$$\ln M0 = 2.975 + 0.003 \ln w - 0.035 R_0, \quad R^2 = 0.921.$$
$$[3.017] \quad [27.7] \quad [-3.735]$$
$$(0.04) \quad (0.000) \quad (0.007)$$

$$\ln M2 = 4.857 + 0.0121 \ln w - 0.077 R_0 \quad R^2 = 0.917.$$
$$[1.384] \quad [35.090] \quad [-2.589]$$
$$(0.176) \quad (0.000) \quad (0.014)$$

Вопрос изучения спроса на деньги становится одним из актуальных вопросов макроэкономики.

В данной работе рассматривалось влияние трех экономических показателей – претендентов на роль переменной y в уравнении (1):

- ВВП;
- средней зарплаты;
- объема промышленного производства.

Дополнение модели (2) факторами увеличило объясняющую долю вариации (R^2).

Коэффициенты при факторах в формуле (3) – суть коэффициенты эластичности изменения показателя m_t в зависимости от изменения регрессоров. В частности из формулы (3) следует, что при изменении уровня w на 1% M2 возрастает на 0.13%, при увеличении ставки по депозитам на 1% M2 уменьшится на 0.075%.

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ON APPROXIMATION OF THE SOLUTIONS OF STOCHASTIC EQUATIONS WITH θ-INTEGRALS

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Abstract. The paper investigates a problem of approximation of stochastic θ-integrals and the solutions of stochastic differential equations. It is proposed complete classifications of the ways of approximations stochastic θ-integrals and the solutions of stochastic integral equations with θ-integral in the convolution algebra.

Key words: standard process of Brownian motion, stochastic θ-integral, stochastic differential equation, convolution algebra, generalized random process, equations in differentials.

The classical methods can't be applied to differential equations including generalized stochastic processes of the "white noise" type. For these equations, a special theory of stochastic differential equations was developed, which is based on definitions of Itô integral [1], Stratonovich integral [2], stochastic θ-integral [3], and others.

In papers [4, 5] the algebra of generalized stochastic processes was introduced based on the Egorov algebra [6] of generalized functions.

The main idea of the construction of algebra of generalized stochastic processes is the approximation of the stochastic processes by smooth functions and multiplying these functions. Of course, the result will depend on the ways of approximations and the right strategy is to include approximate sequence (or equivalent sequence in this or that meaning) into the definition of a generalized stochastic process. That why the problem of classifications of the ways of approximations is raised.

In this subsection we propose complete classifications of the ways of approximations stochastic θ-integrals in the convolution algebra (it is a sample of algebra of generalized stochastic processes). Concurrently we estimate the speed of approximations. These estimates are optimal.

Let $(\Omega, \mathrm{A}, \mathbf{P})$ be a complete probability space, $T=[0, a] \subset \mathbf{R}$, $\{\Phi_t\}_{t\in T}$ a standard flow of σ-algebras, $\Phi_a \subset \mathrm{A}$. Let $\{B(t), t\in T\}$ be a one-dimensional standard process of Φ_t-Brownian motion [7].

As a representative of generalized process of Brownian motion [5] we choose

$$B_n(t)=(B*\rho_n)(t)= \int_0^{1/n} B(t+s)\rho_n(s)ds, \quad (1)$$

where $\rho_n(t)\in D(\mathbf{R})$, $\rho_n(t)\geq 0$, $supp\ \rho_n(t) \subset [0,1/n]$ and $\int_0^{1/n} \rho_n(s)ds=1$.

Lemma 1. For any $t \geq 0$, $h_n > 0$, $n\in\mathbf{N}$ the following equality is valid:

$$\mathbf{E}[B_n(t+h_n) - B_n(t)]^2 = h_n \iint\limits_{\substack{0\leq s,\tau\leq 1/n \\ |s-\tau|\leq h_n}} \left(1-\frac{|s-\tau|}{h_n}\right)\rho_n(s)\rho_n(\tau)ds\,d\tau = h_n K(n, h_n).$$

Remark 1. It is evident, that $0 \leq K(n, h_n) \leq 1$ for any $h_n > 0$, $n\in\mathbf{N}$.

In what follows, C is an absolute constant, independent from n, h_n, t.

As a representative of the generalized function $\tilde{f}$ [6], associating function f:**R**→**R**, we choose

$$f_n = f * \rho_n, \tag{2}$$

where ρ_n from equality (1).

The following theorems give complete classification of ways of approximations of stochastic θ-integrals, $0 \le \theta \le 1$ in the convolution algebra.

Theorem 1. *Let* $f \in C^2_B(\mathbf{R})$, $f \not\equiv const$. *The finite sum* $\sum_{k=1}^{m_t} f_n(B_n(\tau_t+(k-1)h_n)) \times$ $\times[B_n(\tau_t+kh_n) - B_n(\tau_t+(k-1)h_n)]$ *converges in* $L^2(\Omega, A, \mathbf{P})$ *and uniformly in* $t \in T$ *as* $n \to \infty$, $h_n \to 0$ *if, and only if numerical sequence* $K(n,h_n)$ *converges as* $n \to \infty$, $h_n \to 0$.

Theorem 2. *Let* $f \in C^2_B(\mathbf{R})$, $\theta \in [0;1/2]$, *then the inequality is valid*

$$\sup_{t\in T} \mathbf{E}\Big[\sum_{k=1}^{m_t} f_n(B_n(\tau_t+(k-1)h_n))[B_n(\tau_t+kh_n) - B_n(\tau_t+(k-1)h_n)] - (\theta)\int_0^t f(B(s))dB(s)\Big]^2 \le$$
$$\le Ch_n + C/n + C(K(n,h_n) - (1-2\theta))^2.$$

Theorem 3. *Let* $f \in C^2_B(\mathbf{R})$, $f \not\equiv const$. *The finite sum* $\sum_{k=1}^{m_t} f_n(B_n(\tau_t+kh_n)) \times$ $\times [B_n(\tau_t+kh_n) - B_n(\tau_t+(k-1)h_n)]$ *converges in* $L^2(\Omega, A, \mathbf{P})$ *and uniformly in* $t \in T$ *as* $n \to \infty$, $h_n \to 0$ *if, and only if numerical sequence* $K(n,h_n)$ *converges as* $n \to \infty$, $h_n \to 0$.

Theorem 4. *Let* $f \in C^2_B(\mathbf{R})$, $\theta \in [1/2;1]$, *then the following inequality is valid*

$$\sup_{t\in T} \mathbf{E}\Big[\sum_{k=1}^{m_t} f_n(B_n(\tau_t+kh_n))[B_n(\tau_t+kh_n) - B_n(\tau_t+(k-1)h_n)] - (\theta)\int_0^t f(B(s))dB(s)\Big]^2 \le$$
$$\le Ch_n + C/n + C(K(n,h_n) - (2\theta-1))^2.$$

We consider equation

$$X(t) = x + (\theta)\int_0^t f(X(s))dB(s) + \int_0^t g(X(s))ds, \; t \in T, \tag{3}$$

where $x \in \mathbf{R}$, $\theta \in [0;1]$, $f \in C^2_B(\mathbf{R})$, $g \in C^1_B(\mathbf{R})$ and the stochastic integral in the right-hand side of (3) is an θ-integral.

We investigate the problem of approximation of the solution of equation (3) by the following Cauchy problem

$$\begin{cases} X_n(t+h_n)-X_n(t)=f_n(X_n(t))[B_n(t+h_n)-B_n(t)]+g_n(X_n(t))h_n \\ X_n(t)\big|_{[0,h_n)}=X_{n0}(t),\ t\in T, \end{cases} \quad (4)$$

where "initial value" $X_{n0}(t)$ is square integrable and $\Phi_{t+1/n}$ measurable for all $t\in[0;h_n)$, B_n and f_n from representations (1) and (2) respectively, $g_n=g*\rho_n$.

Theorem 5. *Suppose* $\theta\in[0;1/2]$, $f\in C_B^2(\mathbf{R})$ *and* $g\in C_B^1(\mathbf{R})$. *Then*

$$\sup_{t\in T}\mathbf{E}[X_n(t)-X(t)]^2\le C\sup_{t\in[0,h_n)}\mathbf{E}[X_{n0}(t)-x]^2+C/(n^{2/3}h_n^{1/3})+C(K(n,h_n)-(1-2\theta))^2,$$

where $X_n(t)$ *and* $X(t)$ *are the solutions of the equations* (5) *and* (3) *respectively.*

Theorem 6. *Suppose* $\theta\in[0;1/2]$, $f\in C_B^2(\mathbf{R})$, $f\neq const$, $g\in C_B^1(\mathbf{R})$, $n^2h_n\to\infty$ *and* $\sup_{t\in[0,h_n)}\mathbf{E}[X_{n0}(t)-x]^2\to 0$ *as* $n\to\infty$, $h_n\to 0$. *Then the solution of the Cauchy problem* (4) $X_n(t)$ *converges in* $L^2(\Omega,A,\mathbf{P})$ *and uniformly in* $t\in T$ *as* $n\to\infty$, $h_n\to 0$ *if, and only if numerical sequence* $K(n,h_n)$ *converges as* $n\to\infty$, $h_n\to 0$.

For approximation the solution of equation (3) when $\theta\in(1/2;1]$ we should consider the following finite-difference equation with outstrip

$$\begin{cases} Y_n(t+h_n)-Y_n(t)=f_n(Y_n(t+h_n))[B_n(t+h_n)-B_n(t)]-g_n(Y_n(t+h_n))h_n \\ Y_n(t)\big|_{[t,h_n[}=Y_{n0}(t),\ t\in T. \end{cases} \quad (5)$$

We use the same notation as in Cauchy problem (4).

Theorem 7. *Suppose* $\theta\in[1/2;1]$, $f\in C_B^2(\mathbf{R})$ *and* $g\in C_B^1(\mathbf{R})$. *Then*

$$\sup_{t\in T}\mathbf{E}[Y_n(t)-X(t)]^2\le C\sup_{t\in[0,h_n)}\mathbf{E}[Y_{n0}(t)-x]^2+C/(n^{2/3}h_n^{1/3})+C(K(n,h_n)-(2\theta-1))^2,$$

where $Y_n(t)$ *and* $X(t)$ *are the solutions of the equations* (5) *and* (3) *respectively.*

Theorem 8. *Suppose* $\theta\in[1/2;1]$, $f\in C_B^2(\mathbf{R})$, $f\neq const$, $g\in C_B^1(\mathbf{R})$ $n^2h_n\to\infty$ *and* $\sup_{t\in[0,h_n)}\mathbf{E}[Y_{n0}(t)-x]^2\to 0$ *as* $n\to\infty$, $h_n\to 0$. *Then the solution of the Cauchy problem* (5) $Y_n(t)$ *converges in* $L^2(\Omega,A,\mathbf{P})$ *and uniformly in* $t\in T$ *as* $n\to\infty$, $h_n\to 0$ *if, and only if numerical sequence* $K(n,h_n)$ *converges as* $n\to\infty$, $h_n\to 0$.

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ПОЛУБРАУДЕРОВЫ ОПЕРАТОРЫ И СВЯЗАННЫЕ С НИМИ СПЕКТРЫ.

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Аннотация. В данной работе рассматриваются полубраудеровские операторы и спектры, порожденные этими операторами, имеющие приложения в теории устойчивости и теории возмущений дифференциальных операторов.

Ключевые слова. Полубраудеровы операторы, полубраудеровы спектры, операторы взвешенного среднего.

Пусть $R(T)$ - область значений оператора $T{:}X\to X$, где X-банахово пространство; $N(T)$ – его ядро. Обозначим числовые характеристики линейного оператора T следующим образом: $\mathrm{nul}(T){:=}\dim B(T)$; $\mathrm{def}(T){:=}\mathrm{codim}R(T){=}\dim X/R(T)$; $\mathrm{ind}(T){:=}\mathrm{def}(T){-}\mathrm{nul}(T)$, $a(T)$ – подъем оператора T, т.е. наименьшее число $n\in\wedge\cup\{0\}$ такое, что $N(T^n)=N(T^{n+1})$; $d(T)$ – спуск оператора T, т.е. наименьшее число $n\in\wedge\cup\{0\}$ такое, что $R(T^n)=R(T^{n+1})$.

Оператор $T\in B(X)$ называется верхним полубраудеровым, если $T\in\{T\in B(X){:}R(T)=\overline{R(T)},\mathrm{nul}(T)<\infty,a(T)<\infty\}$; оператор $T\in B(X)$ называется нижним полубраудеровым, если $T\in\{T\in B(X){:}R(T)=\overline{R(T)},\mathrm{def}(T)<\infty,d(T)<\infty\}$; оператор T называется браудеровым, если он является одновременно верхним полубраудеровым и нижним полубраудеровым. Впервые название «полубраудеровы операторы» было введено R. Harte в книге [1] (определение 7.9.1).

Приведем некоторые свойства полубраудеровых операторов:

Если X – банахово пространство и $S,T\in BL(X,X)$, $ST=TS$, то

1. S,T – верхние полубраудеровы $\Leftrightarrow$ ST – верхний полубраудеров;