j\upsilon)$ when the frequency changes from zero to infinity, is shown in accordance with Figure 2. It was obtained for the FBD of the RUVI kitchen machine with the parameters: J_1 = 0,009 Нмс²; J_2 = 0,004 Нмс²; D_1 = 18,5 мм; D_2 = 82,5 мм.

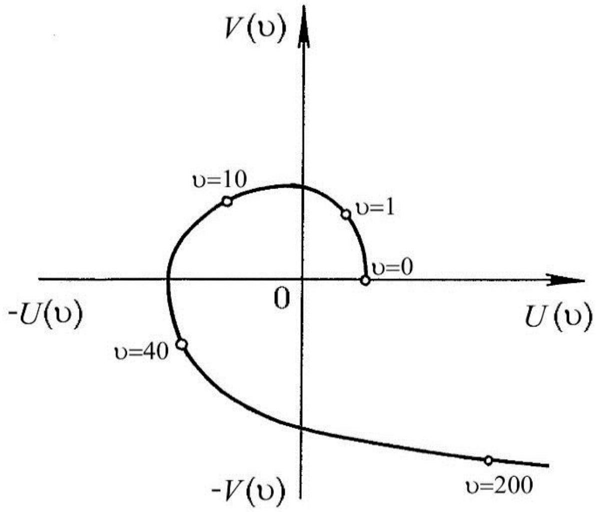

Figure 2 – Mihajlov Frequency hodograph for the transmission of power by friction type flexible link

Conclusions

The frequency hodograph begins on the positive part of the real axis and traverses four quadrants sequentially in the positive direction. Therefore, in accordance with Mikhailov's criterion, the dynamic system of the transmission under consideration is stable. The boundaries of the stability regions of the stationary motion of the FBD can be estimated by the decrease (loss) in the angular velocity of the driven pulley from the load. If, as is customary in the theory of belt drives [7-10], the loss of angular velocity is estimated by the value of the resilient slip coefficient, then a stable operation of the considered FBD under the condition $\xi \le 5\%$ will be observed at $2F_0 \ge 53{,}5\ N$. Thus, for increasing the stability of movement, the value of the belt pre-tension should not be less than the permissible values.

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