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# SIMULATION OF BENDING REINFORCED CONCRETE ELEMENTS WITH CRACKS

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#### **Abstract**

On the basis of deformational design analytical model of the cross section and block model of reinforced concrete element, a new analytical model is proposed by the authors. This model takes into account the work of tensioned concrete between cracks by applying additional stress in the reinforcement (steel bars) due to the difference in relative deformations between the tension reinforcement and concrete during the formation of a crack.

The developed model of analysis of reinforced concrete with cracks makes it possible to obtain the parameters of stress-strain state af the element in any cross-section along the length under the action of a bending moment and a longitudinal force.

**Key words:** reinforced concrete, crack, analytical model, deformational model, block model.

## МОДЕЛИРОВАНИЕ РАБОТЫ ЖЕЛЕЗОБЕТОНА С ТРЕЩИНАМИ ПРИ ИЗГИБЕ

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### Реферат

На основе расчетной деформационной аналитической модели поперечного сечения и блочной модели железобетонного элемента с трещиными авторами предложена новая расчетная модель. Эта модель учитывает работу растянутого бетона

между трещинами за счет приложения дополнительных напряжений в арматуре (стальных стержнях), возникающих за счет разницы относительных деформаций между растянутой арматурой и бетоном в сечении с трещиной при образовании трещины. Разработанная модель расчета железобетона с трещинами позволяет получить параметры напряженно-деформированного состояния элемента в любом поперечном сечении по длине элемента под действием изгибающего момента и продольной силы.

**Ключевые слова:** Железобетон, трещина, расчетная модель, деформационная модель, блочная модель.

Introduction. In a bent reinforced concrete element, before the appearance of cracks in concrete, tensile strength is perceived by concrete and longitudinal reinforcement. The relative deformations of tensioned concrete and reinforcement due to the adhesion between them are equal to each other. With an increase in the load in the zone of maximum bending moments in the most weakened areas (due to the heterogeneity of the concrete structure and partially reinforcement), the tensile relative deformations in concrete approach the limit values and cracks appear. In the section with a crack and near it, the adhesion is broken, the tensile forces are perceived only by the reinforcement. After the formation of a crack, stresses (relative deformations) at the edges of a crack in concrete become equal to zero, and a difference in the relative deformations of concrete and reinforcement  $\Delta \varepsilon$  arises. In accordance with the relations of adhesion [1, 2], this difference in deformations at a distance L from the crack decreases to zero or to the region where the relative deformations in concrete reach the limiting values for tension (Figure 1). If the shear section of the reinforcement and concrete of the tension zone is divided into a number of elementary fragments, then on each i-th fragment of the shear section, the difference in relative deformations is  $\Delta \varepsilon_i$ , which determines the additional stress  $\Delta \sigma_i$  in the reinforcement. The amount of additional stress varies along the length of the stretched concrete block depending on its value in the cross section with a crack and the value of the mutual shear of reinforcement and concrete.

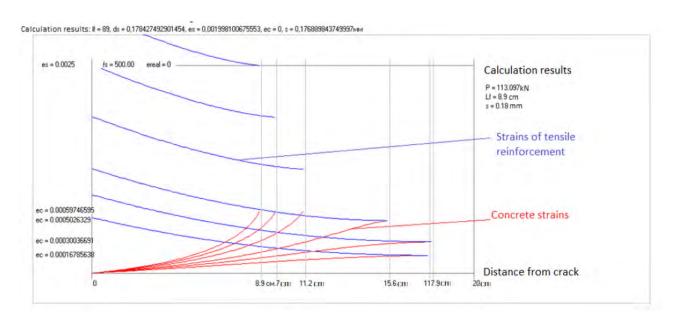


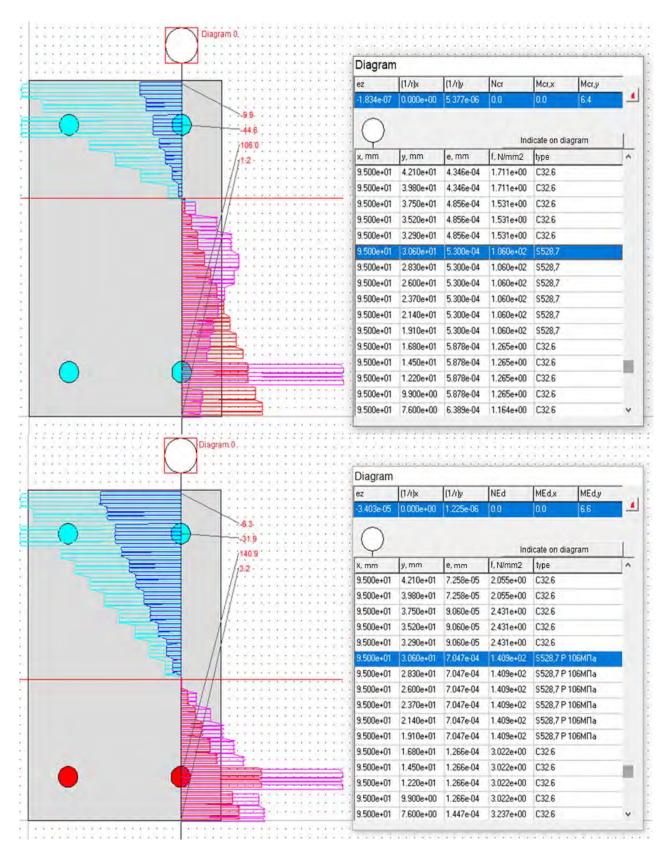
Figure 1. Change in relative strains in tension reinforcement and concrete depending on the distance from the crack at the corresponding stress values in tension reinforcement in a cross section with a crack

Simulation of the stress-strain state of reinforced concrete after the formation of cracks. Additional stress in the reinforcement, due to the resulting difference in relative deformations between the tension reinforcement and concrete, leads to a new equilibrium state of the cross section under load. In [3], this phenomenon is called the "additional state". The increase in stresses in the tensile reinforcement after the compression in concrete is extinguished (immediately after the appearance of a crack) is due to the transfer of additional force to the reinforcement from the cracked concrete [4].

In the deformation model, the effect of additional stress in the reinforcement is proposed to be taken into account by setting the corresponding additional stress (relative deformations) to the reinforcing bars (similar to modeling the prestress of the reinforcement) and analyzing the state of the cross section at the previous moment of cracking (Figure 2). An important effect in non-cracked cross-sections with additional stress in tensile reinforcement is an increase in the bending moment of cracking with increasing load (Figure 3) and, accordingly, an increase in the bending stiffness of areas adjacent to the crack (Figure 4).

Let us consider the stress-strain state of cross sections with a crack and along the length of a concrete block between cracks. Figure 5a shows the distribution of relative strains and stresses in the cross section where crack may appear at the second stage of crack formation (the appearance of cracks in a concrete block between adjacent cracks). The crack appears in the tension block in the cross section between the already existing cracks, where the additional stress as a result of the mutual shear of the reinforcement and the tension concrete is not equal to zero. For example, the additional stress in the reinforcement from the nearest crack is 130 MPa, which corresponds to the bending moment before cracking 10.6 kNm and the total stress in the reinforcement 120.4+130=250.4 MPa. Further, with an increase in load at a bending moment of 10.8 kNm, a secondary crack appears, the stresses in the reinforcement in this cross section increase sharply to 141.8 + 130 = 271.8 MPa, the neutral axis shifts towards the compressed zone (Figure 5, b).

In the cross section passing along the edge of the crack, stresses of 250.4 MPa in the crack due to mutual shear, due to the loss of adhesion of reinforcement and concrete, become additional stress (similar to the prestressing of reinforcement, before tempering), and the relative deformations in concrete are sharply reduced by the value of elastic deformations to zero stress values. There is no crack in this cross section and is not expected until the bending moment in it is 14.4 kNm (Figure 6). As a result of the redistribution of forces (the reinforcement is deformed together in relaxing concrete), the stress in the reinforcement increased to 306.5 MPa (Figure 3). The increase in stress in the reinforcement by 306.5-250.4=56.1 MPa occurs as a result of compression (relaxation) of the stretched concrete block after the formation of a crack. The same stress (306.5 MPa) in the cross section with a crack. If we consider this state from the point of view of modeling the operation of tensile concrete by transforming the reinforcement deformation diagram using the coefficient  $\psi_s$ , similarly to [5, 6], then for the same relative deformations in the reinforcement, the stress is greater in the section with a crack. This stress value (56.1 MPa) in the reinforcement in the proposed model is taken into account by an increase in the bending moment of cracking (the force was transferred to the reinforcement from cracked tensile concrete and its relaxation to zero stresses).



a) before the formation of the first crack; b) after applying additional force from the mutual shear of concrete and reinforcement (numbers on extension lines are normal stresses in MPa) (blue and red colors show the distribution of relative deformations along the height of the cross section, respectively, in the compressed and tension zones; blue and pink - similarly, normal stress)

Figure 2. Modeling the effect of additional stress in the reinforcement on the equilibrium state of the section of the beam block without a crack

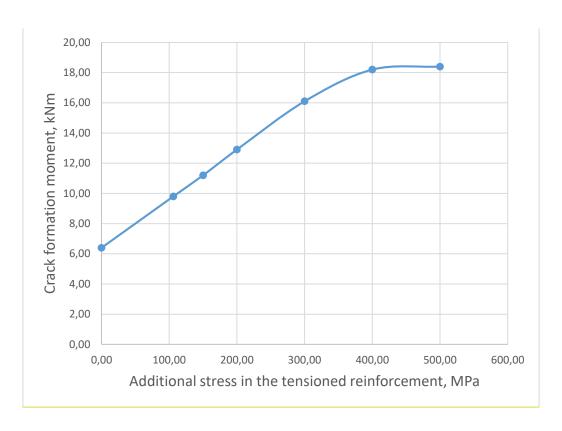


Figure 3. Dependence of the bending moment of crack formation on additional stress in tensile reinforcement

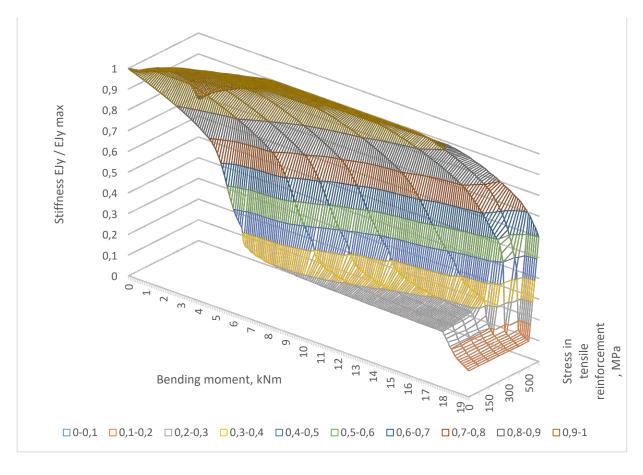
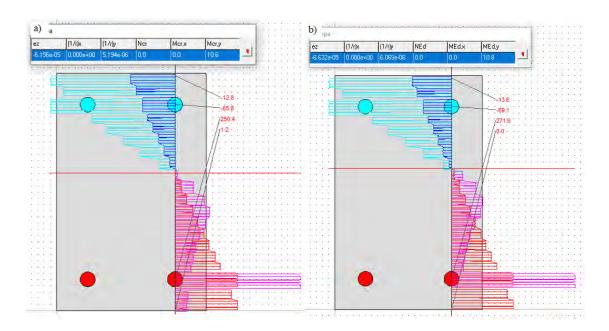


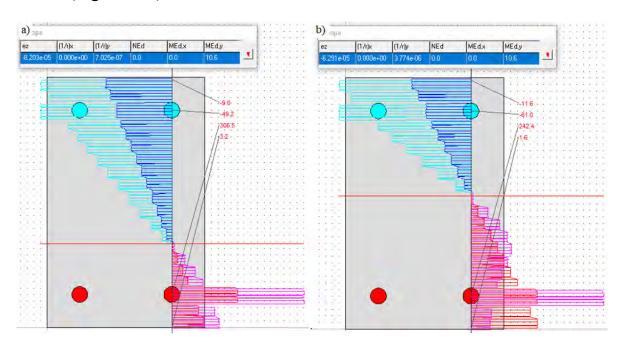
Figure 4. Effect of additional stress in tensile reinforcement in a section with a crack on the change in the flexural stiffness of a finite element



a) - before the formation of a crack in a tensioned block between cracks; b) - after the formation of a crack; (numbers on extension lines are normal stresses in MPa) (blue and red colors show the distribution of relative strains along the height of the cross section, respectively, in the compressed and tension zones; blue and pink, similarly, normal stresses)

Figure 5. Simulation of the stress-strain state in the cross section of a bent element

In the cross section along the length of the concrete block between the cracks, the additional stress of the reinforcement from shear is 150 MPa and the stresses in the reinforcement at the same bending moment of 10.6 kNm are 242.4 MPa in the absence of a crack (Figure 6, b).



a) - along the edge of the crack; b) - along the length of the concrete block between the cracks (numbers on extension lines - normal stresses in MPa) (blue and red colors show the distribution of relative deformations along the height of the cross section, respectively, in the compressed and tension zones; blue and pink - similarly, normal stresses)

Figure 6. Simulation of the stress-strain state of a bent element in cross section

Having calculated, according to the block model, additional relative deformations (stresses) in the reinforcement along the length of its shear section in the concrete of the tension zone (Figure 1), taking into account, according to the deformation model, the new equilibrium state of any cross section (including sections with a crack), the correspondence of the distribution of relative deformations of concrete and reinforcement along the height of the sections to the hypothesis of flat sections is established. Thus, the work of tensile reinforced concrete with cracks in the deformation model of the cross section with a crack is modeled without applying the coefficient  $\psi_s$ . Having obtained as a result of the calculation the distribution of stresses in tension reinforcement along the length of a bent reinforced concrete structure with cracks, the very value of the coefficient  $\psi_s$  for analyzing the participation of the tension zone of concrete in the work of a bent reinforced concrete element can be calculated as the ratio of the area of the diagram of reinforcement stresses along the length of the block separated by cracks to the area of the trapezoid diagrams with ordinates-stresses in cross sections with cracks at its ends, as it was determined empirically [7, 8].

Having determined, according to the deformation model, the distribution of stiffness along the length of a statically indeterminate (continuous) bending structure, by the methods of structural mechanics (finite element method), the distribution of internal forces (bending moments) and deformations (deflections) along its length is calculated.

**Conclusion.** On the basis of deformation design models of the cross section and block reinforced concrete element, a model is proposed for taking into account the work of tensioned concrete between cracks by applying additional stress in the reinforcement due to the difference in relative deformations between the tension reinforcement and concrete during the formation of a crack.

The developed model of operation of reinforced concrete with cracks makes it possible to obtain the parameters of its stress-strain state in any cross section along the length under the action of a bending moment and a longitudinal force.

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## ОСОБЕННОСТИ РАСЧЕТА И КОНСТРУИРОВАНИЯ ЖЕЛЕЗОБЕТОННЫХ ЭЛЕМЕНТОВ С ПЕРЕМЕННОЙ ВЫСОТОЙ СЕЧЕНИЯ

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### Реферат

В элементах криволинейного очертания в местах перелома граней возникаю локальные поля напряжений, которые при определенных условиях могут привести к образованию наклонных трещин и разрушению элемента по одной из таких трещин. Несмотря на то, что в современной практике строительства применяется большое количество балок ломаного очертания, методики для учета локальных полей напряжения не существует, а поперечная арматура устанавливается на основании конструктивных требований и опыта применения подобных конструкций.

**Ключевые слова:** переменная высота сечения, изополя напряжений, железобетонная балка, трещиностойкость, касательные напряжения.

## FEATURES OF CALCULATION AND DESIGN OF REINFORCED CONCRETE ELEMENTS WITH VARIABLE SECTION HEIGHT

N.V.Matweenko, V.N.Malinovskiy, E.S.Matweenko

#### **Abstract**

In the curved elements, in the places of fracture of the faces, local stress fields arise, which, under certain conditions, can lead to the formation of shear cracks and the destruction of the element along one of these cracks. Despite the fact that a large number of corved beams are used in modern construction practice, there is no methodology for taking into account local stress fields, and transverse reinforcement is installed on the basis of design requirements and experience in the use of such structures.