The Regularities of the Development of the Nonlinear Dynamics of the Control Systems with Pulse-Width Modulation

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Abstract. The paper reports on the dynamics of the control systems with pulse-width modulation of the second kind. The bifurcations of the stationary motions are studied. The regularities of occurrence of chaotic processes are revealed.

Key words. Pulse control systems, nonlinear dynamics, bifurcation, chaos.

1. Introduction

The control systems with pulse-width modulation (PWM) belong to the class of essentially nonlinear dynamic systems of variable structure. The use of the control systems with pulse-width modulation in many directions of human practice such as transport, mining industry, metallurgical and coal industry, etc. are related with the solution of the problems of qualitative transformation of great quantities of energy (from one or even less then one kW to hundreds thousands kW). In this case the compulsory requirements to the dynamics of the process of transformation of energy are as follows:

- ensuring of efficiency;
- safeguarding of security, namely, the forecast and elimination of possible emergency (catastrophic) situations;
- ensuring of compatibility (including electromagnetic) with environment. But the efficiency of the process of transformation is in contradiction with the requirements of security and compatibility.

The realizations of these requirements complicated by the fact that many control systems with PWM according to the request for the proposal run under conditions of variation in their own parameters or in the parameters of conjugate systems over a wide range.

It is known [1,2] that for different combinations of parameters the dynamics of the control systems with PWM may be regular or chaotic. In this case stationary motions, different from the predetermined one while designing, may occur both "mildly" and catastrophically. The study of the regularities of the development of the dynamics of the control systems with PWM, that is, the analysis of bifurcation boundaries in the parameter space of the control systems is of great importance for the problems of parametric and structural optimization, identification of stationary motions as well as for the choice of optimal control in such systems.

This research has been carried out with the scope of the approach, which is being formed by the authors, to the analysis of the dynamics of the pulse control systems. The aims of the research is to show characteristic regularities of occurrence of chaotic phenomena for the typical structure of the control systems with PWM-2 and to find the intercommunication between the revealed regularities and the principles of the constructions of the control systems under considerations.

2. Mathematical model

The mathematical model of the standard structure of control system with PWM [1,2] has the form

$$\frac{dX}{dt} = G(X) + B(K_F), \qquad (1)$$

where X is the vector of state variables of the control system; the vector function G(X) and the vector $B(K_F)$ describe the elements of the control system which are invariant in time; K_F is the pulse function, which determines the state of the key element of the control system in accordance with the sign of the function of commutation

$$\xi(t, X(t)) = \alpha \cdot (U_c - S' \cdot X(t)) - U_0 \left(\frac{t}{a} - E_1\left(\frac{t}{a}\right)\right), \quad (2)$$

accordance to the algorithm

$$K_F = \begin{cases} 1, & \xi(t, X(t)) \geq 0, \\ 0, & \xi(t, X(t)) < 0. \end{cases} \quad (3)$$

In equations (2) and (3) α is the coefficient of amplification of the error signal; U_C is the set point; S' is the vector line, which establishes the agreement between the state variables of the control system and the value at the input of the pulse modulator; U_0 is the amplitude of saw tooth voltage; a is the timing period of the pulse-width modulation; $E_1(.)$ is the function whose result represent a whole part of the argument.

The moment of the change of the structure of the control system t_k (the commutation time) during the period of the PWM $(k-1) \cdot a<t<k\,a$ is determined in accordance with the algorithm [1]

$$t_k = \begin{cases} (k-1) \cdot a, \xi((k-1) \cdot a, X_{k-1}) \leq 0 \\ k \cdot a, \xi(t, X(t)) > 0, (k-1)\ a < t < k\ a \\ t_k \rightarrow \xi(t_k, X(t_k)) = 0, \\ \xi((k-1) \cdot a, X_{k-1}) > 0, \\ \xi(k \cdot a, X_k) < 0, k = 1, 2, \ldots \end{cases} \quad (4)$$

Where X_K and X_{K-1} are the values of the vector of state variables of the control system, taken in discrete time $k \cdot a$ and $(k-1) \cdot a$; the third line means that t_k is determined as the least root of the corresponding equation.

3. Descriptions of the results

As the subject of investigations is the D.C. traction motor drive with the pulse regulation of its current and PI-stage in the current regulator, whose equivalent scheme is shown in Fig.1. Using the physical principles of functioning of the system under consideration as the base we can present the system in the form of two interacted oscillators.

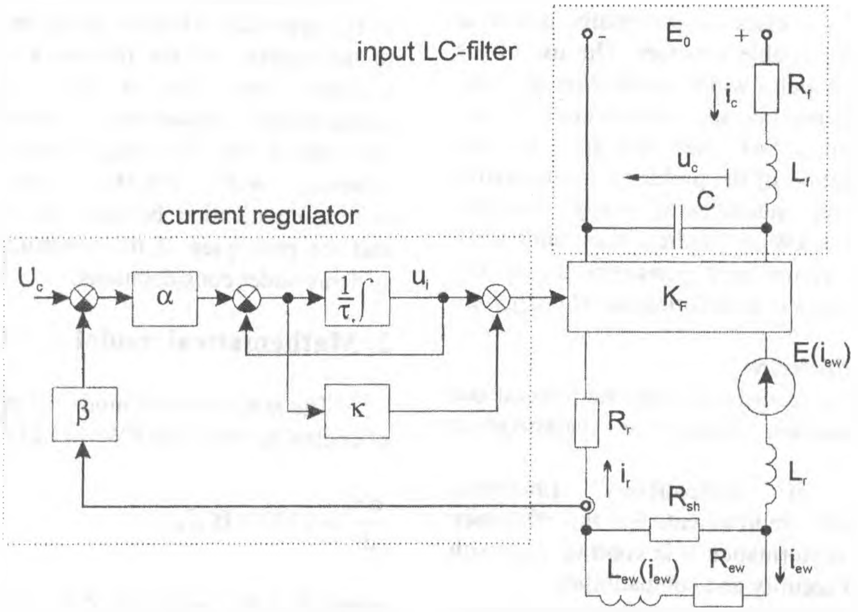

Fig. 1. The equivalent scheme of the D.C. traction motor drive with PWM

The first oscillator is the input LC-filter of the system and as the second oscillator may be considered the control system with PWM itself (the system with outer synchronizing impact, whose period is $T=a$) as a combination of the subject of regulation (of the motor), the regulating element (of the pulse converter) and the current regulator with PI-stage.

It is quite evident that the dynamics of such a system will be highly complicated. Here you may see both the phenomena cased by the interaction of two oscillators with irrational (in the general case) ratio of their frequencies and the phenomena which are due to the structure and the principle of operation of the regulator (especially the type and kind of modulation). To make a qualitative analysis of the regularities of the development of the dynamics of the system under consideration we shall carry on research in two stages. First, let us consider the control system with PWM-2 without taking into account the input LC-filter (that is, we proceed from the assumption that the input of the pulse element is connected to a source of unlimited power). The aim on consideration of such a simplified scheme is to reveal characteristic regularities of the development of the dynamics stipulated by the structure and the principles of operation of the regulator of the control system.

The mathematical model of the simplified system has the form

$$\frac{dX}{dt}=A\cdot X+B(K_F),\ X=\{i_r,i_{ew},u_i\}', \qquad (5)$$

$$A=\begin{vmatrix} -\frac{(R_r+R_{sh})}{L_r} & \frac{(R_{sh}-k_r n)}{L_r} & 0 \\ \frac{R_{sh}}{L_{ew}} & -\frac{(R_{sh}+R_{ew})}{L_{ew}} & 0 \\ \frac{\alpha\beta}{\tau} & 0 & -\frac{1}{\tau} \end{vmatrix},$$

$$B(1)=\begin{vmatrix} \frac{(E_0-k_e n)}{L_r} \\ 0 \\ -\frac{\alpha U_c}{\tau} \end{vmatrix},\ B(0)=\begin{vmatrix} -\frac{k_e n}{L_r} \\ 0 \\ -\frac{\alpha U_c}{\tau} \end{vmatrix},$$

$$\xi(t,X(t))=\alpha\cdot(\kappa\cdot U_c-S'\cdot X(t))-U_0\cdot\left(\frac{t}{a}-E_1\left(\frac{t}{a}\right)\right),$$

$$S'=\left\{\kappa\cdot\beta,0,\frac{1-\kappa}{\alpha}\right\},$$

where R_r, R_{sh}, R_{ew}, L_{ew}, L_r, α, β, τ, κ, U_c, U_0 - are the parameters of the elements of the control system, n is the velocity of the shaft of electric motor, E_0 is a power supply voltage. The nonlinear dependence of magnetic flux of the motor on the excitation winding current was approximated by the dependence $c_e\hat{O}(i_{ew})=k_r\cdot i_{ew}+k_e$. The commutation of the pulse converter is accomplished in accordance with equations (3)-(4).

Let the parameters α and n to be varied. The rest parameters are assumed to be fixed and having the following values $R_r=0{,}0427\ \Omega$; $R_{sh}=0{,}615\ \Omega$; $R_{ew}=0{,}027\ \Omega$; $L_{ew}=0{,}98\ mH$; $L_r=3{,}32\ mH$; $\beta=0{,}01$; $\tau=0.01\ s$; $\kappa=0.1$; $U_c=3\ V$; $U_0=1{,}5\ V$; $E_0=550\ V$; $a=0{,}0025\ s$, $k_r=0{,}0005$; $k_e=0{,}146$.

The analysis of the system (5) was made in the terms of the shift mapping [2].

$$X_k=C_kX_{k-1}+V_k, \qquad (6)$$

$$C_k=C^{(0)}C^{(1)},\ V_k=C^{(0)}V^{(1)}+V^{(0)},$$

$$C^{(1)}=e^{A\,a\,z_k},\ C^{(0)}=e^{A\,a(1-z_k)},$$

$$V^{(1)}=-[E-C^{(1)}]A^{-1}B(1),$$

$$V^{(0)}=-[E-C^{(0)}]A^{-1}B(0),\ k=1,2,\dots.$$

The results of the analysis of the dynamics of the model (5) in the space of parameters $\Pi=\{\alpha, n; 3<\alpha<20, 200<n<2000\}$ are in Fig.2a as a two parametric bifurcation diagram. To understand these results better, in Fig.2b-2c we represent one parametric bifurcation diagrams built up for the sections $\alpha=7$ and $\alpha=18$ respectively. The domains of existence of different periodic motions are denoted in Fig.2a as Π with index, were the index determines the period ratio of the corresponding motion relative to the period of synchronizing impact of PWM (the number of m-cycle [1,2]). The domain of existence of chaotic motion is denoted as Π_{chaos} (see Fig.2a). The characteristic bifurcation curves are denoted as N with two indices (Fig.2a). Here the lower index "-" determines the curve of classic "mild" period doubling, and the lower index "c" determines the C-bifurcation curve [3]. The upper index of N determines the number of m-cycle that undergo a bifurcation. In Fig.2a Γ_0 denotes the curve of break of the conditions of existence of periodic motions in the system.

Even superficial study of the diagram in Fig.2 enables to notice that, first, the domain of periodicity are alternated with the domains of chaotic motion and, second, the domains of periodicity are in the order of increasing of their periods in two times (see Fig.2a from below to upward). Such a superficial view would be sufficient for the continuous dynamic system to come to the conclusion, that the characteristic scenario of occurrence of chaotic motion is the scenario of period doubling. But for the discrete system (5) it is not sufficient. The main difference of the dynamics of the system (5) from "canonic" scenario of period doubling is the presence of C-bifurcation boundaries N_c in the plane Π. For more detailed study of bifurcations of dynamic motions of the system (5)

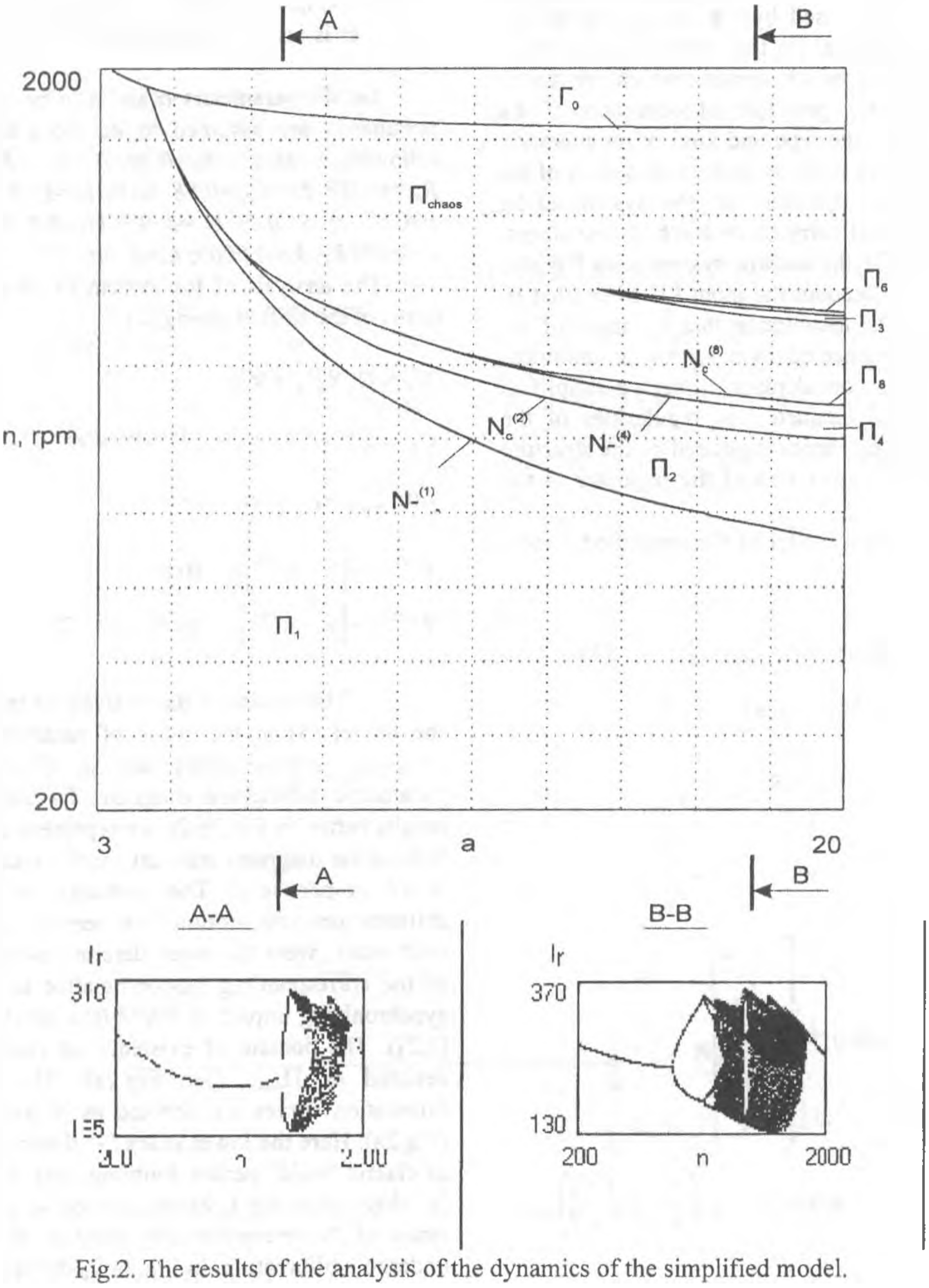

Fig.2. The results of the analysis of the dynamics of the simplified model.

Fig.3 shows a fragment of one parametric bifurcation diagram for the section *α=9,1; 1200 rpm<n<1460 rpm* and the diagram of greatest multiplicator of the corresponding stationary motions. The values of the *i_r[k] k=0,1,...* (corresponding to the shift mapping) are plotted an the axis Y (see Fig.3a).The unstable stationary motions are marked by a dotted line in Fig.3.

Over the range $n < n_-^{(1)}, \left(\alpha = 9{,}1; n_-^{(1)}\right) \in N_-^{(1)}$ there exists a unique stable motion- 1-cycle. When n reaches the value $n_-^{(1)}$ the 1-cycle looses its stability as a result of bifurcation of period doubling and a stable 2-cycle appears in the

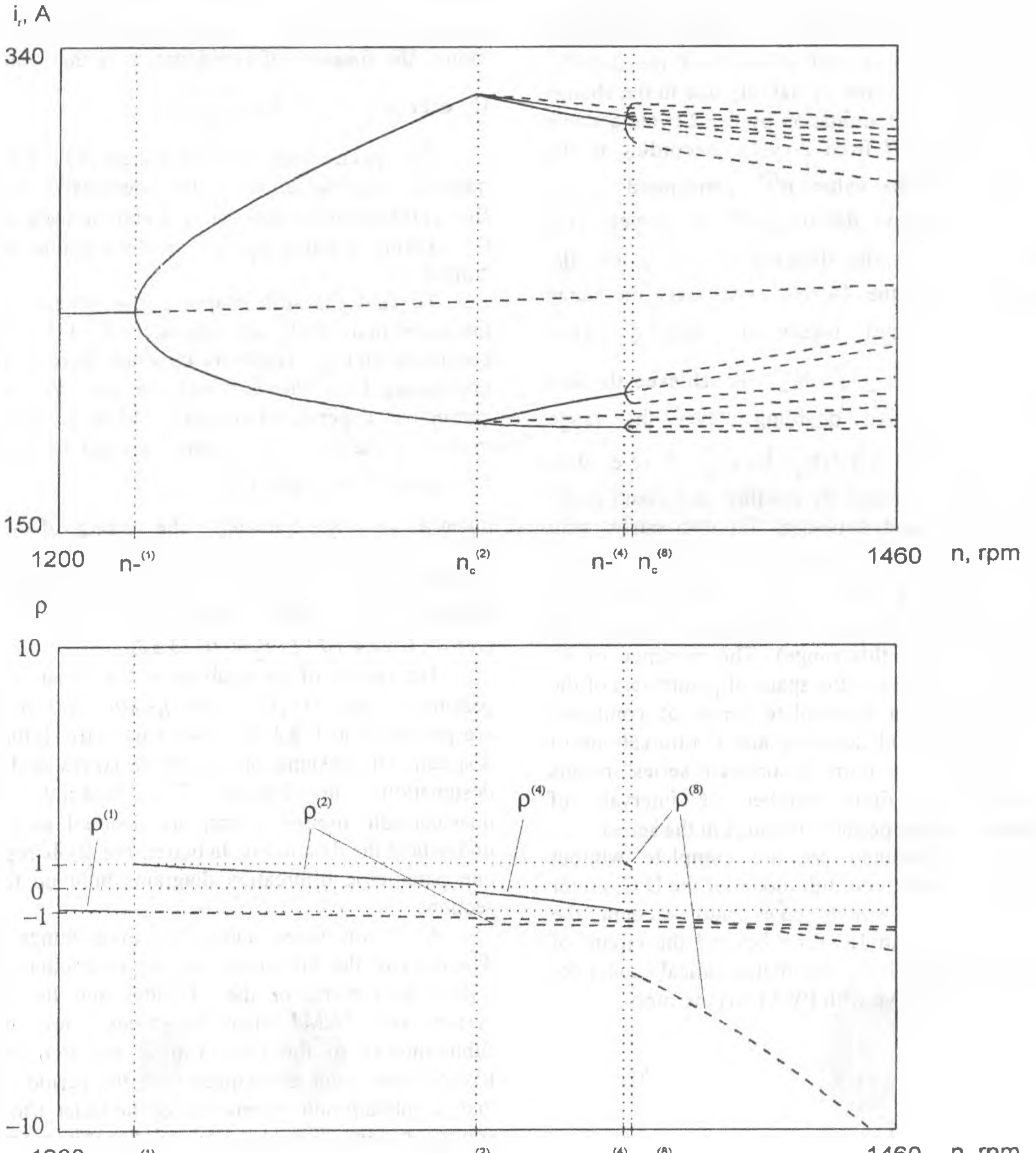

Fig.3. A fragment of one parametric bifurcation diagram and the diagram of greatest multiplicator of the corresponding stationary motions

system, which in its turn is a unique stable stationary motion in the system over a range $n_{-}^{(1)} < n < n_{c}^{(2)}, (\alpha = 9{,}1; n_{c}^{(2)}) \in N_{c}^{(2)}$. When $n = n_{c}^{(2)}$ is achieved, the domain of determination of the 2-cycle is changed (that is, the quantity of intervals of structure constancy of the system is changed which form a stationary motion) and the multiplicators of the 2-cycle are changed abruptly. In this case the greatest multiplicator becomes more than *-1* that determines the break of the conditions of stability of the 2-cycle. Simultaneously, with loss of stability due to the change of the domain of determination by the 2-cycle a stable 4-cycle branches off from 2-cycle. According to the classification [3] the value $n_{c}^{(2)}$ corresponds to C-bifurcation of period doubling of the 2-cycle (see boundary $N_{c}^{(2)}$ on the diagram of Fig.2). On the diagram of Fig.3a the 4-cycle exists over the range $n_{c}^{(2)}$ to $n_{-}^{(4)}$ and looses its stability when $n = n_{-}^{(4)}, (\alpha = 9{,}1; n_{-}^{(4)}) \in N_{-}^{(4)}$ is achieved through bifurcation of period doubling. Over the range $n_{-}^{(4)} < n < n_{c}^{(8)}, (\alpha = 9{,}1; n_{c}^{(8)}) \in N_{c}^{(8)}$ there exists a stable 8-cycle that looses its stability as a result of C-bifurcation of period doubling. Simultaneously, with loss of stability of the 8-cycle an unstable 16-cycle branches off from it. With $n > n_{c}^{(8)}$ there is no stable stationary motions in the system (a chaotic motion exist in the system over this range). The presence of C-bifurcation boundaries in the space of parameters of the system stipulates an incomplete series of combined bifurcations of period doubling and C-bifurcations of period doubling. The term "incomplete series" means that there is a finite number of intervals of corresponding stable periodic motions in the series.

The data obtained are not complete without consideration of relatively full model of the D.C. motor drive with PWM, which takes into account the influence of the input LC-filter beyond the circuit of regulation. The relatively full mathematical model for the electric motor drive with PWM has the form

$$\frac{dx_1}{dt} = -\frac{R_f}{L_f}x_1 - \frac{1}{L_f}x_2 + \frac{E_0}{L_f}, \qquad (6)$$

$$\frac{dx_2}{dt} = \frac{1}{C}x_1 - \frac{K_F(\xi)}{C}x_3,$$

$$\frac{dx_3}{dt} = -\frac{R_r + R_{sh}}{L_r}x_3 + \frac{R_{sh}}{L_r}x_4 + \frac{K_F(\xi)}{L_r}x_2 - \frac{E(x_4)}{L_r},$$

$$\frac{dx_4}{dt} = \frac{R_{sh}}{L_{ew}(x_4)}x_3 - \frac{R_{ew} + R_{sh}}{L_{ew}(x_4)}x_4,$$

$$\frac{dx_5}{dt} = \frac{\alpha \cdot \beta}{\tau}x_3 - \frac{1}{\tau}x_5 - \frac{\alpha \cdot U_c}{\tau},$$

$$E(x_4) = n\, c_e\, \hat{O}(x_4), \quad L_{ew}(x_4) = 2 \cdot p \cdot w\frac{\partial \Phi(x_4)}{\partial x_4},$$

where $X = \{i_c, u_c, i_r, i_{ew}, u_i\}^t$ the vector of state variables; p, w are the constructive parameters of the motor; the function of commutation is the same as in (5) with $S' = \left\{0, 0, \kappa \cdot \beta, 0, \frac{1-\kappa}{\alpha}\right\}$

The specific feature of the system (6) is the use of nonlinear representation of dependencies of magnetic flux of the electric motor $c_e\Phi(i_{ew})$ and the inductance of the existing winding $L_{ew}(i_{ew})$ on the existing winding current.

To find the shift mapping the system (6) was integrated numerically over the intervals of the structure constancy and the solutions obtained were combined proceeding from the fact that the state vector is in continuous dependence on time. While analyzing the model (6) the parameter *n* and the value of resonance frequency of the input filter $f_r = 1/(2 \cdot \pi \cdot \sqrt{L_f \cdot C})$ are taken as varying parameters. The value f_r of the input filter was varied by change of L_f over the range $[0{,}066 \cdot 10^{-3}$ to $4{,}2 \cdot 10^{-3}]$ H. The rest parameters were assumed to be fixed as well as for the simplified model ($\alpha = 10$; $C = 2{,}4 \cdot 10^{-3}$ *F*; $R_f = 0{,}0123\ \Omega$).

The results of the analysis of the dynamic in the parameter space $\Pi = \{f_r, n;\ 50 < f_r < 400,\ 200 < n < 2000\}$ are presented in Fig.4 as a two parametric bifurcation diagram. The designations in Fig.4a correspond to the designations in Fig.2a. The domains, where quasiperiodic motion exists, are denoted as Π_q. To understand the data in Fig.4a better, Fig.4b-4c represent one parametric bifurcation diagrams built up for two sections.

As it was noted above, the main things in the dynamics of the full model are the interaction of two related oscillators: of the LC-filter and the control system with PWM. When the values f_r are close to subharmonics of the ratio 1/m of the frequency of PWM, there occur oscillations with the period $T = m\ a$, that is, subharmonic resonances of the order 1/m in the control system. The irrational relationship between f_r and the timing frequency of PWM leads to the appearance of extensive domains of existence of quasiperiodic motions (toroidal manifolds) Π_q in the plane Π. In spite of the fact that these domains are shown in Fig.4 as uniform while studying them in detail one can observe numerous narrow "windows" of synchronization which are characterized by the ratio of f_r to the frequency of PWM as rational fraction p/q (resonance p/q). The periodic motions from the "windows" of synchronization can undergo numerous bifurcations (and C-bifurcations, in particular) which lead to the appearance of chaotic motion, as well as one can observe the loss of synchronization, etc. As the value of the domains of resonance p/a is quite small and the regularities of change of dynamic motions in resonance domains typical, the paper analyze only the domains of subharmonic resonances of the order 1/m.

In Fig.4 the domains of subharmonic resonances of the order 1/m (Π_2, Π_3, Π_4, Π_5, etc.) are shown in gray color. These domains Fig.4a are located in some vicinity of corresponding values f_r (200 Hz, 133 Hz, 100 Hz, 80 Hz etc.) and can shift slightly with variation in the parameters of the control system. The width of the domain of subharmonic resonance number m Δf_m is not equal to zero for any number m of resonance (it is stipulated by dissipativeness of the system) and becomes less with increase in m.

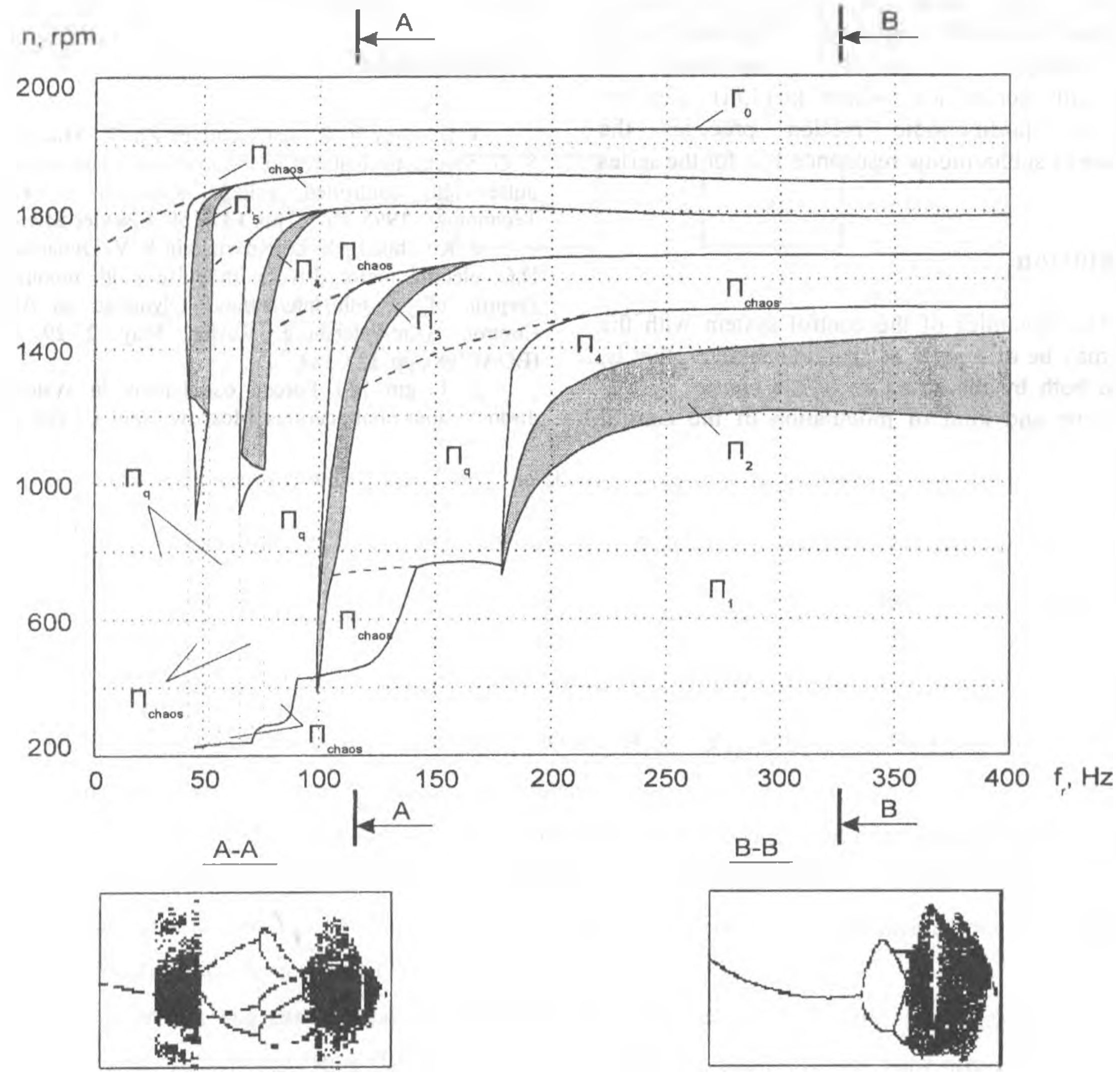

Fig.4. The results of the analysis of the dynamics of the full model.

The quasistatic growth of n from the initial point in the domain of the preset periodic motion Π_1 for a fixed f_r in the vicinity of the domain of subharmonic resonance 1/m leads to the combined series of bifurcations of period doubling and C-bifurcations of period doubling, which can be represented a symbolical series

$$a \to m \cdot a \to 2\, m \cdot a \to 4 \cdot m \cdot a \to \ldots, \qquad (7a)$$

$$a \to \ldots \to m \cdot a \to 2\, m\, a \to 4 \cdot m \cdot a \to \ldots. \qquad (7b)$$

The analysis of simplified model shown that these series may be incomplete, that is, determined by the revealed regularities of occurrence of C-bifurcation boundaries in the parameter space of the control system. The characteristic property of the series of the stationary motions (7a) (or 7b) is the absence of motions with period k a, where $k \in (1,m)$. Besides, chaotic or quasiperiodic motion precedes the appearance of subharmonic resonance 1/m for the series (7b).

4. Conclusion

1. The dynamics of the control system with the PWM-2 may be of regular or chaotic character, that is, stipulated both by the structure of the control system and the type and kind of modulation of the control system. In particular, the peculiarity of the structure of the considered D.C. electric drive with PWM-2 is the presence of two interacted oscillations - the input LC-filter and the control system with PWM (the oscillator with outer synchronizing impact). In this case one of the scenarios of chaotization of motions in the control system under consideration is the disturbance of synchronized resonance motions on toroidal manifolds.

2. The characteristic scenario of chaotization of oscillations in the control system with PWM-2 is a combined series of a bifurcations of period doubling and C-bifurcations of period doubling. It is find out, that this series may be incomplete, that is, stipulated by C-bifurcation phenomena.

5. References

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